Integrals

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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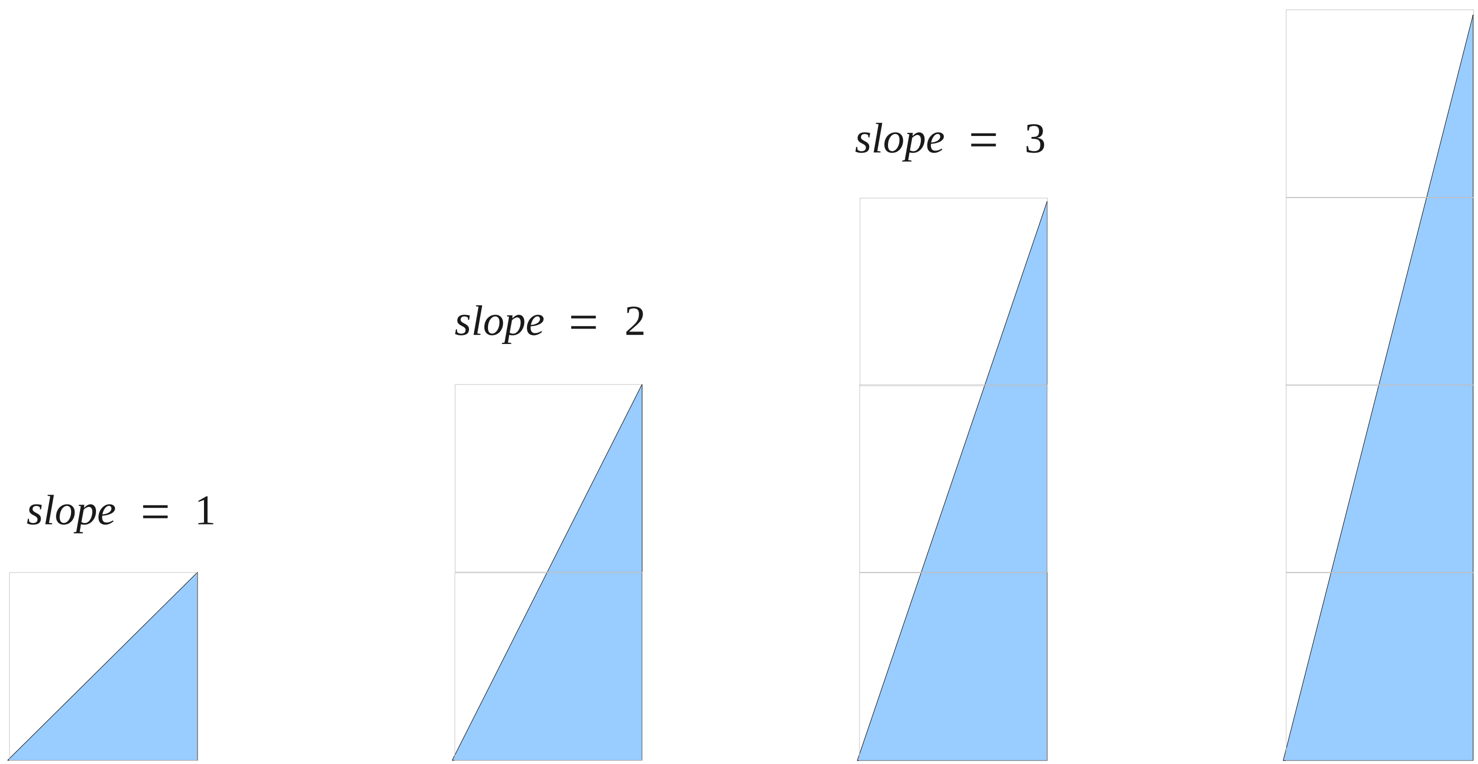
$$f(x) = x$$

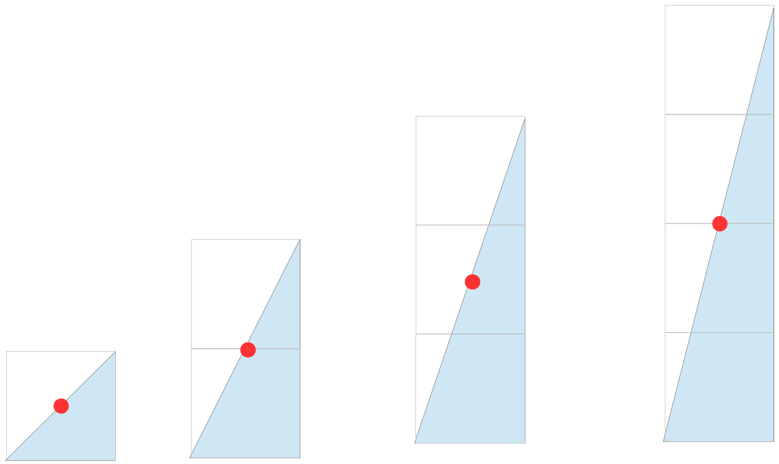
$slope = 1$ $slope = 2$ $slope = 3$ $slope = 4$

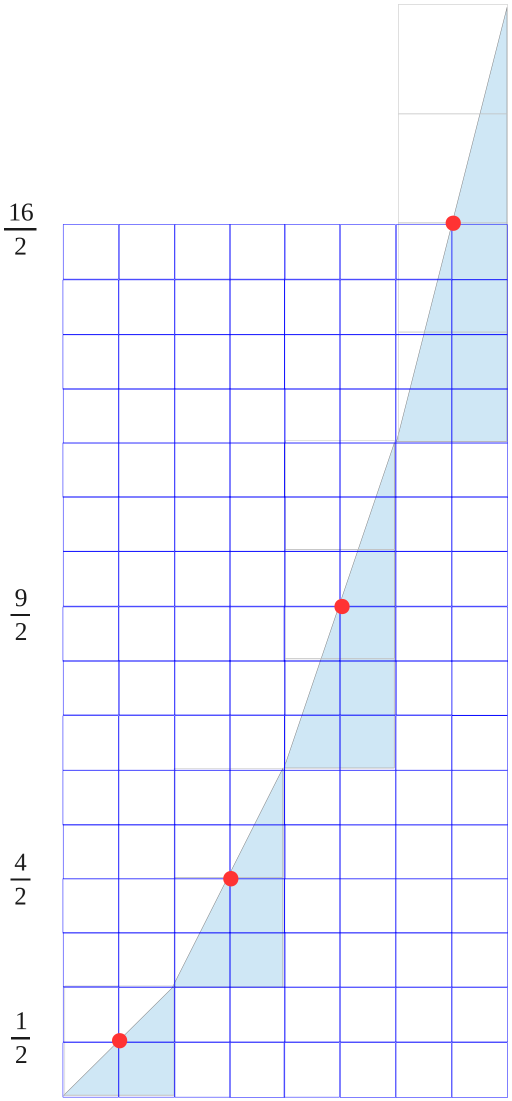


$f(1) = 1$ $f(2) = 2$ $f(3) = 3$ $f(4) = 4$

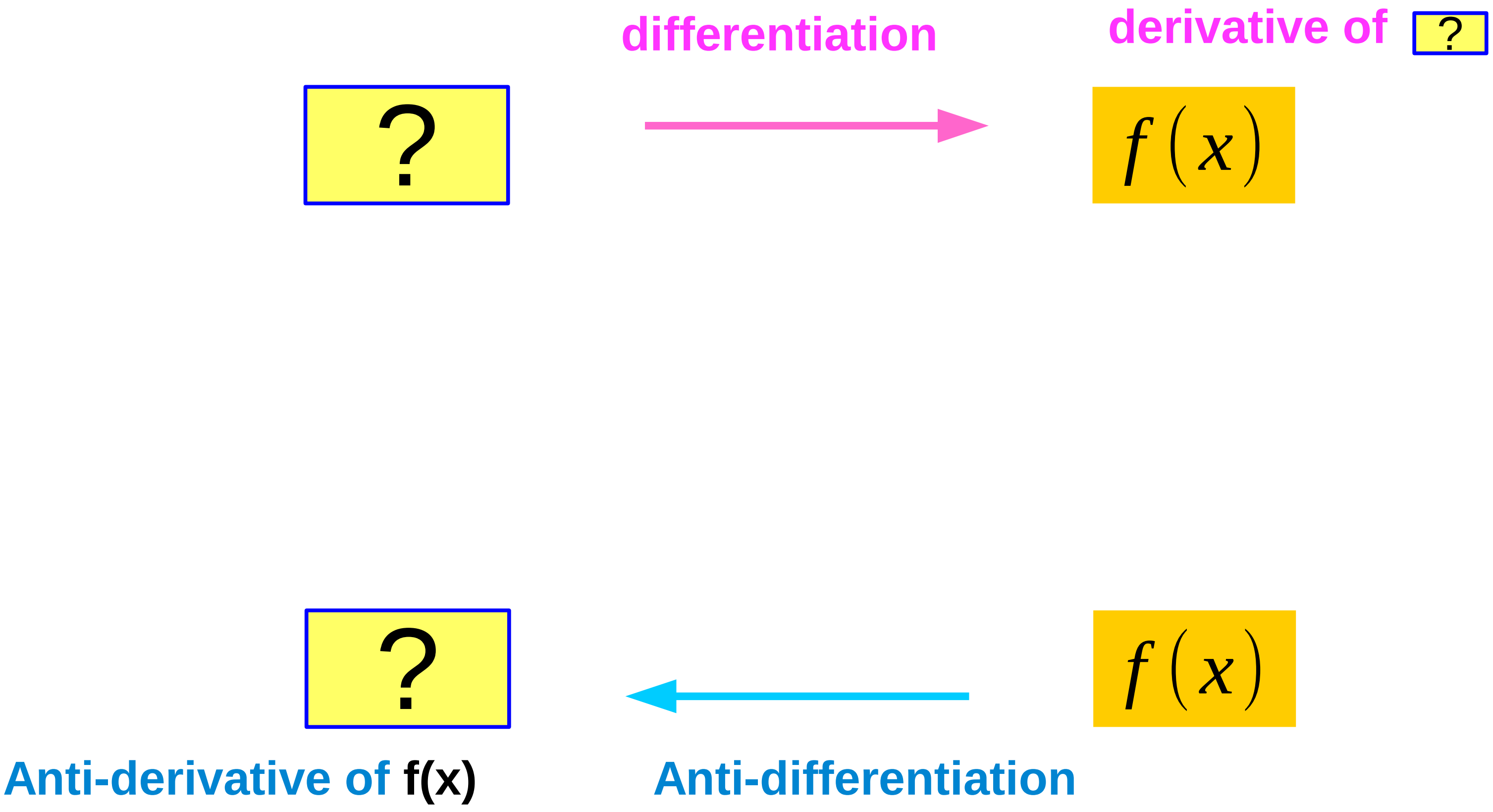
B

$$\int f(x)\,dx = \frac{1}{2}x^2 + C$$





Anti-derivative



Anti-derivative and Indefinite Integral

$$F'(x) = f(x)$$

$$F(x)$$

Anti-derivative without constant
the most *simple* anti-derivative

$$F(x) + C$$

the most <u>*general*</u> anti-derivative

$$\int f(x)\,dx$$

Indefinite Integral **: a function of x**

$$\int f(x)\,dx = F(x) + C$$

Anti-derivative Examples

All are Anti-derivative of f(x)

$$F_1(x) = \frac{1}{3}x^3$$

$$F_2(x) = \frac{1}{3}x^3 + 100$$

$$F_3(x) = \frac{1}{3}x^3 - 49$$

differentiation →

← **Anti-differentiation**

$$f(x) = x^2$$

the most *general* anti-derivative of f(x)

$$\frac{1}{3}x^3 + C$$

indefinite Integral of f(x)

$$\equiv \int x^2 dx$$

Indefinite Integrals

$\int_a^{x_1} 1\, dx$	→	$\int_a^{x} 1\, dx$	→	$\int dx$		$\int dy$
$=$		$=$		$=$		$=$
$x_1 - a$	→	$x - a$	→	$x + C$		$y + C$
given x_1		a variable x		indefinite integral		

$\int_a^{x_1} \frac{df}{dx}\, dx$	→	$\int_{-c}^{x} \frac{df}{dx}\, dx$	→	$\int \frac{df}{dx}\, dx$
$=$		$=$		$=$
$f(x_1) - f(a)$	→	$f(x) - f(a)$	→	$f(x) + C$
given x_1		a variable x		indefinite integral

Indefinite Integrals via the Definite Integral $\int_a^x f(t)\,dt$

definite integral $\int_a^x f(t)\,dt$ ← anti-derivative $f(x)$

indefinite integral $\int f(x)dx$ ← anti-derivative $f(x)$

$$\int f(x)\,dx = F(x) + C$$

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

a common reference point : arbitrary

definite integral $\int_a^x f(t)\,dt$ ← **anti-derivative** $f(x)$

indefinite integral $\int f(x)\,dx$ ← **anti-derivative** $f(x)$

$$\int_{x_1}^{x_2} f(t)\,dt = \int_a^{x_1} f(t)\,dt + \int_a^{x_2} f(t)\,dt$$

a common reference point : arbitrary

$$\left[F(x) + c\right]_{x_1}^{x_2} = F(x_2) - F(x_1)$$

↔

$$\left[F(x)\right]_{x_1}^{x_2} = F(x_1) - F(x_2)$$

Anti-derivative without constant

Indefinite Integral Examples

$$\int_0^x f(x)\,dx = \left[\frac{1}{3}x^3\right]_0^x = \frac{1}{3}x^3$$

$$f(x) = x^2$$

$$\int_a^x f(x)\,dx = \left[\frac{1}{3}x^3\right]_a^x = \frac{1}{3}x^3 - \frac{1}{3}a^3$$

$$\int_a^x f(t)\,dt = \left[\frac{1}{3}t^3\right]_a^x = \frac{1}{3}x^3 - \frac{1}{3}a^3$$

anti-derivative by the definite integral of f(x)

$$\int_a^x t^2\,dt = \frac{1}{3}x^3 - \frac{1}{3}a^2$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) = x^2$$

indefinite integral of f(x)

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

Definite Integrals on [a, x_1]

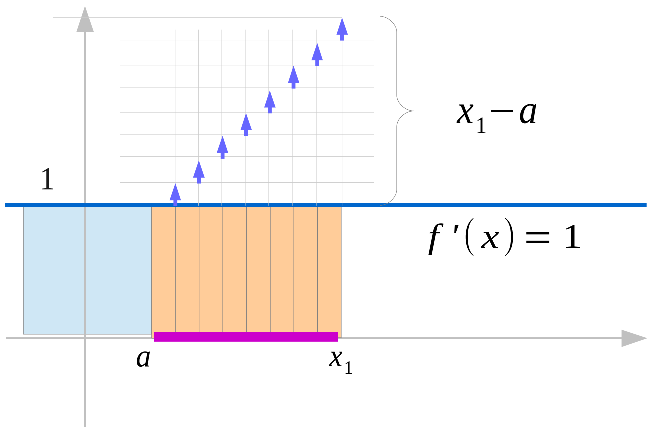
$$\int_a^{x_1} 1\,dx \Rightarrow \int_a^{x_1} \boxed{f'(x)}\,dx \qquad f'(x) = 1 \qquad \textit{view (I)}$$

$$\int_a^{x_1} 1\,dx \Rightarrow \int_a^{x_1} \boxed{g(x)}\,dx \qquad g(x) = 1 \qquad \textit{view (II)}$$

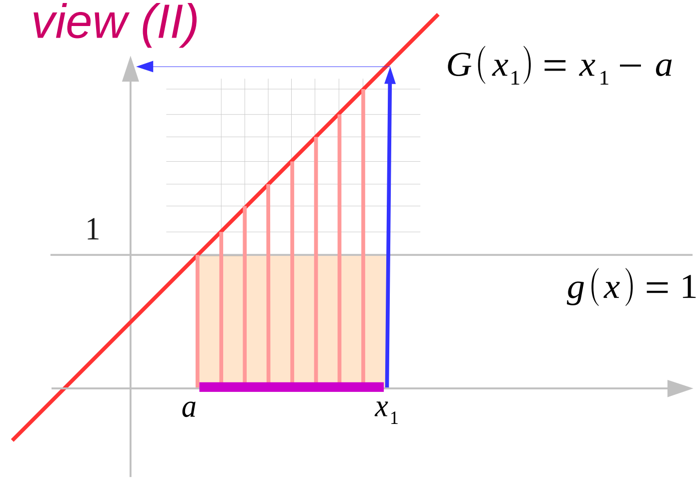
view (I)	$\int_a^{x_1} f'(x)\,dx$	$[f(x)]_a^{x_1} = f(x_1) - f(a)$
view (II)	$\int_a^{x_1} g(x)\,dx$	$[G(x)]_a^{x_1} = G(x_1) - G(a)$

Definite Integrals on [a, x_1]

view (I)



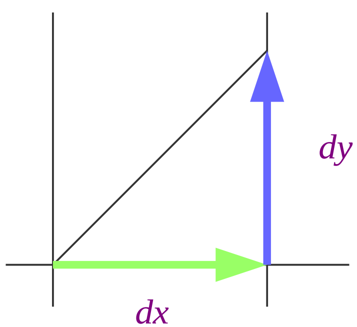
view (II)



$$\int_a^{x_1} 1\, dx = \int_a^{x_1} f'(x)\, dx$$

$$\int_a^{x_1} 1\, dx = \int_a^{x_1} g(x)\, dx$$

$$= [x]_a^{x_1} = x_1 - a$$

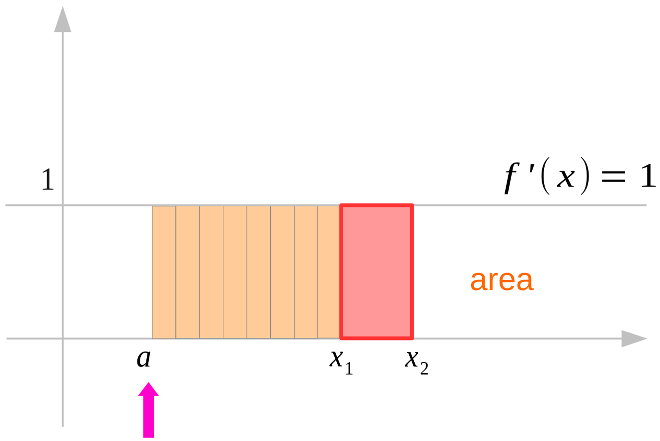


$$dy = \frac{dy}{dx} dx = f'(x)\, dx$$

$$G(x) = x$$

Definite Integrals over an interval [x_1, x_2]

view (I)

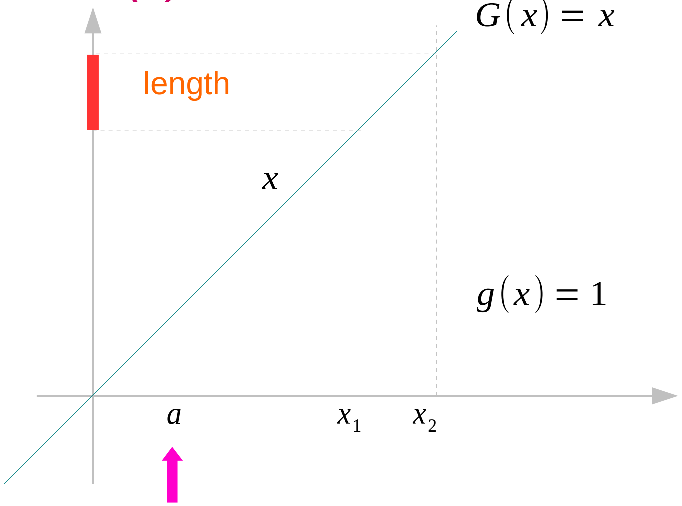


arbitrary reference point (a, f(a))

$$\int_{x_1}^{x_2} f'(x)\, dx =$$

$$[f(x)]_{x_1}^{x_2} = f(x_2) - f(x_1)$$

view (II)



arbitrary reference point (a, G(a))

$$\int_{x_1}^{x_2} g(x)\, dx =$$

$$[G(x)]_{x_1}^{x_2} = G(x_2) - G(x_1)$$

A reference point : integration constant C

view (I)

$$\int_{x_1}^{x_2} 1 \, dx$$

$f'(x)$

Anti-derivative without a constant

$$f(x) = x$$

$$x_2 - x_1$$

$$= \big[f(x) - f(a) \big]_{x_1}^{x_2}$$

arbitrary reference point (a, f(a))

$$= \big[f(x) + C \big]_{x_1}^{x_2}$$

$$= \big[f(x) \big]_{x_1}^{x_2}$$

$$= \int_c^{x_2} f'(x)\,dx - \int_c^{x_1} f'(x)\,dx$$

view (II)

$$\int_{x_1}^{x_2} 1 \, dx$$

$g(x)$

Anti-derivative without a constant

$$G(x) = x$$

$$x_2 - x_1$$

$$= \big[G(x) - G(a) \big]_{x_1}^{x_2}$$

arbitrary reference point (a, G(a))

$$= \big[G(x) + C \big]_{x_1}^{x_2}$$

$$= \big[G(x) \big]_{x_1}^{x_2}$$

$$= \int_c^{x_2} g(x)\,dx - \int_c^{x_1} g(x)\,dx$$

Indefinite Integrals through Definite Integrals

view (I)

$$\int 1\, dx \Leftarrow \int_a^{x_1} f'(x)\, dx$$

$$= f(x) - f(a) = x - a$$

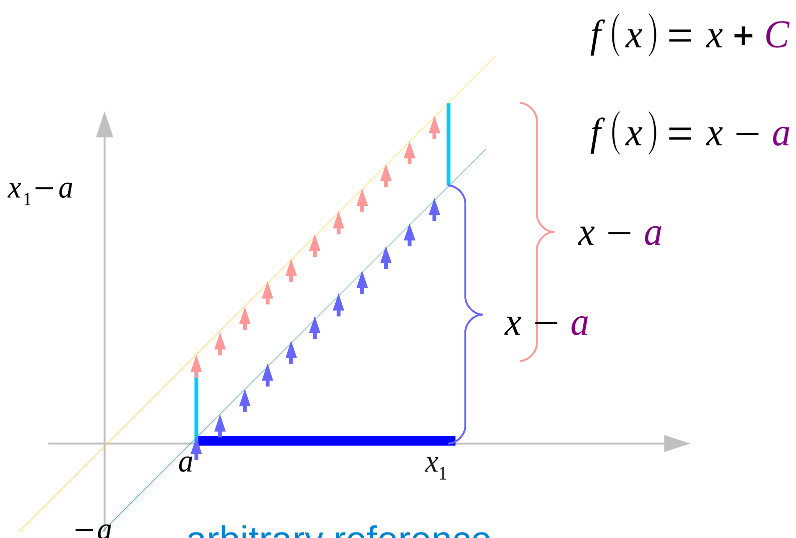
$$= f(x) + C$$

view (II)

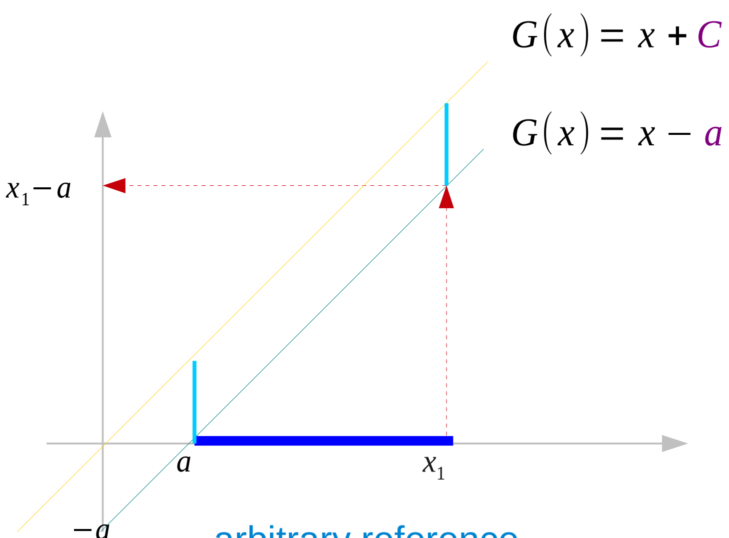
$$\int 1\, dx \Leftarrow \int_a^{x_1} g(x)\, dx$$

$$= G(x) - G(a) = x - a$$

$$= G(x) + C$$

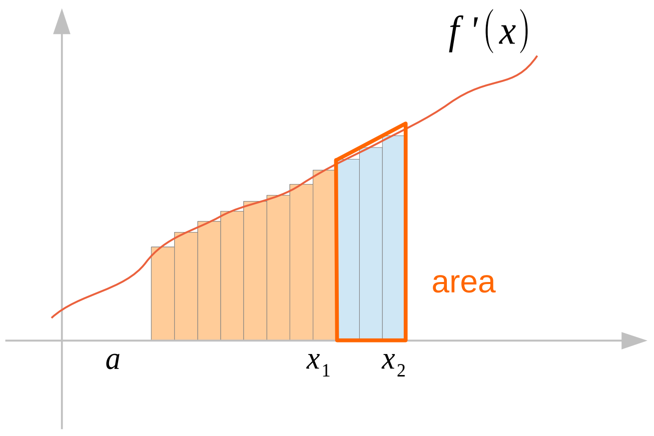


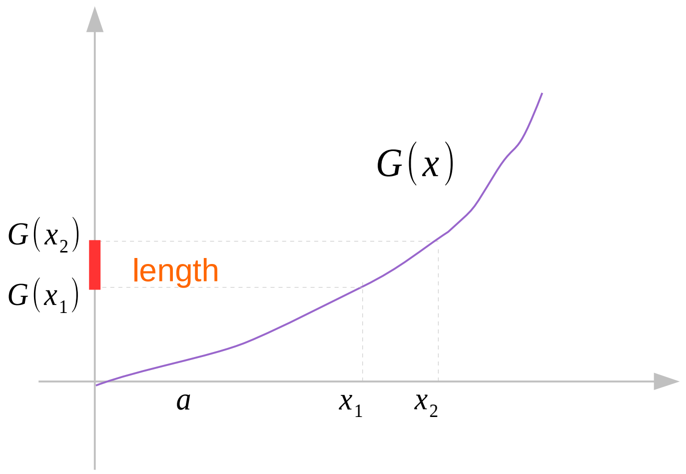
arbitrary reference point (a, f(a))



arbitrary reference point (a, G(a))

Definite Integrals on $[x_1, x_2]$

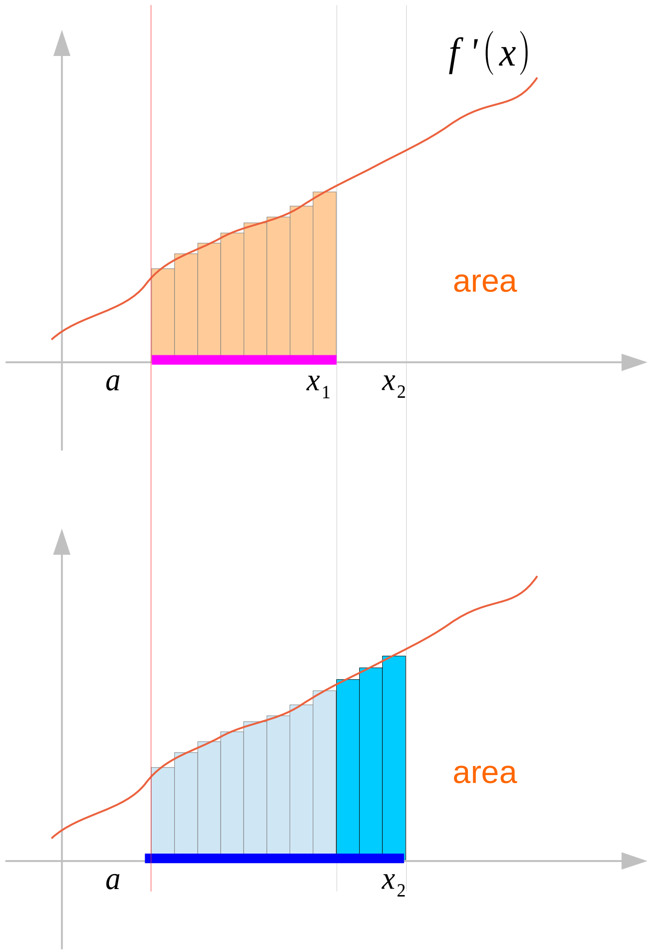


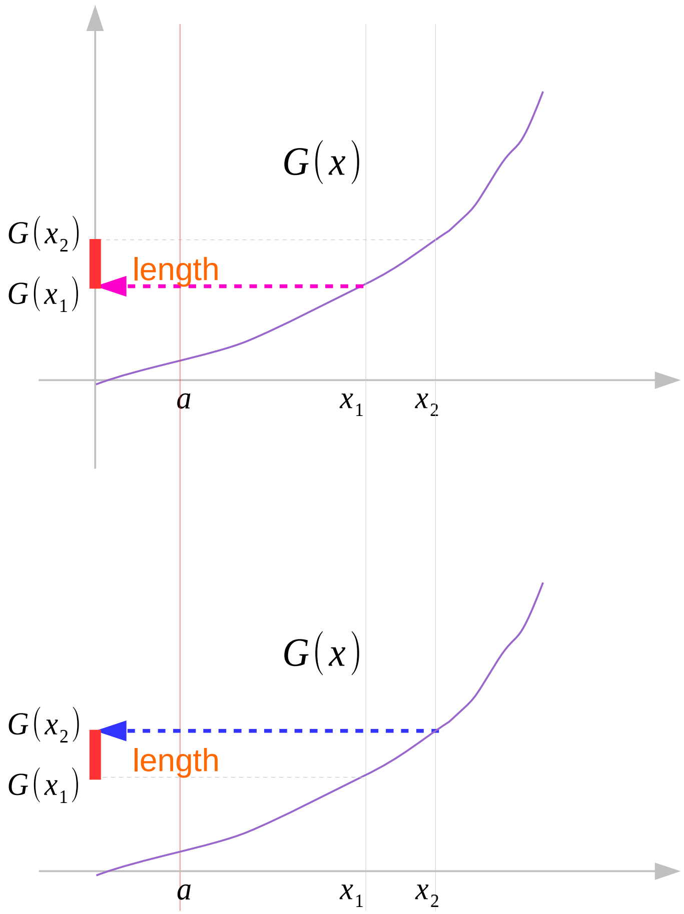


view (I) $\int_{x_1}^{x_2} f'(x)\, dx$

view (II) $\int_{x_1}^{x_2} g(x)\, dx$

Definite Integrals on [a, x_1] and [a, x_2]

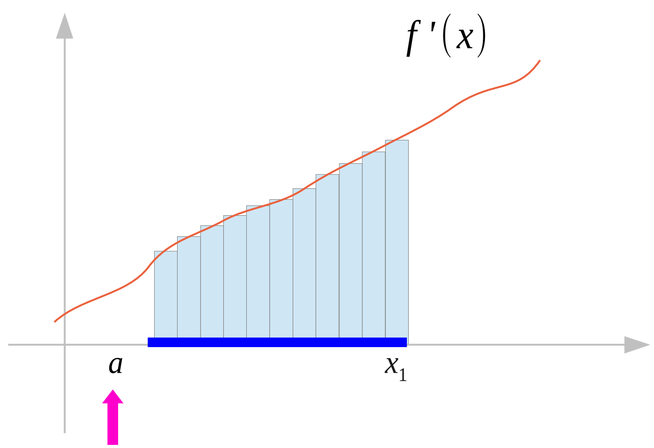




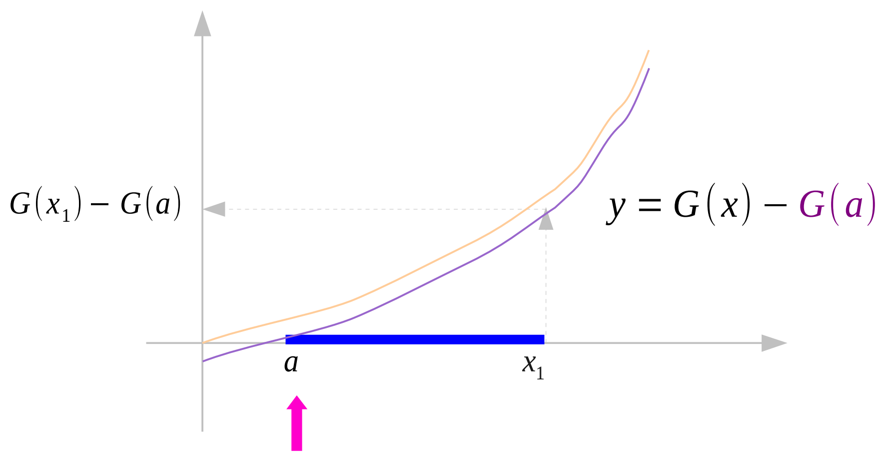
$$\int_c^{x_2} f'(x)\,dx - \int_c^{x_1} f'(x)\,dx$$

$$\int_c^{x_2} g(x)\,dx - \int_c^{x_1} g(x)\,dx$$

Indefinite Integrals through Definite Integrals



arbitrary reference point (a, f(a))



arbitrary reference point (a, G(a))

$\int_a^{x_1} f'(x)\,dx$ *view (I)*

$= f(x) - f(a) = x - a$

$= f(x) + C$

$\int_a^{x_1} g(x)\,dx$ *view (II)*

$= G(x) - G(a) = x - a$

$= G(x) + C$

Derivative Function and Indefinite Integrals

$$f'(x_1) \Rightarrow \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

$$f'(x_2) \Rightarrow \lim_{h \to 0} \frac{f(x_2 + h) - f(x_2)}{h}$$

$$f'(x_3) \Rightarrow \lim_{h \to 0} \frac{f(x_3 + h) - f(x_3)}{h}$$

$$\int_{x_1}^{x_2} f(x)\,dx$$

$$\int_{x_3}^{x_4} f(x)\,dx$$

$$\int_{x_5}^{x_6} f(x)\,dx$$

$x_1, \; x_2, \; x_3$

$$f'(\boldsymbol{x}) = \lim_{h \to 0} \frac{f(\boldsymbol{x} + h) - f(\boldsymbol{x})}{h}$$

$f'(x_1), \; f'(x_2), \; f'(x_3)$

function of x

$[x_1, \; x_2], [x_3, \; x_4], [x_5, \; x_6]$

$$F(\boldsymbol{x}) + C = \int_a^{\boldsymbol{x}} f(x)\,dx$$

$[\,F(x)\,]_{x_1}^{x_2}, \; [\,F(x)\,]_{x_3}^{x_4}, \; [\,F(x)\,]_{x_5}^{x_6}$

function of x

Differentiation & Integration of sinusoidal functions

$$\frac{d}{dx} f(x) = \cos(x)$$ **leads** $$f(x) = \sin(x)$$

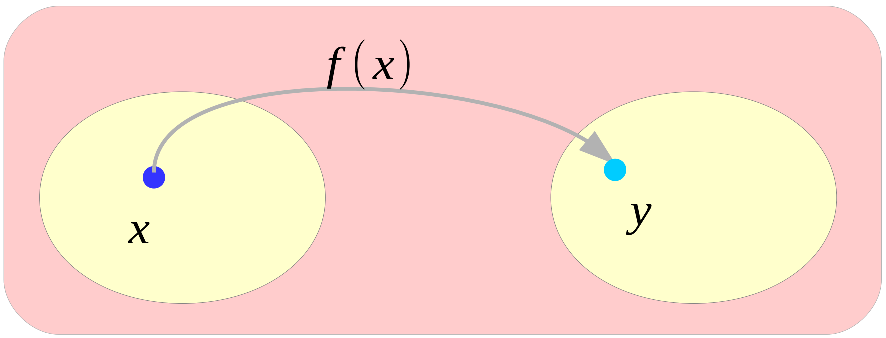
$$\frac{d}{dx} g(x) = -\sin(x)$$ **leads** $$g(x) = \cos(x)$$

$$\int f(x)\, dx = -\cos(x) + C$$ **lags** $$f(x) = \sin(x)$$

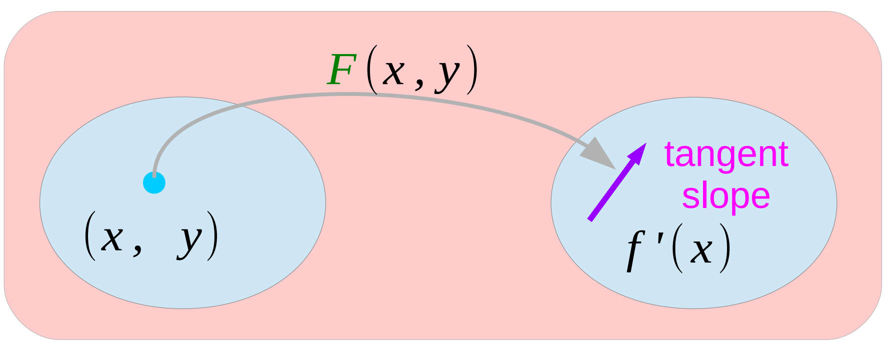
$$\int g(x)\, dx = \sin(x) + C$$ **lags** $$g(x) = \cos(x)$$

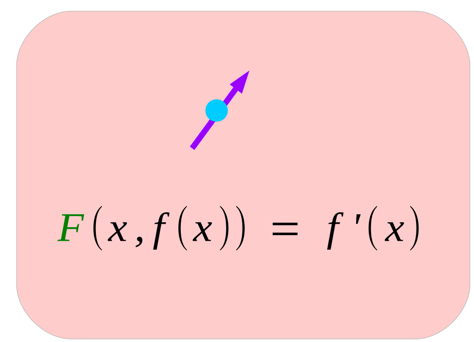
Plotting Lineal Elements

a single variable function



a two variable function

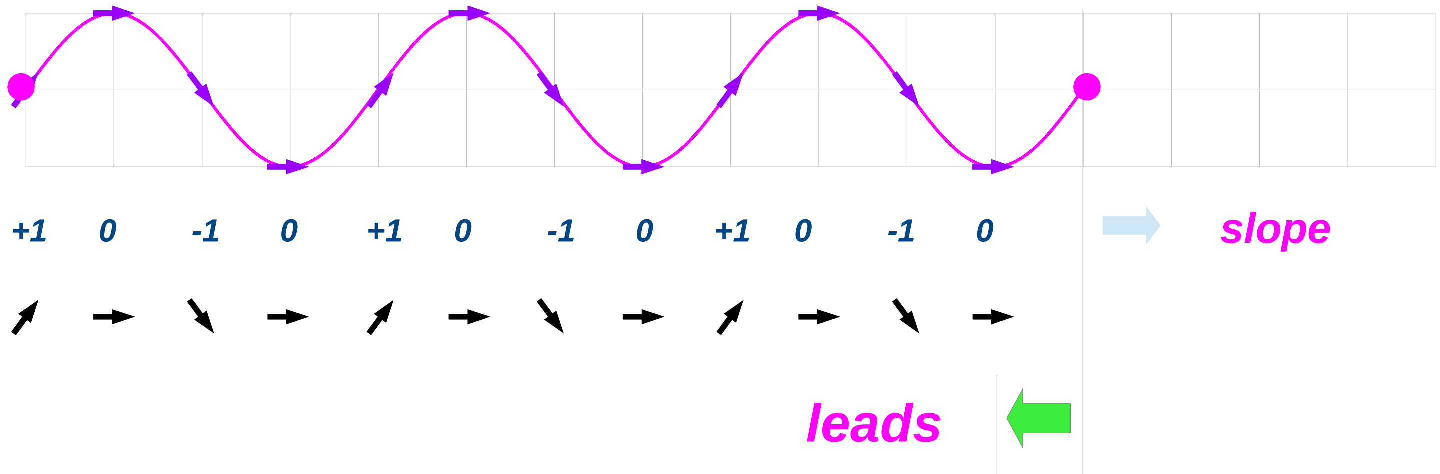




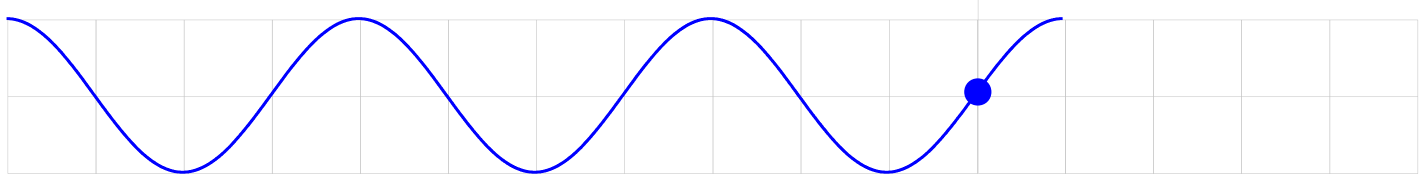
Derivative of sin(x)

A1

$$f(x) = \sin(x)$$



$$\frac{d}{dx} f(x) = \cos(x)$$

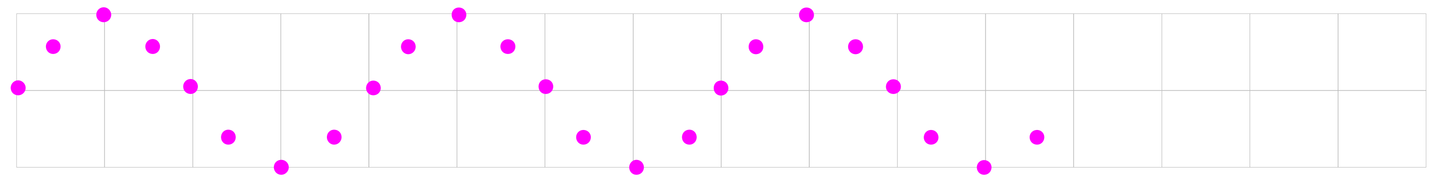


Young Won Lim
12/29/15

Plot of F(x,y) = f'(x) (= cos(x))

A2

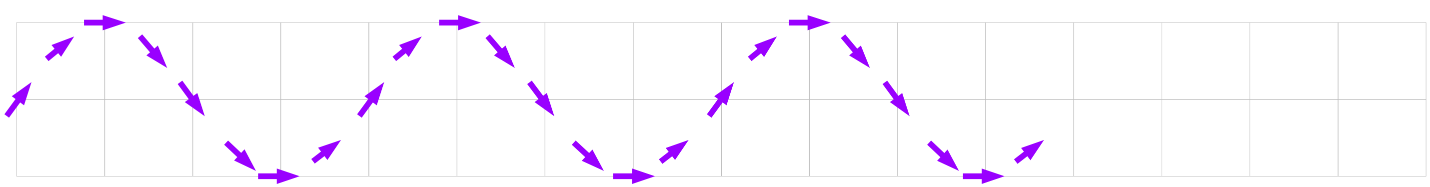
$(x, y) = (x, f(x)) = (x, \sin(x))$ $x \Rightarrow y$



$f(x) = \sin(x)$

+1 0 -1 0 +1 0 -1 0 +1 0 -1 0 ***slope***

$(x, y) \Rightarrow y'$



$F(x, y) = f'(x)$

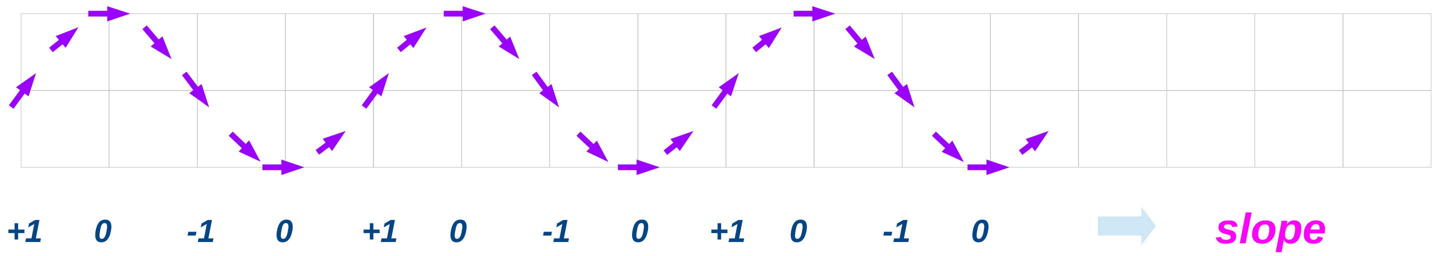
+1 0 -1 0 +1 0 -1 0 +1 0 -1 0 ***slope m***

$(x, y) \Rightarrow$ ***m*** *= slope of a tangent* $f'(x)$

$F(x, \sin(x)) = \cos(x)$

Plot of f'(x)=cos(x) from a lineal element plot

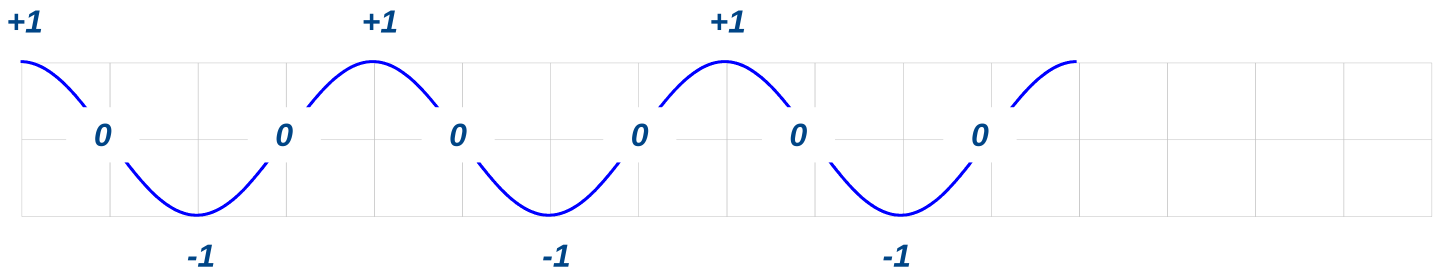
A3



$$F(x,\ y) = f'(x)$$

$$f(x) = \sin(x)$$

$$f'(x) = \cos(x)$$

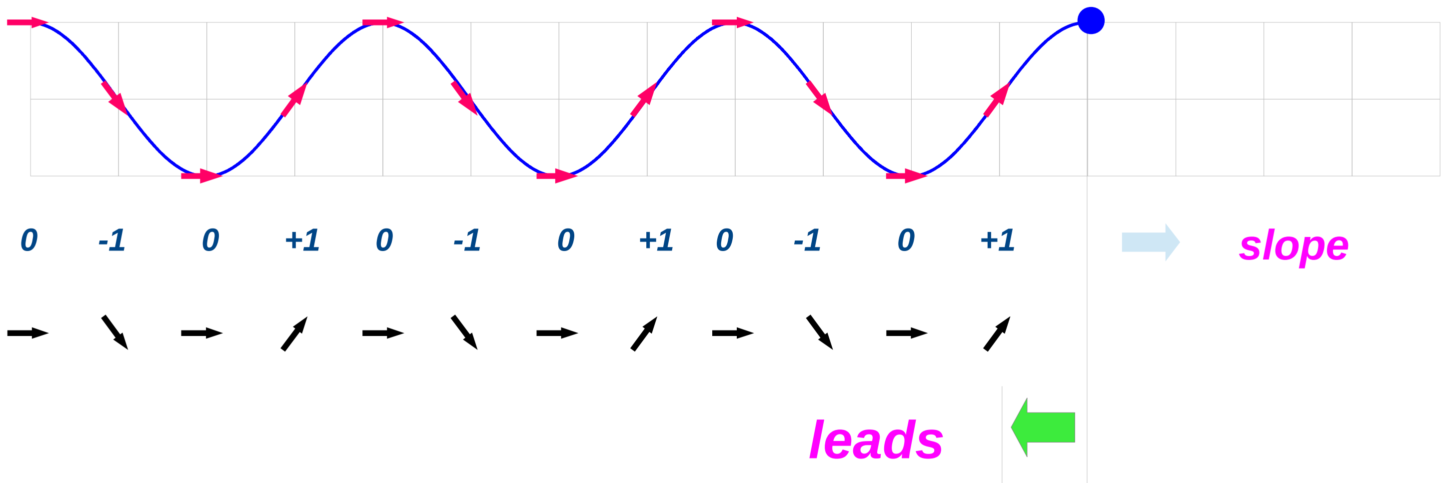


$$f(x) = \cos(x)$$

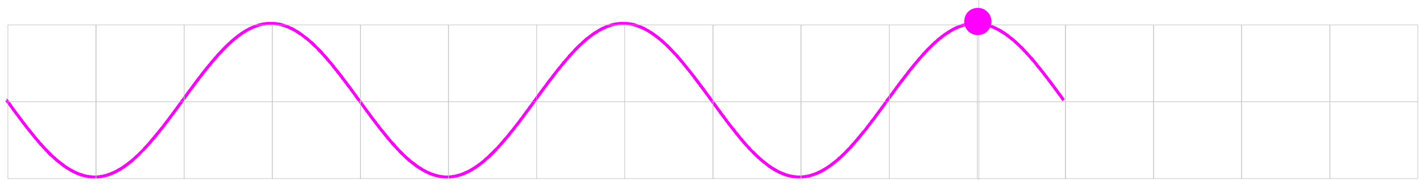
Derivative of cos(x)

B1

$$f(x) = \cos(x)$$



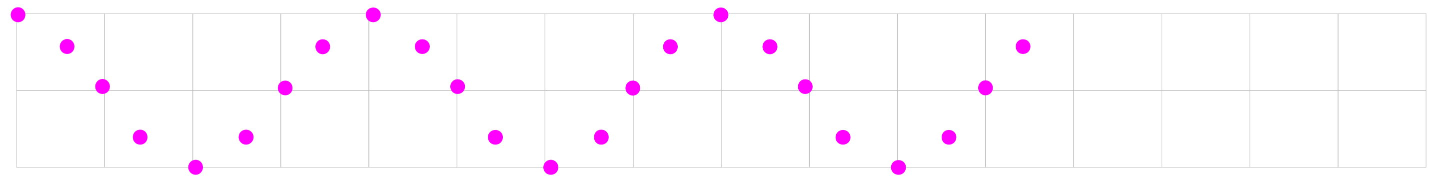
$$\frac{d}{dx} f(x) = -\sin(x)$$



Plot of F(x,y) = f'(x) (= -sin(x))

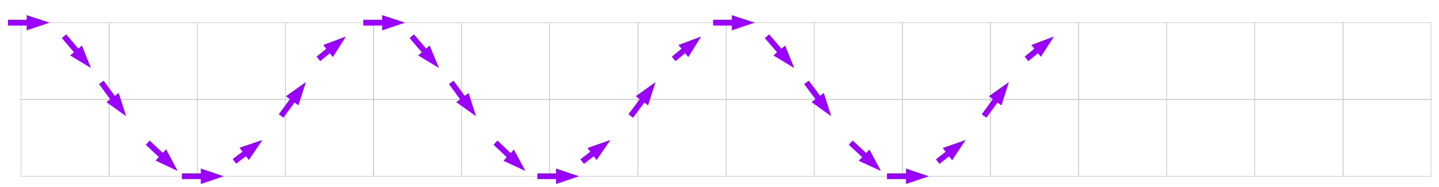
B2

$(x,\ y)\ =\ (x, f(x))\ =\ (x, \cos(x))$ $\quad x \Rightarrow y$



$f(x)\ =\ \cos(x)$

$(x,\ y) \Rightarrow y'$



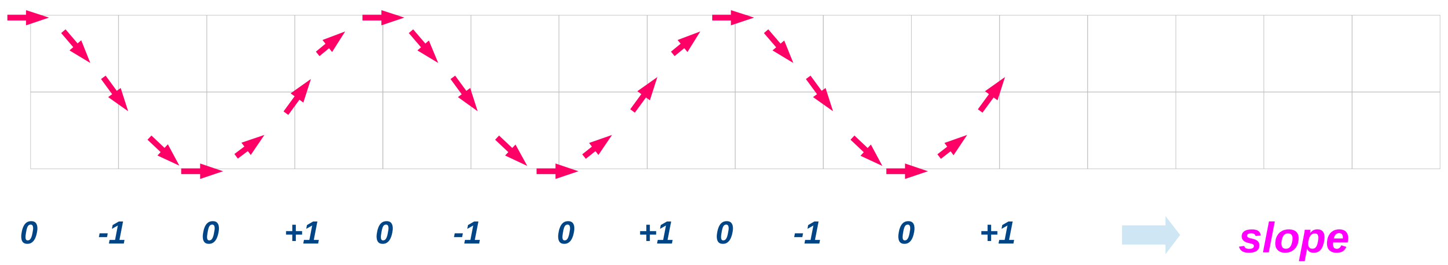
$F(x,\ y)\ =\ f'(x)$

$(x,\ y) \Rightarrow$ ***m*** *= slope of a tangent* $f'(x)$

$F(x,\ \cos(x))\ =\ -\sin(x)$

Plot of f'(x)=-sin(x) from a lineal element plot

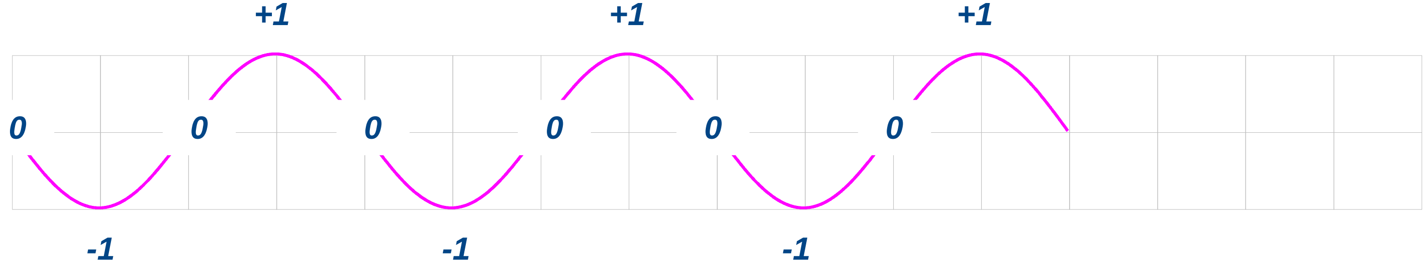
B3



$\cos(x)$

$$f(x) = \cos(x)$$

$$f'(x) = -\sin(x)$$

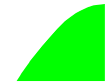


$f(x) = -\sin(x)$

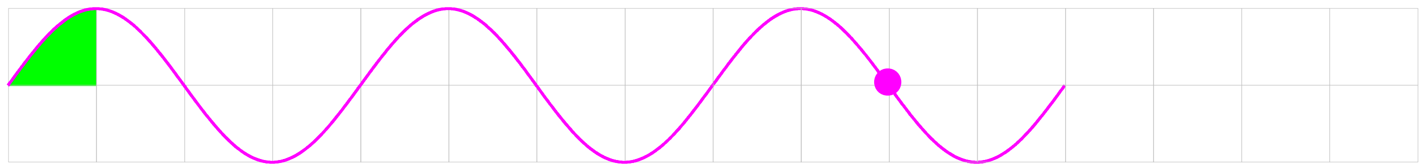
Definite Integrals of sin(x)

C1

$$f(x) = \sin(x)$$



$$\int_0^{\pi/2} \sin(t)\, dt = 1$$

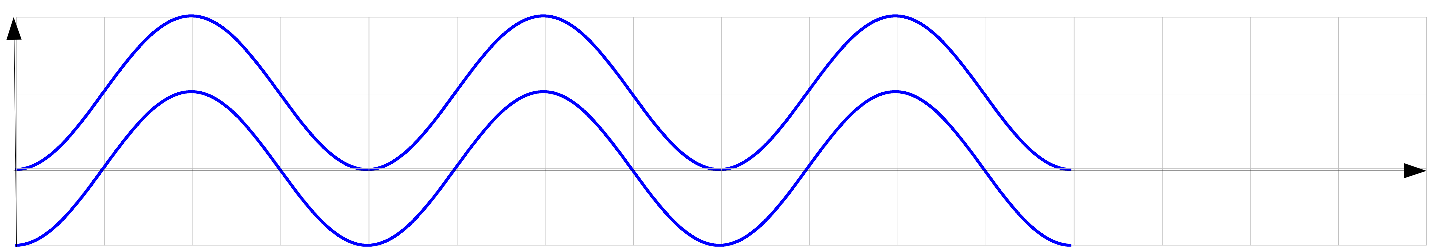


$\int_0^x \sin(t)\, dt$ 0 1 2 1 0 1 2 1 0 1 2 1 ⇨ **area + 0**

$= [-\cos(t)]_0^x$ $= -\cos(x) + 1$

$\int_{-\pi/2}^x \sin(t)\, dt$ -1 0 +1 0 -1 0 +1 0 -1 0 +1 0 ⇨ **area - 1**

$= [-\cos(t)]_{-\pi/2}^x$ $= -\cos(x) + 0$

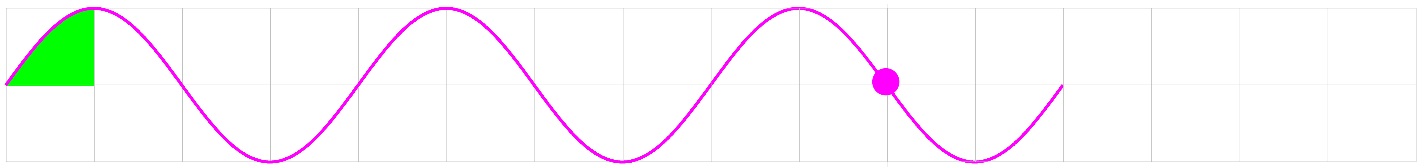


Indefinite Integrals of sin(x)

C2

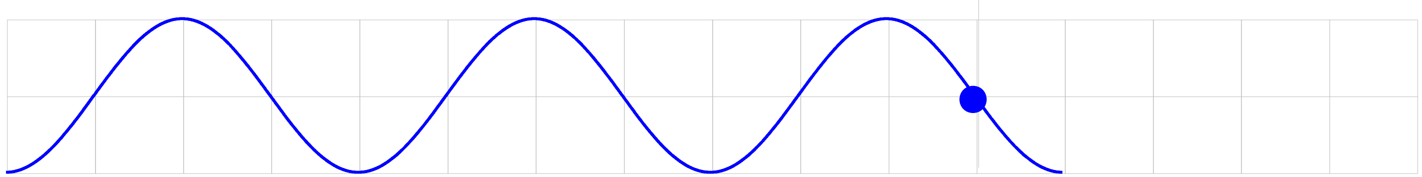
$$f(x) = \sin(x)$$

$$\int_0^{\pi/2} \sin(t)\, dt = 1$$



lags

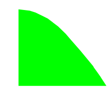
$$\int f(x)\, dx = -\cos(x) + C$$



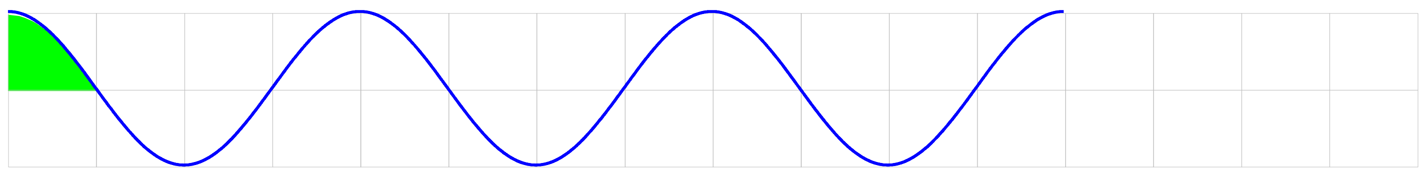
Definite Integrals of cos(x)

D1

$$f(x) = \cos(x)$$



$$\int_0^{\pi/2} \cos(x)\, dx = 1$$

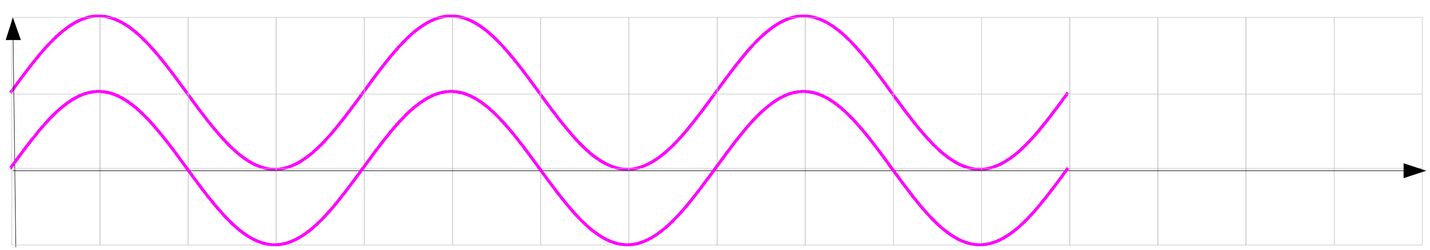


$\int_0^x \cos(t)\, dt$ 0 1 0 -1 0 1 0 -1 0 1 0 -1 **area - 0**

$= [\sin(t)]_0^x$ $= \sin(x) - 0$

$\int_{-\pi/2}^x \cos(t)\, dt$ 1 2 1 0 1 2 1 0 1 2 1 0 **area + 1**

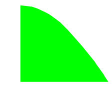
$= [\sin(t)]_{-\pi/2}^x$ $= \sin(x) + 1$



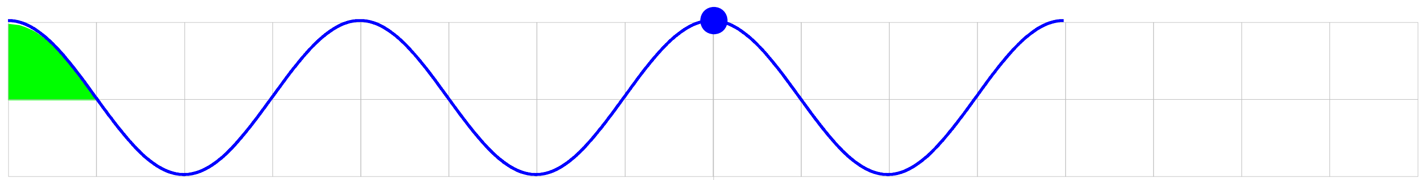
Indefinite Integrals of cos(x)

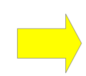
D2

$$f(x) = \cos(x)$$



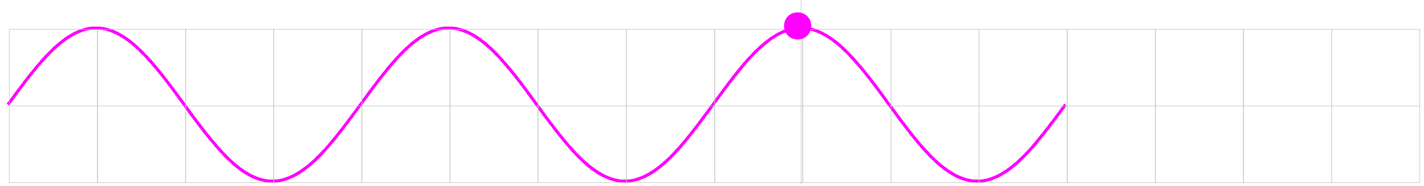
$$\int_0^{\pi/2} \cos(x)\, dx = 1$$





lags

$$\int f(x)\, dx = \sin(x) + C$$



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