

Expectation

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May 6, 2020

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

- 1 Expected Value of Random Variables
- 2 Expected Values of a Function of Random Variables
- 3 Conditional Expected Value

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Expectation

Definition

For a random variable X :

$$E[X]$$

- the mathematical expectation of X
- the expected value of X
- the mean value of X
- the statistical average of X

Computing $E[X]$

Definitions

for a continuous random variable X

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x) dx$$

for a discrete random variable X

$$E[X] = \sum_{i=1}^N x_i P(x_i)$$

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- 1 Expected Value of Random Variables
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Computing $E[g(X)]$

Definitions

for a continuous random variable X

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x)f_X(x)dx$$

for a discrete random variable X

$$E[g(X)] = \sum_{i=1}^N g(x_i)P(x_i)$$

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- 1 Expected Value of Random Variables
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Computing $E[X|B]$

Definitions

for a continuous random variable X
and for a conditional density $f_X(x|B)$

$$E[X|B] = \int_{-\infty}^{+\infty} x f_X(x|B) dx$$

Defining Conditioning Event B

event B is defined in terms of the random variable X

let $B = \{X \leq b\}$, where b is a real number $-\infty < b < \infty$, then

$$F_X(x | X \leq b) = P\{X \leq x | X \leq b\} = \frac{P\{X \leq x \cap X \leq b\}}{P\{X \leq b\}}$$

consider the case $x < b$ then $\{X \leq x\} \cap \{X \leq b\} = \{X \leq x\}$

$$f_X(x | X \leq b) = \begin{cases} \frac{f_X(x)}{F_X(b)} = \frac{f_X(x)}{\int_{-\infty}^b f_X(x) dx} & x < b \\ 0 & b \leq x \end{cases}$$

Computing $E[X|B]$ with $B = \{x < b\}$

Definitions

for a continuous random variable X
and for a conditional density $f_X(x|B)$

$$E[X|B] = \int_{-\infty}^{+\infty} x f_X(x|B) dx$$

$$E[X|X \leq b] = \begin{cases} \frac{\int_{-\infty}^b x f_X(x) dx}{\int_{-\infty}^b f_X(x) dx} & x < b \\ 0 & b \leq x \end{cases}$$

