Logic (H.2)

20180308

P Q

T/F T/F

1/0 1/0

and · conjunction

$P \wedge Q = f(P, Q)$

function

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

all inputs are true

P	Q	$P \wedge Q$
T	T	
T	F	F
F	T	F
F	F	F

at least one input is false

	P	Q	$P \wedge Q$
case 1 ⇒	T	T	T
case 2 ⇒	T	F	F
case 3 ⇒	F	T	F
case 4 ⇒	F	F	F

Case 1: all inputs are true

Cases 2–4: at least one input is false

All possible combination of inputs

T	T	T
T	T	F
T	F	T
T	F	T
F	T	T
F	T	F
F	F	T
F	F	T

T	T
T	F
F	T
F	F

T→1
F→0

	2^2	2^1	2^0			
7	1	1	1	T	T	T
6	1	1	0	T	T	F
5	1	0	1	T	F	T
4	1	0	0	T	F	F
3	0	1	1	F	T	T
2	0	1	0	F	T	F
1	0	0	1	F	F	T
0	0	0	0	F	F	F

Three input AND

2^2	2^1	2^0	
p	q	r	$p \wedge q \wedge r$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

$(p \wedge q) \wedge r$

$p \wedge (q \wedge r)$

$(p \wedge q) \wedge r$

$p \wedge (q \wedge r)$

associative law

	p	q	r	$p \wedge q \wedge r$
7	1	1	1	T
6	1	1	0	F
5	1	0	1	F
4	1	0	0	F
3	0	1	1	F
2	0	1	0	F
1	0	0	1	F
0 ←	0	0	0	F

10^2	10^1	10^0
1	2	3

2^2	2^1	2^0
1	1	1

T → 1

F → 0

bit-wise operators in C

$p \wedge q$ $\quad\quad$ $p \,\&\, q$

$p \vee q$ $\quad\quad$ $p \mid q$

$\neg p$ $\quad\quad$ $\sim p$

OR ..... disjunction

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

$P \vee Q = f(P, Q)$ function

at least one input is true

all inputs are false

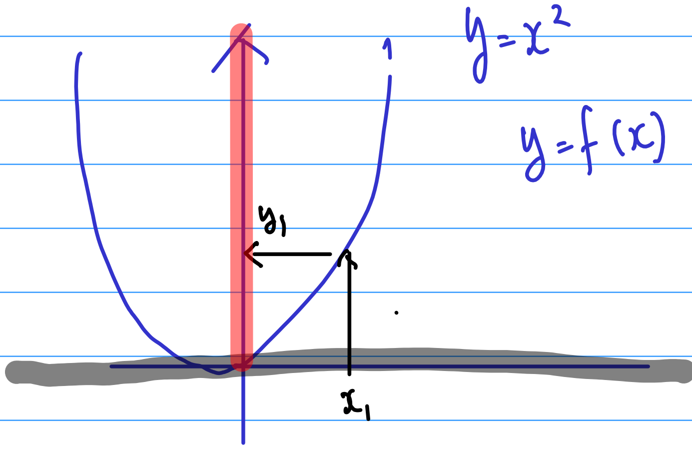
all ~~to~~

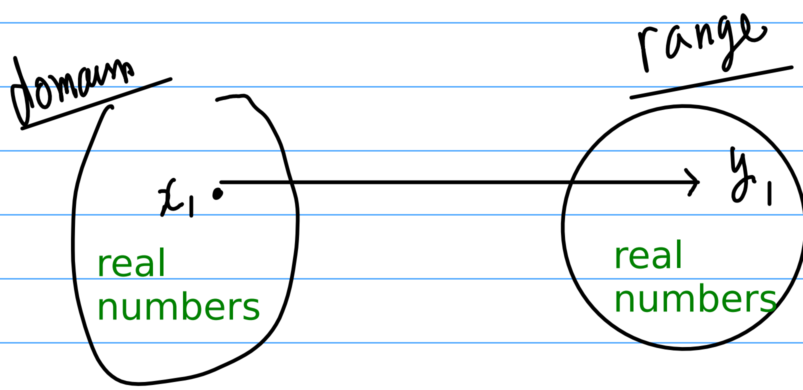
any

Function

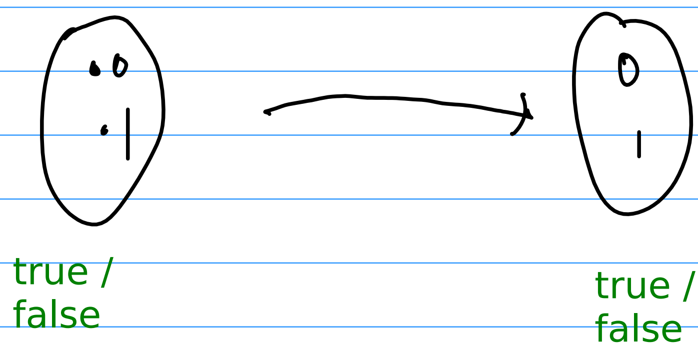
$y = x^2$

$y = f(x)$





boolean algebra

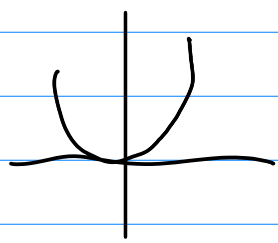


$$P \wedge Q = f(P, Q)$$

1-variable function

$$y = x^2$$

$$y = f(x)$$

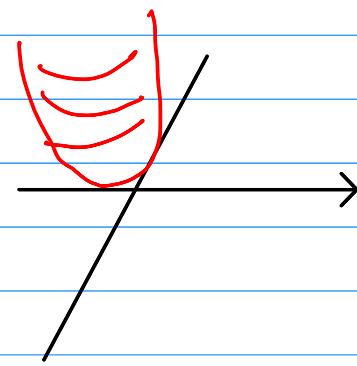


2-dimensional graph

2-variable function

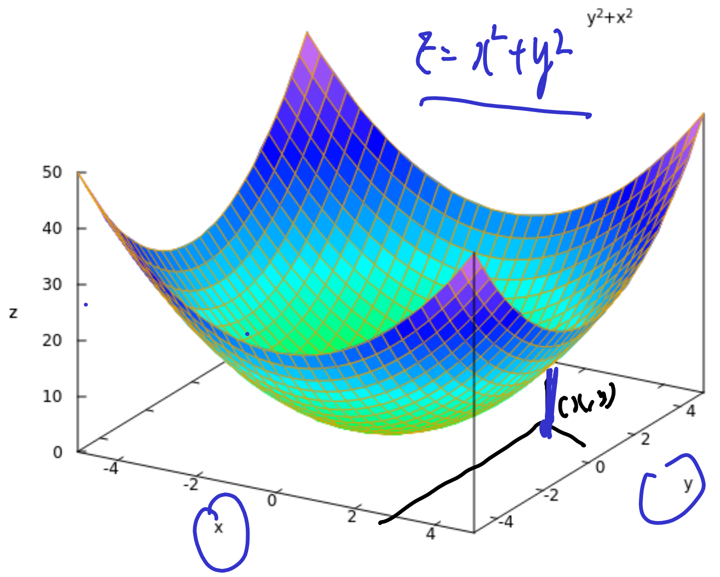
$$z = x^2 + y^2$$

$$z = f(x, y)$$



3-dimensional graph

2-variable real function



$z = x^2 + y^2$

2-variable boolean function

$f(x, y)$

$\underbrace{f(P, Q)}_{0/1} = \overset{0/1}{P} \wedge \overset{0/1}{Q}$

$P \vee Q$

	P	Q	
0	F	F	
1	F	T	
2	T	F	
3	T	T	

Material Implication

¬P ∨ Q

P Q P → Q

hypothesis (가설) conclusion (결론)

antecedent (전제) consequence (결과)

수학과에 6만 달러를 지원한다 P

수학과에 신임교수를 한명 고용한다 Q

	P	Q	$\boxed{P \to Q}$	~P	Q	$\boxed{\sim P \vee Q}$
case 1	T	T	T	F	T	T
case 2	T	F	F	F	F	F
case 3	F	T	T	T	T	T
case 4	F	F	T	T	F	T

$p \to Q$ 동치 $\equiv$

$\sim p \vee Q$

$\boxed{\text{logical equivalent}}$

Material Implicaton Example

$p \to q \equiv -p \vee q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$(p \wedge q) \to r$

$\neg(p \wedge q) \vee r$

$(\neg p \vee \neg q) \vee r$

$\boxed{\neg p} \vee \boxed{\neg q} \vee \boxed{r} \Rightarrow F$

false only when all inputs are false
true otherwise

$F \quad F \quad F$

$p = T \quad q = T \quad r = F$

De Morgan's Law

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\square \to \bigcirc$

$\equiv \neg \square \vee \bigcirc$

Verification by a truth table

$p \land q \to r$

$p=T \quad q=T \quad r=F$

	p	q	r	$p \land q$	r	$p \land q \to r$	$\neg p \lor \neg q \lor r$
case 1	0	0	0	0	0	1	1
case 2	0	0	1	0	1	1	1
...	0	1	0	0	0	1	1
	0	1	1	0	1	1	1
	1	0	0	0	0	1	1
	1	0	1	0	1	1	1
	1	1	0	1	0	0	0
	1	1	1	1	1	1	1

logical equivalence

Precedence

Order of precedence [edit]

As a way of reducing the number of necessary parentheses, one may introduce precedence rules: ¬ has higher precedence than ∧, ∧ higher than ∨, and ∨ higher than →. So for example, $P \vee Q \wedge \neg R \rightarrow S$ is short for $(P \vee (Q \wedge (\neg R))) \rightarrow S$.

Here is a table that shows a commonly used precedence of logical operators.[15]

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$p \wedge q \rightarrow r$

$(p \wedge q) \rightarrow (r)$

https://en.wikipedia.org/wiki/Logical_connective

Associativity

Truth functional connectives [edit]

Associativity is a property of some logical connectives of truth-functional propositional logic. The following logical equivalences demonstrate that associativity is a property of particular connectives. The following are truth-functional tautologies.

Associativity of disjunction:

$$((P \vee Q) \vee R) \leftrightarrow (P \vee (Q \vee R))$$
$$(P \vee (Q \vee R)) \leftrightarrow ((P \vee Q) \vee R)$$

Associativity of conjunction:

$$((P \wedge Q) \wedge R) \leftrightarrow (P \wedge (Q \wedge R))$$
$$(P \wedge (Q \wedge R)) \leftrightarrow ((P \wedge Q) \wedge R)$$

Associativity of equivalence:

$$((P \leftrightarrow Q) \leftrightarrow R) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow R))$$
$$(P \leftrightarrow (Q \leftrightarrow R)) \leftrightarrow ((P \leftrightarrow Q) \leftrightarrow R)$$

https://en.wikipedia.org/wiki/Associative_property

Commutativity

Truth functional connectives [edit]

Commutativity is a property of some logical connectives of truth functional propositional logic. The following logical equivalences demonstrate that commutativity is a property of particular connectives. The following are truth-functional tautologies.

Commutativity of conjunction

$$(P \land Q) \leftrightarrow (Q \land P)$$

Commutativity of disjunction

$$(P \lor Q) \leftrightarrow (Q \lor P)$$

Commutativity of implication (also called the law of permutation)

$$(P \to (Q \to R)) \leftrightarrow (Q \to (P \to R))$$

Commutativity of equivalence (also called the complete commutative law of equivalence)

$$(P \leftrightarrow Q) \leftrightarrow (Q \leftrightarrow P)$$

https://en.wikipedia.org/wiki/Commutative_property

Distributivity (1)

Truth functional connectives [edit]

Distributivity is a property of some logical connectives of truth-functional propositional logic. The following logical equivalences demonstrate that distributivity is a property of particular connectives. The following are truth-functional tautologies.

Distribution of conjunction over conjunction

$$(P \land (Q \land R)) \leftrightarrow ((P \land Q) \land (P \land R))$$

Distribution of conjunction over disjunction

$$(P \land (Q \lor R)) \leftrightarrow ((P \land Q) \lor (P \land R))$$

Distribution of disjunction over conjunction

$$(P \lor (Q \land R)) \leftrightarrow ((P \lor Q) \land (P \lor R))$$

Distribution of disjunction over disjunction

$$(P \lor (Q \lor R)) \leftrightarrow ((P \lor Q) \lor (P \lor R))$$

$3 \cdot (4 + 7)$

$3 \cdot 4 + 3 \cdot 7$

https://en.wikipedia.org/wiki/Distributive_property

Laws of logical equivalence (1)

Equivalence	***Name***
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$ $p \mid p \wedge T \equiv p$: T T, F F	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$ $p \mid p \vee F \equiv p$: T T, F F	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws

$(q \wedge r) \vee p \equiv (q \vee p) \wedge (r \vee p)$

$(q \vee r) \wedge p \equiv (q \wedge p) \vee (r \wedge p)$

https://en.wikipedia.org/wiki/Logical_equivalence

Laws of logical equivalence (2)

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

p	~p	p∨¬p
T	F	T
F	T	T

https://en.wikipedia.org/wiki/Logical_equivalence

argument
(논영)

bug는 module 17 또는 module 81에 있다 — $p \vee q$
bug는 numerical error이다 — r
module 81에는 numerical error가 없다 — $r \to \neg q$

p: bug는 module 17에 있다
q: bug는 module 81에 있다
r: bug는 numerical error이다

$$\frac{(p \vee q) \wedge (r) \wedge (r \to \neg q)}{p \ ?}$$

논영 argument

$\boxed{r} \to \neg q$

$\neg \boxed{r} \vee \neg q$

$(p \vee q) \wedge$
$(r \quad) \wedge$
$(r \to \neg q) \quad \neg r \vee \neg q$

p	q	r	$p \vee q$	r	$\neg r \vee \neg q$
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	0	1
0	1	1	1	1	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	0

p: bug는 module 17에 있다
q: bug는 module 81에 있다
r: bug는 numerical error이다

$p \vee q$ ✓
r ✓
$r \rightarrow \neg q$ ✓

p

논증
argument

H1 $p \vee q$
H2 r
H3 $r \rightarrow \neg q$

p

$\boxed{r} \rightarrow \neg q$

$\neg \boxed{r} \vee \neg q$

$p \vee q$

r

$r \to \neg q$

$\neg r \vee \neg q$

$\neg q$

p

$p \vee q$ T

$\neg q$ T

$p \vee F \equiv T$

$q \equiv F$

$p \equiv T$

$(A \vee B) \wedge$
$(\neg A)$

$(A \vee B) \wedge (\neg A)$

$(A \wedge (\neg A)) \vee (B \wedge (\neg A))$

$F \vee (B \wedge (\neg A))$

$(B \wedge (\neg A))$

$(A \vee B) \wedge (\neg A)$

① $A = T$ $\quad (T \vee B) \wedge (F) \equiv B \wedge F = F$ ✓

② $A = F$ $\quad (F \vee B) \wedge (T) \equiv B \wedge T = B$

B

$$\cancel{A} \lor B$$
$$\neg \cancel{A}$$
$$\overline{\qquad\qquad\qquad}$$
$$B$$

$$(A \lor B) \land (\neg A)$$

$$(A \lor B) \land (\neg A)$$

① case $A = T$ $\quad (T \lor B) \land (F) \quad B \land F = F$

or

② case $A = F$ $\quad (F \lor B) \land (T) \quad B \land T = B$

$$\overline{\qquad\qquad\qquad}$$
$$F \lor B$$
$$= B$$

$$\begin{array}{c} p \lor q \\ \neg p \lor r \\ \hline q \lor r \end{array}$$

Case 1: p is false.

$$\begin{array}{c} F \lor q \equiv q \\ T \lor r \equiv T \\ \hline q \end{array}$$

Case 2: p is true.

$$\begin{array}{c} T \lor q \equiv T \\ F \lor r \equiv r \\ \hline r \end{array}$$

$(p \vee q) \wedge$
$(r) \wedge$
$(r \to \neg q)$ $\quad \neg r \vee \neg q$

$$\begin{array}{c} p \vee q \\ r \\ r \to \neg q \\ \hline ? \end{array} \qquad \begin{array}{c} p \vee q \\ \cancel{r} \\ \cancel{\neg r} \vee \neg q \\ \hline \end{array} \qquad \begin{array}{c} p \vee \cancel{q} \\ \cancel{\neg q} \\ \hline p \end{array}$$

resolution 방식