# Applicative Methods (3B)

Young Won Lim 5/5/18 Copyright (c) 2016 - 2018 Young W. Lim.

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# The definition of Applicative

class (Functor f) => Applicative f where pure :: a -> f a (<\*>) :: f (a -> b) -> f a -> f b

The class has a two methods :

pure brings arbitrary values into the functor

(<\*>) takes a function wrapped in a functor f and a value wrapped in a functor f and returns the result of the application which is also wrapped in a functor f f (a -> b) :: a function <u>wrapped in f</u>

f a :: a value wrapped in f

# The Maybe instance of Applicative

instance Applicative Maybe where		
pure	= Just	
(Just f) <*> (Just x)	= Just (f x)	
_ <*> _	= Nothing	

pure wraps the value with Just;

(<\*>) applies

the <u>function</u> wrapped in **Just** to the <u>value</u> wrapped in **Just** if both exist, and results in **Nothing** otherwise.

https://en.wikibooks.org/wiki/Haskell/Applicative\_functors

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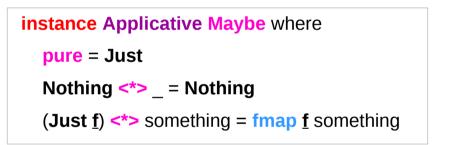
### An Instance of the Applicative Typeclass

class (Functor f) => Applicative f where pure :: a -> f a (<\*>) :: f (a -> b) -> f a -> f b

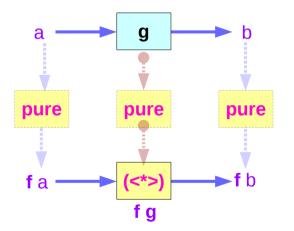


```
(Functor f) => Applicative f
```

**f** : Functor, Applicative

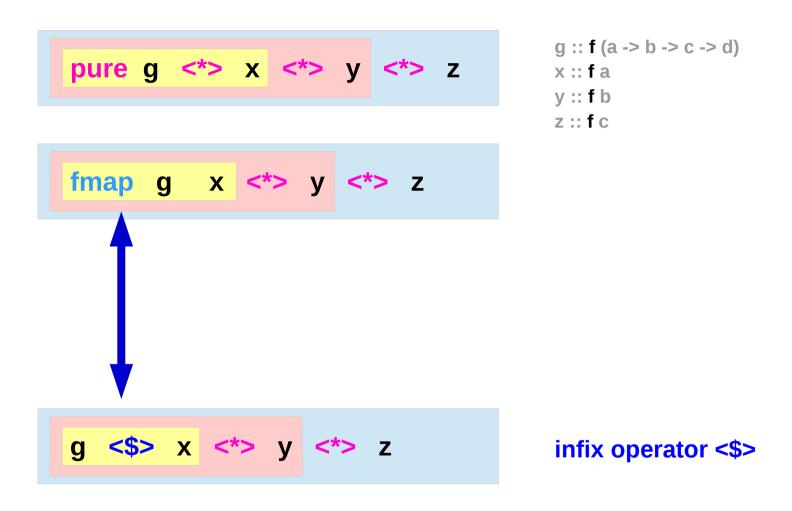


#### $\underline{f}$ : function in a context



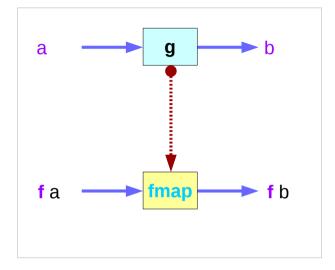
(Functor f) => Applicative f

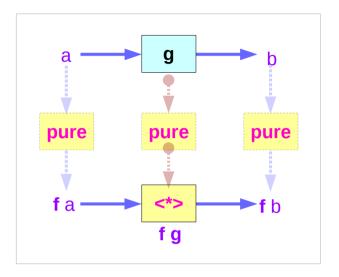
### Left associative <\*>, fmap, and <\$>



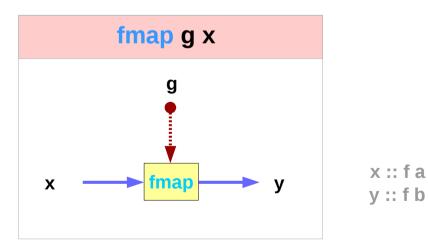
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

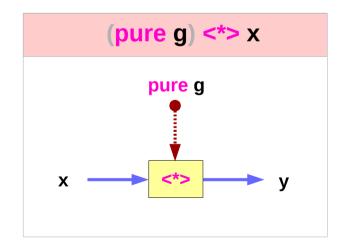
### fmap g x = (pure g) < > x





pure = f





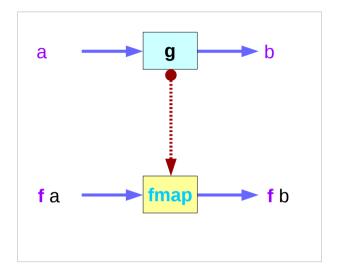
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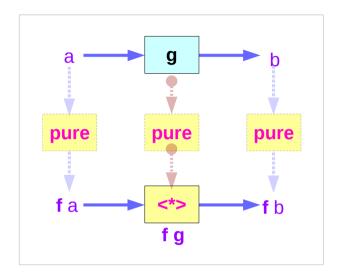
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#### Applicatives Methods (3B)

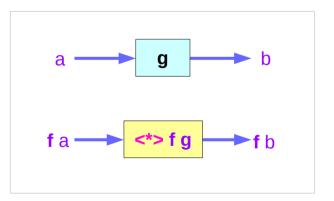
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## f a <\*> f g





pure = f

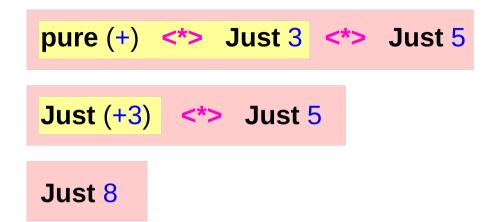


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### Left associative <\*> examples

ghci> pure (+) <\*> Just 3 <\*> Just 5

Just 8

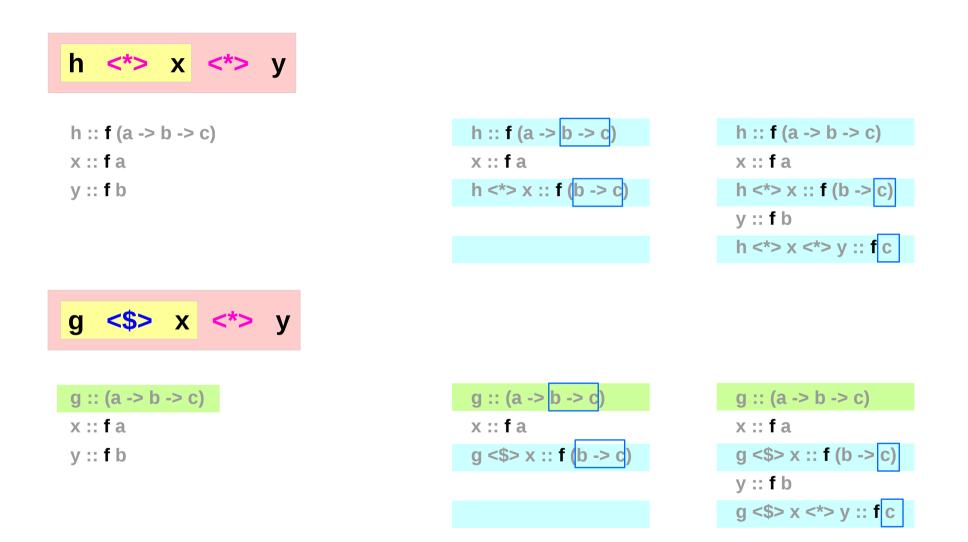


ghci> pure (+) <\*> Just 3 <\*> Nothing Nothing

ghci> pure (+) <\*> Nothing <\*> Just 5 Nothing

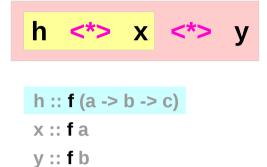
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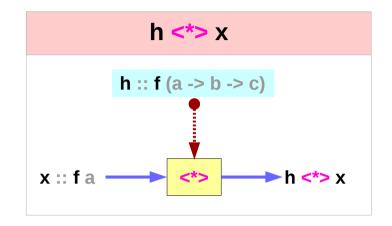
Infix Operators <\*> vs <\$> - a type view

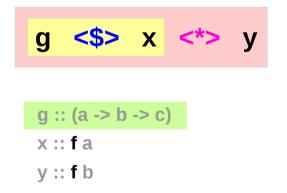


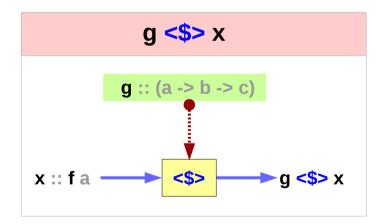
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### Infix Operators <\*> vs <\$> - a curried function view







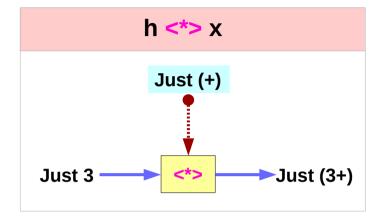


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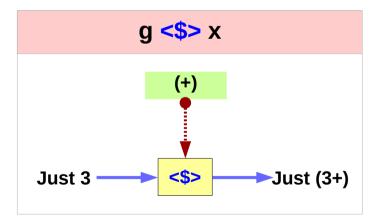
### Infix Operators <\*> vs <\$> examples



Just (+) <\*> Just 3 <\*> Just 2 Just (+3) <\*> Just 2 Just 5



(+) <\$> Just 3 <\*> Just 2 Just (+3) <\*> Just 2 Just 5



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# the minimal complete definition

class (Functor f) => Applicative f where
 pure :: a -> f a
 (<\*>) :: f (a -> b) -> f a -> f b

the minimal complete definition

(<\$>) :: (Functor f) => (a -> b) -> f a -> f b

g **<\$>** x = fmap g x

Not in the minimal complete definition

g::a->b, x::fa

instance Applicative Maybe where
pure = Just
Nothing <\*> \_ = Nothing
(Just g) <\*> something = fmap g something

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

# The Applicative Typeclass

Applicative is a <u>superclass</u> of Monad. every Monad is also a Functor and an Applicative fmap, pure, (<\*>) can all be used with monads.

#### a **Monad** instance

requires **Functor** and **Applicative** instances. defines the types and roles of **return** and (>>)

- fmap : defined in Functors
- pure, (<\*>) : defined in Applicatives
- return, (>>) : defined in Monads

(<\$>) infix operator

(<\$>) :: (Functor f) => (a -> b) -> f a -> f b g <\$> x = fmap g x

The **\$ operator** is for avoiding parentheses

putStrLn (show (1 + 1)) putStrLn \$ show (1 + 1) putStrLn \$ show \$ 1 + 1 - right associative

(\$) calls the <u>function</u> which is its left-hand argument of \$ on the <u>value</u> which is its right-hand argument of \$

### The Applicative Laws

The identity law:	pure id <*> v = v	id :: a -> a v :: f a
Homomorphism:	pure g <*> pure x = pure (g x)	g :: a -> b x :: a
Interchange:	u <*> pure y = pure (\$ y) <*> u	u :: f (a -> b) y :: a
Composition:	u <*> (v <*> w) = pure (.) <*> u <*> v <*> w	w :: f a v :: f (a -> b) u :: f (b -> c)
Left associative		u = f(a > b > a)
Lett associative	u <*> v <*> w = (u <*> v) <*> w	u :: f (c -> b -> a) v :: f c
		u <*> v :: f (b -> a) w :: f b
		u <*> v <*> w = f a

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# The Identity Law

pure to inject <u>values</u> into the <u>functor</u> in a default, featureless way, so that the result is as close as possible to the <u>plain</u> value.

applying the **pure id** morphism does nothing, exactly like with the plain **id** function.

# The Homomorphism Law

pure g <*> pure x = pure (g x)	g :: a -> b	x :: a	
	pure g <*> pure x = pure (g x)	pure g <*> pure x = pure (g x) g :: a -> b	pure g <*> pure x = pure (g x) g :: a -> b x :: a

applying a "**pure**" <u>function</u> to a "**pure**" <u>value</u> is the same as applying the <u>function</u> to the <u>value</u> in the *ordinary way* and then using **pure** on the result. means **pure** <u>preserves</u> function application.

applying a non-effectful function g
to a non-effectful argument x in an effectful context pure
is the same as just applying the function g to the argument x
and then injecting the result (f x)
into the effectual context with pure.

### The Interchange Law

The interchange law	u <*> pure y = pure (\$ y) <*> u	u :: f (a -> b) y	':: a
( <b>\$ y</b> ) is the <i>function</i> that <u>suppl</u> as <u>argument</u> to another function – a higher order function		Function \$ Argument \$ y ( y ) as a sing	gle argument
applying a <u>morphism</u> <b>u</b> to a " <b>p</b> is the same as applying <mark>pure</mark>		Just ( <b>+3</b> ) <*> Just <b>2</b> Just ( <b>\$ 2</b> ) <*> Just ( <b>+3</b> )	)
when evaluating the application an <u>effectful function</u> ( <b>u)</b> to a <u>pr</u> the <u>order doesn't matter</u> – con	<u>ure argument</u> ( <mark>pure y</mark> ),	u :: f (a -> b) u <*> pure y :: f b pure ( <b>\$</b> y) <*> u :: f b	

# The Composition Law

The composition law pure (.) <\*> u <\*> v <\*> w = u <\*> (v <\*> w) w :: f a v :: f (a -> b) u :: f (b -> c)

pure (.) <u>composes</u> morphisms similarly to how (.) <u>composes</u> functions:

applying the <u>composed</u> mourphism **pure (.) <\*> u <\*> v** to w gives the same <u>result</u> (**u <\*> (v <\*> w**)) as applying **u** to the <u>result</u> (**v <\*> w**) of applying **v** to **w** 

it is expressing a sort of <u>associativity</u> property of (<\*>).

w :: f a	value
v :: f (a -> b)	func1
u : f (b -> c)	func2

v <\*> w :: f b u <\*> (v <\*> w) :: f c

pure (.) <\*> u <\*> v :: f (a -> c ) pure (.) <\*> u <\*> v <\*> w :: f c

https://en.wikibooks.org/wiki/Haskell/Applicative\_functors

# The Composition Law and Left Associativity

The composition law pure (.) <*> u <*> v <*> w = u <*	> (v <*> w) w ::: f a v ::: f (a -> b) u ::: f (b -> c)
<pre>pure (.) &lt;*&gt; pure g &lt;*&gt; pure h &lt;*&gt; pure x</pre>	f (b -> c) f (a -> b) pure g pure h pure (.) f (a -> c)
= pure g <*> (pure h <*> pure x) g (h x)	u = pure g :: f (b -> c) g :: (b -> c) v = pure h :: f (a -> b) h :: (a -> b) w = pure x :: f a x :: a
Left associative u <*> v <*> w = (u <*> v) <*> w	u :: f (c -> b -> a) v :: f c w :: f b
https://en.wikibooks.org/wiki/Haskell/Applicative_functors	u :: f (c -> b -> a) v :: f c u <*> v :: f (b -> a) w :: f b u <*> v <*> w = f a
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### liftA2

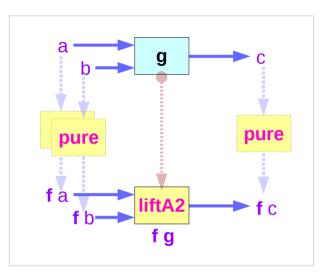
#### liftA2 :: (a -> b -> c) -> f a -> f b -> f c

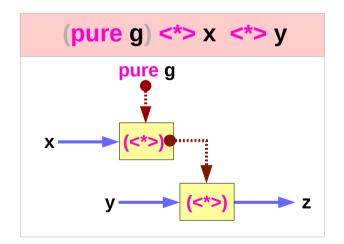
<u>lift</u> a <u>binary</u> <u>function</u> (**a->b->c**) to actions.

Some functors support an implementation of liftA2 that is more efficient than the default one.

**liftA2** may have an <u>efficient</u> implementation whereas **fmap** is an <u>expensive</u> operation,

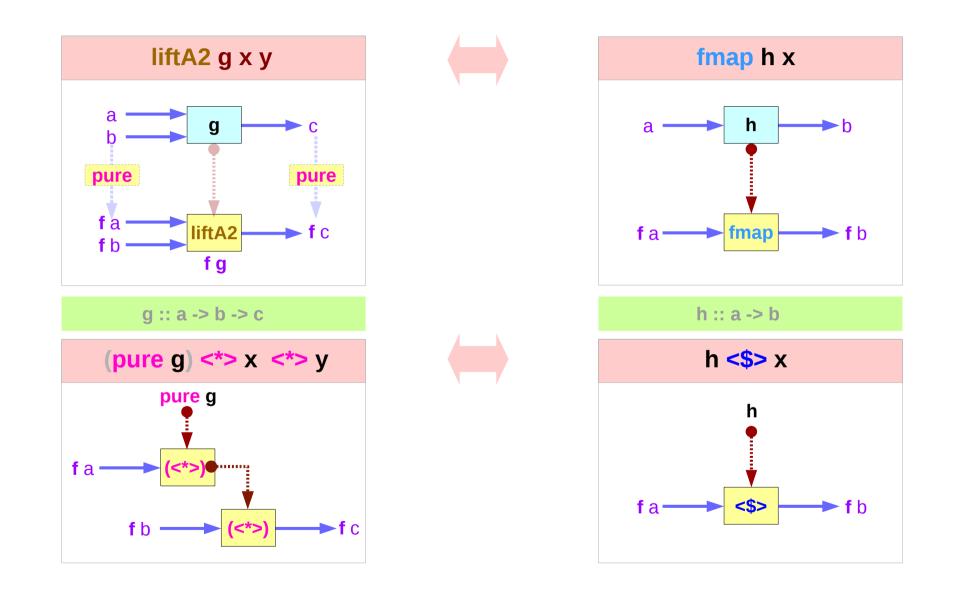
sometimes better to use **liftA2** than to use **fmap** over the structure and then use **<\*>**.



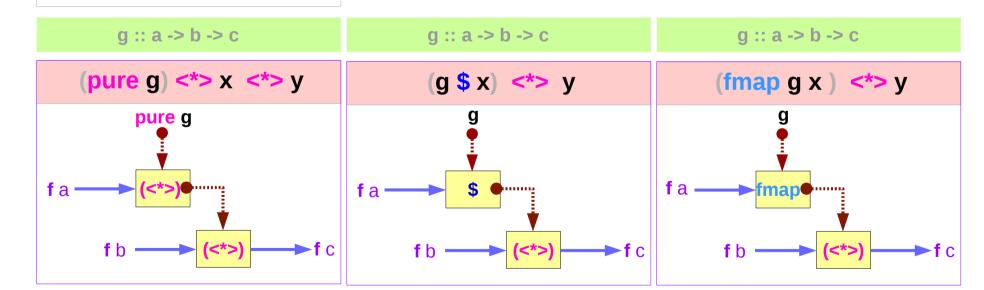


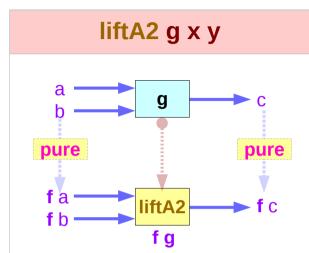
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

### liftA2, <\*>, fmap, <\$>



### pure g <\*> x <\*> y equivalent





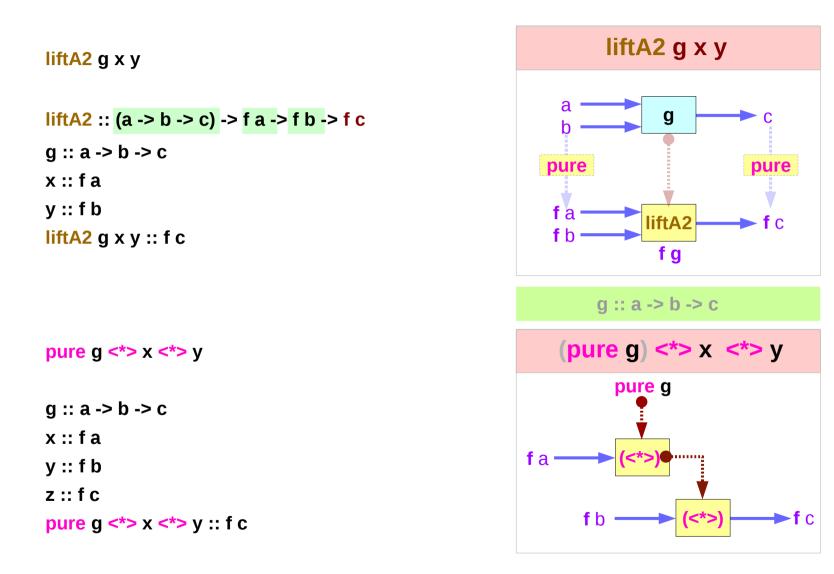
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liftA2



https://wiki.haskell.org/Applicative\_functor

### Limitations of Functors

(a -> b -> c) -> (f a -> f b -> f c) - let fmap2 Functor as an extension of fmap

```
fmap :: (a -> b) -> (f a -> f b)
fmap2 :: Functor f => (a -> b -> c) -> (f a -> f b -> f c)
```

```
fmap2 h fa fb = undefined
h :: a -> b -> c
fa :: f a
fb :: f b
```

```
h :: a -> (b -> c)

fmap h :: f a -> f (b -> c)

fmap h fa :: f (b -> c) - now f (b -> c) must be applied to f b
```

**fmap** gives us a way to <u>apply</u> functions  $(a \rightarrow b)$  to <u>values</u> (f a) inside a <u>Functor</u> context, but **fmap** cannot be used to <u>apply</u> a <u>functions</u>  $f (b \rightarrow c)$  which are themselves <u>in a Functor context</u> to <u>values</u> f b in a <u>Functor</u> context.

http://www.openhaskell.com/lectures/applicative.html

# pure, fmap, and liftA2

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

pure	:: a -> <mark>f a</mark>	– fmap0 → pure
fmap	:: (a -> b) -> f a -> <mark>f b</mark>	– fmap1 → fmap
fmap2	:: (a -> b -> c) -> f a -> f b -> <mark>f c</mark>	– fmap2 $\rightarrow$ liftA2

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 h fa fb = (h `fmap` fa) <*> fb
liftA2 h fa fb = h <$> fa <*> fb
```

```
(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

```
liftA2 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc
```

http://www.openhaskell.com/lectures/applicative.html

### liftA2 examples

liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

liftA2 (+) (Just 5) (Just 6) = Just 11

liftA2 h fa fb = (h `fmap` fa) <\*> fb liftA2 h fa fb = h <\$> fa <\*> fb

fmap (+) (Just 5) = Just (+5) (+) <\$> (Just 5) = Just (+5)

<\*> :: Applicative f => f (a -> b) -> f a -> f b (Just (+5)) <\*> (Just 6) = Just 11 let v1 = IO (Just (+5)) let v2 = IO (Just 6)

liftA2 (<\*>) v1 v2 = IO (Just 11)

https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html

## <\*> or liftA2 implementations

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

A <u>minimal complete</u> <u>definition</u> :

either one of the two

pure and <\*>
 pure and liftA2

If it defines <u>both</u>, then they must behave the same as their default definitions:

1) pure g <\*> x <\*> y

2) liftA2 g x y

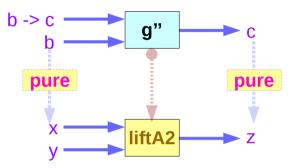
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

# (<\*>) = liftA2 id

**liftA2 id** x y = **id** <\$> x <\*> y = x <\*> y

liftA2 id x y = x <\*> y





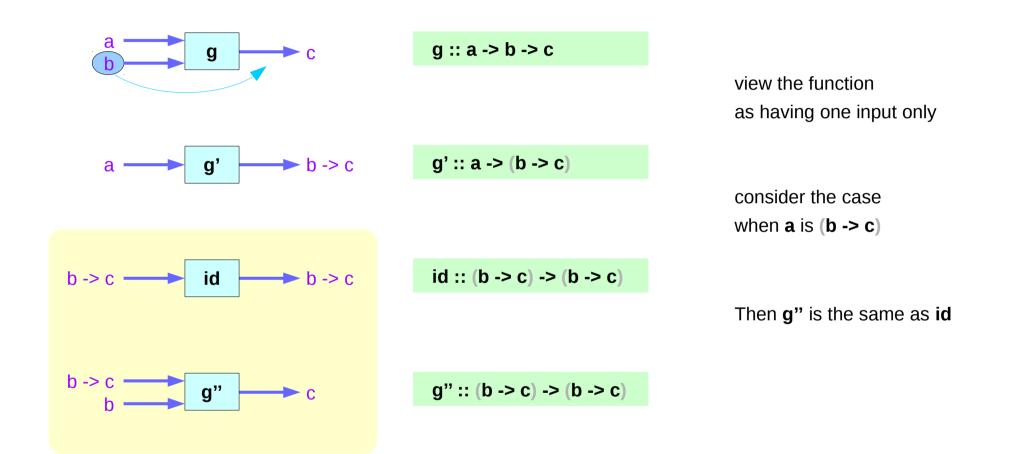
 ->c) y::fb
-> c) y :: f b
· · · · · · · · · · · · · · · · · · ·

(<\*>) = liftA2 id

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

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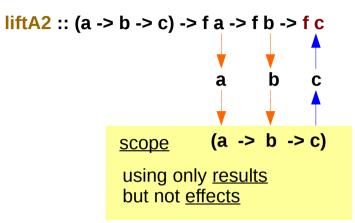
### g"::(b->c)->b->c



# Results and effects in a scope

Actually, using the liftA commands				
we can pull results of applicative functors				
into a <u>scope</u> where we can talk				
exclusively about functor results	С			
and not about <u>effects</u> .	fc			
Note that functor results can also be functions.	С			
This scope is simply a function,				

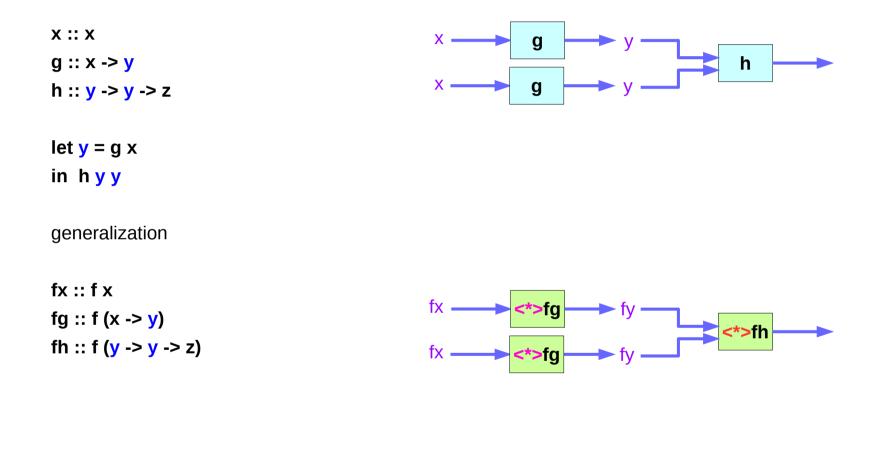
which contains the code that we used in the non-functorial setting.



http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

# liftA3 – a non-functorial expression

Consider the non-functorial expression:



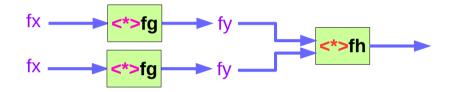
https://wiki.haskell.org/Applicative\_functor

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### liftA3 - using <\*> only

let fy = fg <\*> fx in fh <\*> fy <\*> fy

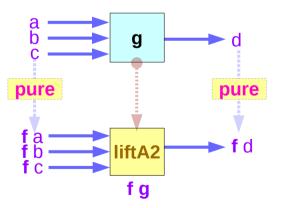
if **fy** writes something to the terminal then **fh <\*> fy <\*> fy** writes twice.



this runs the effect of fy twice.

How the effect is run only once and the result is used twice?

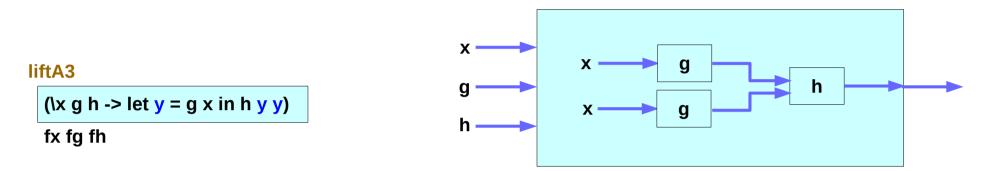
→ utilize liftA3

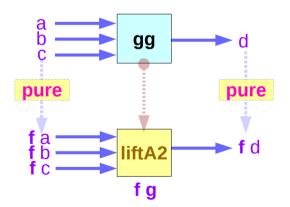


https://wiki.haskell.org/Applicative\_functor

# liftA3 – using three input function

liftA3 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d liftA3 h fa fb fc = ((h <\$> fa) <\*> fb) <\*> fc





http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2



# liftA3 – effects, results and scopes

Actually, using the liftA commands				
we can pull <u>results</u> of applicative functors	<b>y</b> from <b>fy</b>			
into a <u>scope</u> where we can talk	y -> y -> z			
exclusively about functor results	У			
and not about <u>effects</u> .	fy			
Note that functor results can also be functions.	У			
This scope is simply a function,	<mark>y -&gt; y</mark> -> z			
which contains the code that we used in the non-functorial setting.				

#### liftA3

(\x g h -> let y = g x in h y y) fx fg fh

The order of effects is entirely determined by the order of arguments to liftA3

https://wiki.haskell.org/Applicative\_functor



10 down vote accepted

The wiki article says that **liftA2 (<\*>)** can be used to <u>compose applicative functors</u>. It's easy to see how to use it from its type:

o :: (Applicative f, Applicative f1) =>
 f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)
o = liftA2 (<\*>)

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

## liftA2 (<\*>) for composite applicative functors

o :: (Applicative f, Applicative f1) => f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)

o = liftA2 (<\*>)

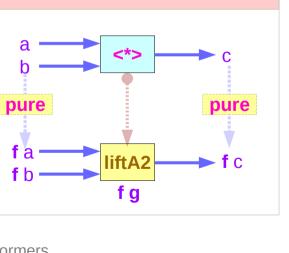
**Applicatives** 

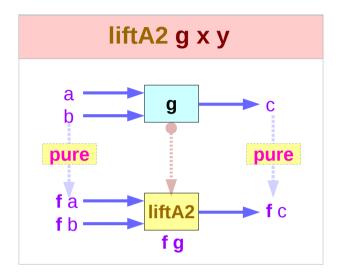
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f1 (a -> b) <\*> f1 a <\*> f1 b

liftA2 (<\*>) can be used to <u>compose applicative functors</u>.

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers





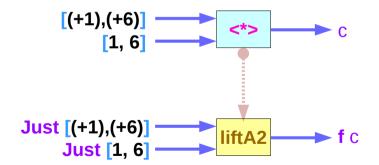
liftA2 (<\*>) x y

## liftA2 (<\*>) Examples (1)

if <b>f</b> is <b>Maybe</b> and <b>f1</b> is <b>[ ]</b> we get:	[(+1), (+6)]	[1, 6]
	(+1) [1, 6]	
Just [(+1),(+6)] `o` Just [1, 6]	(+6) [1, 6]	
Just [2,7,7,12]		

```
liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]
Just ( [(+1),(+6)] <*> [1, 6] )
Just [2, 7, 7, 12] )
```

# liftA2 (<\*>) Just [(+1),(+6)] Just [1, 6]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

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## liftA2 (<\*>) Examples (2)

if **f** is **Maybe** and **f1** is **[**] we get:

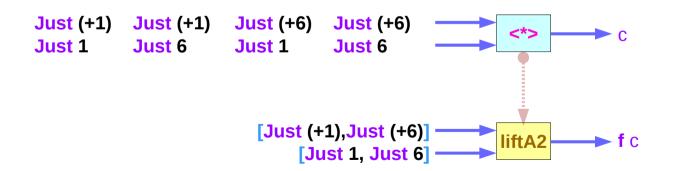
[Just (+1),Just (+6)] `o` [Just 1, Just 6] [Just 2, Just 7, Just 7, Just 12] [Just (+1),Just (+6)]

#### [Just 1, Just 6]

Just (+1) [Just 1, Just 6] Just (+6) [Just 1, Just 6]

liftA2 (<\*>) [Just (+1),Just (+6)] [Just 1, Just 6] [Just (+1) <\*> Just 1, Just (+1) <\*> Just 6, Just (+6) <\*> Just 1, Just (+6) <\*> Just 6] [Just 2, Just 7, Just 7, Just 12]

### liftA2 (<\*>) [Just (+1), Just (+6)] [Just 1, Just 6]

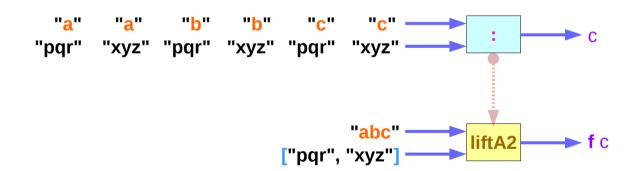


https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

## liftA2 (:)

liftA2 (:) "abc" ["pqr", "xyz"] ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]

### liftA2 (:) "abc" ["pqr", "xyz"]



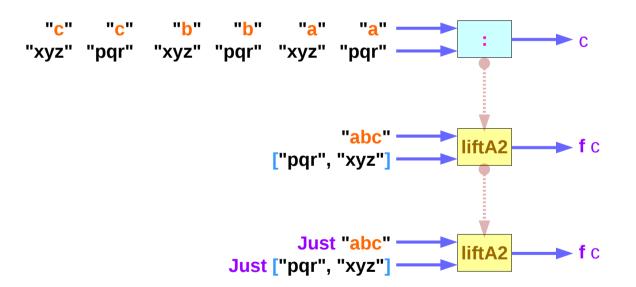
https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers



## liftA2 (:)

(liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]

(liftA2 . liftA2) (:) "abc" ["pqr", "xyz"]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers



# liftA2 (:)

```
liftA2 (:) "abc" ["pqr", "xyz"]
["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]
```

To use (:) with deeper applicative stack you need multiple applications of **liftA2**:

```
(liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]
```

However it only works when both operands are equally deep. So besides double liftA2 you should use pure to fix the level:

```
(liftA2 . liftA2) (:) (pure "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers



## <\$> related operators

Functor map <\$>

- (<\$>) :: Functor f => (a -> b) -> f a -> f b
- (<\$) :: Functor f => a -> f b -> f a
- (\$>) :: Functor f => f a -> b -> f b

The **<\$>** operator is just a synonym for the **fmap** function from the Functor typeclass.

This function generalizes the **map** function for lists to many other data types, such as **Maybe**, **IO**, and **Map**.

### <\$> examples

#!/usr/bin/env stack

-- stack --resolver ghc-7.10.3 runghc

```
import Data.Monoid ((<>))
```

main :: IO ()

main = do

putStrLn "Enter your year of birth"

year <- read <\$> getLine

let age :: Int

age = 2020 - year

putStrLn \$ "Age in 2020: " <> show age

## <\$, \$> operators

In addition, there are two additional operators provided which replace a value inside a Functor instead of applying a function. This can be both more convenient in some cases, as well as for some Functors be more efficient.

value <\$ functor = const value <\$> functor
functor \$> value = const value <\$> functor

x <\$ y = y \$> x x \$> y = y <\$ x

### <\*> related operators

Applicative function application <\*>

(<\*>) :: Applicative f => f (a -> b) -> f a -> f b
(\*>) :: Applicative f => f a -> f b -> f b
(<\*) :: Applicative f => f a -> f b -> f a

Commonly seen with **<\$>**, **<\*>** is an operator that applies a wrapped function to a wrapped value. It is part of the Applicative typeclass, and is very often seen in code like the following:

foo <\$> bar <\*> baz

### <\*> examples

For cases when you're dealing with a Monad, this is equivalent to:

do x <- bar

y <- baz

return (foo x y)

Other common examples including parsers and serialization libraries. Here's an example you might see using the aeson package:

data Person = Person { name :: Text, age :: Int } deriving Show

-- We expect a JSON object, so we fail at any non-Object value.

instance FromJSON Person where

parseJSON (Object v) = Person <\$> v .: "name" <\*> v .: "age"
parseJSON \_ = empty

**Applicatives** Methods (3B)

### **\*>** operator

To go along with this, we have two helper operators that are less frequently used:

**\*>** ignores the value from the first argument. It can be defined as:

a1 \*> a2 = (id <\$ a1) <\*> a2

Or in do-notation:

a1 \*> a2 = do \_ <- a1 a2

For Monads, this is completely equivalent to >>.

### <\* operator

<\* is the same thing in reverse: perform the first action then the second, but only take the value from the first action. Again, definitions in terms of <\*> and do-notation:

(<\*) = liftA2 const

a1 <\* a2 = do

res <- a1

\_<- a2

return res

## (\*> v.s. >>) and (pure v.s. return)

- (\*>) :: Applicative f => fa -> fb -> fb
- (>>) :: Monad m => m a -> m b -> m b

pure :: Applicative f => a -> f a

return :: Monad m => a -> m a

the constraint changes from Applicative to Monad.

(\*>) in Applicative(>>) in Monadpure in Applicativereturn in Monad

https://en.wikibooks.org/wiki/Haskell/Applicative\_functors

#### References

- [1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
- [2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf