ST Monad – Introduction (5A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

A State Transformer

A State Transformer ST Example

in https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A good example to learn **State** monad and similar monads

do not be confused with **monad transformers**

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A State Transformer (ST)

```
type State = ...

type ST = State -> State                    -- a function type
```

about functions that manipulate some kind of state

this state can be represented by a type (State)

a **state transformer** (**ST**) a state manipulating function

takes the current state as its **argument**

produces a modified state as its **result**

which reflects any side effects performed by the function:

a **State Transformer** (**ST**)
not **Monad Transformer**

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A Generalized State Transformer

```
type State = ...

type ST = State -> State

type ST a = State -> (a, State)
```

generalized state transformers

return a <u>result value</u> in addition to the <u>modified state</u>

specify the <u>result type</u> as a parameter of the **ST** type

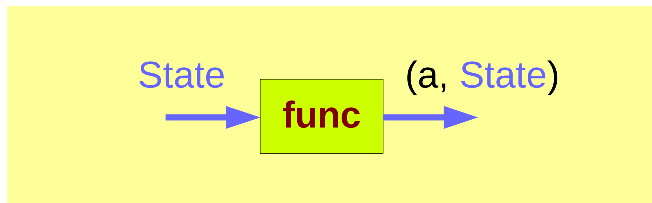
https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Types and Values

type ST a = State -> (a, State)

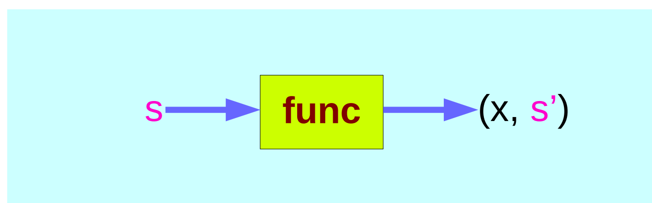
Types

State -> (a, State)



Values

s (x, s')



func :: ST a

func :: State -> (a, State)

x :: a	**the result value**
s :: State	**input state**
s' :: State	**output state**

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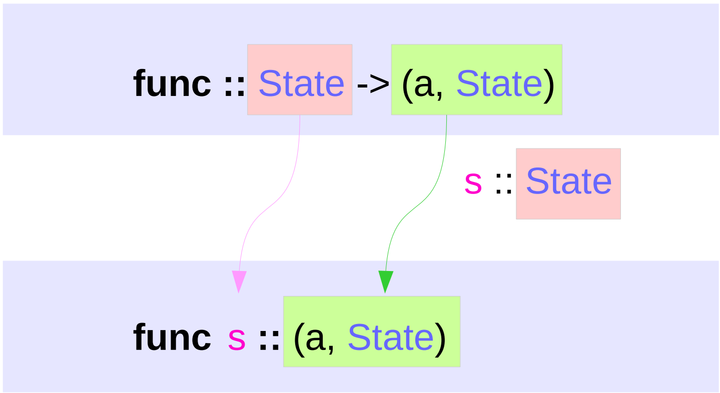
func and **func** s type signatures

type **ST** a = State -> (a, State)

func :: **ST** a



func s ➡ (x, s')

application of
input s gives
output (x, s')

func :: **ST** a
x :: a
s :: State
s' :: State

func :: **ST** a
func :: State -> (a, State)
func s ➡ (x, s')
func s :: (a, State)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

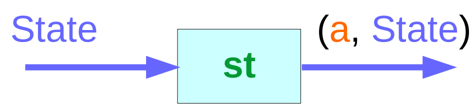
Function input and output types

type **ST** a = State -> (a, State)

st s (x, s')

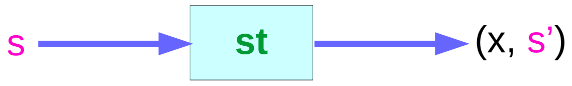
generalized ST

(a_result, updated_state) :: (a, State)



x :: a
s :: State
(x, s') :: (a, State)

st :: **ST** a
s :: State
st s :: (a, State)



type **ST** a State = (a, State)

st s (x, s')

application of **input** s gives **output** (x, s')

st s :: ~~**ST** a State~~ **st** s :: (a, State)

(x, s') :: ~~**ST** a State~~ (x, s') :: (a, State)

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Taking an additional argument

type ST Int = State -> (Int, State)

How to convert **ST Int** into a state transformer

that takes a character and returns an integer ?

further generalization of the state transformer **ST**

which takes an argument of type b

- no need to use more generalized ST type
- instead, use currying.

type ST2 a b

type ST3 b a

type ST2 a b = b -> State -> (a, State)

type ST3 b a = b -> State -> (a, State)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A Curried Generalized State Transformer

type ST a = State -> (a, State)	generalized ST
type ST3 b a = b -> State -> (a, State)	further generalized ST
b -> ST a = b -> State -> (a, State)	think currying

a state transformer
that takes a character
and returns an integer
would have type **Char -> ST Int**

Char -> State -> (**Int**, State)	**curried form**

* Curried Function

f x y	**f :: a -> b -> c**
(f x) y	**f :: a -> (b -> c)**

f x returns a function of type **b -> c**

g y	**g :: b -> c**

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST Monad Instance – return

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

ST : an instance of a monadic type

return converts a value (x)

into a state transformer (s ->(x,s))

that simply *returns that value* (x)

without modifying the state (s → s)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST Monad Instance – >>=

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

```
st >>= f  =  \s -> f x s'
        where (x,s') = st s

st >>= f  =  \s -> (y,s')
        where (x,s') = st s
              (y,s') = f x s'
```

sequencing state transformers: st >>= f

- the 1st state transformer **st** — st s ➡ (x,s') — (1) **input monad** (update + compute)
- the 2nd state transformer (f x) — f x s' ➡ (y,s') — (2) **return monad** (result argument)

1) apply **st** to an initial state s, to get (x,s')
2) apply the function **f** to the x, the value of result
3) apply (**f** x) to the updated state s'

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

The types of the sequencer >>=

```
instance Monad ST where
   -- return :: a -> ST a
   return x  =  \s -> (x,s)

   -- (>>=)  :: ST a -> (a -> ST b) -> ST b
   st >>= f  =  \s -> let (x,s') = st s in f x s'
```

st :: ST a
f :: a -> ST b
(>>=) :: ST a -> (a -> ST b) -> ST b

(x,s') = st s   s → (x,s')
(y,s')= f x s'   s' → (y,s')

st :: State -> (a, State)
f :: a -> State -> (b, State)
(>>=) :: State -> (a, State) -> (a -> State -> (b, State)) -> State -> (b, State)

type ST a = State -> (a, State)

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The type of st s and f x s’

st :: State -> (a, State)

f :: a -> State -> (b, State)

(>>=) :: State -> (a, State) -> (a -> State -> (b, State)) -> State -> (b, State)

st :: State -> (a, State)

st s :: (a, State)

st s Ü (x,s') s -> (x,s')

f :: a -> State -> (b, State)

f x :: State -> (b, State)

f x s’ :: (b, State)

f x s' Ü (y,s') s' -> (y,s')

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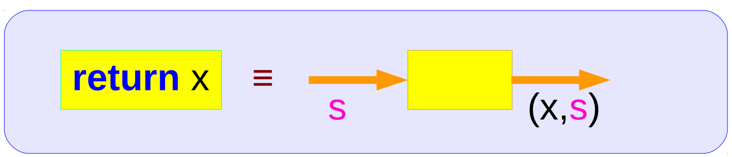
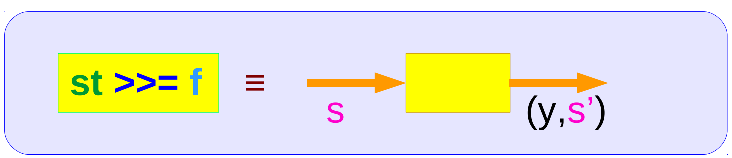
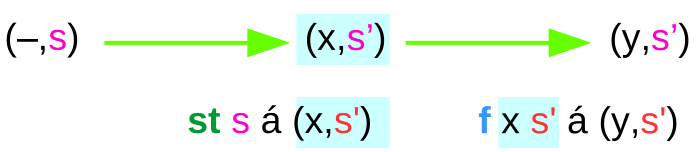
ST Monad – return and >>=

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```







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List, Maybe, and ST Monads

```
instance Monad [] where
  -- return :: a -> [a]
  return x  =  [x]

  -- (>>=)  :: [a] -> (a -> [b]) -> [b]
  xs >>= f  =  concat (map f xs)
```

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

```
instance Monad Maybe where
  -- return:: a->Maybe a
  return x       =  Just x
  -- (>>=) ::
  Maybe a -> (a -> Maybe b) -> M aybe b
  Nothing  >>= _ =  Nothing
  (Just x)  >>= f =  f x
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Dummy Constructor **DC**

```
type ST a = State -> (a, State)
```

~~instances~~ (X)

```
data ST0 a = DC (State -> (a, State))
```

instances (O)

to make instances

use the **data** mechanism

with a dummy constructor (DC)

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The application function **apply0**

```
type ST a = State -> (a, State)

data ST0 a = DC (State -> (a, State))
```

TYPE – NO INSTANCE is allowed

DATA – INSTANCE is allowed

to remove (unwrap) the dummy constructor,

the application function **apply0** is defined

```
apply0  :: ST0 a -> State -> (a, State)
           input    output
```

an accessor function

like a **runState** function

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

apply0 and DC

```
type ST a = State -> (a, State)

data ST0 a = DC (State -> (a, State))


apply0  :: ST0 a -> State -> (a, State)
           input    output

apply0  ST0 a  ::  State -> (a, State)          unwrapping


DC (State -> (a, State)) :: ST0 a               wrapping
```

TYPE – NO INSTANCE is allowed

DATA – INSTANCE is allowed

an accessor function

like a **runState** function

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Unwrapping Data Constructor in (DC g)

```
data ST0 a = DC (State -> (a, State))          Data Constructor

DC :: (State -> (a, State)) -> ST0 a
apply0  :: ST0 a -> State -> (a, State)        Application Function

    s :: State
    g :: State -> (a, State)
    g s :: (a, State)
    (DC  g)  :: ST0 a                          State Transformer

apply0 (DC  g) :: State -> (b, State)
apply0 (DC  g) = g

apply0 (DC  g) s = g s                         Definition to remove DC
```

```
(.)     :: (b->c) -> (a->b) -> (a->c)
f . g   = \x -> f (g x)
f . g  x = f (g x)

(DC . f) x = DC  (f x)
```

~~(DC . f) x = DC (f x)~~

not a composite function

but a function argument

```
(DC  g) :: DC (State -> (b, State))
(DC  g) :: ST0 a

apply0 (DC  g) s =  g s        -- definition
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST a and ST0 a

```
type ST a = State -> (a, State)

st :: State -> (a, State)
st = \s -> (s, s+1)

st s ::  (b, State)

f    ::  a -> ST a

f  x :: State -> (b, State)
f  x s :: (b, State)
```

```
data ST0 a = DC (State -> (a, State))

         st0 ::  DC (State -> (a, State))
         st0 =  DC (\s -> (s, s+1))

apply0 st0 ::  State -> (a, State)
apply0 st0 s ::  (b, State)

         f    ::  a -> ST0 a
         f  x ::  ST0 a
         f  x ::  DC (State -> (a, State))
apply0 f  x ::  State -> (a, State)
apply0 f  x s ::  (b, State)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST a and ST0 a Examples

t.hs

```
type ST a = Int -> (a, Int)
data ST0 a = DC (Int->(a, Int))

st0 :: ST0 Int
st0 = DC(\s -> (s, s+1))

apply0 :: ST0 a -> Int -> (a, Int)
apply0 (DC f)  = f


st :: ST Int
st = (\s -> (s, s+1))
```

```
:load t.hs
 …
*Main> :t st
st :: ST Int
*Main> :t st0
st0 :: ST0 Int
*Main> :t st 3
st 3 :: (Int, Int)
*Main> :t apply0 st0 3
apply0 st0 3 :: (Int, Int)
*Main>
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

apply0 st0 s and apply0 f x s’

```
data ST0 a = DC (State -> (a, State))

apply0  :: ST0 a -> State -> (a, State)
apply0 (DC f) x = f x
```

apply0 st0 s	= (x,s')	s → (x,s')
apply0 f x s’	= (y,s')	s' → (y,s')

(–,s) ⟶ (x,s’) ⟶ (y,s’)

st s á (x,s')   f x s' á (y,s')

```
st0 :: ST0 a
st0 ::  DC (State -> (a, State))
st0 = DC (\s -> (s, s+1))
apply0 st0 s ::  (a, State)

f :: a -> ST0 a
f :: a -> DC (State -> (b, State))
f  x :: DC (State -> (b, State))
apply0 f  x s’ :: (b, State)
```

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st0 >> f using apply0

```
st >>= f  =  \s -> let (x,s') = st s in f x s'
```

```
type ST a = State -> (a, State))
```

binding variables

```
st0 >>= f  = DC (   \s -> let (x, s') = apply0 st s in apply0 f  x  s'   )
```

```
data ST0 a = DC (State -> (a, State))
```

apply0 st0 s	á (x,s')	s → (x,s')
apply0 f x s	á (y,s')	s' → (y,s')

st s	á (x,s')	s → (x,s')
f x s'	á (y,s')	s' → (y,s')

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ST0 Monad Instance

```
instance Monad ST0 where
  -- return :: a -> ST0 a
 return x   = DC( \s -> (x,s) )

  -- (>>=)  :: ST0 a -> (a -> ST0 b) -> ST0 b
  st >>= f   = DC(   \s -> let (x, s') = apply0 st s in apply0 (f x) s'   )
```

```
instance Monad ST where
   -- return :: a -> ST a
   return x  =  \s -> (x,s)

   -- (>>=)  :: ST a -> (a -> ST b) -> ST b
   st >>= f  =  \s -> let (x,s') = st s in f x s'
```

the runtime overhead of manipulating the dummy constructor **DC**

can be eliminated by defining **ST0** using the **newtype** mechanism

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A value of type **ST0** **a**

a value of type **ST** **a** (or **ST0** **a**) is simply

an action that returns an **a** value.

(like state processor function of **State** Monad)

The sequencing combinators **(>>)** allow us

to combine simple actions to get bigger actions,

the **apply0** allows us

to **execute** an action from some initial state.

(like **runState** accessor function of **State** Monad)

action

function

connecting

executing an action

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Sequencing Combinator (>>)

consider the simple **sequencing combinator**

(>>) :: Monad m => m a -> m b -> m b;

a1 >> a2 takes the actions **a1** and **a2** and returns the mega action which is **a1**-then-**a2**-returning-the-value-returned-by-**a2**.

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Sequencer (>>=) and return

the >>= sequencer is kind of like >>

only it allows you to "remember" intermediate values
that may have been returned.

return :: a -> ST0 a

takes a value **x** and yields an action
that doesn't actually change the state,
but just returns the same value **x**

remember

intermediate

return

action

the same state

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Pairs Example (1)

```
pairs :: [a] -> [b] -> [(a,b)]
pairs xs ys =  do x <- xs
                  y <- ys
                  return (x, y)
```

do method

this function returns *all* possible ways of pairing elements from two lists

each possible value x from the list **xs** — x <- **xs**

each possible value y from the list **ys** — y <- **ys**

return the pair (x, y).

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Pairs Example (2)

```
pairs :: [a] -> [b] -> [(a,b)]
pairs xs ys =  do x <- xs
                  y <- ys
                  return (x, y)
```

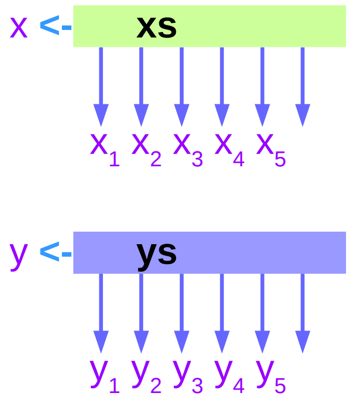
do method

```
pairs xs ys = [(x, y) | x <- xs, y <- ys]
```

comprehension notation

In fact, there is a formal connection between the **do** notation and the **comprehension** notation.

simply different shorthands for repeated use of the **>>=** operator for lists.



Generators

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Counter Example (1)

the state processing function can be defined

using the notion of a state transformer,

in which the internal state is simply the next fresh integer

```
type State = Int

fresh :: ST0 Int
fresh =  DC (\n -> (n, n+1))
```

return   next state

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Counter Example (2)

```
type State = Int

fresh :: ST0 Int
fresh =  DC (\n -> (n, n+1))
```

In order to generate a **fresh** integer,
we define a special state transformer
that simply returns the **current state** as its **result**,
and the **next integer** as the **new state**:

Note that **fresh** is a state transformer
(where the State is itself just **Int**),
that is an action that happens to **return** integer values.

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf1 (1)

```
type State = Int

fresh :: ST0 Int
fresh =  DC (\n -> (n, n+1))

wtf1 = fresh >>
       fresh >>
       fresh >>
       fresh

ghci> apply0 wtf1 0
```

```
wtf1 = DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1))
```

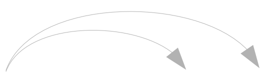
https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing **wtf1** (2)

```
data ST0 a = DC (State -> (a, State))
data ST0 a = DC (Int -> (a, Int))
data ST0 Int = DC (Int -> (Int, Int))

apply0  :: ST0 a -> State -> (a, State)
apply0  :: ST0 a -> Int -> (a, Int)
apply0  :: ST0 Int -> Int -> (Int, Int)

apply0     fresh        0    ( 0 ,   1)
```



```
apply0 fresh 0    á     ( 0 ,   1)
```

```
fresh :: ST0 Int
fresh =  DC (\n -> (n, n+1))

apply0 st s =  (x,s')      s → (x,s')
apply0 f x s =  (y,s')     s' → (y,s')
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf1 (3)

```
apply0 wtf1 0

apply0 (fresh >> fresh >> fresh >> fresh) 0      á   ( 0 ,  1)

apply0 (         fresh >> fresh >> fresh) 1      á   ( 1 ,  2)

apply0 (                  fresh >> fresh) 2      á   ( 2 ,  3)

apply0 (                        >> fresh) 3      á   ( 3 ,  4)
```

Not used

```
wtf1 = fresh >>
       fresh >>
       fresh >>
       fresh
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf1 (4)

```
type State = Int

fresh :: ST0 Int
fresh                                    = DC (\n -> (n+0, n+1))
fresh >> fresh                           = DC (\n -> (n+1, n+2))
fresh >> fresh >> fresh                  = DC (\n -> (n+2, n+3))
fresh >> fresh >> fresh >> fresh         = DC (\n -> (n+3, n+4))


                            wtf1 = DC (\n+3 -> (n, n+4))
wtf1 = fresh >>
       fresh >>
       fresh >>
       fresh
```

```
wtf1 = DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1)) >>
       DC (\n -> (n, n+1))
```

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Executing wtf1 (5)

```
wtf1 0= DC (0 -> (0, 1)) >>        -> Not used

        DC (1 -> (1, 2)) >>        -> Not used

        DC (2 -> (2, 3)) >>        -> Not used

        DC (3 -> (3, 4))           -> Not used
```

wtf1 0= DC (**0** -> (0, **1**)) **>>**
DC (**1** -> (1, **2**)) **>>**
DC (**2** -> (2, **3**)) **>>**
DC (**3** -> (3, **4**))

internal state s

external output x

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Executing wtf2

```
wtf2 = fresh >>= \n1 ->                    n1 = 0
       fresh >>= \n2 ->                    n2 = 1
       fresh >>
       fresh >>
       return [n1, n2]

wtf2 = fresh >>=
       ( \n1 -> fresh >>=
           (\n2 ->  fresh >> fresh >>  return [n1, n2])  )



*Main> apply0 wtf2 0
([0,1],4)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf2'

```
wtf2' = do  { n1 <- fresh;                    n1 = 0
              n2 <- fresh;                    n2 = 1
              fresh ;
              fresh ;
              return [n1, n2];
            }
```

```
*Main> apply0 wtf2' 0
([0,1],4)
```

```
wtf2 = fresh >>= \n1 ->
       fresh >>= \n2 ->
       fresh >>
       fresh >>
       return [n1, n2]
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf3

```
wtf3 = do  n1 <- fresh                    n1=0
           fresh
           fresh
           fresh
           return n1                      3 → (0, 4)   instead of (3, 4)

*Main> apply0 wtf3 0
(0,4)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Executing wtf4

```
wtf4 = fresh >>= \n1 ->                    n1 = 0
          fresh >>= \n2 ->                 n2 = 1
          fresh >>= \n3 ->                 n3 = 2
          fresh

*Main> apply0 wtf4 0
(3,4)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Make Functor and Applicative Instances

```
import Control.Applicative
import Control.Monad (liftM, ap)


instance Functor ST0 where
  fmap = liftM


instance Applicative ST0 where
  pure  = return
  (<*>) = ap
```

```
newtype ST0 a = DC (Int -> (a, Int))


instance Monad ST0 where
  return x   = DC( \s -> (x,s) )
  st >>= f   = DC( \s -> let (x, s') = apply0 st s
                         in apply0 (f x) s' )
```

https://stackoverflow.com/questions/31652475/defining-a-new-monad-in-haskell-raises-no-instance-for-applicative

Example Code Listing

```
apply0 :: ST0 a -> Int -> (a, Int)
apply0 (DC f)   = f

fresh :: ST0 Int
fresh = DC (\n -> (n, n+1))

wtf1 = fresh >>
       fresh >>
       fresh >>
       fresh

wtf2 =  fresh >>= \n1 ->
        fresh >>= \n2 ->
        fresh >>
        fresh >>
        return [n1, n2]
```

```
wtf2' = do {  n1 <- fresh
              n2 <- fresh
              fresh
              fresh
              return [n1, n2]
            }

wtf3 = do   n1 <- fresh
            fresh
            fresh
            fresh
            return n1

wtf4 = fresh >>= \n1 ->
       fresh >>= \n2 ->
       fresh >>= \n3 ->
       fresh
```

Results

```
*Main> :load st.hs
[1 of 1] Compiling Main            ( st.hs, interpreted )
Ok, modules loaded: Main.


*Main> apply0 (fresh) 0
(0,1)
*Main> apply0 (fresh >> fresh) 0
(1,2)
*Main> apply0 (fresh >> fresh >> fresh) 0
(2,3)
*Main> apply0 (fresh >> fresh >> fresh >> fresh) 0
(3,4)
```

```
*Main> apply0 wtf1 0
(3,4)

*Main> apply0 wtf2 0
([0,1],4)

*Main> apply0 wtf2' 0
([0,1],4)

*Main> apply0 wtf3 0
(0,4)

*Main> apply0 wtf4 0
(3,4)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Transformer Stacks

making a double, triple, quadruple, ... monad

by wrapping around existing monads

that provide wanted functionality.

You have an innermost monad (usually Identity or IO

but you can use any monad). You then wrap monad transformers

around this monad to make bigger, better monads.

a ➡ M a ➡ N M a ➡ O N M a

To do stuff in an inner monad → cumbersome → monad transformers

lift $ lift $ lift $ foo

https://wiki.haskell.org/Monad_Transformers_Explained

Monad Transformers

Precursor	Transformer	Original Type	Combined Type
Writer	WriterT	(a, w)	m (a, w)
Reader	ReaderT	r -> a	r -> m a
State	StateT	s -> (a, s)	s -> m (a, s)
Cont	ContT	(a -> r) -> r	(a -> m r) -> m r

https://wiki.haskell.org/Monad_Transformers_Explained

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf