Laurent Series and z-Transform Examples case 4.B

20171016

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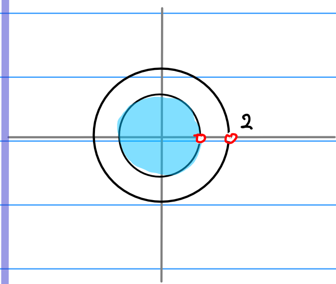

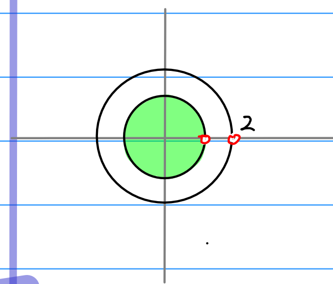

$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^n$$

$$X(z) = \sum_{n=-1}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

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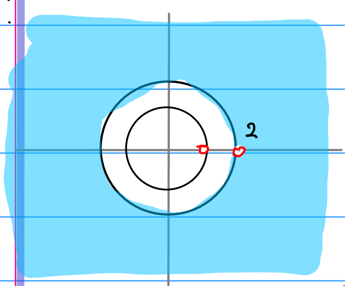

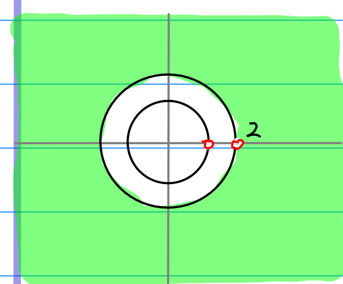

$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left[2^{n-1} - 1 \right] z^n$$

$$X(z) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$

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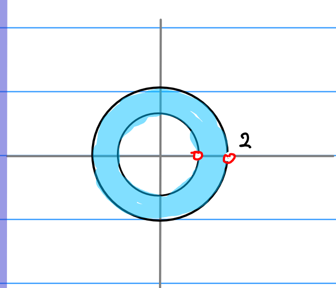
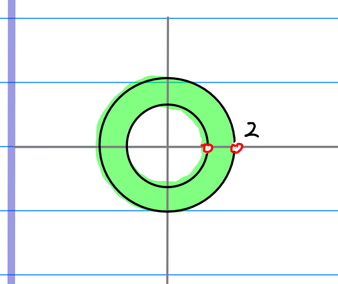




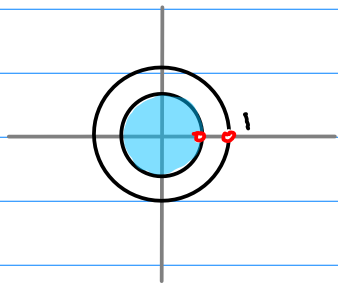
$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

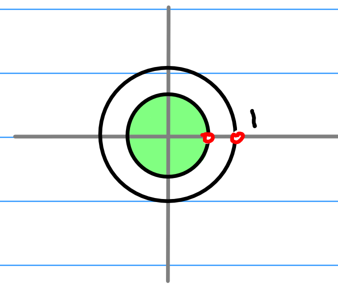
$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = + \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

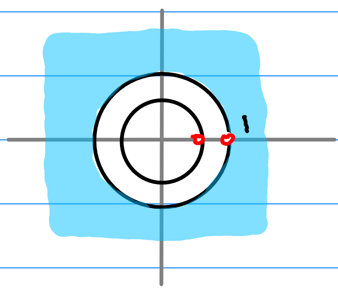
$$X(z) = + \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

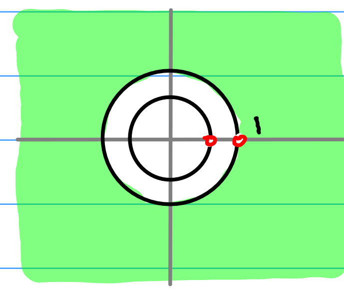
4.B $f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$



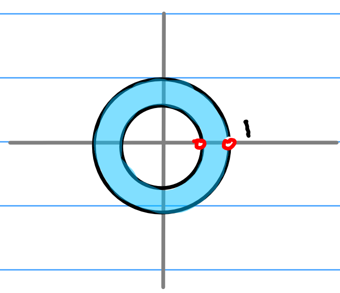


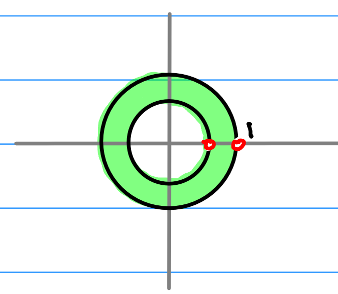
$$\sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n \xleftarrow{-n} \sum_{n=-1}^{\infty} [1 - (\tfrac{1}{2})^{n+1}] z^{-n}$$





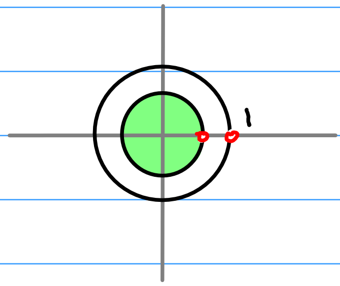
$$\sum_{n=-1}^{-\infty} [2^{n-1} - 1] z^n \xleftarrow{-n} \sum_{n=1}^{\infty} [(\tfrac{1}{2})^{n+1} - 1] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \xleftarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1} z^{-n}$$

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



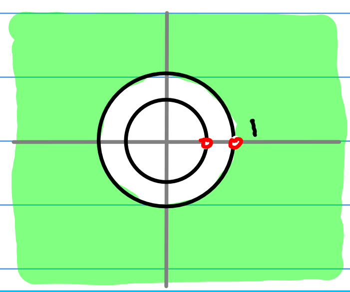
$$+ \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= + \sum_{n=0}^{\infty} (z)(z)^n - \sum_{n=0}^{\infty} (z)(2z)^n$$

$$= \sum_{n=0}^{\infty} \left[1 - 2^n\right] z^{n+1}$$

$$= \sum_{n=-1}^{-\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

$$\boxed{\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n}$$



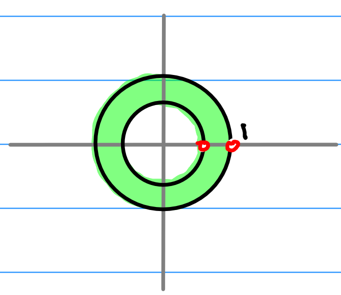
$$- \frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= - \sum_{n=0}^{\infty} (1)\left(\tfrac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2z}\right)^n$$

$$= - \sum_{n=0}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

$$\boxed{\sum_{n=-1}^{-\infty} \left[2^{n-1} - 1\right] z^n}$$



$$- \frac{(1)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

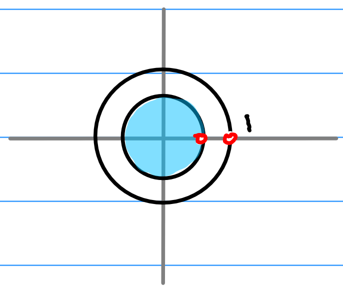
$$= - \sum_{n=0}^{\infty} (1)\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\tfrac{z}{1}\right)\left(\tfrac{2z}{1}\right)^n$$

$$= - \sum_{n=0}^{\infty} z^{-n} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= - \sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{-n-1} z^{-n}$$

$$\boxed{+ \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n}$$

$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5\,z}{z-0.5}$$

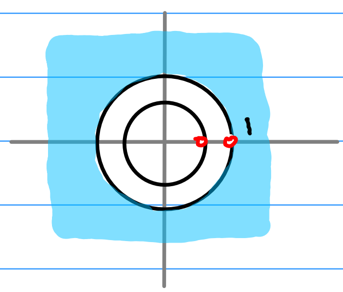


$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[1 - 2^{n-1}\right] z^n$$

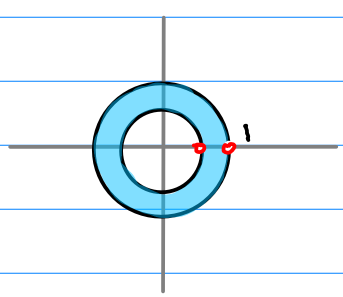


$$-\frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1} - 1\right] z^{-n}$$

$$= \sum_{n=-1}^{-\infty}\left[2^{n-1} - 1\right] z^n$$



$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$