Laurent Series and z-Transform Examples case 1.B

20171213

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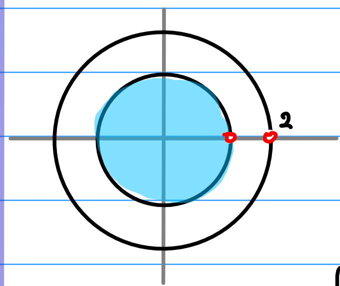

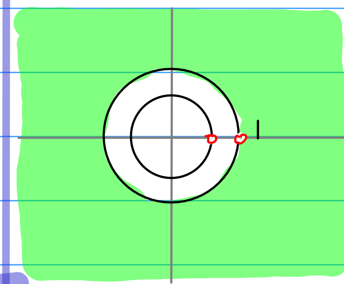

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

$$X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

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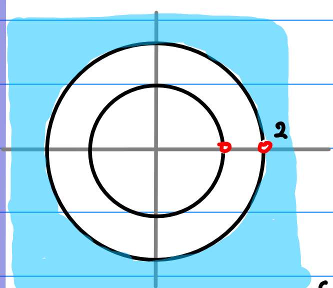

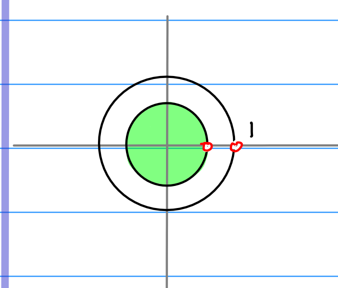

$$a_n = \begin{cases} 0 & (n \geq 0) \\ \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n < 0) \end{cases}$$

$$x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$

$$X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$

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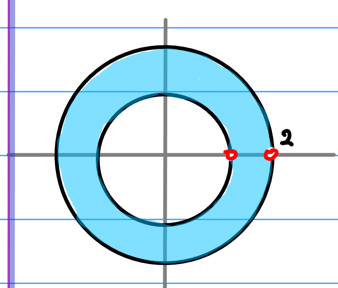
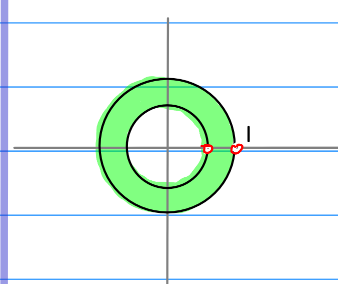




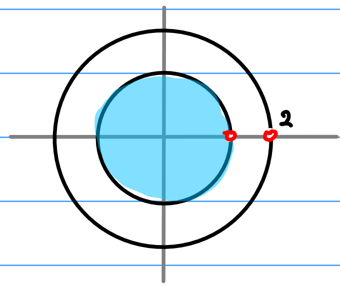
$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

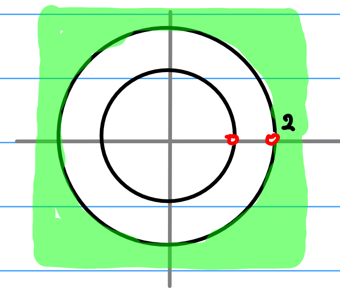
$$x_n = \begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

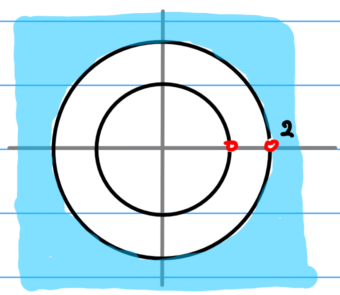
$$X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

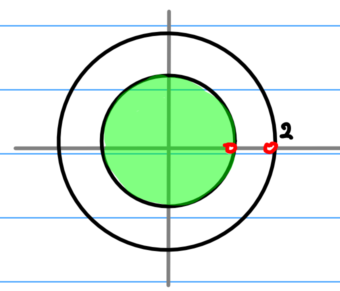
1. A $f(z) = \dfrac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} X(z) = \dfrac{-z^2}{(z-2)(z-0.5)}$



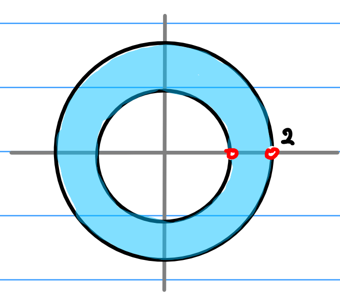


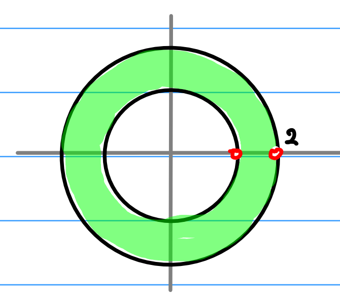
$$\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}\right] z^n = \sum_{n=1}^{\infty}\left[1-2^{-n+1}\right] z^{-n}$$





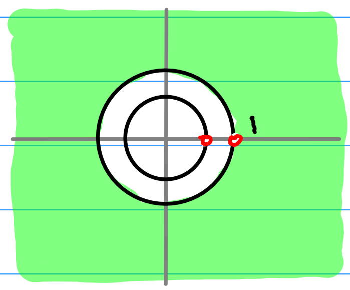
$$\sum_{n=-1}^{-\infty}\left[1-\left(\frac{1}{2}\right)^{n+1}\right] z^n = \sum_{n=-1}^{-\infty}\left[-1+2^{n-1}\right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} 1\cdot z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

$$X(z) = \frac{-z^2}{(z-2)(z-0.5)} = \frac{2}{3}\left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$



$\frac{1}{z}$

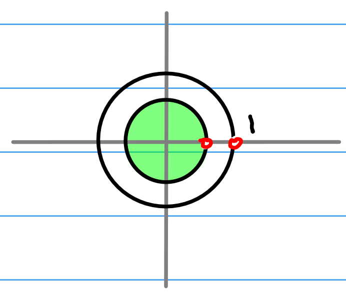
$$+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1} - 2^{n-1}\right] z^{-n}$$

$$= \sum_{n=-1}^{-\infty}\left[2^{n-1} - 1\right] z^{n}$$

$$\frac{2}{3} \quad \boxed{\sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n}}$$



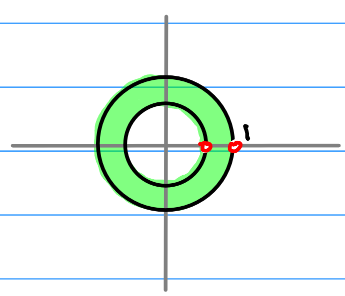
$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[1 - 2^{n-1}\right] z^{n}$$

$$\boxed{\sum_{n=-1}^{\infty}\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] z^{-n}}$$



$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

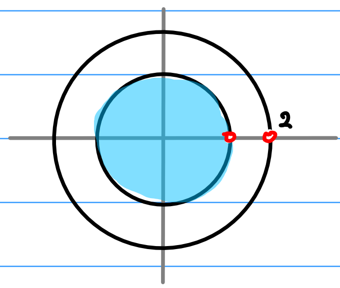
$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= +\sum_{n=1}^{\infty} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n}$$

$$\boxed{+\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}}$$

$$f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$

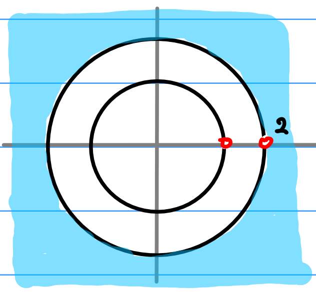


$$- \frac{\left(\frac{1}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} (1)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$

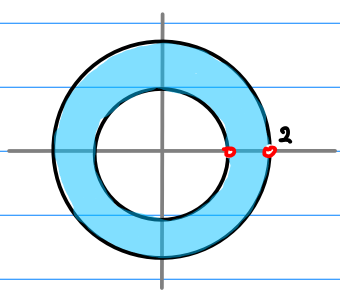


$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty} \left[1 - 2^n \right] z^{-n-1}$$

$$= \sum_{n=-1}^{-\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$+ \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

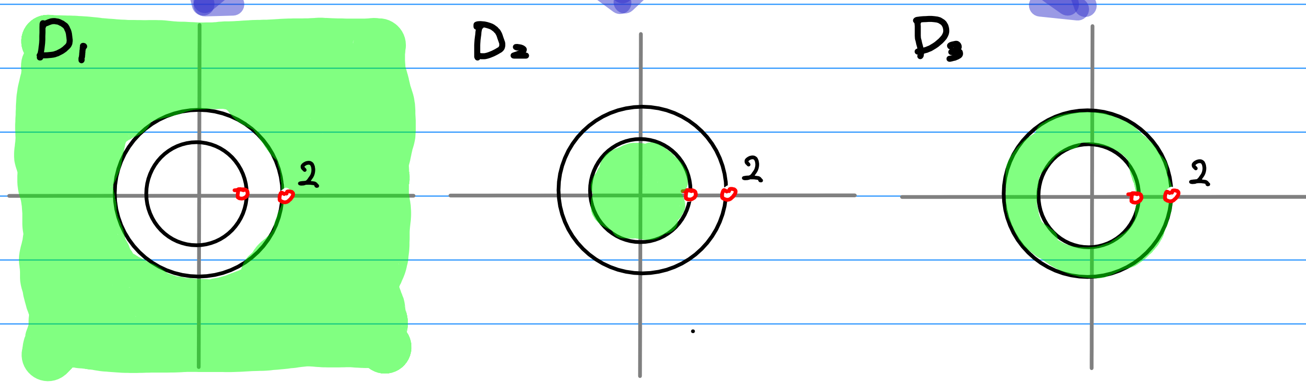
$$= + \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$= + \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$X(z) = \frac{-1}{(z-1)(z-2)}$$

Z.T. first

$$\boxed{X(z) = \frac{-1}{(z-1)(z-2)}}$$



x_n? $\|$ a_n

x_n? $\|$ a_n

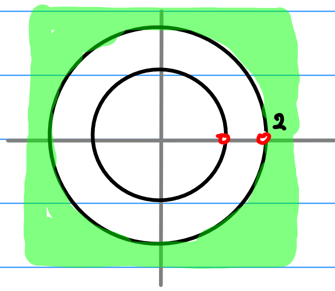
x_n? $\|$ a_n

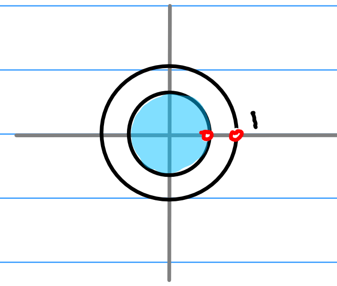
f(z)?

f(z)?

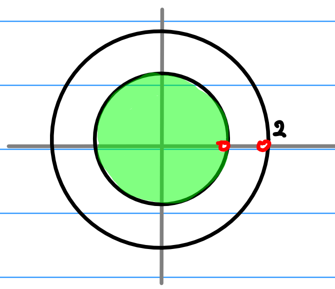
f(z)?

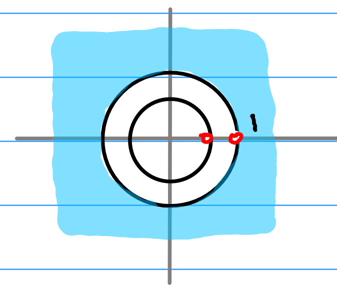
1. B $X(z) = \dfrac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} f(z) = \dfrac{-0.5\, z^2}{(z-1)(z-0.5)}$



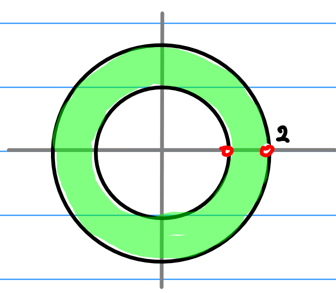


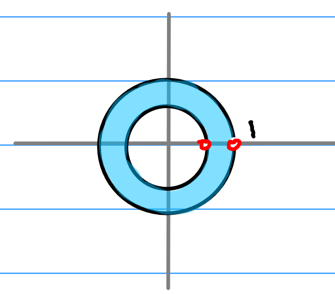
$$\sum_{n=1}^{\infty} \left[1 - 2^{-n+1}\right] z^{-n} = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{n}$$





$$\sum_{n=-1}^{-\infty} \left[-1 + 2^{n-1}\right] z^{-n} = \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^{n}$$



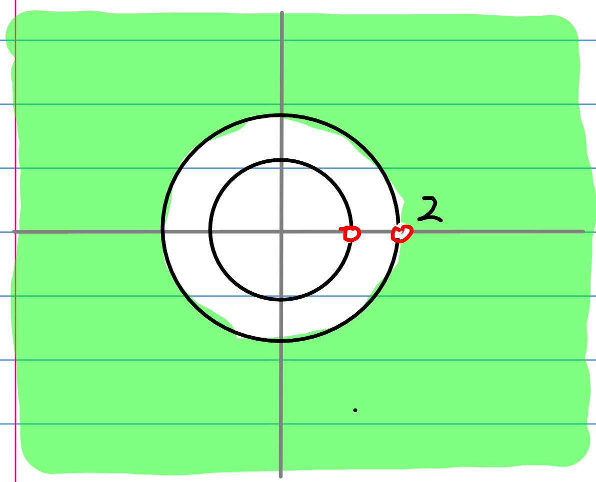


$$\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} = \sum_{n=-1}^{-\infty} z^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \cdot z^{n}$$

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$$

Z.T. first

① D_1 $|z| > 2$ $\left[\left|\frac{1}{z}\right| < 1 , \left|\frac{2}{z}\right| < 1 \right]$



ROC (Region of Convergence)

$|z| > 2 \Rightarrow \frac{2}{|z|} < 1$

$\left(\frac{2}{z}\right)^0 + \left(\frac{2}{z}\right)^1 + \left(\frac{2}{z}\right)^2 + \cdots \xrightarrow{\text{converge}} \frac{1}{1-\frac{2}{z}}$

$|z| > 1 \Rightarrow \frac{1}{|z|} < 1$

$\left(\frac{1}{z}\right)^0 + \left(\frac{1}{z}\right)^1 + \left(\frac{1}{z}\right)^2 + \cdots \xrightarrow{\text{converge}} \frac{1}{1-\frac{1}{z}}$

$$f(z) = \frac{1}{z-1} - \frac{1}{z-2} = \frac{1}{z}\frac{z}{z-1} - \frac{1}{z}\frac{z}{z-2} = \frac{1}{z}\frac{1}{1-\left(\frac{1}{z}\right)} - \frac{1}{z}\frac{1}{1-\left(\frac{2}{z}\right)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} - \sum_{n=0}^{\infty} \frac{2^n}{z^{n+1}} = \sum_{n=0}^{\infty} \frac{1-2^n}{z^{n+1}}$$

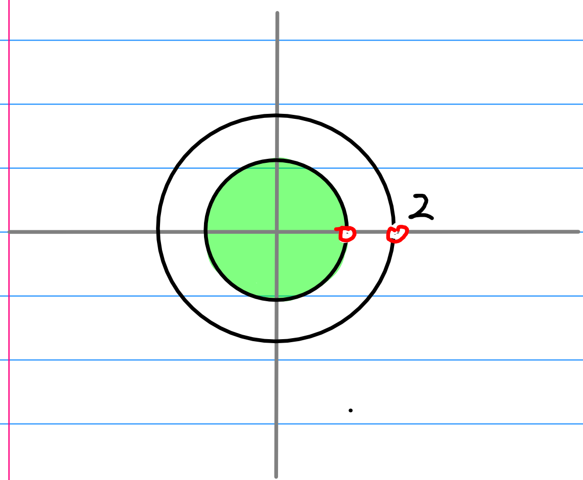
$$= \sum_{n=1}^{\infty} \frac{1-2^{n-1}}{z^n} = \sum_{n=1}^{\infty} (1-2^{n-1}) z^{-n}$$

$$\left(\frac{1}{z}\right) + \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 + \cdots$$
$$+ \frac{1}{2}\left\{ \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + \cdots \right\} \xrightarrow{\text{converge}} \frac{1}{z-1} - \frac{1}{z-2} = \frac{-1}{(z-1)(z-2)}$$

$$(1-2^0)z^{-1} + (1-2^1)z^{-2} + (1-2^2)z^{-3} + \cdots \xrightarrow{\text{converge}} \frac{-1}{(z-1)(z-2)} \quad (|z| > 2)$$

$$x[n] = 1 - 2^{n-1} \longleftrightarrow X(z) = \frac{-1}{(z-1)(z-2)} \quad (|z| > 2)$$

$\textcircled{II}$ D_2 $|z|<1$ $\left[\,\left|\frac{z}{1}\right|<1\,,\quad \left|\frac{z}{2}\right|<1\,\right]$



$|z|<2 \Rightarrow \frac{|z|}{2}<1$

$\left(\frac{z}{2}\right)^0+\left(\frac{z}{2}\right)^1+\left(\frac{z}{2}\right)^2+\cdots \xrightarrow[\text{converge}]{} \frac{1}{1-\frac{z}{2}}$

$|z|<1 \Rightarrow \frac{|z|}{1}<1$

$\left(\frac{z}{1}\right)^0+\left(\frac{z}{1}\right)^1+\left(\frac{z}{1}\right)^2+\cdots \xrightarrow[\text{converge}]{} \frac{1}{1-\frac{z}{1}}$

ROC (Region of onvergence)

$$f(z)=\frac{1}{z-1}-\frac{1}{z-2}=-\frac{1}{1}\frac{1}{1-z}+\frac{1}{2}\frac{2}{2-z}=-\frac{1}{1}\frac{1}{1-\left(\frac{z}{1}\right)}+\frac{1}{2}\frac{1}{1-\left(\frac{z}{2}\right)}$$

$$=-\sum_{n=0}^{\infty}z^n+\frac{1}{2}\sum_{n=0}^{\infty}\frac{z^n}{2^n}=\sum_{n=0}^{\infty}\left(-1+2^{-n-1}\right)z^n$$

$$=\sum_{k=0}^{-\infty}\left(-1+2^{k-1}\right)z^{-k}=\sum_{n=0}^{-\infty}\left(-1+2^{n-1}\right)z^{-n}$$

$$\begin{aligned}&-\left\{\left(\frac{z}{1}\right)+\left(\frac{z}{1}\right)^2+\left(\frac{z}{1}\right)^3+\cdots\right\}\\&+\frac{1}{2}\left\{\left(\frac{z}{2}\right)+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\cdots\right\}\end{aligned} \xrightarrow[\text{converge}]{} \frac{1}{z-1}-\frac{1}{z-2}=\frac{-1}{(z-1)(z-2)}$$

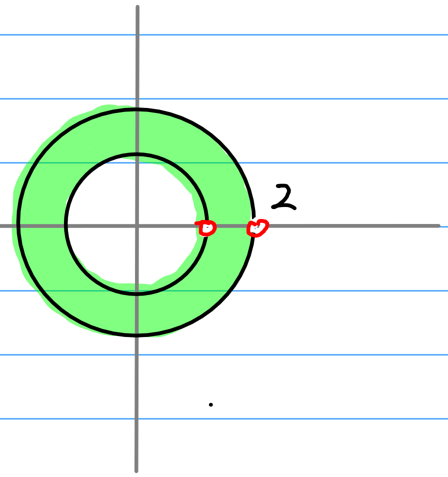
$$(-1+2^{-2})z^1+(-1+2^{-3})z^2+(-1+2^{-4})z^3+\cdots \xrightarrow[\text{converge}]{} \frac{-1}{(z-1)(z-2)} \quad (|z|<2)$$

$$x[n]=-1+2^{n-1} \longleftrightarrow X(z)=\frac{-1}{(z-1)(z-2)} \quad (|z|>2)$$

$n \leq 0$

Z.T. first

Ⅲ D3 $\quad 1<|z|<2 \qquad \left[\left|\frac{1}{z}\right|<1,\quad \left|\frac{z}{2}\right|<1\right]$



$$|z|<2 \Rightarrow \frac{|z|}{2}<1$$

$$\left(\frac{z}{2}\right)^0+\left(\frac{z}{2}\right)^1+\left(\frac{z}{2}\right)^2+\cdots \longrightarrow \frac{1}{1-\frac{z}{2}}$$

Converge

$$|z|>1 \Rightarrow \frac{1}{|z|}<1$$

$$\left(\frac{1}{z}\right)^0+\left(\frac{1}{z}\right)^1+\left(\frac{1}{z}\right)^2+\cdots \longrightarrow \frac{1}{1-\frac{1}{z}}$$

Converge

ROC (Region of Convergence)

$$f(z)=\frac{1}{z-1}-\frac{1}{z-2}=\frac{1}{z}\frac{z}{z-1}+\frac{1}{2}\frac{2}{z-2}=\frac{1}{z}\frac{1}{1-\left(\frac{1}{z}\right)}+\frac{1}{2}\frac{1}{1-\left(\frac{z}{2}\right)}$$

$$=\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}+\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}=\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}+\sum_{k=0}^{-\infty}\frac{z^{-k}}{2^{-k+1}}=\sum_{n=1}^{\infty}z^{-n}+\sum_{n=0}^{-\infty}2^{n-1}z^{-n}$$

$$x_n=\begin{cases}1 & (n>0)\\ 2^{n-1} & (n\leq 0)\end{cases}$$

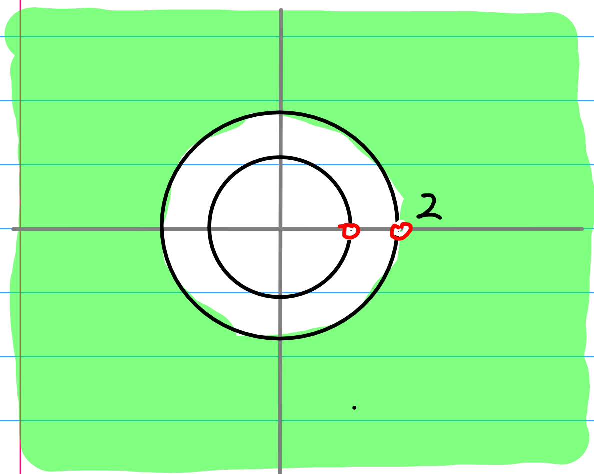
$$\left(\frac{1}{z}\right)+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3+\cdots$$
$$+\frac{1}{2}\left\{\left(\frac{z}{2}\right)+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\cdots\right\} \longrightarrow \frac{1}{z-1}-\frac{1}{z-2}=\frac{-1}{(z-1)(z-2)}$$

Converge

Z.T. first

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$$

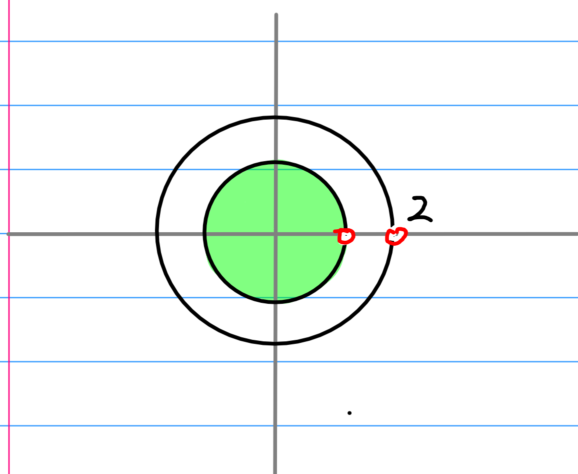
Ⅰ D_1 $|z| > 2$ causal $x_n = 0 \quad (n \leq 0)$



$$\boxed{X(z) = \frac{-1}{(z-1)(z-2)}}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

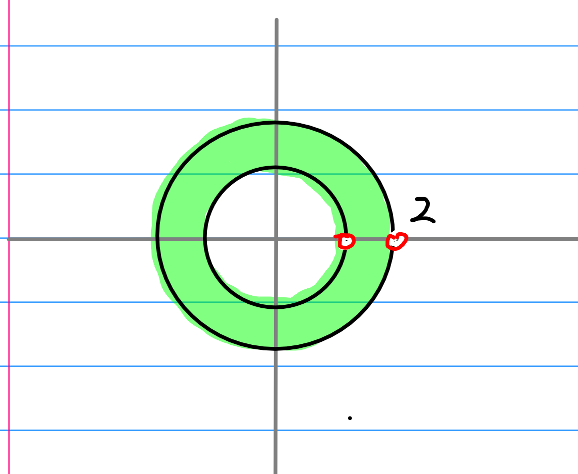
Ⅱ D_3 $|z| < 1$ anti-causal $x_n = 0 \quad (n \geq 0)$



$$\boxed{X(z) = \frac{-1}{(z-1)(z-2)}}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⅲ D_2 $1 < |z| < 2$ two-sided



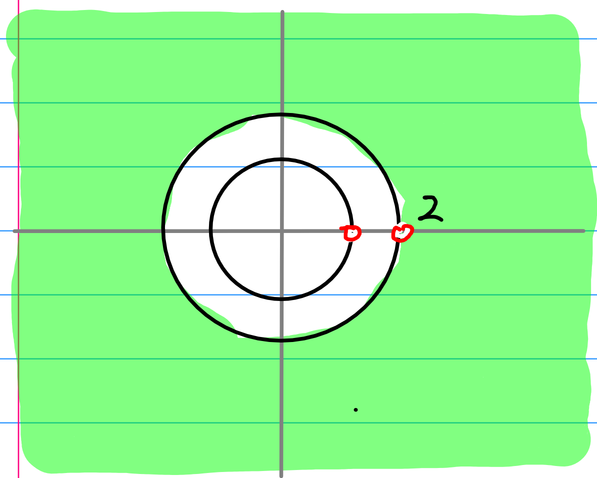
$$\boxed{X(z) = \frac{-1}{(z-1)(z-2)}}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Z.T. first

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$$

① D_1 $\quad |z| > 2$ $\quad$ causal $\quad x_n = 0 \quad (n<0)$



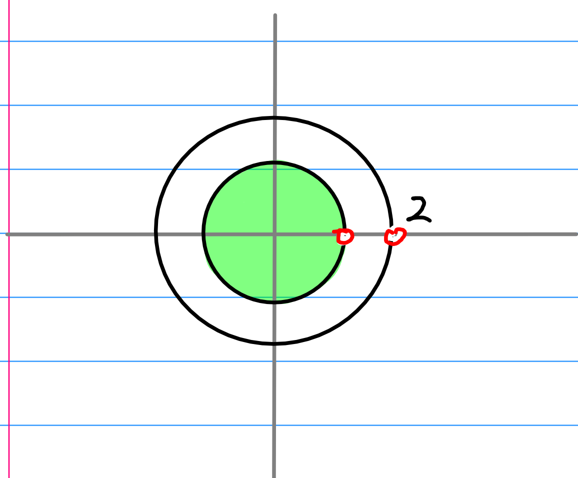
$$\left|\frac{1}{z}\right| < 1 \qquad \left|\frac{2}{z}\right| < 1$$

$$X(z) = \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= \sum_{n=0}^{\infty} 1 \cdot z^{-n-1} - \sum_{n=0}^{\infty} 2^n z^{-n-1}$$

$$= \sum_{n=1}^{\infty} \left[1 - 2^{n+1}\right] z^{-n}$$

② D_3 $\quad |z| < 1$ $\quad$ anti-causal $\quad x_n = 0 \quad (n>0)$



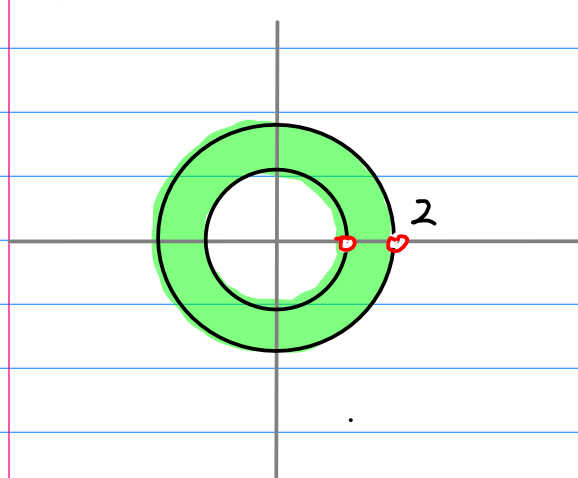
$$\left|\frac{z}{1}\right| < 1 \qquad \left|\frac{z}{2}\right| < 1$$

$$X(z) = \frac{-1}{1-\left(\frac{z}{1}\right)} + \frac{\frac{1}{2}}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty} 1\ z^n - \sum_{n=0}^{\infty} 2^{-n-1} z^n$$

$$= \sum_{n=-1}^{-\infty} \left[-1 + 2^{n-1}\right] z^{-n}$$

③ D_2 $\quad 1 < |z| < 2$ $\quad$ two-sided



$$\left|\frac{1}{z}\right| < 1 \qquad \left|\frac{z}{2}\right| < 1$$

$$X(z) = \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

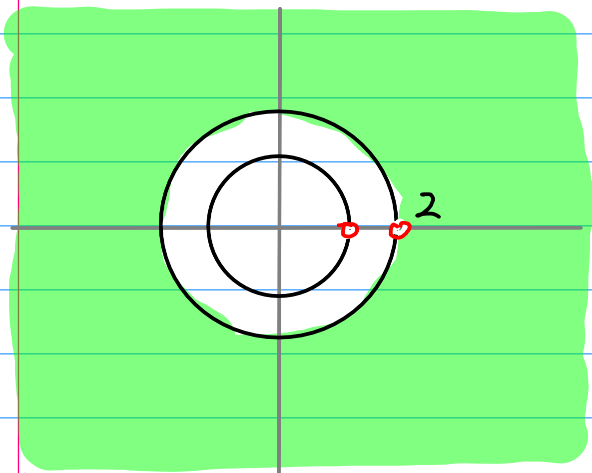
$$= \sum_{n=0}^{\infty} 1 \cdot z^{-n-1} + \sum_{n=0}^{\infty} 2^{-n-1} z^n$$

$$= \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

Z.T. first

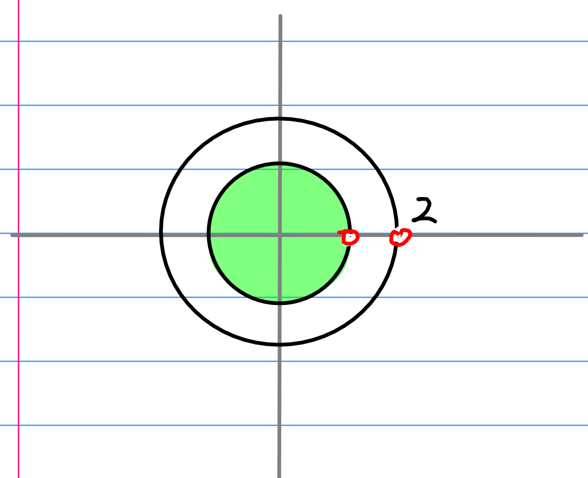
$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$$

Ⅰ D_1 $|z| > 2$ causal $x_n = 0 \quad (n<0)$



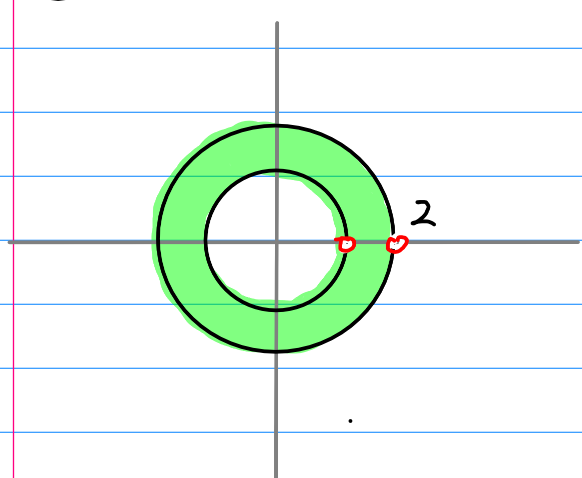
$$X(z) = \sum_{n=1}^{\infty} \left[1 - 2^{-n+1}\right] z^{-n}$$

Ⅱ D_3 $|z| < 1$ anti-causal $x_n = 0 \quad (n > 0)$



$$X(z) = \sum_{n=-1}^{\infty} \left[-1 + 2^{n-1}\right] z^{-n}$$

Ⅲ D_2 $1 < |z| < 2$ two-sided



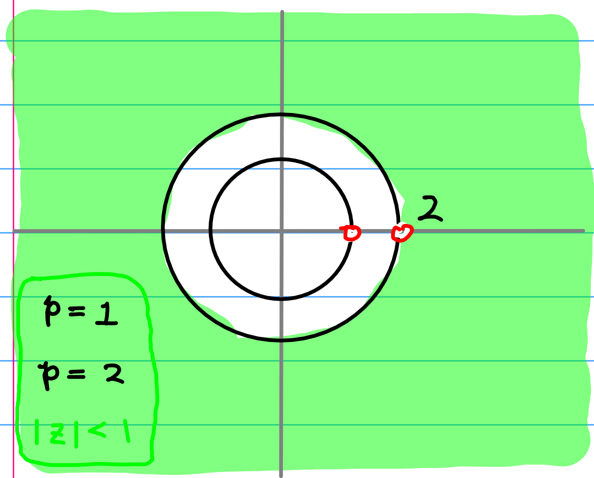
$$X(z) = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

$$X(z) = \frac{-1}{(z-1)(z-2)} \qquad f(z) =$$

Z.T. first

Ⅰ − 1

Ⅰ D_1 $|z| > 2$ $\left[\, \left|\frac{1}{z}\right| < 1 ,\ \left|\frac{2}{z}\right| < 1 \,\right]$

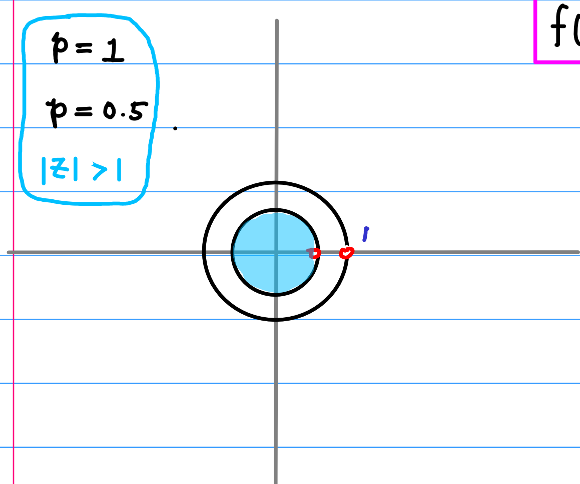


$$X(z) = \frac{1}{z-1} - \frac{1}{z-2}$$

$$= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= \sum_{n=0}^{\infty} 1\, z^{-n-1} - \sum_{n=0}^{\infty} 2^n z^{-n-1}$$

$$= \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{-n}$$



$$\boxed{f(z) = X(z^{-1})} = \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{n}$$

$$= \sum_{n=1}^{\infty} 1 \cdot z^n - 2^{n-1} \cdot z^n$$

$$= \frac{z}{1-\left(\frac{z}{1}\right)} - \frac{z}{1-\left(\frac{2z}{1}\right)}$$

$$= -\frac{z}{z-1} + \frac{0.5z}{(z-0.5)}$$

$$= \frac{-z+0.5+0.5z-0.5}{(z-1)(z-0.5)}\, z$$

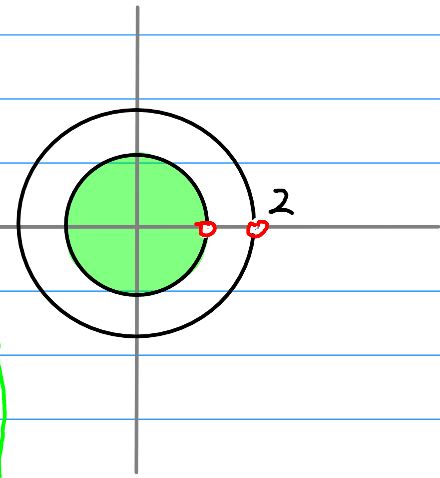
$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$X(z) = \frac{-1}{(z-1)(z-2)}$$

$$X(z^{-1}) = \frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \frac{-z^2}{(1-z)(1-2z)} = \frac{-0.5z^2}{(z-1)(z-0.5)} = f(z)$$

$$X(z) = \frac{-1}{(z-1)(z-2)} \qquad f(z) =$$

Ⅱ D_2 $\quad |z|<1 \quad \left[\left|\frac{z}{1}\right|<1, \ \left|\frac{z}{2}\right|<1 \right]$



$p=1$
$p=2$
$|z|>2$

$$X(z) = \frac{1}{z-1} - \frac{1}{z-2}$$

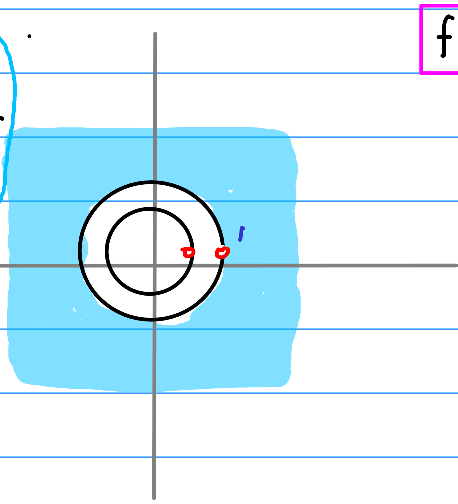
$$= \frac{-1}{1-\left(\frac{z}{1}\right)} + \frac{\frac{1}{2}}{1-\left(\frac{z}{2}\right)}$$

$$= \sum_{n=0}^{\infty} (-1)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} 1\cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} z^n$$

$$= \sum_{n=0}^{-\infty} \left[-1 + 2^{n-1}\right] z^{-n}$$

$p=1$
$p=0.5$
$|z|<\frac{1}{2}$



$$\boxed{f(z) = X(z^{-1})} = \sum_{n=0}^{-\infty} \left[-1 + 2^{n-1}\right] z^{n}$$

$$= \sum_{n=0}^{\infty} -1\cdot z^{-n} + \sum_{n=0}^{\infty} 2^{-n-1}\cdot z^{-n}$$

$$= -\frac{1}{1-\left(\frac{1}{z}\right)} + \frac{\frac{1}{2}}{1-\left(\frac{1}{2z}\right)}$$

$$= -\frac{z}{z-1} + \frac{0.5z}{z-0.5}$$

$$= \frac{-z+0.5+0.5z-0.5}{(z-1)(z-0.5)} z$$

$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$X(z) = \frac{-1}{(z-1)(z-2)}$$

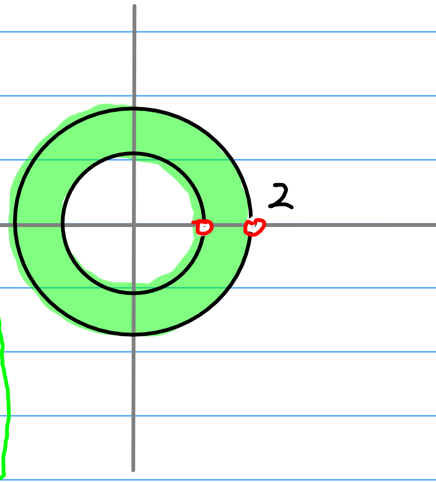
$$X(z^{-1}) = \frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \frac{-z^2}{(1-z)(1-2z)} = \frac{-0.5z^2}{(z-1)(z-0.5)} = f(z)$$

$$X(z) = \frac{-1}{(z-1)(z-2)} \qquad f(z) =$$

Z.T. first

III -1

$$\boxed{\text{III} \quad D_3 \quad 1 < |z| < 2 \qquad \left[\left|\tfrac{1}{z}\right| < 1 , \quad \left|\tfrac{z}{2}\right| < 1 \right]}$$



$p = 1$

$p = 2$

$1<|z|<2$

$$X(z) = \frac{1}{z-1} - \frac{1}{z-2}$$

$$= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{z}\right)\left(\tfrac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)\left(\tfrac{z}{2}\right)^n$$

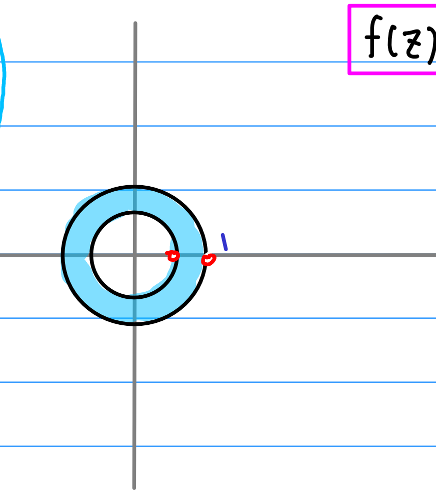
$$= \sum_{n=0}^{\infty} z^{-n-1} + \sum_{n=0}^{\infty} 2^{-n-1} z^n$$

$$= \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

$p = 1$

$p = 0.5$

$\frac{1}{2}<|z|<1$



$$\boxed{f(z) = X(z^{-1})} = \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

$$= \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^{-n}$$

$$= \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$

$$= \frac{-z+0.5+0.5z-0.5}{(z-1)(z-0.5)} z$$

$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$X(z) = \frac{-1}{(z-1)(z-2)}$$

$$X(z^{-1}) = \frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \frac{-z^2}{(1-z)(1-2z)} = \frac{-0.5z^2}{(z-1)(z-0.5)} = f(z)$$