Logic - Introduction (1A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Proposition

proposition (*countable and uncountable*, *plural* **propositions**)

1. (*uncountable*) The act of offering (an idea) for consideration.
2. (*c* Appendix:Glossary ea or a plan offered. [quotations ▼]
3. (*countable*, *business settings*) The terms of a transaction offered.
4. (*countable*, *US*, *politics*) In some states, a proposed statute or constitutional amendment to be voted on by the electorate.
5. (*countable*, *logic*) The content of an assertion that may be taken as being true or false and is considered abstractly without reference to the linguistic sentence that constitutes the assertion.
6. (*countable*, *mathematics*) An assertion so formulated that it can be considered true or false.
7. (*countable*, *mathematics*) An assertion which is provably true, but not important enough to be called a theorem.
8. A statement of religious doctrine; an article of faith; creed. [quotations ▼]

   *the **propositions** of Wyclif and Huss*

9. (*poetry*) The part of a poem in which the author states the subject or matter of it.

http://en.wikipedia.org/wiki/Derivative

Predicate

predicate (*plural* **predicates**)

1. (*grammar*) The part of the sentence (or clause) which states something about the subject or the object of the sentence. [quotations ▼]

   *In "The dog barked very loudly", the subject is "the dog" and the **predicate** is "barked very loudly".*

2. (*logic*) A term of a statement, where the statement may be true or false depending on whether the thing referred to by the values of the statement's variables has the property signified by that (predicative) term. [quotations ▼]

   *A nullary **predicate** is a proposition. Also, an instance of a **predicate** whose terms are all constant — e.g., P(2,3) — acts as a proposition.*

   *A **predicate** can be thought of as either a relation (between elements of the domain of discourse) or as a truth-valued function (of said elements).*

   *A **predicate** is either valid, satisfiable, or unsatisfiable.*

   *There are two ways of binding a **predicate**'s variables: one is to assign constant values to those variables, the other is to quantify over those variables (using universal or existential quantifiers). If all of a **predicate**'s variables are bound, the resulting formula is a proposition.*

3. (*computing*) An operator or function that returns either true or false.

http://en.wikipedia.org/wiki/Derivative

Syllogism

syllogism (*plural* **syllogisms**)

1. (*logic*) An inference in which one proposition (the conclusion) follows necessarily from two other propositions, known as the premises. [quotations ▼]

1. (*obsolete*) A trick, artifice.

http://en.wikipedia.org/wiki/Derivative

Contraposition

Consider the Euler diagram shown. According to this diagram, if something is in A, it must be in B as well. So we can interpret "all of A is in B" as:

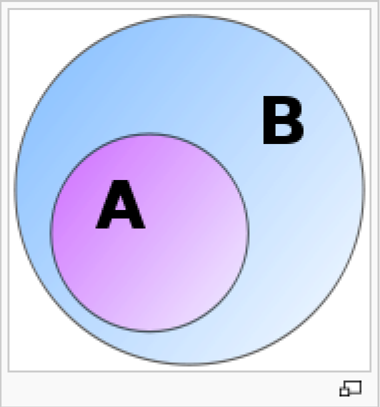
$$A \to B$$

It is also clear that anything that is **not** within B (the white region) **cannot** be within A, either. This statement,

$$\neg B \to \neg A$$

is the contrapositive. Therefore we can say that

$$(A \to B) \to (\neg B \to \neg A).$$



Practically speaking, this may make life much easier when trying to prove something. For example, if we want to prove that every girl in the United States (A) is blonde (B), we can either try to directly prove $A \to B$ by checking all girls in the United States to see if they are all blonde. Alternatively, we can try to prove $\neg B \to \neg A$ by checking all non-blonde girls to see if they are all outside the US. This means that if we find at least one non-blonde girl within the US, we will have disproved $\neg B \to \neg A$, and equivalently $A \to B$.

To conclude, for any statement where A implies B, then *not B* always implies *not A*. Proving or disproving either one of these statements automatically proves or disproves the other. They are fully equivalent.

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

name	form	description
implication	if *P* then *Q*	first statement implies truth of second
inverse	if not *P* then not *Q*	negation of both statements
converse	if *Q* then *P*	reversal of both statements
contrapositive	if not *Q* then not *P*	reversal and negation of both statements
negation	*P* and not *Q*	contradicts the implication

Examples [edit]

Take the statement "*All red objects have color.*" This can be equivalently expressed as "*If an object is red, then it has color.*"

- The **contrapositive** is "*If an object does not have color, then it is not red.*" This follows logically from our initial statement and, like it, it is evidently true.
- The **inverse** is "*If an object is not red, then it does not have color.*" An object which is blue is not red, and still has color. Therefore in this case the inverse is false.
- The **converse** is "*If an object has color, then it is red.*" Objects can have other colors, of course, so, the converse of our statement is false.
- The **negation** is "*There exists a red object that does not have color.*" This statement is false because the initial statement which it negates is true.

http://en.wikipedia.org/wiki/Derivative

Contraposition : Formal Definition

A proposition *Q* is implicated by a proposition *P* when the following relationship holds:

$$(P \to Q)$$

This states that, "if *P*, then *Q*", or, "if *Socrates is a man*, then *Socrates is human*." In a conditional such as this, *P* is the antecedent, and *Q* is the consequent. One statement is the **contrapositive** of the other only when its antecedent is the negated consequent of the other, and vice versa. The contrapositive of the example is

$$(\neg Q \to \neg P).$$

That is, "If not-*Q*, then not-*P*", or, more clearly, "If *Q* is not the case, then *P* is not the case." Using our example, this is rendered "If *Socrates is not human*, then *Socrates is not a man*." This statement is said to be *contraposed* to the original and is logically equivalent to it. Due to their logical equivalence, stating one effectively states the other; when one is true, the other is also true. Likewise with falsity.

Strictly speaking, a contraposition can only exist in two simple conditionals. However, a contraposition may also exist in two complex conditionals, if they are similar. Thus, $\forall x(Px \to Qx)$, or "All *P*s are *Q*s," is contraposed to $\forall x(\neg Qx \to \neg Px)$, or "All non-*Q*s are non-*P*s."

http://en.wikipedia.org/wiki/Derivative

Contraposition : Simple Proof

In first-order logic, the conditional is defined as:

$$A \to B \iff \neg A \lor B$$

We have:

$$\neg A \lor B \iff \neg A \lor (\neg\neg B)$$
$$\iff \neg(\neg B) \lor \neg A$$
$$\iff \neg B \to \neg A$$

http://en.wikipedia.org/wiki/Derivative

Rules for Negation

Rules for negations [edit]

Reductio ad absurdum (or *Negation Introduction*)

$$\frac{\varphi \vdash \psi \qquad \varphi \vdash \neg\psi}{\neg\varphi}$$

Reductio ad absurdum (related to the law of excluded middle)

$$\frac{\neg\varphi \vdash \psi \qquad \neg\varphi \vdash \neg\psi}{\varphi}$$

Noncontradiction (or *Negation Elimination*)

$$\frac{\varphi \qquad \neg\varphi}{\psi}$$

Double negation elimination

$$\frac{\neg\neg\varphi}{\varphi}$$

Double negation introduction

$$\frac{\varphi}{\neg\neg\varphi}$$

http://en.wikipedia.org/wiki/Derivative

Rules for Conditionals

Rules for conditionals [edit]

Deduction theorem (or *Conditional Introduction*)

$$\frac{\varphi \vdash \psi}{\varphi \rightarrow \psi}$$

Modus ponens (or *Conditional Elimination*)

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi}$$

Modus tollens

$$\frac{\varphi \rightarrow \psi \quad \neg\psi}{\neg\varphi}$$

http://en.wikipedia.org/wiki/Derivative

Rules for Conjunction

Rules for conjunctions [edit]

Adjunction (or ***Conjunction Introduction***)

$$\frac{\begin{array}{l}\varphi \\ \psi\end{array}}{\varphi \wedge \psi}$$

Simplification (or ***Conjunction Elimination***)

$$\frac{\varphi \wedge \psi}{\varphi}$$

$$\frac{\varphi \wedge \psi}{\psi}$$

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

Rules for disjunctions [edit]

Addition (or *Disjunction Introduction*)

$$\frac{\varphi}{\varphi \lor \psi}$$

$$\frac{\psi}{\varphi \lor \psi}$$

Case analysis

$$\frac{\begin{array}{l}\varphi \lor \psi \\ \varphi \to \chi \\ \psi \to \chi\end{array}}{\chi}$$

Disjunctive syllogism

$$\frac{\begin{array}{l}\varphi \lor \psi \\ \neg\varphi\end{array}}{\psi}$$

$$\frac{\begin{array}{l}\varphi \lor \psi \\ \neg\psi\end{array}}{\varphi}$$

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

Rules for biconditionals [edit]

Biconditional introduction

$$\frac{\varphi \to \psi \qquad \psi \to \varphi}{\varphi \leftrightarrow \psi}$$

Biconditional Elimination

$$\frac{\varphi \leftrightarrow \psi \qquad \varphi}{\psi}$$

$$\frac{\varphi \leftrightarrow \psi \qquad \psi}{\varphi}$$

$$\frac{\varphi \leftrightarrow \psi \qquad \neg\varphi}{\neg\psi}$$

$$\frac{\varphi \leftrightarrow \psi \qquad \neg\psi}{\neg\varphi}$$

$$\frac{\varphi \leftrightarrow \psi \qquad \psi \lor \varphi}{\psi \land \varphi}$$

$$\frac{\varphi \leftrightarrow \psi \qquad \neg\psi \lor \neg\varphi}{\neg\psi \land \neg\varphi}$$

http://en.wikipedia.org/wiki/Derivative

First Order Logic

First-order logic is a formal system used in mathematics, philosophy, linguistics, and computer science. It is also known as **first-order predicate calculus**, the **lower predicate calculus**, **quantification theory**, and predicate logic. First-order logic uses quantified variables over (non-logical) objects. This distinguishes it from propositional logic which does not use quantifiers.

A theory about some topic is usually first-order logic together with a specified domain of discourse over which the quantified variables range, finitely many functions which map from that domain into it, finitely many predicates defined on that domain, and a recursive set of axioms which are believed to hold for those things. Sometimes "theory" is understood in a more formal sense, which is just a set of sentences in first-order logic.

The adjective "first-order" distinguishes first-order logic from higher-order logic in which there are predicates having predicates or functions as arguments, or in which one or both of predicate quantifiers or function quantifiers are permitted.[1] In first-order theories, predicates are often associated with sets. In interpreted higher-order theories, predicates may be interpreted as sets of sets.

http://en.wikipedia.org/wiki/Derivative

First Order Logic

There are many deductive systems for first-order logic that are sound (all provable statements are true in all models) and complete (all statements which are true in all models are provable). Although the logical consequence relation is only semidecidable, much progress has been made in automated theorem proving in first-order logic. First-order logic also satisfies several metalogical theorems that make it amenable to analysis in proof theory, such as the Löwenheim–Skolem theorem and the compactness theorem.

First-order logic is the standard for the formalization of mathematics into axioms and is studied in the foundations of mathematics. Mathematical theories, such as number theory and set theory, have been formalized into first-order axiom schemas such as Peano arithmetic and Zermelo–Fraenkel set theory (ZF) respectively.

No first-order theory, however, has the strength to describe fully and categorically structures with an infinite domain, such as the natural numbers or the real line. Categorical axiom systems for these structures can be obtained in stronger logics such as second-order logic.

For a history of first-order logic and how it came to dominate formal lo Second-order logic rreirós (2001).

http://en.wikipedia.org/wiki/Derivative

Logical Symbol

There are many deductive systems for first-order logic that are sound (all provable statements are true in all models) and complete (all statements which are true in all models are provable). Although the logical consequence relation is only semidecidable, much progress has been made in automated theorem proving in first-order logic. First-order logic also satisfies several metalogical theorems that make it amenable to analysis in proof theory, such as the Löwenheim–Skolem theorem and the compactness theorem.

First-order logic is the standard for the formalization of mathematics into axioms and is studied in the foundations of mathematics. Mathematical theories, such as number theory and set theory, have been formalized into first-order axiom schemas such as Peano arithmetic and Zermelo–Fraenkel set theory (ZF) respectively.

No first-order theory, however, has the strength to describe fully and categorically structures with an infinite domain, such as the natural numbers or the real line. Categorical axiom systems for these structures can be obtained in stronger logics such as second-order logic.

For a history of first-order logic and how it came to dominate formal lo Second-order logic rreirós (2001).

http://en.wikipedia.org/wiki/Derivative

Predicate

In mathematics, a **predicate** is commonly understood to be a Boolean-valued function P: $X \to$ {true, false}, called the predicate on X. However, predicates have many different uses and interpretations in mathematics and logic, and their precise definition, meaning and use will vary from theory to theory. So, for example, when a theory defines the concept of a relation, then a predicate is simply the characteristic function or the indicator function of a relation. However, not all theories have relations, or are founded on set theory, and so one must be careful with the proper definition and semantic interpretation of a predicate.

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

The precise semantic interpretation of an atomic formula and an atomic sentence will vary from theory to theory.

- In propositional logic, atomic formulas are called propositional variables.[3] In a sense, these are nullary (i.e. 0-arity) predicates.
- In first-order logic, an atomic formula consists of a predicate symbol applied to an appropriate number of terms.
- In set theory, predicates are understood to be characteristic functions or set indicator functions, *i.e.* functions from a set element to a truth value. Set-builder notation makes use of predicates to define sets.
- In autoepistemic logic, which rejects the law of excluded middle, predicates may be true, false, or simply *unknown*; *i.e.* a given collection of facts may be insufficient to determine the truth or falsehood of a predicate.
- In fuzzy logic, predicates are the characteristic functions of a probability distribution. That is, the strict true/false valuation of the predicate is replaced by a quantity interpreted as the degree of truth.

http://en.wikipedia.org/wiki/Derivative

Quantifier

In logic, **quantification** is a construct that specifies the quantity of specimens in the domain of discourse that satisfy an open formula. For example, in arithmetic, it allows the expression of the statement that every natural number has a successor. A language element which generates a quantification (such as "every") is called a **quantifier**. The resulting expression is a quantified expression, it is said to be **quantified** over the predicate (such as "the natural number *x* has a successor") whose free variable is bound by the quantifier. In formal languages, quantification is a formula constructor that produces new formulas from old ones. The semantics of the language specifies how the constructor is interpreted. Two fundamental kinds of quantification in predicate logic are universal quantification and existential quantification. The traditional symbol for the universal quantifier "all" is "∀", a rotated letter "A", and for the existential quantifier "exists" is "∃", a rotated letter "E". These quantifiers have been generalized beginning with the work of Mostowski and Lindström.

Quantification is used as well in natural languages; examples of quantifiers in English are *for all*, *for some*, *many*, *few*, *a lot*, and *no*; see Quantifier (linguistics) for details.

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

If *D* is a domain of *x* and *P*(*x*) is a predicate dependent on *x*, then the universal proposition can be expressed as

$$\forall x \in D \; P(x)$$

This notation is known as restricted or relativized or bounded quantification. Equivalently,

$$\forall x \; (x \in D \to P(x))$$

The existential proposition can be expressed with bounded quantification as

$$\exists x \in D \; P(x)$$

or equivalently

$$\exists x \; (x \in D \land P(x))$$

Together with negation, only one of either the universal or existential quantifier is needed to perform both tasks:

$$\neg(\forall x \in D \; P(x)) \equiv \exists x \in D \; \neg P(x),$$

which shows that to disprove a "for all *x*" proposition, one needs no more than to find an *x* for which the predicate is false. Similarly,

$$\neg(\exists x \in D \; P(x)) \equiv \forall x \in D \; \neg P(x),$$

to disprove a "there exists an *x*" proposition, one needs to show that the predicate is false for all *x*.

http://en.wikipedia.org/wiki/Derivative

A triangle and its slope

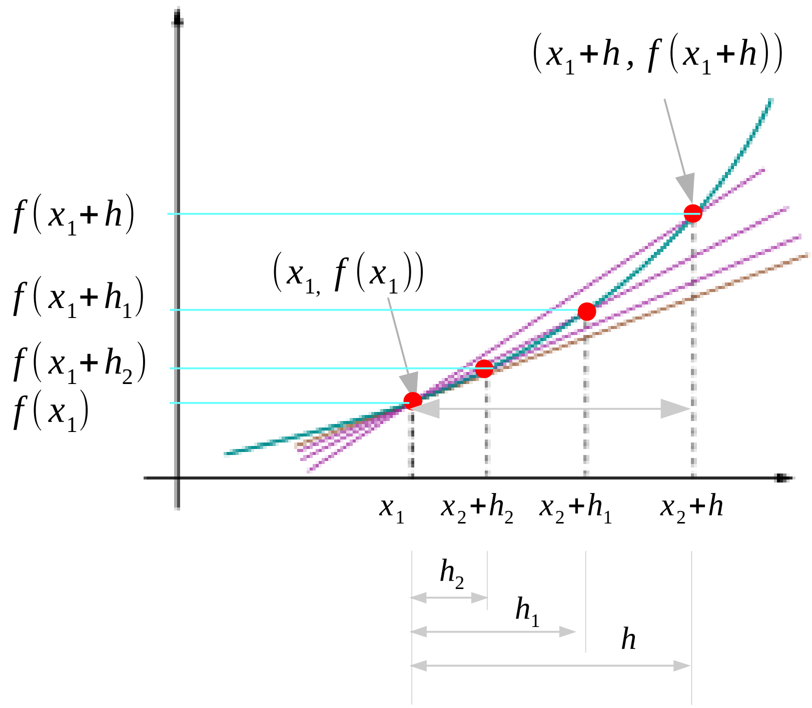
http://en.wikipedia.org/wiki/Derivative

Many smaller triangles and their slopes

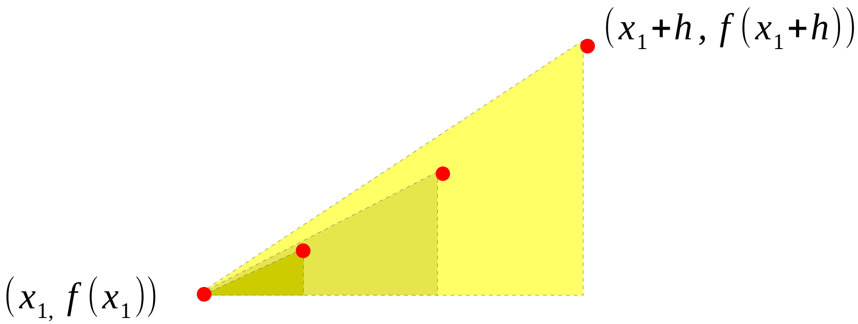
$$\frac{f(x_1 + h) - f(x_1)}{h}$$

$$\frac{f(x_1 + h_1) - f(x_1)}{h_1}$$

$$\frac{f(x_1 + h_2) - f(x_1)}{h_2}$$



$$\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$



http://en.wikipedia.org/wiki/Derivative

References

[1] http://en.wikipedia.org/
[2]