

FPGA Carry Chain Adder (1A)

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FPGA Carry Chain Cell



$$s_i = (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i$$

$$c_{i+1} = (a_i \cdot b_i) + (a_i \oplus b_i) c_i = \bar{p}_i \cdot g_i + p_i \cdot c_i = \bar{p}_i \cdot a_i + p_i \cdot c_i = \bar{p}_i \cdot b_i + p_i \cdot c_i$$

when $\bar{p}_i = 1$, then $a_i = b_i$

when $g_i = 1$, then $a_i = b_i = 1$

$p(i)$	0	1
0	0	1
1	1	0

$g(i)$	0	1
0	0	0
1	0	1

FPGA Carry Chain Cell



Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al

FPGA Carry Chain

FPGAs generally contain dedicated computation resources for generating fast adders

The Virtex family programmable arrays include logic gates (**XOR**) and **multiplexers** that along with the general purpose **lookup tables** allow one to build effective carry-chain adders

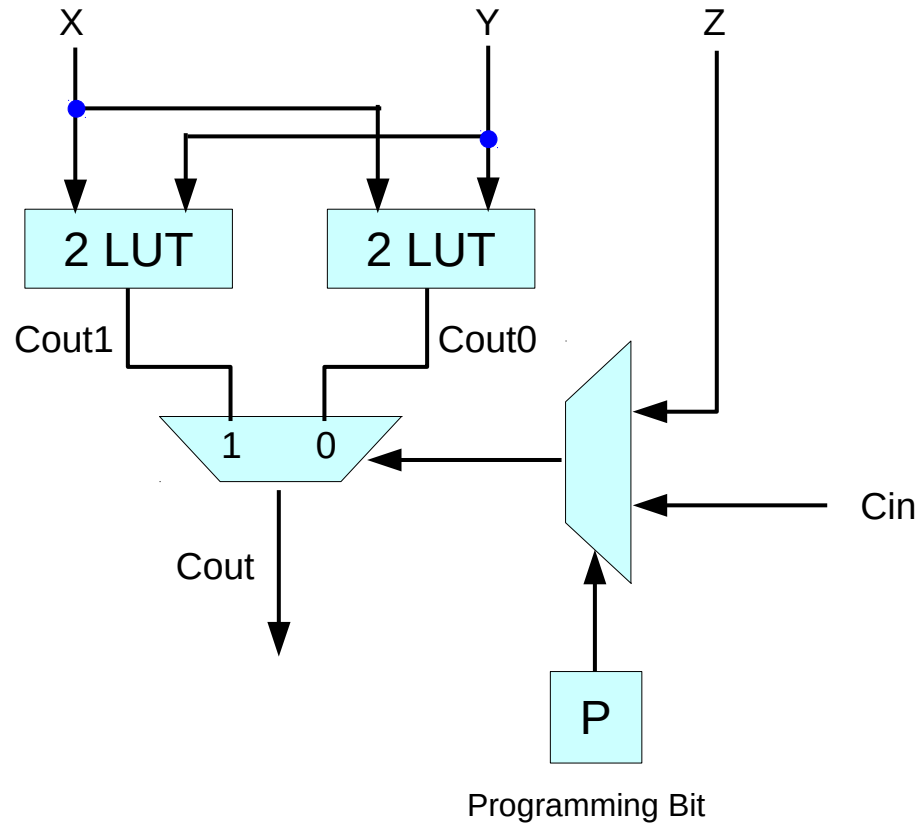
The carry chain is made up of multiplexers belonging to adjacent configurable blocks

the lookup table is used for implementing the exclusive or function

$$p(i) = x(i) \text{ xor } y(i)$$

https://en.wikipedia.org/wiki/Carry-lookahead_adder

FPGA Carry Chain Cell



Cout1, Cout2 : functions of X, Y, Cin

Cout1 = X+Y when Cin=1

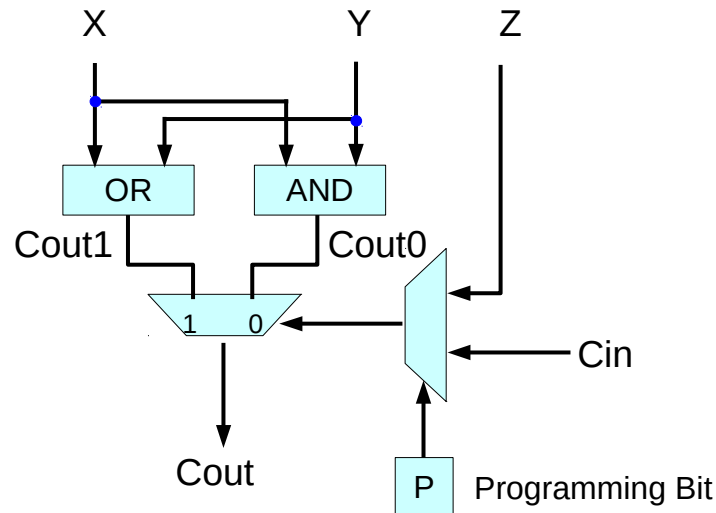
Cout0 = X Y when Cin=0

Cout = (X + Y) Cin + X Y $\overline{\text{Cin}}$

Cout = P' Cin + G $\overline{\text{Cin}}$... P' = relaxed P

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

FPGA Carry Chain Cell



X	Y	Cin	$\overline{\text{Cin}}$	$\overline{X} \overline{Y}$
		Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X} Y$
1	0	1	0	$X \overline{Y}$
1	1	1	1	$X Y$

Cout : functions of X, Y, Cin

$$\text{Cout}(X, Y, 1) = \text{Cout1} = X + Y$$

$$\text{Cout}(X, Y, 0) = \text{Cout0} = X Y$$

$$\text{Cout1} = X + Y \text{ when Cin}=1$$

$$\text{Cout0} = X Y \text{ when Cin}=0$$

$$\text{Cout1} = P' \text{Cin} \dots P' = \text{relaxed } P$$

$$\text{Cout0} = G \overline{\text{Cin}}$$

$$\text{If Cin, then Cout} = (\overline{X} Y + X \overline{Y} + X Y)$$

$$\text{If } \overline{\text{Cin}}, \text{ then Cout} = X Y$$

$$\text{Cin} (X + Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin} (\overline{X} Y + X \overline{Y} + X Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin} (\overline{X} Y + X \overline{Y}) + (\text{Cin} + \overline{\text{Cin}}) X Y$$

$$P \text{Cin} + G$$

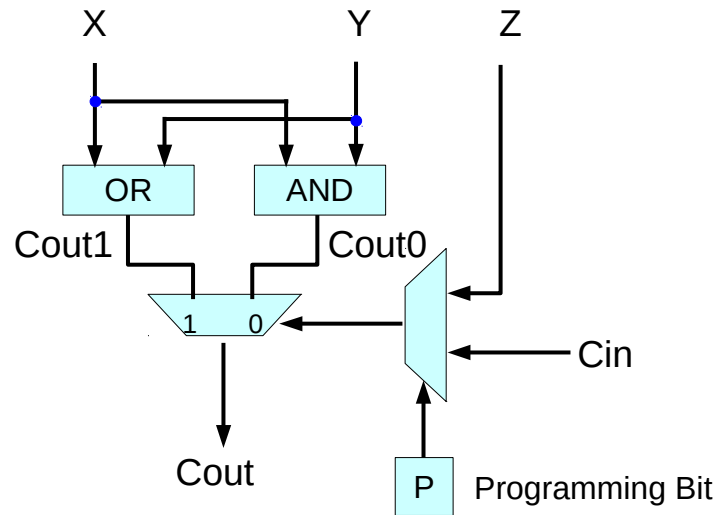
$$\text{Cin} (X + Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin } P' + \overline{\text{Cin}} G$$

$$\dots P' : \text{relaxed } P$$

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FPGA Carry Chain Cell

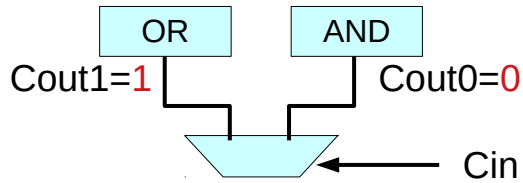


X	Y	Cin	$\overline{\text{Cin}}$	$\overline{X} \overline{Y}$
		Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X} Y$
1	0	1	0	$X \overline{Y}$
1	1	1	1	$X Y$

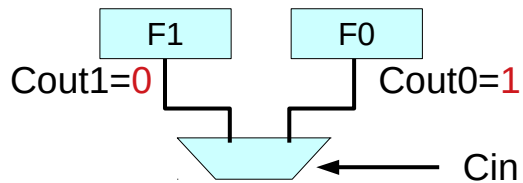
X	Y	Cin	Cout
0	0	0	Cout0
0	1	0	Cout0
1	0	0	Cout0
1	1	0	Cout0
0	0	1	Cout1
0	1	1	Cout1
1	0	1	Cout1
1	1	1	Cout1

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FPGA Carry Chain Cell



Cout1=1 when Cin=1
 Cout0=0 when Cin=0
 Cout = Cin



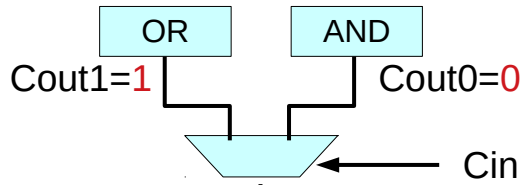
Cout1=0 when Cin=1
 Cout0=1 when Cin=0
 Cout = $\overline{\text{Cin}}$

Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Propagate
1	0	Cin	Inverse Propagate
1	1	1	Generate

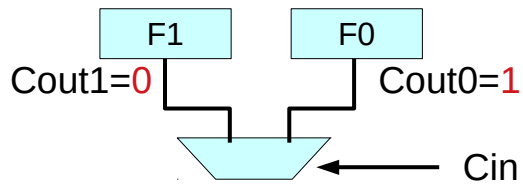
Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

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FPGA Carry Chain Cell



Cout1=1 when Cin=1
 Cout0=0 when Cin=0
 Cout = Cin



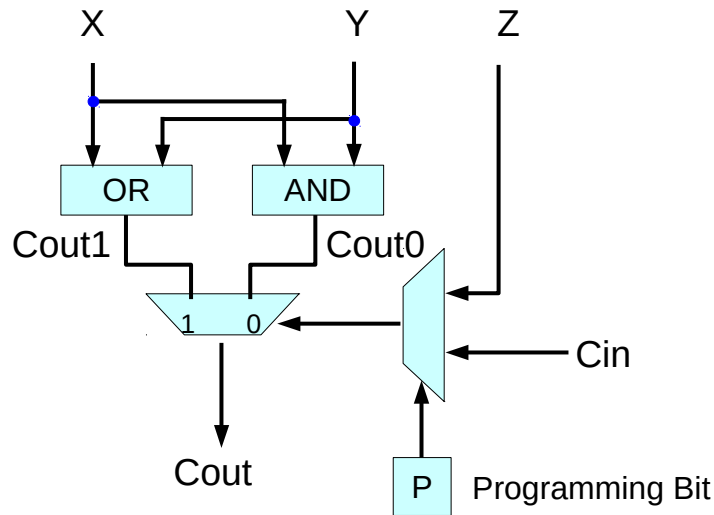
Cout1=0 when Cin=1
 Cout0=1 when Cin=0
 Cout = $\overline{\text{Cin}}$

Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Propagate
1	0	$\overline{\text{Cin}}$	Inverse Propagate
1	1	1	Generate

X	Y	Cin	Cout		Cout1	Cout0
0	0	0	0	Cout0	0	0
0	1	0	0	Cout0	1	0
1	0	0	0	Cout0	1	0
1	1	0	1	Cout0	1	1
0	0	1	0	Cout1	0	0
0	1	1	1	Cout1	1	0
1	0	1	1	Cout1	1	0
1	1	1	1	Cout1	1	1

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Carry Chain



Carry Out

X	Y	Cin	Cout
0	0	Cin	$\overline{\text{Cin}}$
0	1	Cin	$\overline{\text{Cin}}$
1	0	Cin	$\overline{\text{Cin}}$
1	1	Cin	Cin

X	Y	Cin	$\overline{\text{Cin}}$	$\overline{\text{X}} \overline{\text{Y}}$
		Cout1	Cout0	
0	0	0	0	$\overline{\text{X}} \overline{\text{Y}}$
0	1	1	0	$\overline{\text{X}} \text{Y}$
1	0	1	0	$\text{X} \overline{\text{Y}}$
1	1	1	1	$\text{X} \text{Y}$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Cout1=1 when Cin=1

Cout0=0 when Cin=0

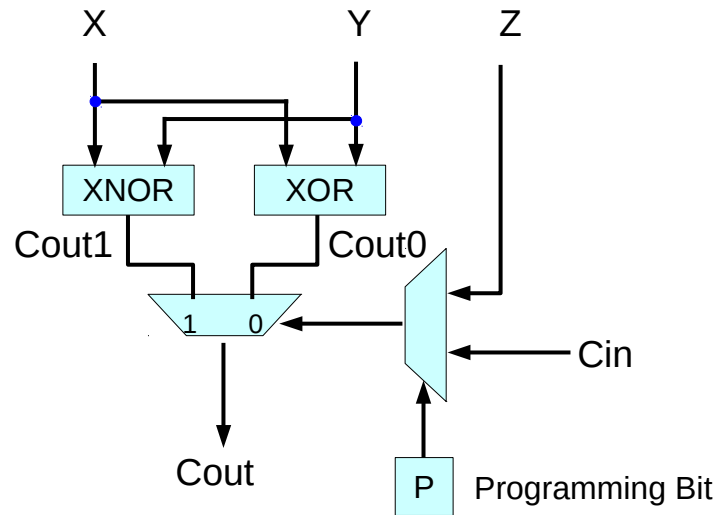
Cout = Cin propagate

Cout1=0 when Cin=1

Cout0=1 when Cin=0

Cout = $\overline{\text{Cin}}$ inverse propagate

Parity Checker



X	Y	Cin	$\overline{\text{Cin}}$	$\overline{X} \overline{Y}$
		Cout1	Cout0	
0	0	1	0	$\overline{X} \overline{Y}$
0	1	0	1	$\overline{X} Y$
1	0	0	1	$X \overline{Y}$
1	1	1	0	$X Y$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Computing Parity

$X \oplus Y \oplus \text{Cin}$	
$0 \oplus 0 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$0 \oplus 1 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$1 \oplus 0 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$1 \oplus 1 \oplus \text{Cin}$	Cin

Cout1=1 when Cin=1

Cout0=0 when Cin=0

Cout = Cin propagate

Cout1=0 when Cin=1

Cout0=1 when Cin=0

Cout = $\overline{\text{Cin}}$ inverse propagate

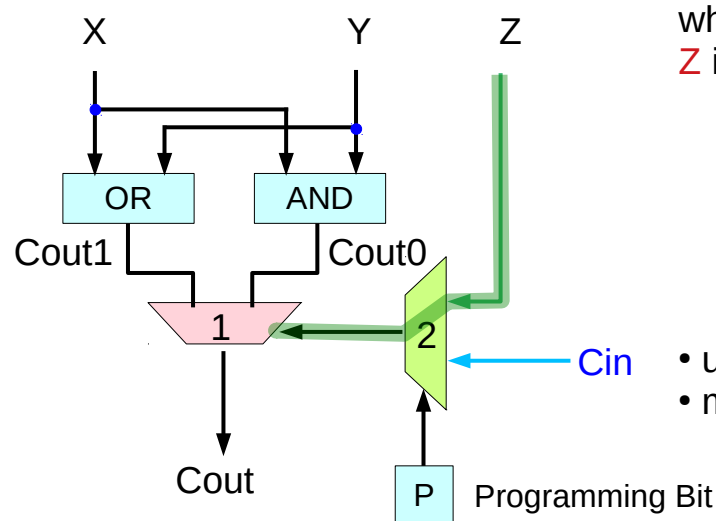
Ripple Carry Chain



the **first** cell in the chain

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FPGA Carry Chain Cell



when **Cin** is ignored,
Z is routed to **mux1**

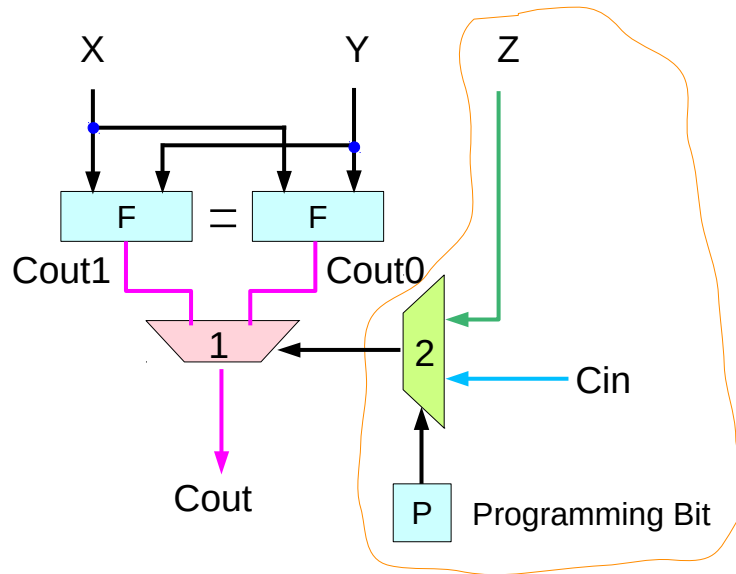
- used in **combined adder/subtractors**
- must be ignored, otherwise

the logic cells - resources to compute a function
the exact location of logic cells depends on the user.
a user can start or end a carry computation
at any place in an fpga.

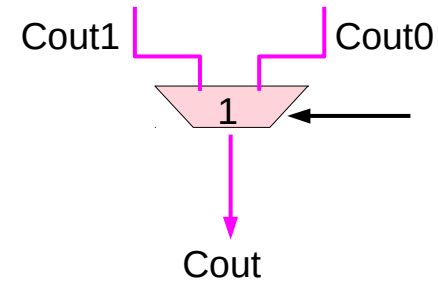
But in many carry computations,
the first cell has only 2 inputs,
and forcing the carry chain
to wait for the arrival of an additional,
unnecessary input **Z** will only needlessly
slow down the circuit's computation.

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FPGA Carry Chain Cell



when **Cin** is ignored,
Z can also be ignored
by having the same LUTs



the **first cell** in the chain

the same LUTs

the same output
regardless of **Z** and **Cin**

$Cout1 = Cout0 = Cout$
regardless of the select

Ripple Carry Chain

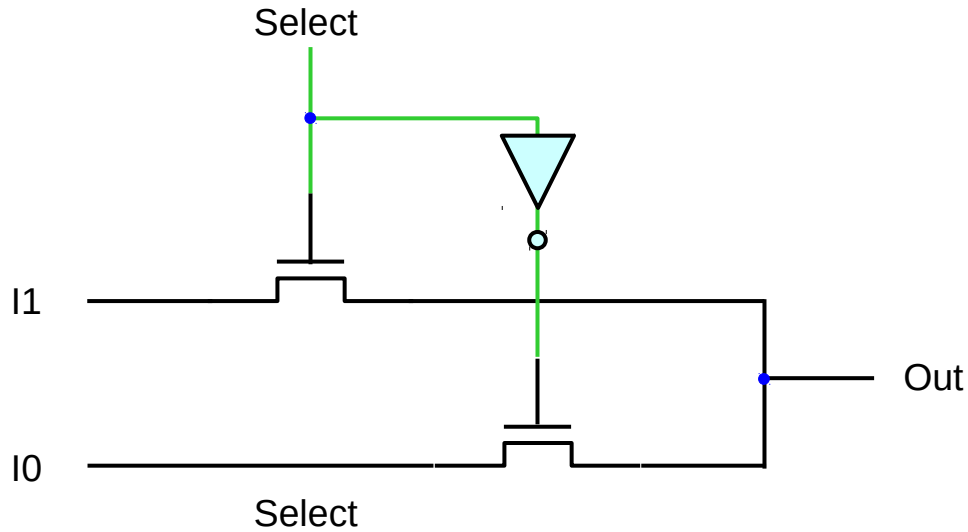
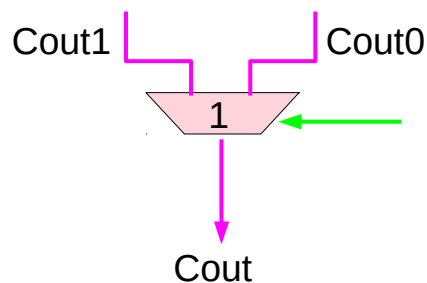


fig1b shows an implementation of a mux that does not obey this requirement

since the carry chain is part of an fpga, the input to this mux could be connected to some **unused logic** in another row which is generating **unknown values**.

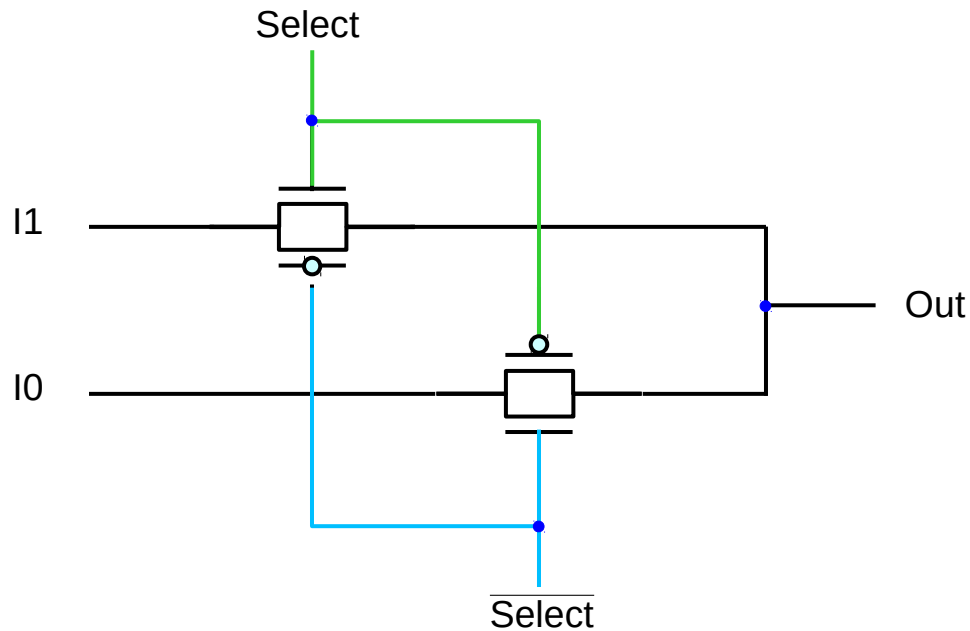
if that unused logic had **multiple transitions** which caused the signal to change **quicker** than the gate could react, then it is possible that **the select signal** to this mux could be stuck midway between true and false (2.5V for 5V CMOS)

in this case, it will not be able to pass a true value from the input to the output and thus will not function properly for this application.



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Ripple Carry Chain



however a mux built with both n-transistor and p-transistor pass gates will operate properly for this case

assume this mux implementation will be used

tristate driver based muxes could be used, which restore signal drive and cut series RC chains

Unit Gate Delay Model

All simple gate of two or three inputs that are directly implementable in **one logic level** in CMOS are considered to have a **delay of one**.

All other gate must be implemented by such gates, and have the delay of the underlying circuit.

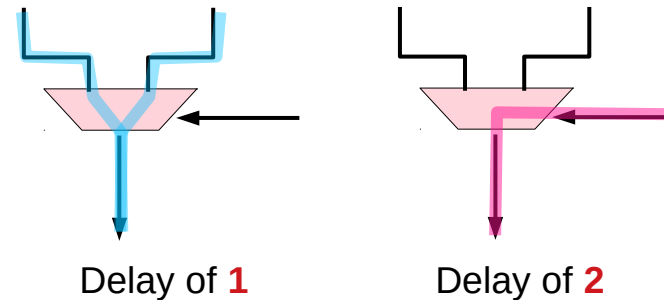
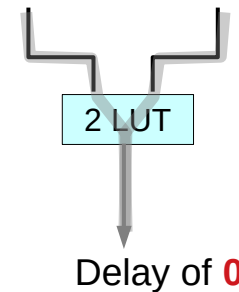
Delay of one

- inverters and
- 2 to 3 input NAND
- 2 to 3 input NOR gates

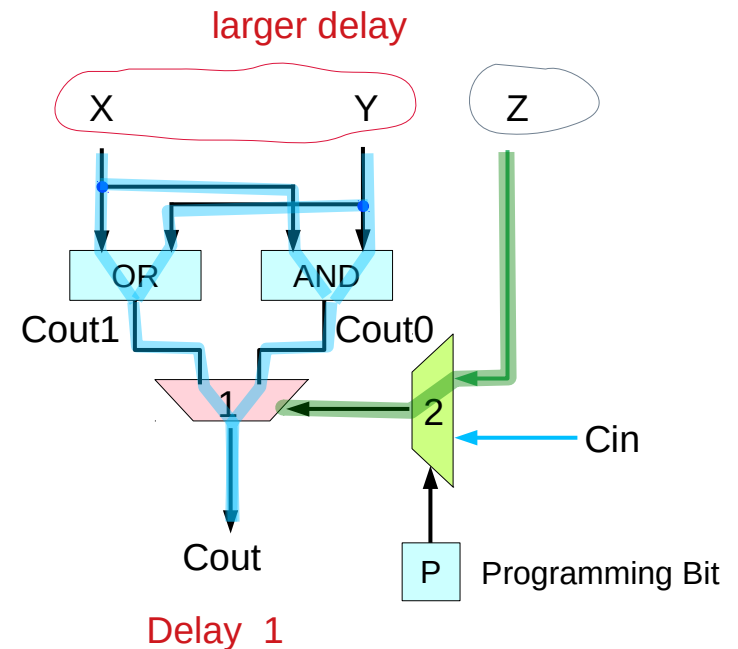
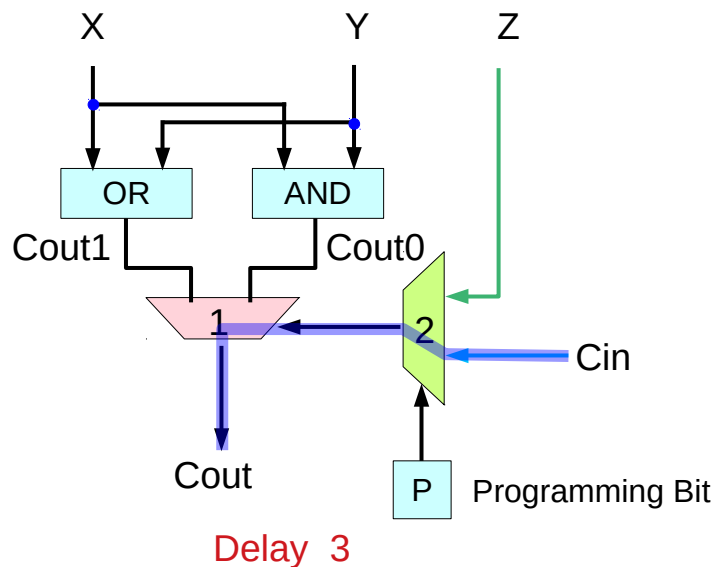
A **2:1 mux** has a **delay of one** from the I0 or I1 inputs to the output, But has a **delay of two** from the select input to the output due to the Inverter delay

Delay of zero (constant delay)

- the delay of the 2-LUTs,
- any routing leading to them,



FPGA Carry Chain Cell



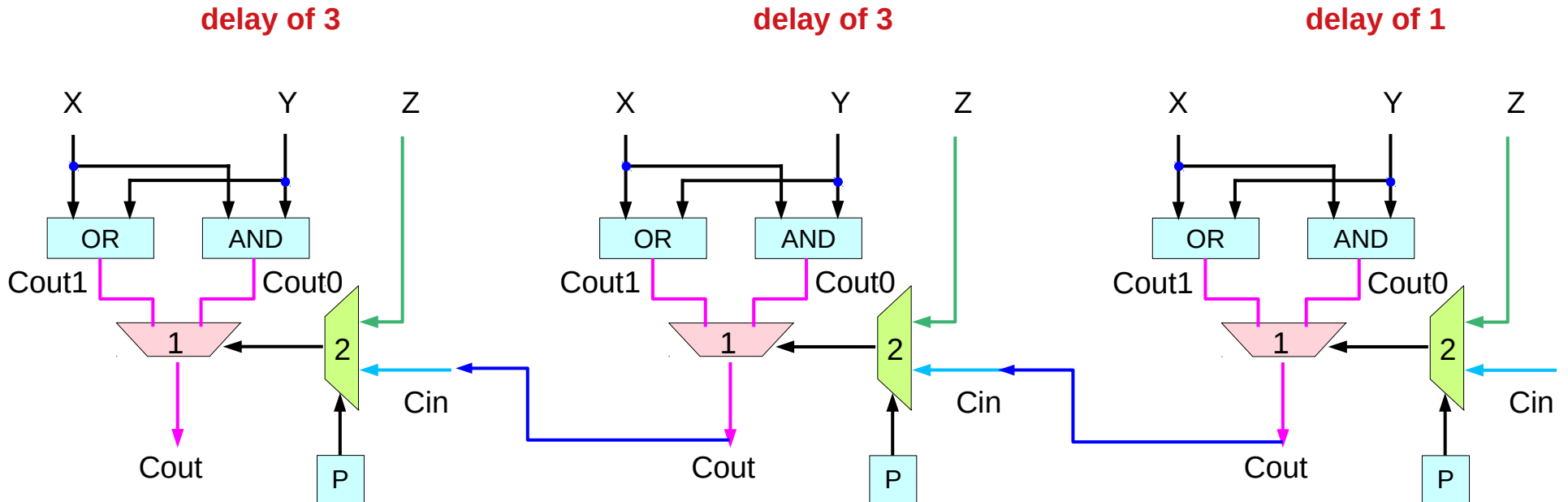
Significantly slower
two muxes on the carry chain in each cell

Delay **1** for first cell
Delay **3** for each additional cell in the carry chain
delay **1** for mux2
delays **2** for mux1

Overall $3n-2$ for an n-cell carry chain

The critical path comes from the 2-LUTs
and not from the input Z
since the delay through the 2-LUTs
will be larger than through mux 2 in the first cell

FPGA Carry Chain Cell



delay of $3n-2$ for an n -bit ripple carry chain

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FPGA Carry Chain Cell

the linear delay growth of ripple carry adders

optimize a ripple carry chain structure for use in FPGAs

while this provides some performance gain
over the basic ripple carry scheme
found in many current FPGAs,

still much slower than what is done in custom logic

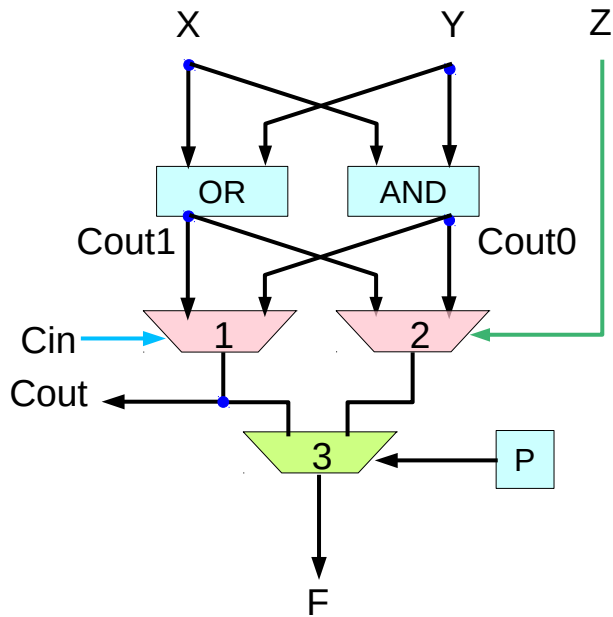
advanced adder techniques in custom logic
can be integrated into reconfigurable logic

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FPGA Carry Chain Cell

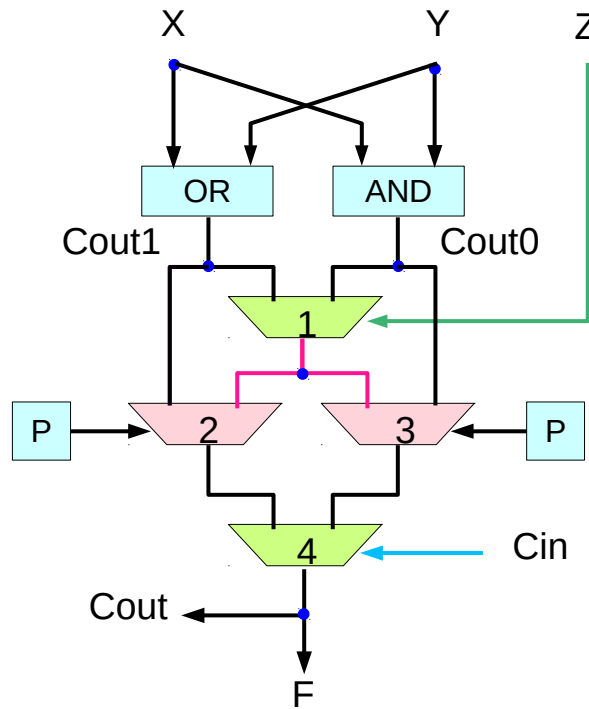
Design A

$$2n / 2n+2$$



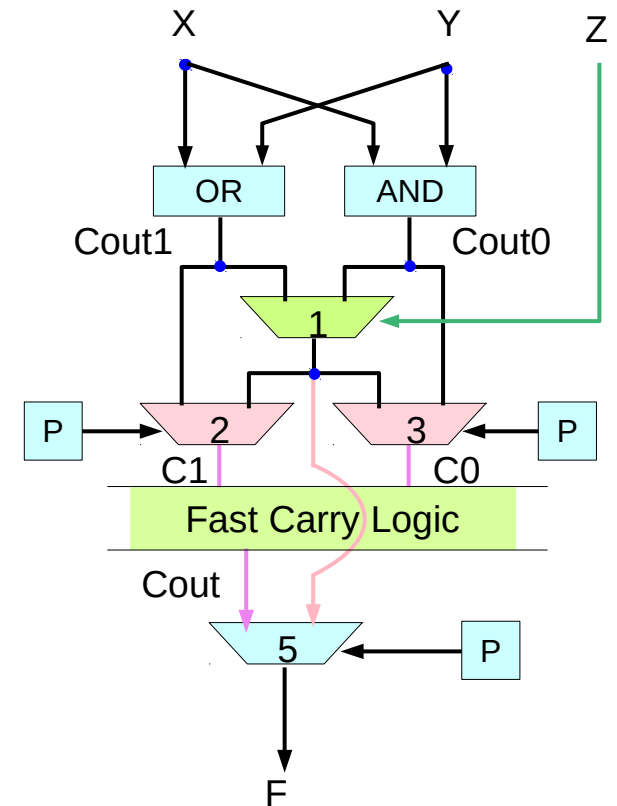
Design B

$$2n / 2n+1$$



Design C

$$2n+2$$



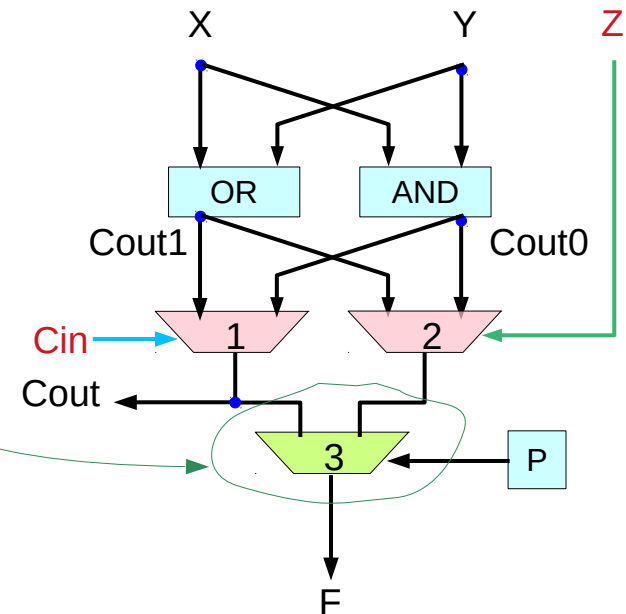
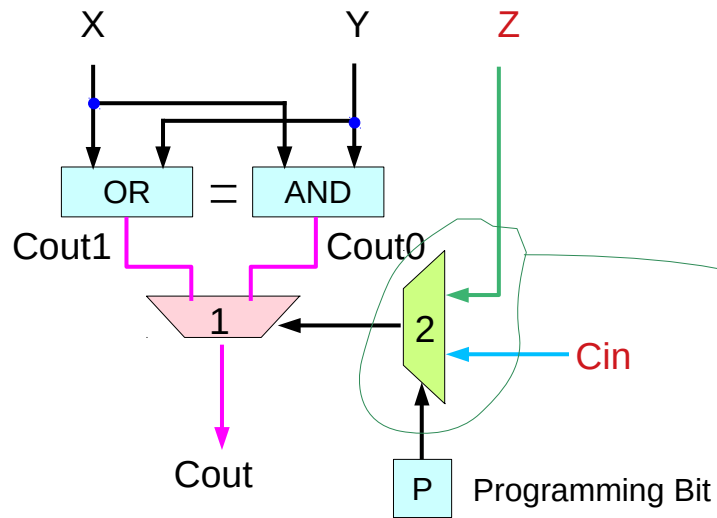
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Design A (1)

to reduce the delay of the ripple carry chain

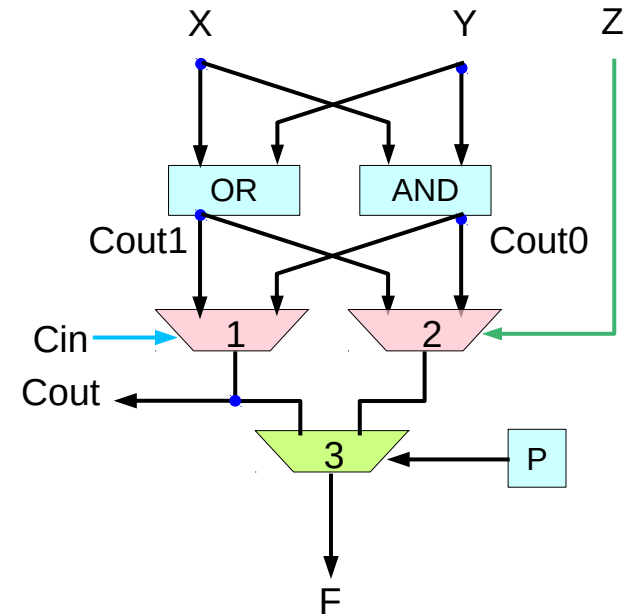
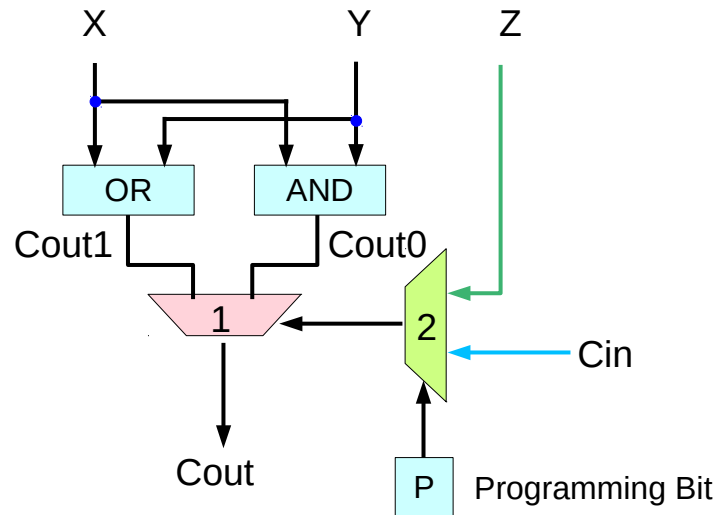
- remove **mux2** from the carry path.
- no need to choose between **Cin** and **Z** for the select line to the output **mux1**

- two separate muxes, **mux1** and **mux2**, controlled by **Cin** and **Z**, respectively.
- the circuit chooses between these outputs with **mux3**.



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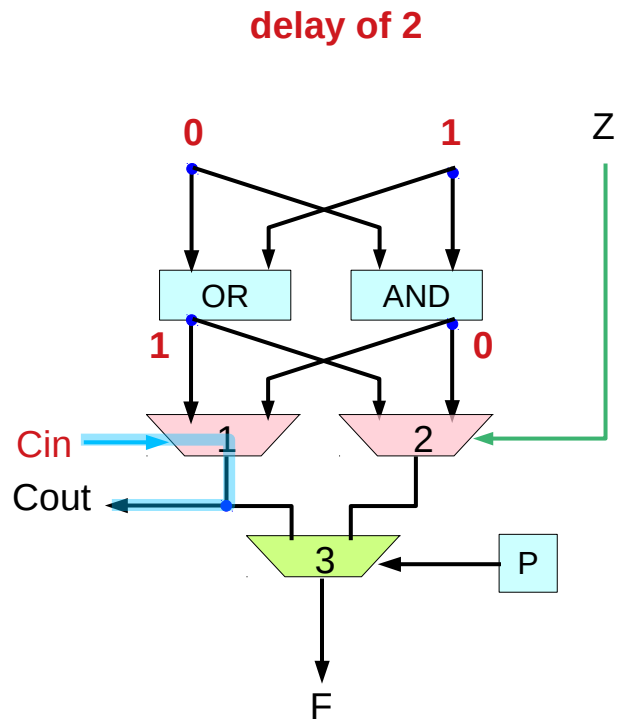
Design A (2)



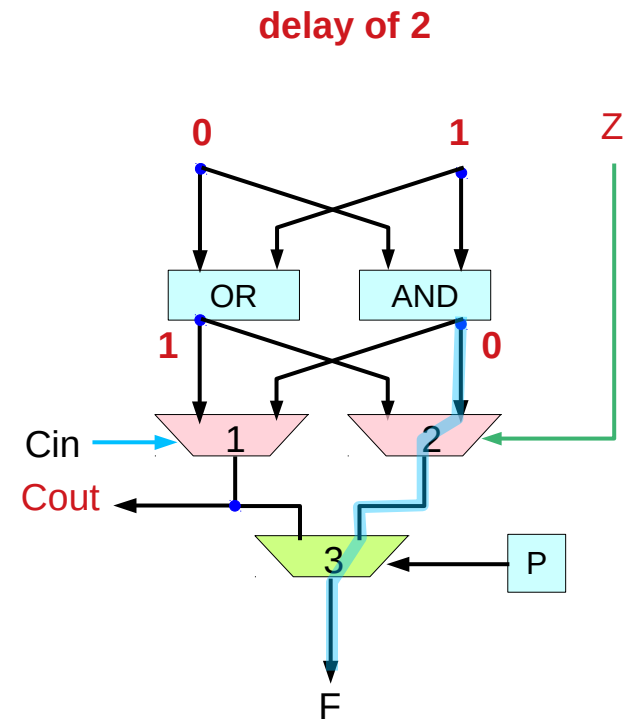
- not logically equivalent
- the **Z** input in the first cell cannot be used
 - **Z** is only attached to **mux2**
 - **mux2** does not lead to the carry cells
 - not connected to **Cout**

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Design A (3)



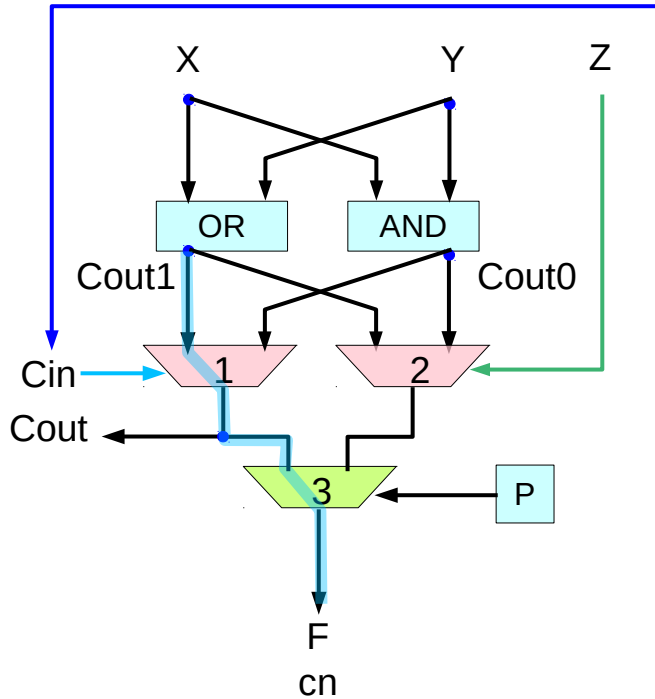
an additional cell
for generating Cin



- need an additional cell to use Z as a carry input

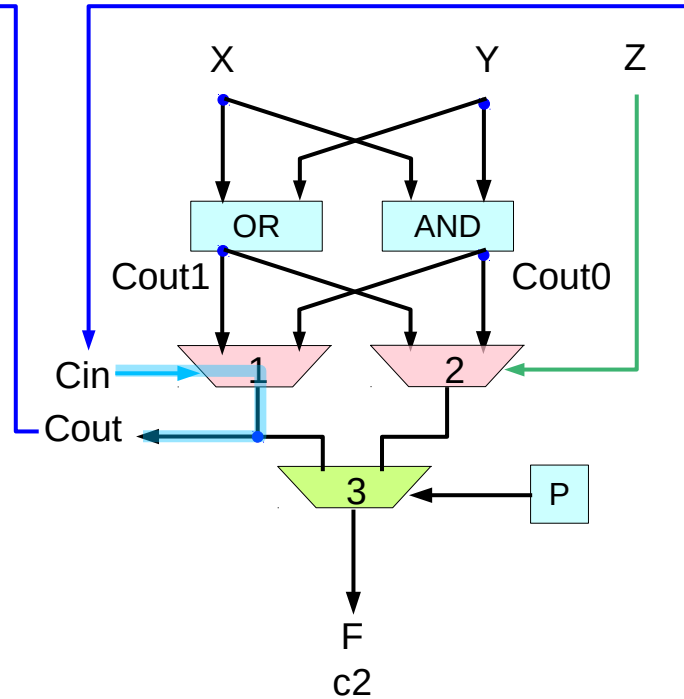
Design A (4)

delay of 3



(2 for mux1, 1 for mux3)

delay of 2



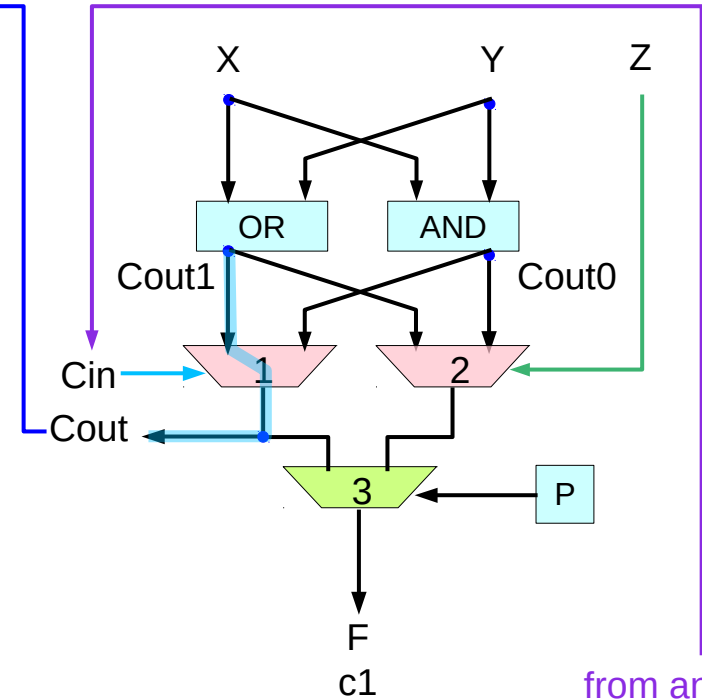
50% faster circuit than the original design

delay of $2n$ for an n -bit ripple carry chain

delay of $2(n+1)$

the first cell

delay of 1



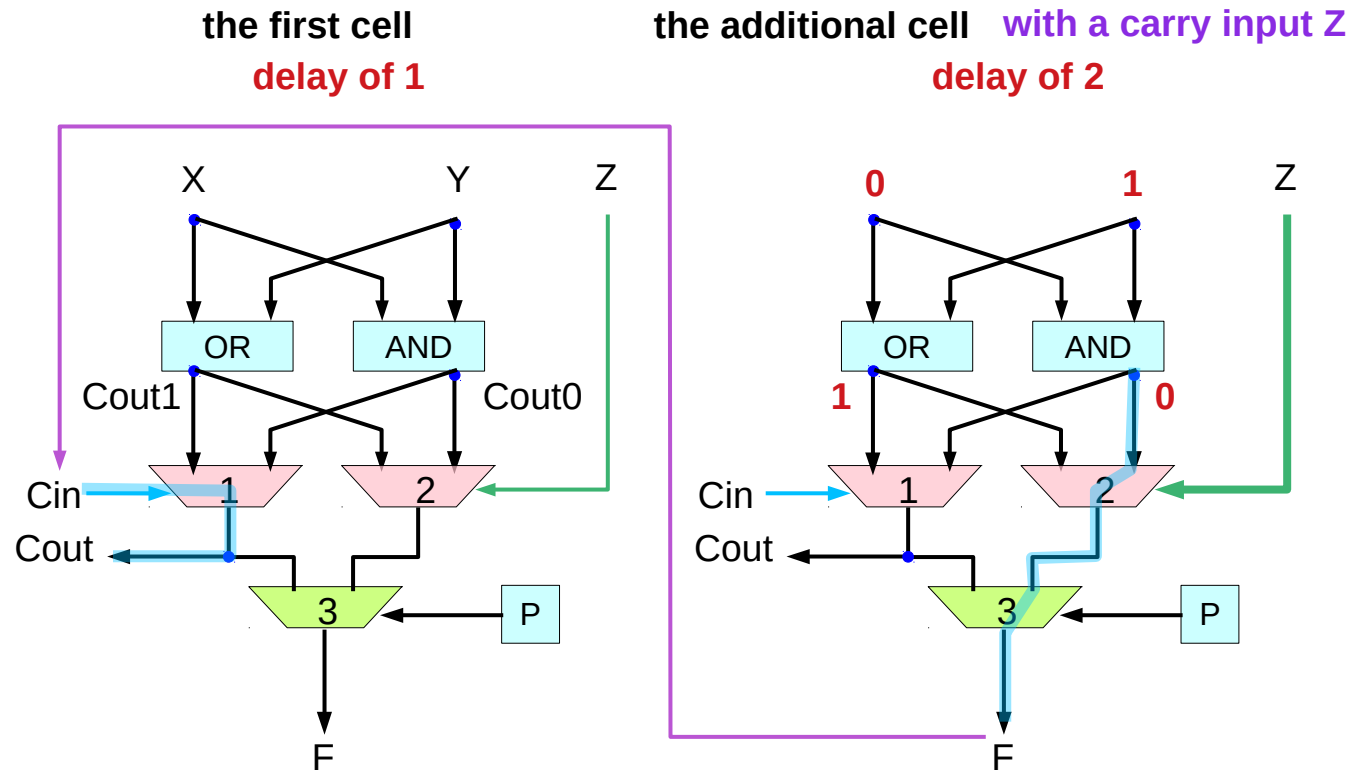
from an additional cell

without a carry input Z

with a carry input Z

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Design A (5)

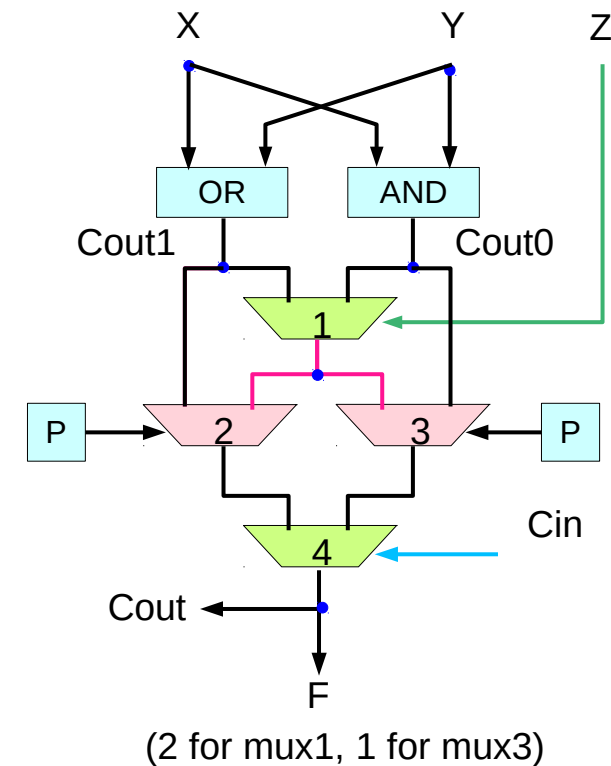


delay of $2(n+1)$ for an n -bit ripple carry chain with a carry input

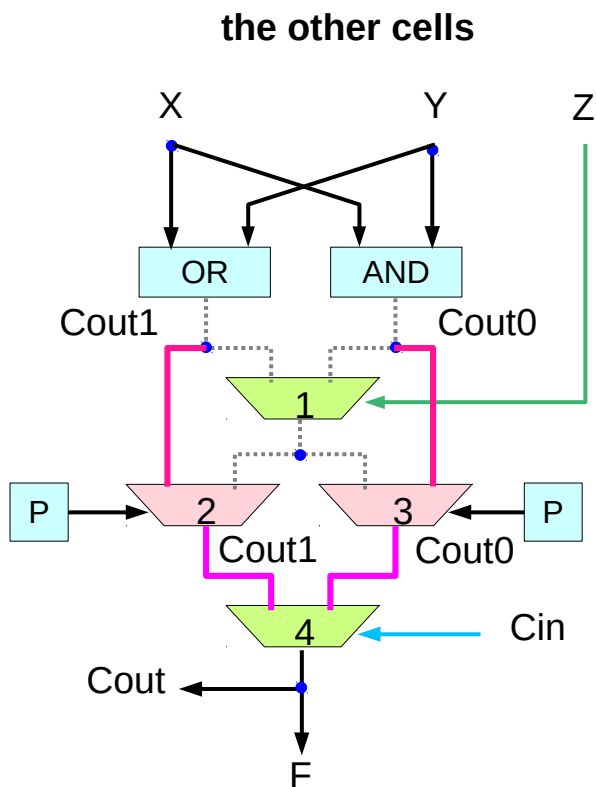
Design B (1)

although this design is 1 gate delay slower than that of fig 2a, it provides the ability to have a **carry input** to the **first cell** in a **carry chain**, something that is important in many computations.

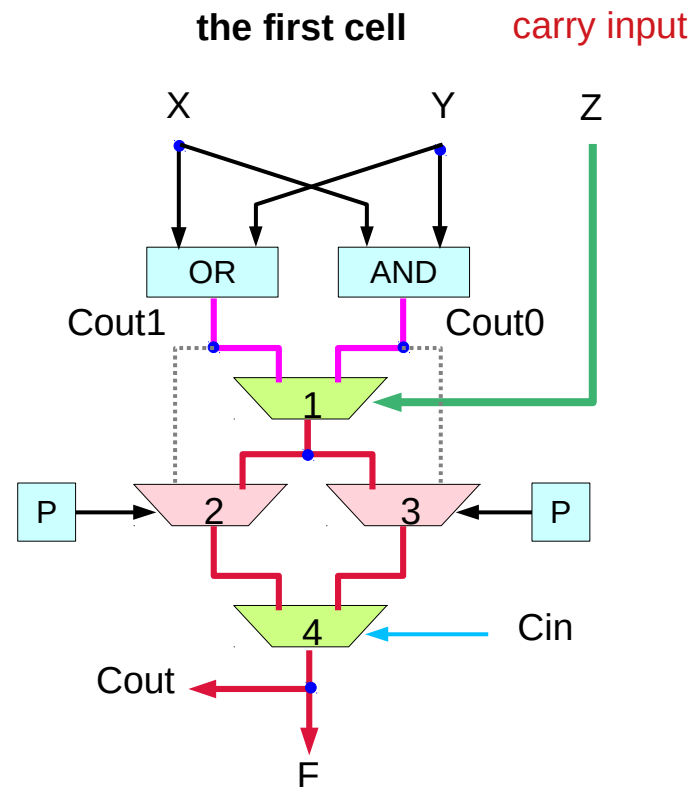
Also, for carry computations that do not need this feature, without a **carry input** the first cell in a **carry chain** built from fig 2b can be configured to bypass **mux1**, reducing the overall delay to $2n$, which is identical to that of fig2a.



Design B (2)



for cells in the middle of a carry chain
 mux2 passes Cout1
 mux3 passes Cout0
 mux4 receives Cout1 and Cout0
 provides a standard ripple carry path.

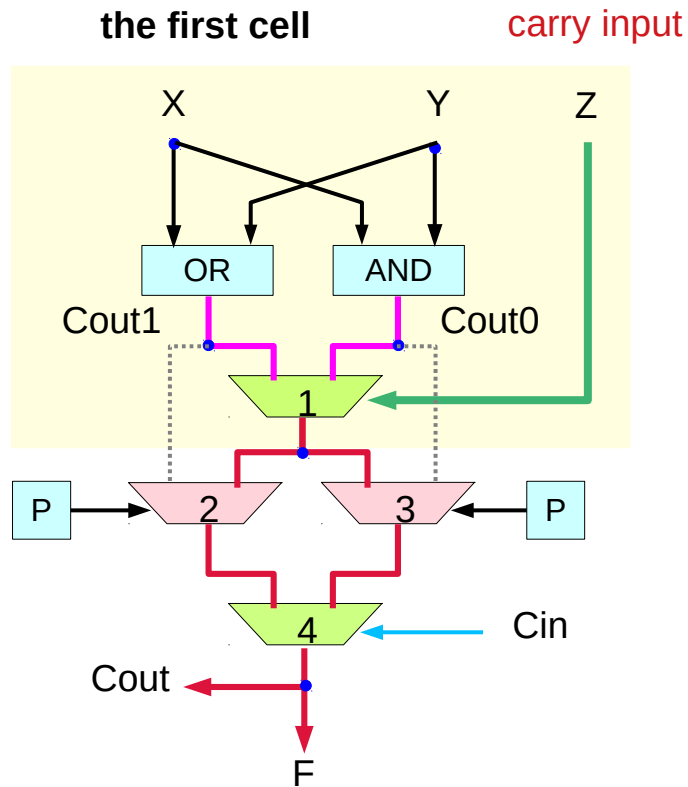


For the first cell in a carry chain
 with a carry input (provided by input Z),
 mux2 and mux3 both pass the value from mux1

the two main inputs to mux4 are identical
 the output of mux4 (Cout) will be the same
 as the output of mux1 (ignoring Cin)

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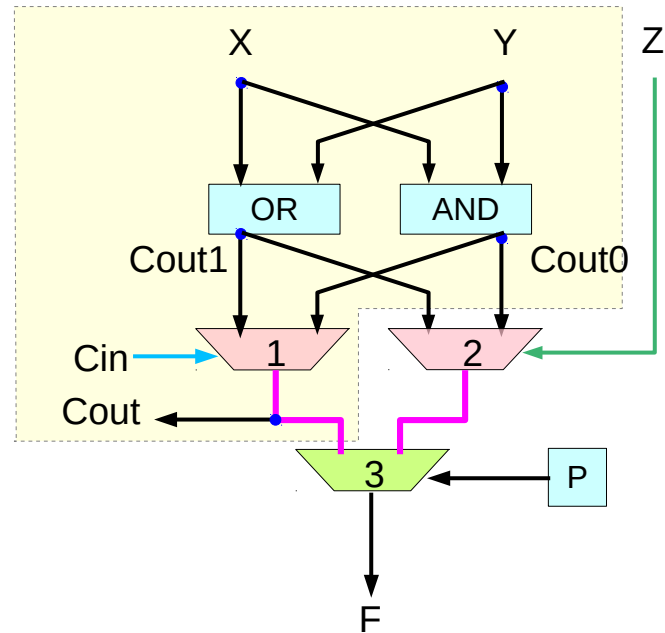
Design B (3)



mux1's main inputs are driven by two 2-LUTs (OR, AND) controlled by **X** and **Y**
mux1 forms a **3-LUT** with the other 2-LUTs

When **mux2** and **mux3** pass the value from **mux1** (**Cout1** and **Cout2** respectively)
the circuit is configured to continue the carry chain

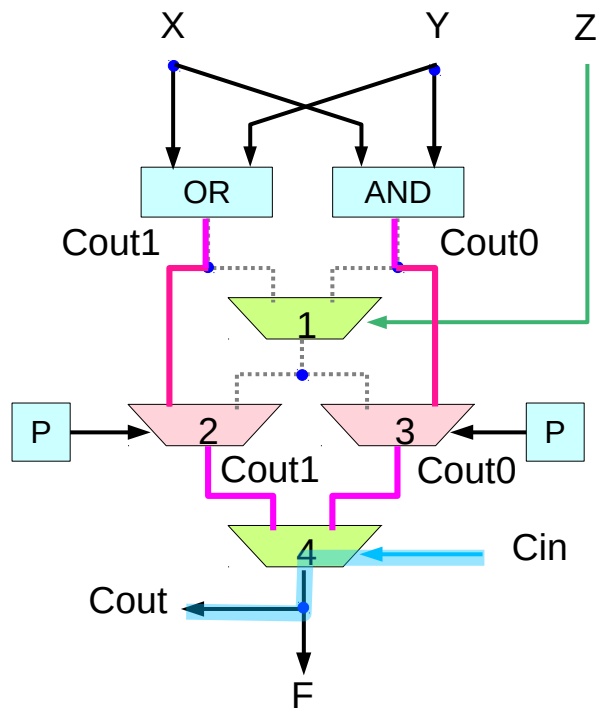
Functionally equivalent



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Design B (4)

delay of 2 the other cells

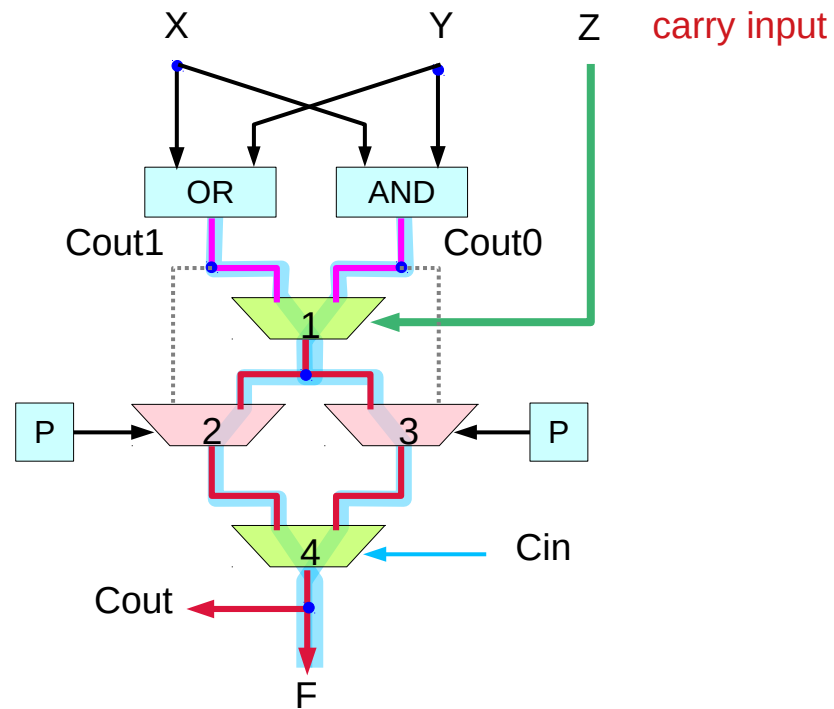


A delay of 2 in all other cells
except the first cell in the carry chain

an total delay of $2n+1$ for an n-bit carry chain
 when a carry input to the first cell is enabled

1 gate delay slower than that of fig 2a,

delay of 3 the first cell with a carry input

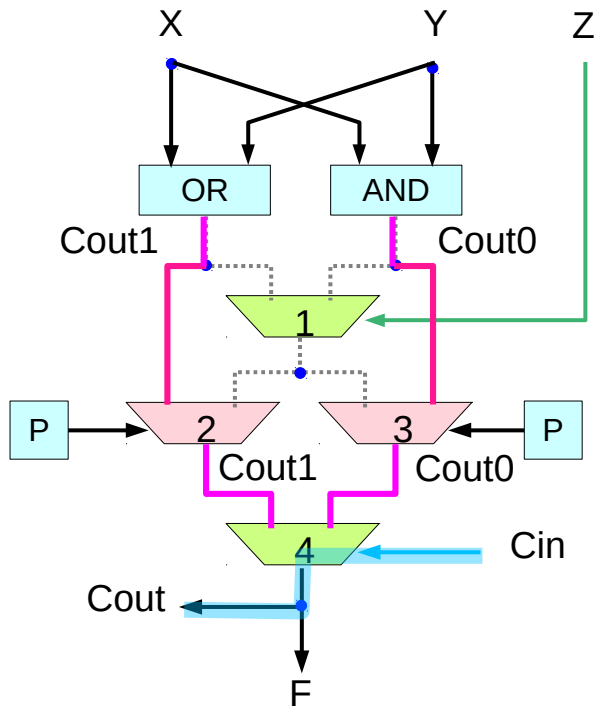


a delay of 3 in the first cell
 1 in mux1, 1 in mux2, 1 in mux4

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Design B (5)

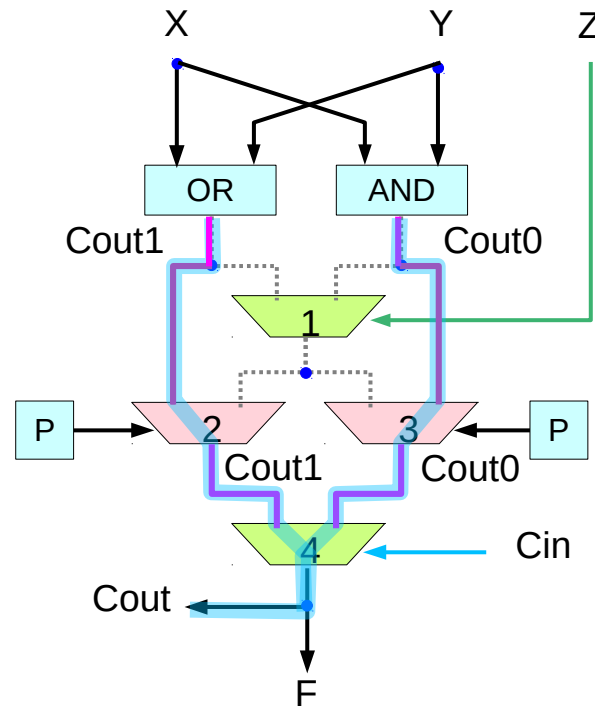
delay of 2 the other cells



A delay of 2 in all other cells
except the first cell in the carry chain

an total delay of **2n** for an n-bit carry chain
when a carry input to the first cell is **disabled**

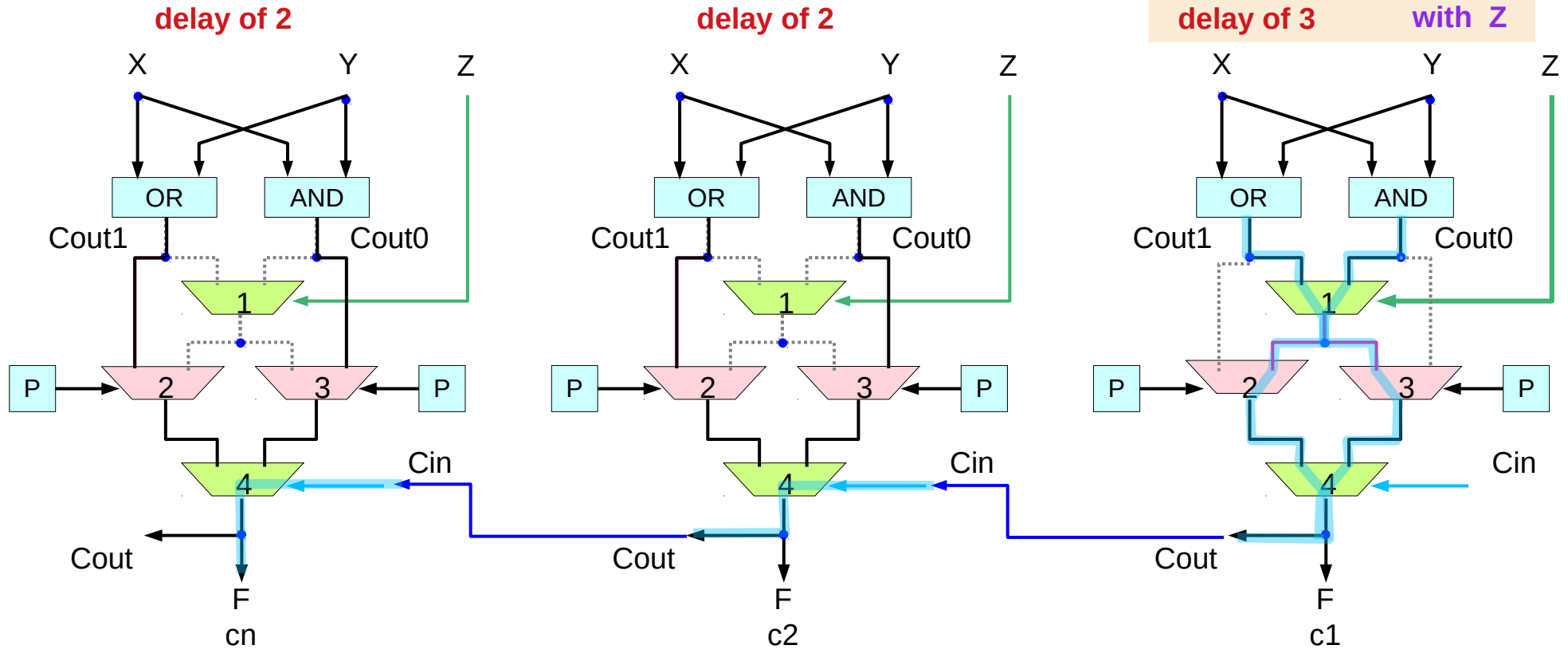
delay of 2 the first cell without a carry input



a delay of 2 in the first cell
when a carry input is not used

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Design B (6)



delay of $2n$ for an n -bit ripple carry chain

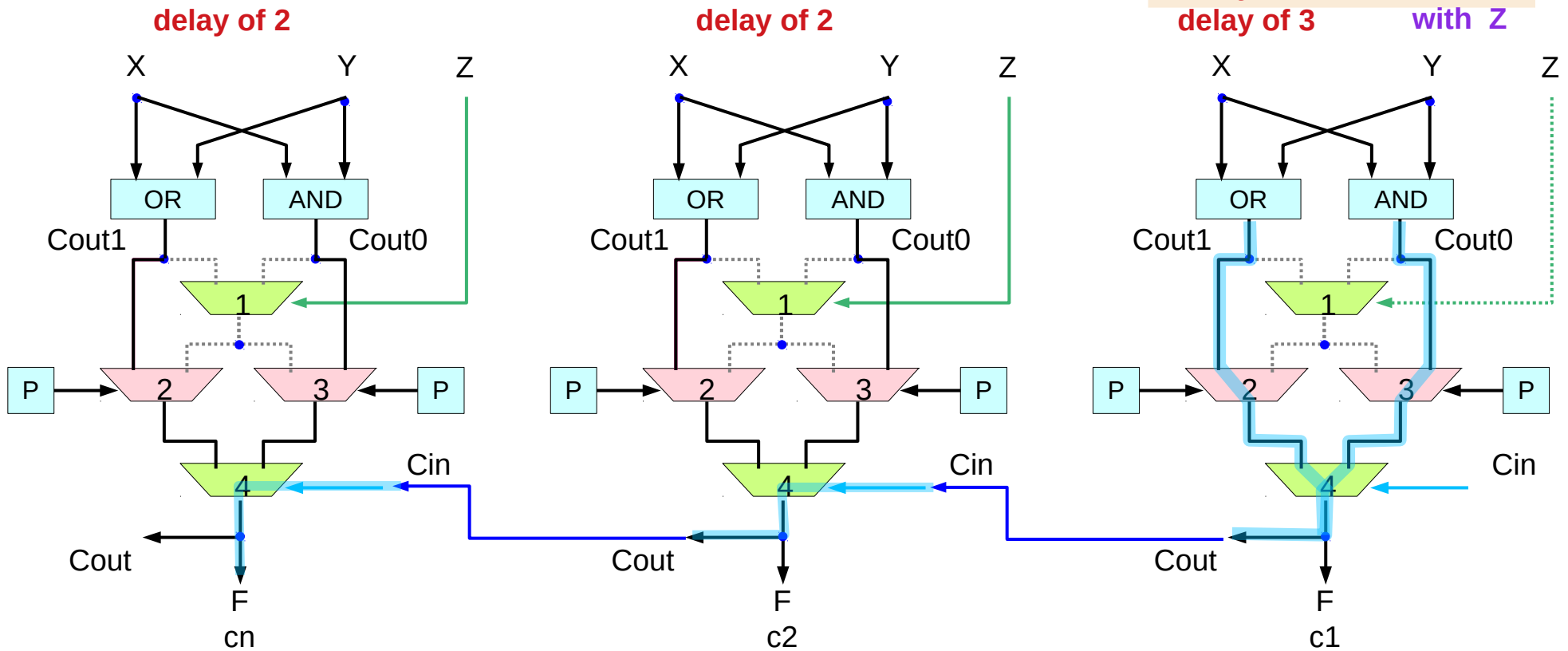
without a carry input Z

delay of $2n+1$

with a carry input Z

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Design B (7)



delay of $2n$ for an n -bit ripple carry chain

without a carry input Z

delay of $2n+1$

with a carry input Z

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Design C (1)

the actual carry chain (mux4) in Design B has been replaced by

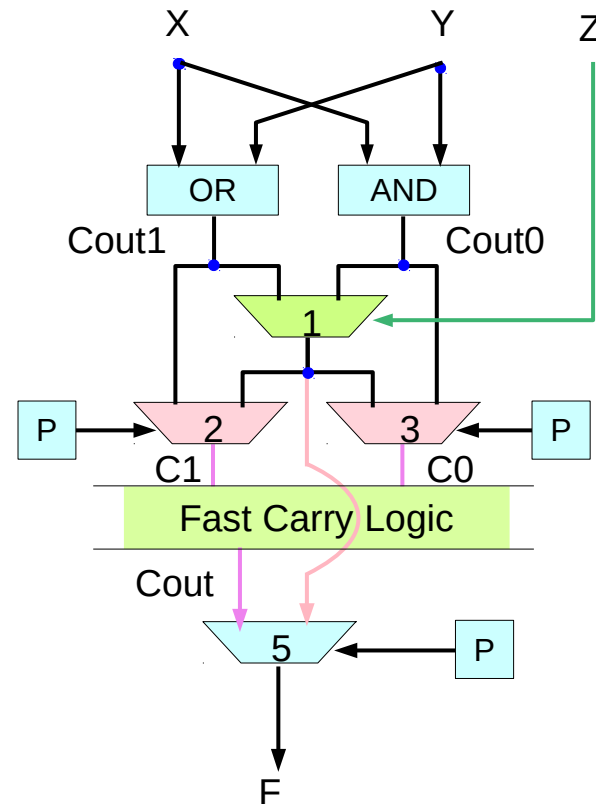
- an **abstract fast carry logic unit**
- **mux5** has been added

to the **abstract fast carry logic units**, various high performance carry chains can be applied

mux5 is present because

- significant delay for **non-carry computations**
- much faster **carry propagation** for long carry chains

when used as a simple normal **3 LUT**, using inputs X, Y, and Z
mux5 allows us to bypass the carry chain by selecting the output of **mux1**



Design C (2)

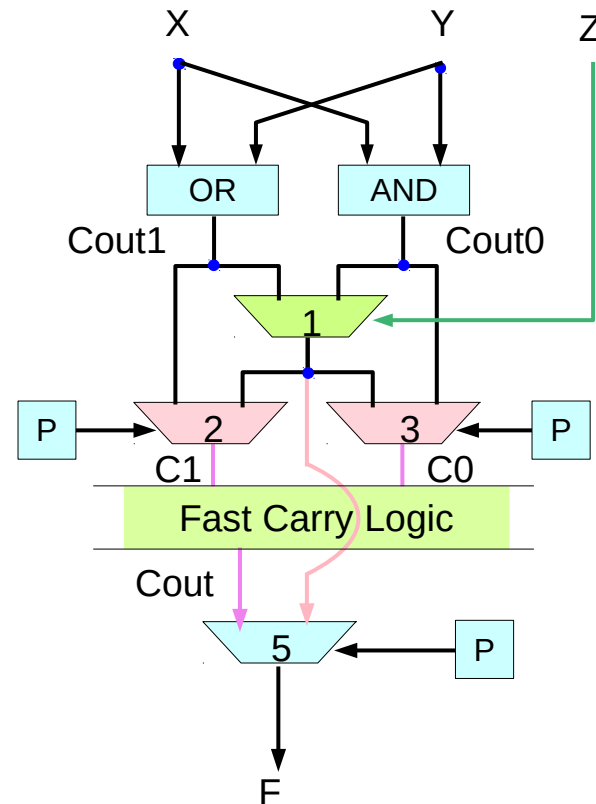
All the developed fast carry logic units in Design C that can compute the following value, can provide the functionality necessary to support the needs of FPGA carry chain computations

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

where i is the position of the cell within the carry chain,

thus, the **fast carry logic unit** can contain any logic structure implementing this equation (including Brent-Kung, Variable Bit, and Ripple Carry).

Note that because of the needs and requirements of carry chains for FPGAs, new circuits are developed, by utilizing the standard adder structures, but which are more appropriate for FPGAs



Design C (3)

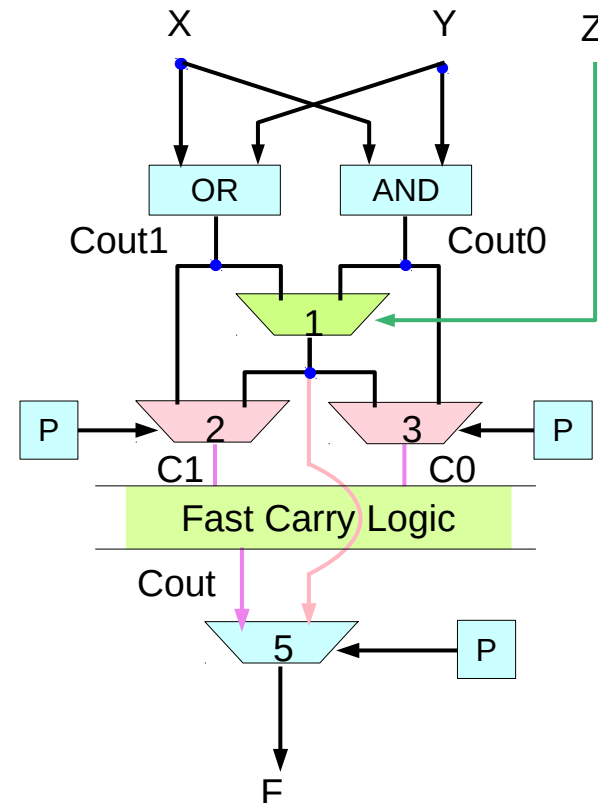
the main difference is to support all states

- Generate
- Propagate
- Kill
- Inverse Propagate

These 4 states are encoded on signals **C1** and **C0**

Also, while standard adders are concerned only with the maximum delay through an entire **n-bit adder** structure, the delay concerns for FPGAs are more complicated

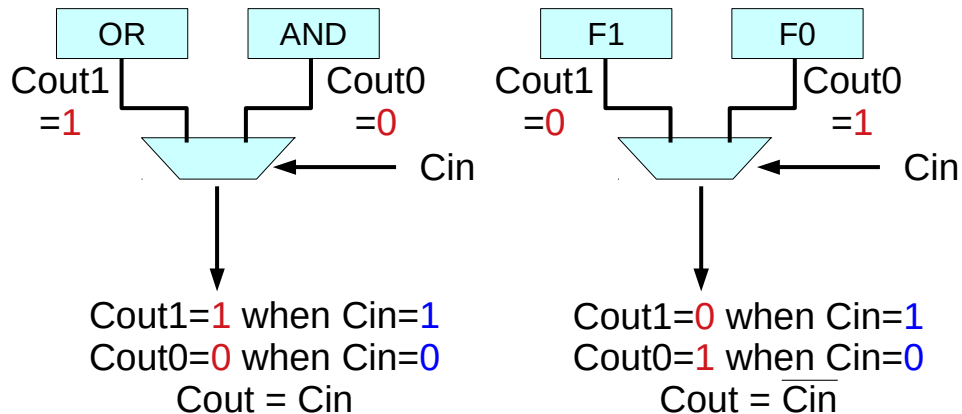
Specifically, when an **n-bit carry chain** is built into the architecture of an FPGA it does not represent an actual computation, but only the potential for a computation.



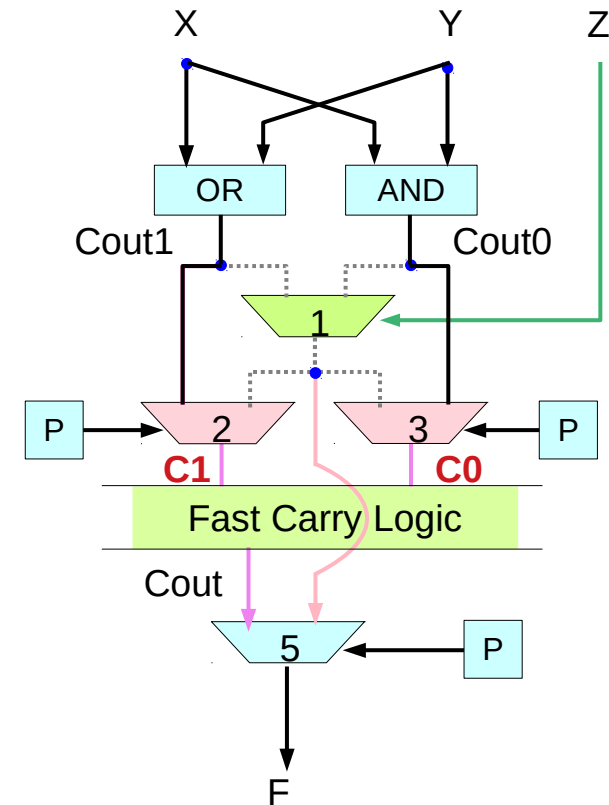
Design C (4)

X	Y	Cin	Cout0	Cin
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X} Y$
1	0	1	0	$X \overline{Y}$
1	1	1	1	$X Y$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	\overline{Cin}	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



C1	C0	Name
0	0	0 Kill
0	1	\overline{Cin} Inverse Propagate
1	0	Cin Propagate
1	1	1 Generate



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Design C (6) - Complements of C0 and C1

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

X	Y	C1
0	0	0
0	1	1
1	0	1
1	1	1

$$C0 = XY$$

X	Y	C0
0	0	0
0	1	0
1	0	0
1	1	1

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

$$C0 = XY$$

$$\bar{C1} = \overline{(\bar{X}Y) + (X\bar{Y}) + (XY)} = \bar{X}\bar{Y}$$

$$\bar{C0} = \bar{X} + \bar{Y} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

$$\bar{C1} = \bar{X}\bar{Y}$$

X	Y	$\bar{C1}$
0	0	0
0	1	1
1	0	1
1	1	1

$$\bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

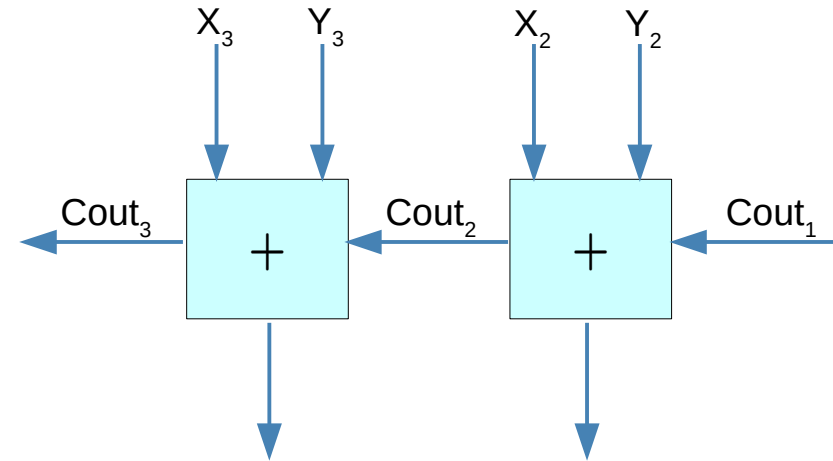
X	Y	$\bar{C0}$
0	0	0
0	1	1
1	0	1
1	1	1

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \bar{C1}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \bar{C0}_2))$$

$$= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + X_2Y_2) + C0_3 \cdot \bar{X}_2\bar{Y}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot X_2Y_2 + C0_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + \bar{X}_2\bar{Y}_2)))$$

Design C (7) - Cout₃ in terms of Cout₁

X ₃ Y ₃	X ₂ Y ₂	Cout ₂	Cout ₃	Cout ₃
0 0	0 0	0	0	0
0 0	0 1	Cout ₁	0	0
0 0	1 0	Cout ₁	0	0
0 0	1 1	1	0	0
0 1	0 0	0	Cout ₃	0
0 1	0 1	Cout ₁	Cout ₃	Cout ₁
0 1	1 0	Cout ₁	Cout ₃	Cout ₁
0 1	1 1	1	Cout ₃	1
1 0	0 0	0	Cout ₃	0
1 0	0 1	Cout ₁	Cout ₃	Cout ₁
1 0	1 0	Cout ₁	Cout ₃	Cout ₁
1 0	1 1	1	Cout ₃	1
1 1	0 0	0	1	1
1 1	0 1	Cout ₁	1	1
1 1	1 0	Cout ₁	1	1
1 1	1 1	1	1	1



$$Cout_3 = (Cout_1 \cdot (C_{13} \cdot C_{12} + C_{03} \cdot \overline{C_{12}})) + (\overline{Cout_1} \cdot (C_{13} \cdot C_{02} + C_{03} \cdot \overline{C_{02}}))$$

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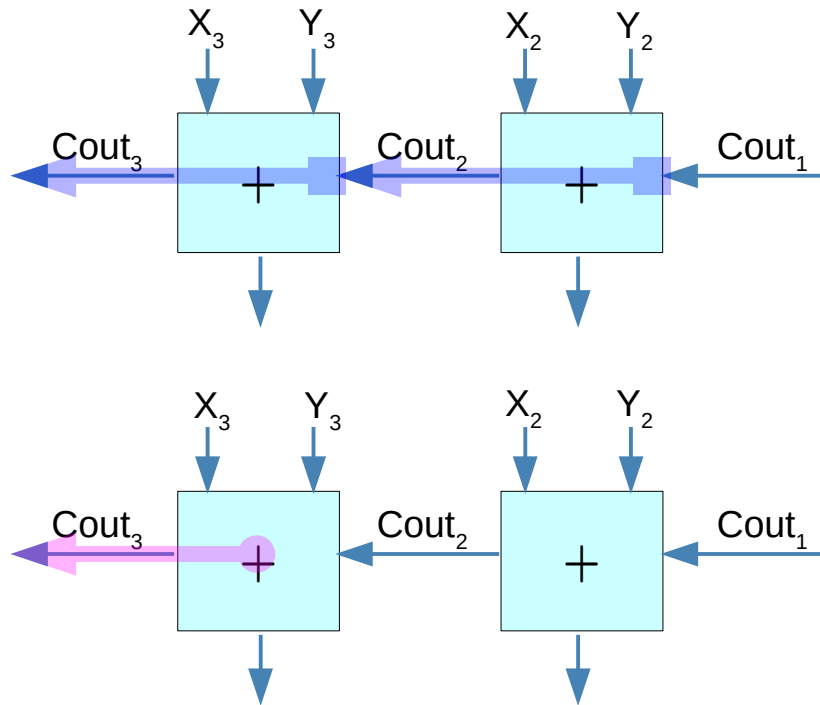
Design C (8) - Cout₃ in terms of Cout₁

X ₃ Y ₃	X ₂ Y ₂	C ₁₃	C ₀₃	C ₁₂	C ₀₂	Cout ₁		$\overline{\text{Cout}}_1$		Cout ³
						C ₁₃ C ₁₂	C ₀₃ $\overline{\text{C}}_1$	C ₁₃ C ₀₂	C ₀₃ $\overline{\text{C}}_0$	
0 0	0 0	0	0	0	0	0	0	0	0	0
0 0	0 1	0	0	1	0	0	0	0	0	0
0 0	1 0	0	0	1	0	0	0	0	0	0
0 0	1 1	0	0	1	1	0	0	0	0	0
0 1	0 0	1	0	0	0	0	0	0	0	0
0 1	0 1	1	0	1	0	1	0	0	0	Cout ₁
0 1	1 0	1	0	1	0	1	0	0	0	Cout ₁
0 1	1 1	1	0	1	1	1	0	1	0	1
1 0	0 0	1	0	0	0	0	0	0	0	0
1 0	0 1	1	0	1	0	1	0	0	0	Cout ₁
1 0	1 0	1	0	1	0	1	0	0	0	Cout ₁
1 0	1 1	1	0	1	1	1	0	1	0	1
1 1	0 0	1	1	0	0	0	1	0	1	1
1 1	0 1	1	1	1	0	1	0	0	1	1
1 1	1 0	1	1	1	0	1	0	0	1	1
1 1	1 1	1	1	1	1	1	0	1	0	1

$$\text{Cout}_3 = (\text{Cout}_1 \cdot (C_{13} \cdot C_{12} + C_{03} \cdot \overline{C_{12}})) + (\overline{\text{Cout}}_1 \cdot (C_{13} \cdot C_{02} + C_{03} \cdot \overline{C_{02}}))$$

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Design C (9) - When Cout1 = 1



$$C1_3 \cdot C1_2 \cdot Cout_1$$

prop

prop

$$\overline{X_3}Y_3$$

$$X_3\overline{Y_3}$$

$$X_3Y_3$$

$$\overline{X_2}Y_2$$

$$X_2\overline{Y_2}$$

$$X_2Y_2$$

$$C0_3 \cdot \overline{C1_2} \cdot Cout_1$$

gen

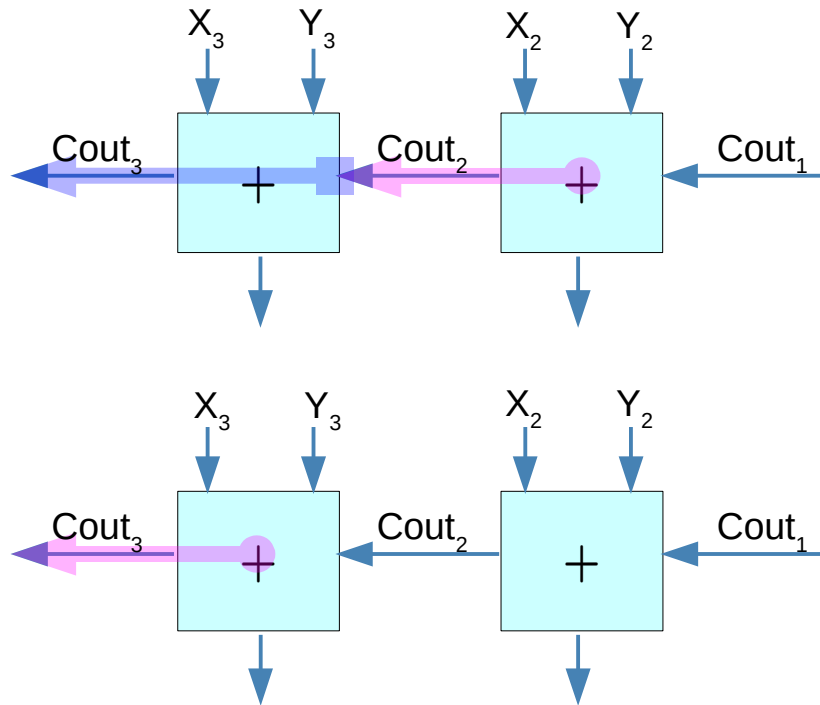
$\overline{\text{prop}}$

$$X_3Y_3$$

$$\overline{X_2}Y_2$$

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) + (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}))$$

Design C (10) - When Cout1 = 0



$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$

prop

gen

$$\overline{X_3}Y_3$$

$$X_2Y_2$$

$$X_3\overline{Y_3}$$

$$X_3Y_3$$

$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$

gen

gen

$$X_3Y_3$$

$$\overline{X_2}Y_2$$

$$X_2\overline{Y_2}$$

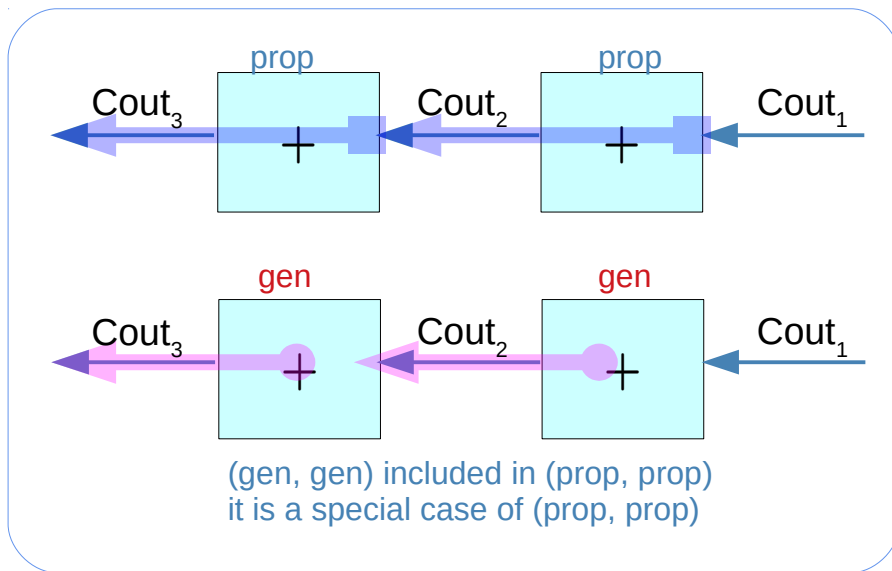
$$X_2Y_2$$

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) + (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}))$$

$$(C1_3 C1_2 + C0_3 \overline{C1_2})Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2})\overline{Cout_1}$$

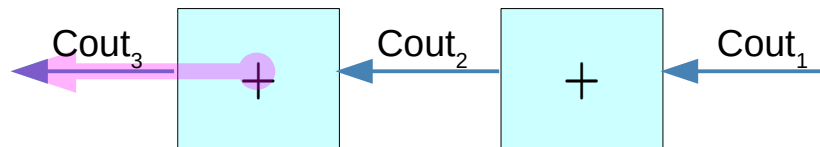
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Design C (11) - When Cout1 = 1



$$C 1_3 \cdot C 1_2 \cdot Cout_1$$

prop	prop
$\bar{X}_3 Y_3$	$\bar{X}_2 Y_2$
$X_3 \bar{Y}_3$	$X_2 \bar{Y}_2$
$X_3 Y_3$	$X_2 Y_2$



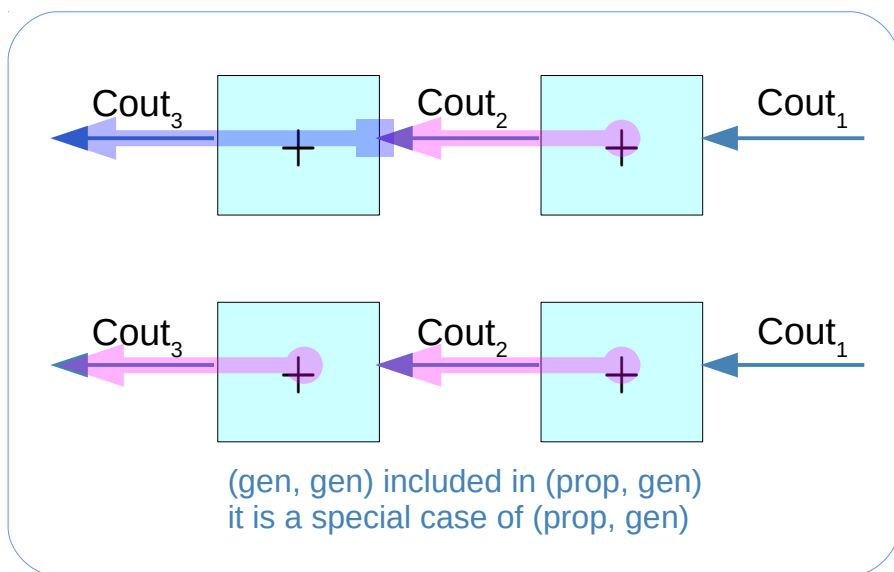
$$C 0_3 \cdot \overline{C 1_2} \cdot Cout_1$$

gen	$\overline{\text{prop}}$
$X_3 Y_3$	$\overline{X_2 Y_2}$

$$Cout_3 = (Cout_1 \cdot (C 1_3 \cdot C 1_2 + C 0_3 \cdot \overline{C 1_2})) + (\overline{Cout_1} \cdot (C 1_3 \cdot C 0_2 + C 0_3 \cdot \overline{C 0_2}))$$

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Design C (12) - When Cout1 = 0



$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$

prop

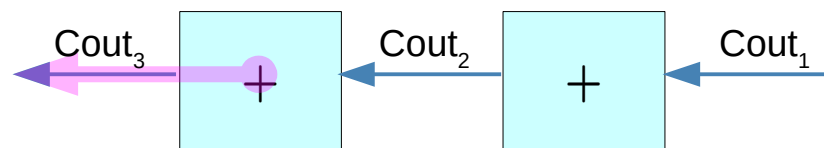
gen

$$\overline{X_3}Y_3$$

$$X_2Y_2$$

$$X_3\overline{Y_3}$$

$$X_3Y_3$$



$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$

gen

gen

$$X_3Y_3$$

$$\overline{X_2}Y_2$$

$$X_2\overline{Y_2}$$

$$X_2Y_2$$

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) + (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}))$$

$$(C1_3 C1_2 + C0_3 \overline{C1_2})Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2})\overline{Cout_1}$$

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FPGA Carry Chain Cell

$$C1 = \bar{X}Y + X\bar{Y} + XY \quad C0 = XY$$

$$\bar{C1} = \bar{X}\bar{Y} \quad \bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

C1 and C0 are not mutually exclusive

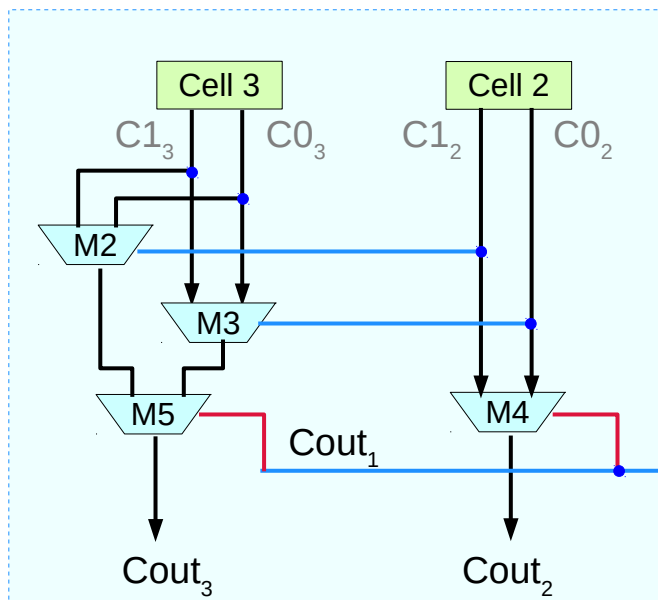
C1 includes C0

$$C1 \cdot C0 = C0 \quad C1 + C0 = C1$$

$$\bar{C1} \cdot \bar{C0} = \bar{C1} \quad \bar{C1} + \bar{C0} = \bar{C0}$$

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Design C - Carry Select (1)



$$C1 = \bar{X}Y + X\bar{Y} + XY \quad C0 = XY$$

$$\bar{C1} = \bar{X}\bar{Y} \quad \bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\bar{Cout}_2 \cdot C0_3)$$

$$Cout_2 = (Cout_1 \cdot C1_2) + (\bar{Cout}_1 \cdot C0_2)$$

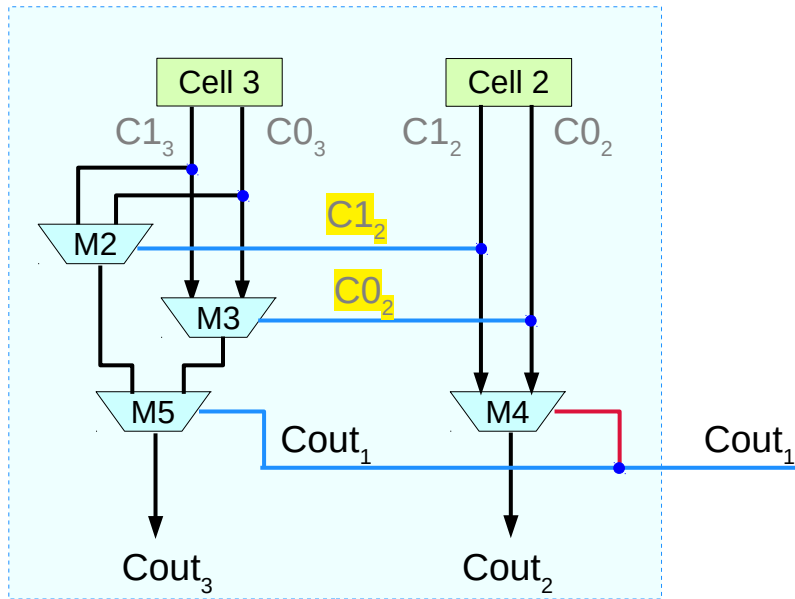
$$Cout_1 = (Cout_0 \cdot C1_1) + (\bar{Cout}_0 \cdot C0_1)$$

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \bar{C1}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \bar{C0}_2))$$

$$= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + X_2 Y_2) + C0_3 \cdot \bar{X}_2 \bar{Y}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot X_2 Y_2 + C0_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + \bar{X}_2 \bar{Y}_2)))$$

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FPGA Carry Chain Cell



$$Cout_2 = (Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\overline{Cout_2} \cdot C0_3)$$

$$= (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3) + (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C0_3)$$

$$= (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3) = (C1_3 C1_2 Cout_1 + C1_3 C0_2 \overline{Cout_1})$$

$$\begin{aligned} & (((\overline{Cout_1} \cdot C1_2) \cdot (\overline{Cout_1} \cdot C0_2)) \cdot C0_3) \\ &= (((\overline{Cout_1} + \overline{C1_2}) \cdot (\overline{Cout_1} + \overline{C0_2})) \cdot C0_3) \\ &= (\overline{Cout_1} \overline{Cout_1} + \overline{C1_2} \overline{Cout_1} + \overline{Cout_1} \overline{C0_2} + \overline{C1_2} \overline{C0_2}) \cdot C0_3 \\ &= (\overline{C1_2} \overline{Cout_1} + \overline{C0_2} \overline{Cout_1}) \cdot C0_3 \\ &= (C0_3 \overline{C1_2} \overline{Cout_1} + C0_3 \overline{C0_2} \overline{Cout_1}) \end{aligned}$$

$$\overline{C1_2} \overline{C0_2} = (\overline{X_2} \overline{Y_2}) \cdot (\overline{X_2} Y_2 + X_2 \overline{Y_2} + \overline{X_2} \overline{Y_2}) = \overline{C1_2}$$

$$\overline{C1_2} \overline{Cout_1} + \overline{C1_2} = \overline{C1_2} (\overline{Cout_1} + 1)$$

$$(C1_3 C1_2 + C0_3 \overline{C1_2}) Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2}) \overline{Cout_1}$$

$$C1 = \overline{X} Y + X \overline{Y} + X Y \quad C0 = X Y$$

$$\overline{C1} = \overline{X} \overline{Y} \quad \overline{C0} = \overline{X} Y + X \overline{Y} + \overline{X} \overline{Y}$$

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FPGA Carry Chain Cell

$$\begin{aligned}
 &= (\overline{Cout_1}Cout_1 + \overline{C1_2}Cout_1 + \overline{Cout_1}C0_2 + \overline{C1_2}C0_2) \cdot C0_3 \\
 &= (\overline{C1_2}Cout_1 + \overline{Cout_1}C0_2 + \overline{C1_2}C0_2) \cdot C0_3 \\
 &= (\overline{C1_2}C0_2Cout_1 + \overline{Cout_1}C0_2 + \overline{C1_2}C0_2) \cdot C0_3
 \end{aligned}$$

$$C1 \cdot C0 = C0$$

$$C1 + C0 = C1$$

$$\overline{C1} \cdot \overline{C0} = \overline{C1}$$

$$\overline{C1} + \overline{C0} = \overline{C0}$$

$$Cout_2 = (Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\overline{Cout_2} \cdot C0_3)$$

$$= (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3)$$

$$+ (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C0_3)$$

$$(((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3)$$

$$= (C1_3C1_2Cout_1 + C1_3C0_2\overline{Cout_1})$$

$$(((\overline{Cout_1} \cdot C1_2) \cdot (\overline{Cout_1} \cdot C0_2)) \cdot C0_3)$$

$$= (((\overline{Cout_1} + \overline{C1_2}) \cdot (\overline{Cout_1} + \overline{C0_2})) \cdot C0_3)$$

$$= (\overline{Cout_1}Cout_1 + \overline{C1_2}Cout_1 + \overline{Cout_1}C0_2 + \overline{C1_2}C0_2) \cdot C0_3$$

$$= (\overline{C1_2}Cout_1 + \overline{C0_2}Cout_1) \cdot C0_3$$

$$= (C0_3\overline{C1_2}Cout_1 + C0_3\overline{C0_2}Cout_1)$$

$$\overline{C1_2}C0_2 = (\overline{X_2}Y_2) \cdot (\overline{X_2}Y_2 + X_2\overline{Y_2} + \overline{X_2}Y_2) = \overline{C1_2}$$

$$\overline{C1_2}Cout_1 + \overline{C1_2} = \overline{C1_2}(Cout_1 + 1)$$

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FPGA Carry Chain Cell

$$C_1 = \bar{X}Y + X\bar{Y} + XY \quad C_0 = XY$$

$$\bar{C}_1 = \bar{X}\bar{Y} \quad \bar{C}_0 = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

C1 and C0 are not mutually exclusive

C1 includes C0

$$C_{out_2} = (C_{out_1} \cdot C_{1_2}) + (\bar{C}_{out_1} \cdot C_{0_2})$$

$$C_{out_3} = (C_{out_2} \cdot C_{1_3}) + (\bar{C}_{out_2} \cdot C_{0_3})$$

$$= (((C_{out_1} \cdot C_{1_2}) + (\bar{C}_{out_1} \cdot C_{0_2})) \cdot C_{1_3})$$

$$+ (((C_{out_1} \cdot C_{1_2}) + (\bar{C}_{out_1} \cdot C_{0_2})) \cdot C_{0_3})$$

$$\overline{((C_{out_1} \cdot C_{1_2}) + (\bar{C}_{out_1} \cdot C_{0_2}))}$$

$$\overline{((C_{out_1} \cdot (C_{1_2} + C_{0_2})) + (\bar{C}_{out_1} \cdot C_{0_2}))}$$

$$\overline{(C_{out_1} \cdot C_{1_2} + (C_{out_1} + \bar{C}_{out_1}) \cdot C_{0_2})}$$

$$\overline{(C_{out_1} \cdot C_{1_2} + C_{0_2})}$$

$$((\bar{C}_{out_1} + \bar{C}_{1_2}) \cdot \bar{C}_{0_2})$$

$$((\bar{C}_{out_1} \cdot \bar{C}_{0_2} + \bar{C}_{1_2}))$$

$$= (\bar{C}_{out_1} C_{out_1} + \bar{C}_{1_2} C_{out_1} + \bar{C}_{out_1} \bar{C}_{0_2} + \bar{C}_{1_2} \bar{C}_{0_2})$$

$$= (\bar{C}_{1_2} C_{out_1} + \bar{C}_{out_1} \bar{C}_{0_2} + \bar{C}_{1_2})$$

$$= (\bar{C}_{1_2} + \bar{C}_{out_1} \bar{C}_{0_2})$$

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Design C - Carry Select (1)

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

$$C0 = XY$$

$$\bar{C1} = \bar{X}\bar{Y}$$

$$\bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

C1	C0	Name
0	0	0 Kill
0	1	\bar{Cin} Inverse Propagate
1	0	Cin Propagate
1	1	1 Generate

$$Cout_1 = (Cout_0 \cdot C1_1) + (\bar{Cout}_0 \cdot C0_1)$$

$$Cout_2 = (Cout_1 \cdot C1_2) + (\bar{Cout}_1 \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\bar{Cout}_2 \cdot C0_3)$$

$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \bar{C1}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \bar{C0}_2))$$

$$= Cout_1 \cdot [(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (\bar{X}Y + X\bar{Y} + XY)_2 + (XY)_3 \cdot (XY)_2] + \bar{Cout}_1 \cdot [(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (XY)_2 + (XY)_3 \cdot (\bar{X}Y + X\bar{Y} + \bar{X}\bar{Y})_2]$$

$$= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + X_2Y_2) + C0_3 \cdot \bar{X}_2\bar{Y}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot X_2Y_2 + C0_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + \bar{X}_2\bar{Y}_2)))$$

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Design C (3)

A **carry chain resource** may span the entire height of a **column** in the FPGA, but a **mapping** to the logic may use only a small portion of this chain, with the **carry logic** in the mapping starting and ending at arbitrary points in the **column**

Must consider

- the **carry delay** from the first to the last position in a **carry chain**,
- the delay for a **carry computation** beginning at any point within this **column**.

For example, even though the FPGA architecture may provide support for **carry chains** of up to **32 bits**, it must also efficiently support **8 bit carry computations** placed at any point within this **carry chain** resource

Design C (4)

Carry Select

the problem with a **ripple carry** structure is that the **computation** of the **Cout** for bit position **i** cannot begin until after the **computation** has been completed in bit positions **0 .. i-1**

A **carry select** structure overcomes this limitation

the main observation is that for any bit position, the only information it received from the previous bit positions is its **Cin** signal, which can be either **true** or **false**.

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Design C (5)

In a **carry select adder** the **carry chain** is broken at a specific **column**, and two separate additions occur

one for the **true Cin** signal
the other for the **false Cin** signal

These computations can take place before the completion of the **previous columns**, since they do not depend on the actual value of the **Cin** signal

This **Cin** signal is instead used to determine which adder's outputs should be used

if the **Cin** signal is **true**, the output of the following stages comes from the adder that assumed that the **Cin** would be **true**

likewise, a **false Cin** chooses the other adder's output

Design C (6)

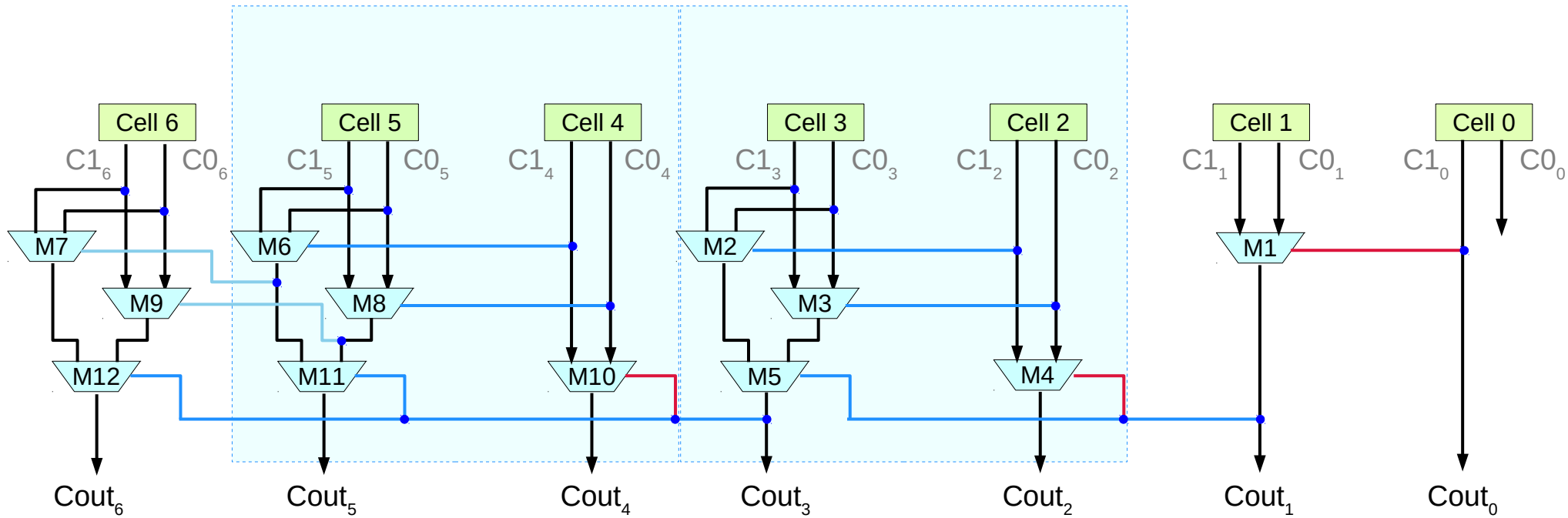
This splitting of the **carry chain** can be done multiple times, breaking the computation into several pairs of **short adders** with **output muxes** choosing which adder's output to **select**

the length of the adders and the breakpoint are carefully chosen such that the **small adders** finish computation just as their **Cin** signals become available

Short adders handle the low-order bits, and the adder length is increased further along the carry chain, since later computations have more time until their **Cin** signal is available

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FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_1 = (C1_0 \cdot C1_1) + (\overline{C1_0} \cdot C0_1)$$

$$Cout_{i+1} = (((Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)) \cdot C1_{i+1}) + (\overline{((Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i))} \cdot C0_{i+1})$$

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Design C - Carry Select (3)

Similarly, **cell4**, **cell5**, **cell6** have

two ripple carry adders

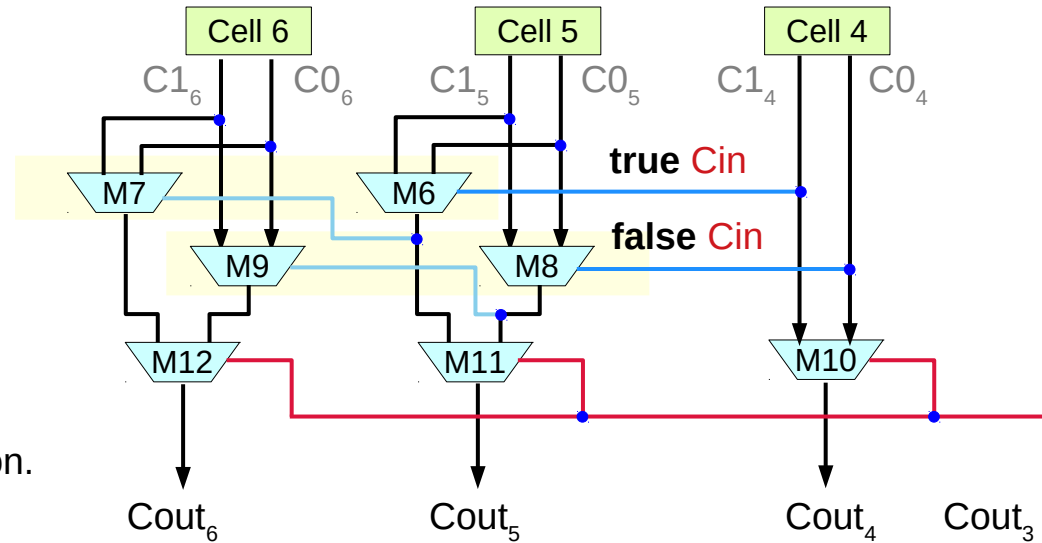
mux6 & **mux7** for a **Cin** of 1

mux8 & **mux9** for a **Cin** of 0

with output muxes (**mux10**, **mux11**, **mux12**)
deciding between the two
based upon the actual **Cin** (from **mux5**).

Subsequent stages will continue
to grow in length by one,
with **cells7**, **cell8**, **cell9**, **cell10** in one block,
cell11, **cell12**, **cell13**, **cell14**, **cell15** in another, and so on.

timing values showing
the delay of the Carry Select carry chain
relative to other carry chain will be presented later



Design C - Carry Select (3)

A Carry Select carry chain structure for use in FPGAs
The carry computation for the first two cells is performed with the simple ripple-carry structure implemented by mux1

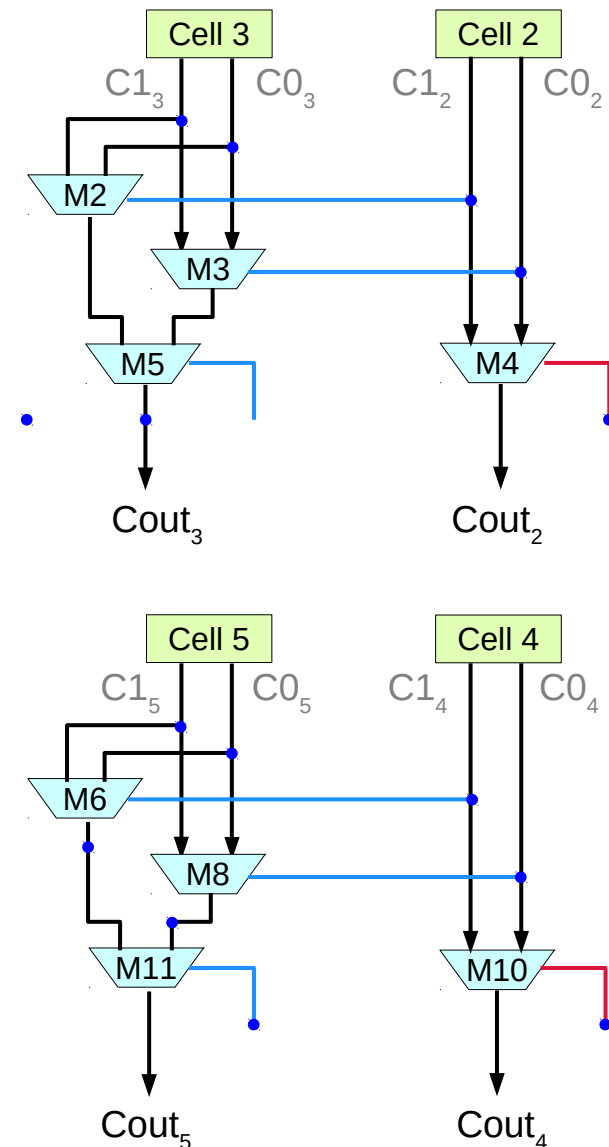
For cells 2 and 3 we use two ripple carry adders, with one adder (implemented by mux2) assuming the C_{in} is true, and the other (mux3) assuming the C_{in} is false

Then muxes 4 and 5 pick between these two adders' outputs based on the actual C_{in} coming from mux1.

Similarly, cells 4-6 have two ripple carry adders (mux6 & mux7 for a C_{in} of 1, mux8 & mux9 for a C_{in} of 0), with output muxes (muxes 10-12) deciding between the two based upon the actual C_{in} (from mux5).

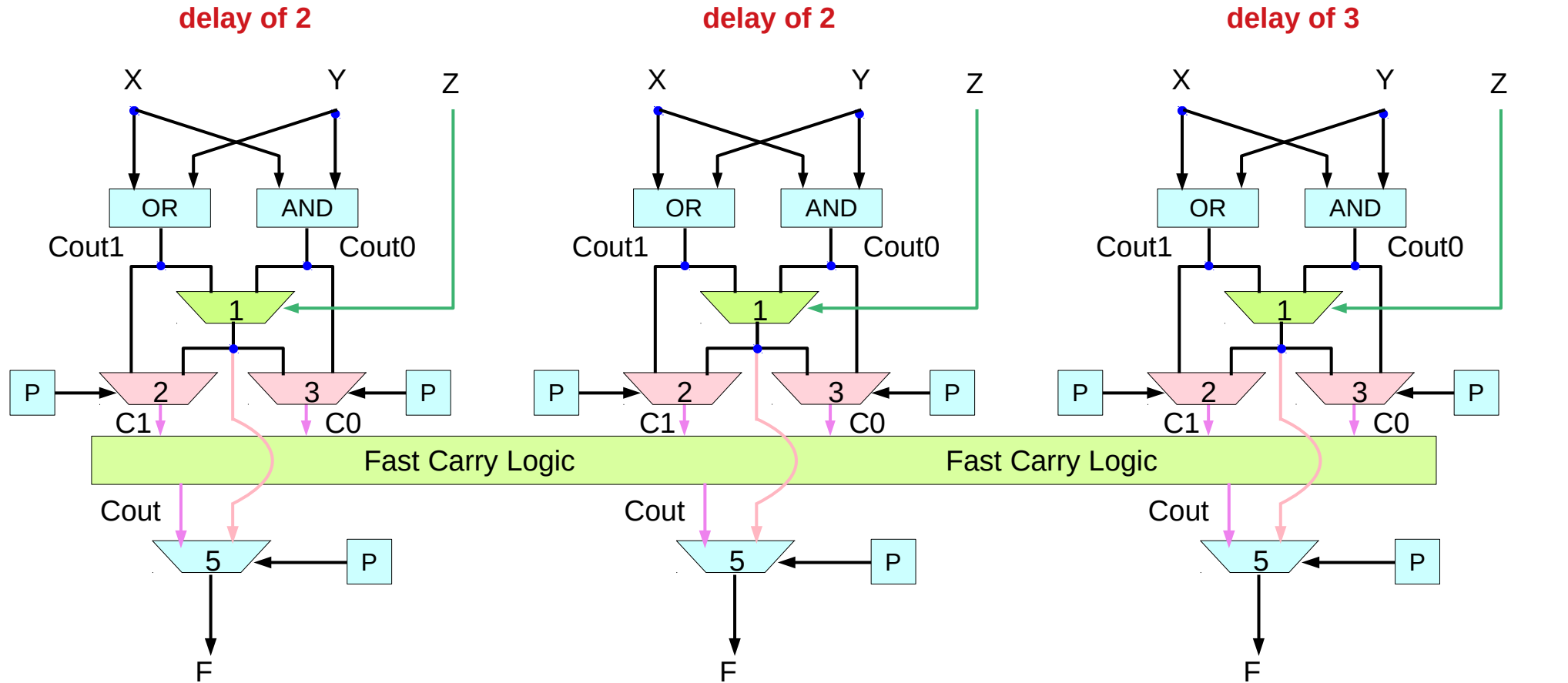
Subsequent stages will continue to grow in length by one, with cells 7-10 in one block, cells 11-15 in another, and so on.

timing values showing the delay of the Carry Select carry chain relative to other carry chain will be presented later



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Design C

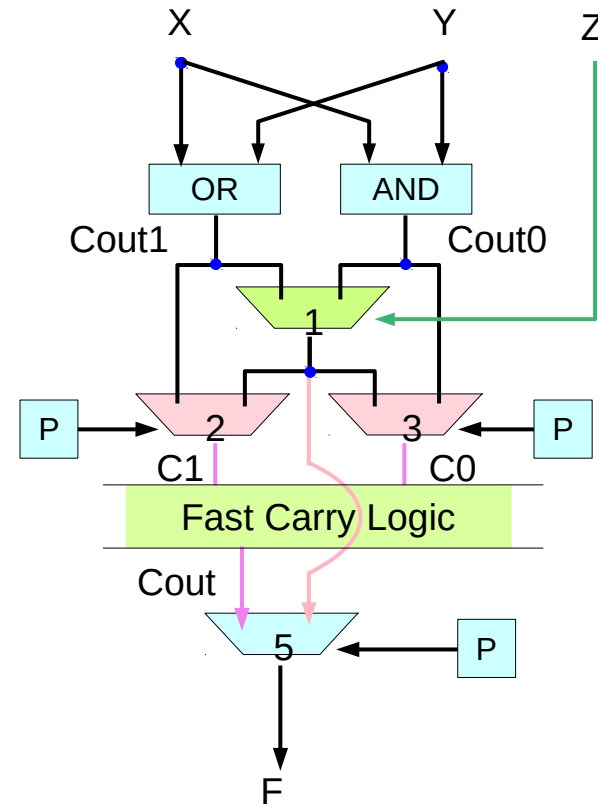
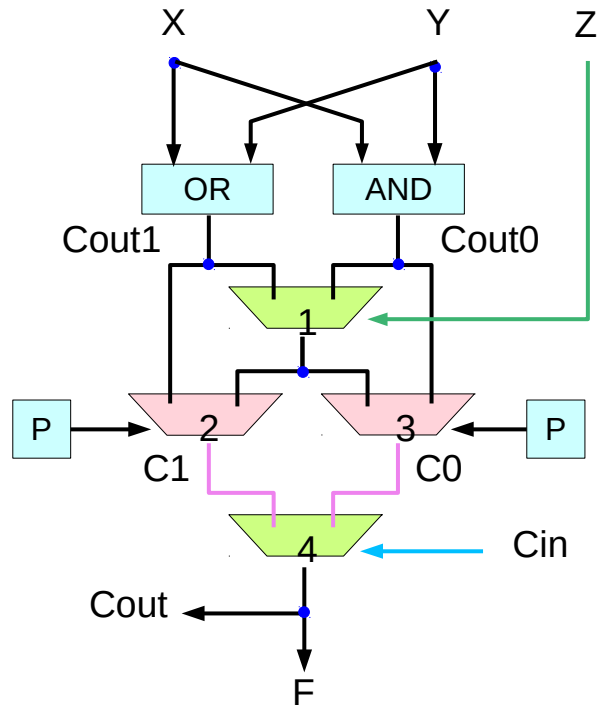


(1 for mux1, 1 for mux2, 1 in mux4)

delay of $2n+2$ for an **n-bit** ripple carry chain

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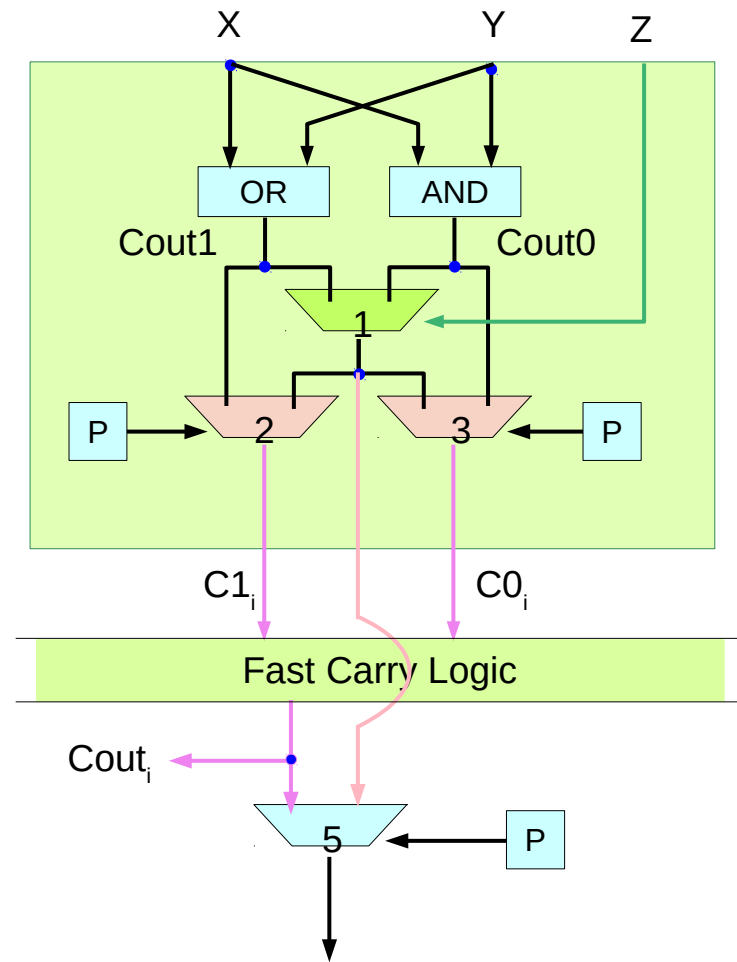
FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

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FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

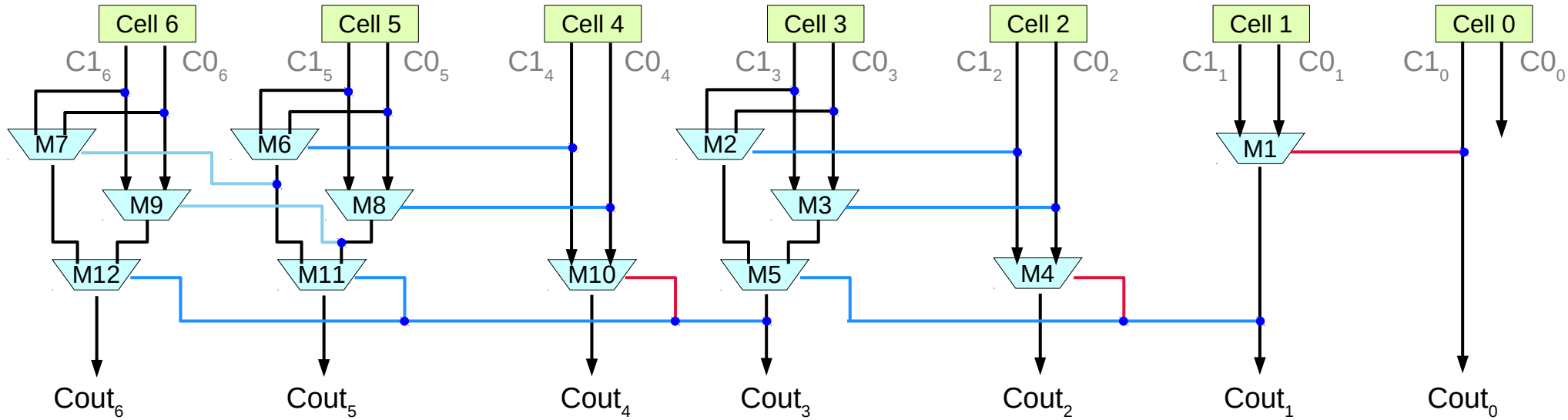
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Fast Carry Logc

Carry Select Adder
Carry Lookahead Adder
 Brent-Kung
Variable Block
Ripple Carry Adder

https://en.wikipedia.org/wiki/Carry-lookahead_adder

FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

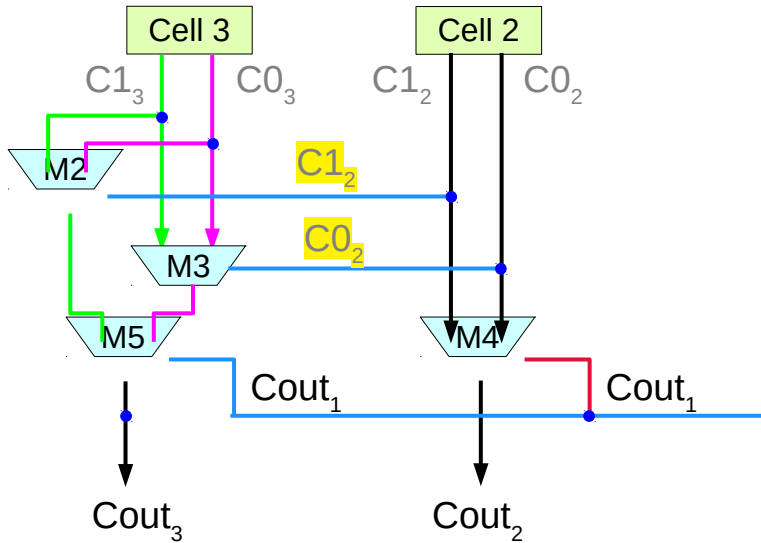
$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_1 = (C1_0 \cdot C1_1) + (\overline{C1_0} \cdot C0_1)$$

$$Cout_{i+1} = (((Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)) \cdot C1_{i+1}) + (\overline{((Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i))} \cdot C0_{i+1})$$

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FPGA Carry Chain Cell

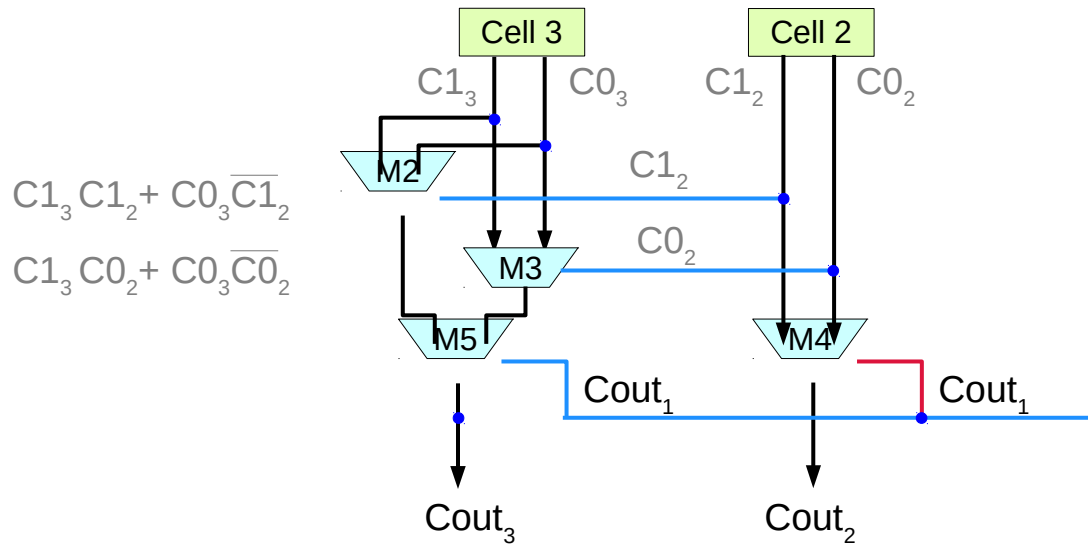


$$\begin{aligned}
 &= (\overline{Cout_1}Cout_1 + \overline{C1_2}Cout_1 + \overline{Cout_1}C0_2 + \overline{C1_2}C0_2) \cdot C0_3 \\
 &= (\overline{C1_2}Cout_1 + \overline{C0_2}Cout_1) \cdot C0_3 \\
 &= (C0_3\overline{C1_2}Cout_1 + C0_3\overline{C0_2}Cout_1)
 \end{aligned}$$

$$(C1_3 C1_2 + C0_3 \overline{C1_2})Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2})\overline{Cout_1}$$

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FPGA Carry Chain Cell



$$C1_3 C1_2 + C0_3 \overline{C1_2}$$

$$C1_3 C0_2 + C0_3 \overline{C0_2}$$

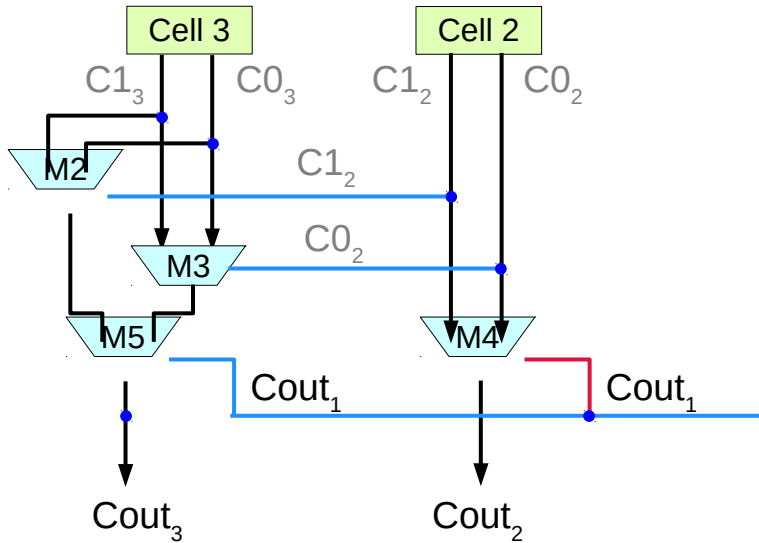
$$(C1_3 C1_2 + C0_3 \overline{C1_2}) Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2}) \overline{Cout_1}$$

$$= C1_3 \cdot (C1_2 Cout_1 + C0_2 \overline{Cout_1})$$

$$+ C0_3 \cdot (\overline{C1_2} Cout_1 + \overline{C0_2} \overline{Cout_1})$$

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FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_{i+1} = \left(\left[(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i) \right] \cdot C1_{i+1} \right) + \left(\left[(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i) \right] \cdot C0_{i+1} \right)$$

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References

[1] <http://en.wikipedia.org/>

[2] J-P Deschamps, et. al., “Synthesis of Arithmetic Circuits”, 2006