Expectation

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

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1. Expected Value of Random Variables
2. Expected Values of a Function of Random Variables
3. Conditional Expected Value

Expectation

Definition

For a random variable X:

$$E[X]$$

- the mathematical expectation of X
- the expected value of X
- the mean value of X
- the statistical average of X

Computing $E[X]$

Definitions

for a continuous random variable X

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x) dx$$

for a discrete random variable X

$$E[X] = \sum_{i=1}^{N} x_i P(x_i)$$

Outline

Computing $E[g(X)]$

Definitions

for a continuous random variable X

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx$$

for a discrete random variable X

$$E[g(X)] = \sum_{i=1}^{N} g(x_i) P(x_i)$$

Outline

Computing $E[X|B]$

Definitions

for a continuous random variable X
and for a conditional density $f_x(x|B)$

$$E[X|B] = \int_{-\infty}^{+\infty} x f_X(x|B) dx$$

Defining Conditioning Event B

event B is defined in terms of the random variable X
let $B = \{X \le b\}$, where b is a real number $-\infty < b < \infty$, then

$$F_X(x \mid X \le b) = P\{X \le x \mid X \le b\} = \frac{P\{X \le x \cap X \le b\}}{P\{X \le b\}}$$

consider the case $x < b$ then $\{X \le x\} \cap \{X \le b\} = \{X \le x\}$

$$f_X(x \mid X \le b) = \begin{cases} \dfrac{f_X(x)}{F_X(b)} = \dfrac{f_X(x)}{\int_{-\infty}^{b} f_X(x)dx} & x < b \\ 0 & b \le x \end{cases}$$

Computing $E[X|B]$ with $B = \{x < b\}$

Definitions

for a continuous random variable X
and for a conditional density $f_x(x|B)$

$$E[X|B] = \int_{-\infty}^{+\infty} x f_X(x|B) dx$$

$$E[X|X \leq b] = \begin{cases} \dfrac{\int_{-\infty}^{b} x f_X(x) dx}{\int_{-\infty}^{b} f_X(x) dx} & x < b \\ 0 & b \leq x \end{cases}$$