Laurent Series and z-Transform - Geometric Series Double Pole Properties A

20181231 Mon

2 formulas of z

1 $$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow \; z^{-1}$$

2 $$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

$$\boxed{① \ \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \qquad ② \ \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}}$$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}} \qquad \boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z| < 0.5} \qquad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z| > 2}$$

causal $f_1(z)$ | anti-causal $g_1(z)$

anti-causal $X_1(z)$ | causal $Y_1(z)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z| > 2} \qquad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z| < 0.5}$$

anti-causal $f_2(z)$ | causal $g_2(z)$

causal $X_2(z)$ | anti-causal $Y_2(z)$

① $\frac{3}{2} \cdot \frac{-1}{(z-0.5)(z-2)}$

② $\frac{3}{2} \cdot \frac{-z^2}{(z-2)(z-0.5)}$

$$+\frac{1}{z-0.5} - \frac{1}{z-2}$$

$$\downarrow \cdot 2 \qquad \downarrow \cdot \frac{1}{2}$$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$\uparrow \cdot \frac{1}{z} \qquad \uparrow \cdot \frac{1}{z}$$

$$+\frac{1}{z-0.5} - \frac{1}{z-2}$$

$$-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\downarrow \cdot \frac{1}{z} \qquad \downarrow \cdot \frac{1}{z}$$

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$$\uparrow \cdot \frac{1}{2} \qquad \uparrow \cdot 2$$

$$-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\boxed{① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$|2z|<1$ $\quad$ $|0.5z|<1$ $\quad$ $|2z^{-1}|<1$ $\quad$ $|0.5z^{-1}|<1$

$|z|<0.5$ $\quad$ $|z|<2$ $\quad$ $2<|z|$ $\quad$ $0.5<|z|$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5} \qquad \boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2} \qquad \boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$|0.5z^{-1}|<1$ $\quad$ $|2z^{-1}|<1$ $\quad$ $|0.5z|<1$ $\quad$ $|2z|<1$

$0.5<|z|$ $\quad$ $2<|z|$ $\quad$ $|z|<2$ $\quad$ $|z|<0.5$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$\Big\downarrow \cdot\frac{1}{2z} \qquad\qquad \Big\downarrow \cdot\frac{2}{z}$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$\Big\downarrow \cdot\frac{z}{2} \qquad\qquad \Big\downarrow \cdot 2z$$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$\Big\uparrow \cdot 2z \qquad\qquad \Big\uparrow \cdot\frac{z}{2}$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$\Big\uparrow \cdot\frac{2}{z} \qquad\qquad \Big\uparrow \cdot\frac{1}{2z}$$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

Causal sequence a_n & x_n

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<0.5}$$

causal $f_1(z) =$

$-[2 + 2^2 z^1 + 2^3 z^2 + \cdots]$ $\quad -2^{n+1}$

$+[(\frac{1}{2}) + (\frac{1}{2})^2 z^1 + (\frac{1}{2})^3 z^2 + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>2}$$

causal $Y_1(z) =$

$-[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots]$ $\quad -2^{n+1}$

$+[(\frac{1}{2})^1 z^0 + (\frac{1}{2})^2 z^{-1} + (\frac{1}{2})^3 z^{-2} + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

causal $X_2(z)$

$+[(\frac{1}{2})^0 z^{-1} + (\frac{1}{2})^1 z^{-2} + (\frac{1}{2})^2 z^{-3} + \cdots]$ $\quad +(\frac{1}{2})^{n-1}$

$-[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots]$ $\quad -2^{n-1}$

1 2 3

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

causal $g_2(z)$

$+[(\frac{1}{2})^0 z^1 + (\frac{1}{2})^1 z^2 + (\frac{1}{2})^2 z^3 + \cdots]$ $\quad +(\frac{1}{2})^{n-1}$

$-[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$ $\quad -2^{n-1}$

1 2 3

Anti-causal sequence a_n & x_n

$$\boxed{2 = \left(\tfrac{1}{2}\right)^{-1}} \quad \boxed{\left(\tfrac{1}{2}\right) = 2^{-1}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \boxed{|z|<0.5}$$

anti-causal $X_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2}z^{1} + \left(\tfrac{1}{2}\right)^{-3}z^{2} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1} + 2^{-2}z^{1} + 2^{-3}z^{2} + \cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \boxed{|z|>2}$$

anti-causal $g_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^{0} + \left(\tfrac{1}{2}\right)^{-2}z^{-1} + \left(\tfrac{1}{2}\right)^{-3}z^{-2} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}z^{0} + 2^{-2}z^{-1} + 2^{-3}z^{-2} + \cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{2 = \left(\tfrac{1}{2}\right)^{-1}} \quad \boxed{\left(\tfrac{1}{2}\right) = 2^{-1}}$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$+\left[2^{0}z^{-1} + 2^{-1}z^{-2} + 2^{-2}z^{-3} + \cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{-1} + \left(\tfrac{1}{2}\right)^{-1}z^{-2} + \left(\tfrac{1}{2}\right)^{-2}z^{-3} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \boxed{|z|<0.5}$$

anti-causal $Y_2(z)$

$$+\left[2^{0}z^{1} + 2^{-1}z^{2} + 2^{-2}z^{3} + \cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{1} + \left(\tfrac{1}{2}\right)^{-1}z^{2} + \left(\tfrac{1}{2}\right)^{-2}z^{3} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{|z|<0.5}$$

causal $f_1(z) =$

$-[2 + 2^2 z^1 + 2^3 z^2 + \cdots]$ $\quad -2^{n+1}$

$+[(\frac{1}{2}) + (\frac{1}{2})^2 z^1 + (\frac{1}{2})^3 z^2 + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

anti-causal $X_1(z)$

$-[(\frac{1}{2})^{-1} + (\frac{1}{2})^{-2} z^1 + (\frac{1}{2})^{-3} z^2 + \cdots]$ $\quad -(\frac{1}{2})^{n-1}$

$+[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots]$ $\quad +2^{n-1}$

0 -1 -2

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad \boxed{|z|>2}$$

anti-causal $g_1(z)$

$-[(\frac{1}{2})^{-1} z^0 + (\frac{1}{2})^{-2} z^{-1} + (\frac{1}{2})^{-3} z^{-2} + \cdots]$ $\quad -(\frac{1}{2})^{n-1}$

$+[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$ $\quad +2^{n-1}$

0 -1 -2

causal $Y_1(z) =$

$-[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots]$ $\quad -2^{n+1}$

$+[(\frac{1}{2})^1 z^0 + (\frac{1}{2})^2 z^{-1} + (\frac{1}{2})^3 z^{-2} + \cdots]$ $\quad +(\frac{1}{2})^{n+1}$

0 1 2

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{|z|>2}$$

anti-causal $f_2(z)$

$+[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots]$ $\quad +2^{n+1}$

$-[(\frac{1}{2})^0 z^{-1} + (\frac{1}{2})^{-1} z^{-2} + (\frac{1}{2})^{-2} z^{-3} + \cdots]$ $\quad -(\frac{1}{2})^{n+1}$

-1 -2 -3

causal $X_2(z)$

$+[(\frac{1}{2})^0 z^{-1} + (\frac{1}{2})^1 z^{-2} + (\frac{1}{2})^2 z^{-3} + \cdots]$ $\quad +(\frac{1}{2})^{n-1}$

$-[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots]$ $\quad -2^{n-1}$

1 2 3

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad \boxed{|z|<0.5}$$

causal $g_2(z)$

$+[(\frac{1}{2})^0 z^1 + (\frac{1}{2})^1 z^2 + (\frac{1}{2})^2 z^3 + \cdots]$ $\quad +(\frac{1}{2})^{n-1}$

$-[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$ $\quad -2^{n-1}$

1 2 3

anti-causal $Y_2(z)$

$+[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots]$ $\quad +2^{n+1}$

$-[(\frac{1}{2})^0 z^1 + (\frac{1}{2})^{-1} z^2 + (\frac{1}{2})^{-2} z^3 + \cdots]$ $\quad -(\frac{1}{2})^{n+1}$

-1 -2 -3

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$f(z) = -\left[2+2^2 z+2^3 z^2+\cdots\right] + \left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2 z+\left(\tfrac{1}{2}\right)^3 z^2+\cdots\right]$$

$$\boxed{a_n = -2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} z^0+\left(\tfrac{1}{2}\right)^{-2} z^{-1}+\left(\tfrac{1}{2}\right)^{-3} z^{-2}+\cdots\right] + \left[2^{-1} z^0+2^{-2} z^{-1}+2^{-3} z^{-2}+\cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \quad (n<1)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2} z^1+\left(\tfrac{1}{2}\right)^{-3} z^2+\cdots\right] + \left[2^{-1}+2^{-2} z^1+2^{-3} z^2+\cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \quad (n<1)}$$

$$X(z) = -\left[2^1 z^0+2^2 z^{-1}+2^3 z^{-2}+\cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0+\left(\tfrac{1}{2}\right)^2 z^{-1}+\left(\tfrac{1}{2}\right)^3 z^{-2}+\cdots\right]$$

$$\boxed{x_n = -2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad |z|<0.5$$

$$f(z) = +\left[2^0 z^{-1}+2^{-1} z^{-2}+2^{-2} z^{-3}+\cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1}+\left(\tfrac{1}{2}\right)^{-1} z^{-2}+\left(\tfrac{1}{2}\right)^{-2} z^{-3}+\cdots\right]$$

$$\boxed{a_n = +2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)}$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^1+\left(\tfrac{1}{2}\right)^1 z^2+\left(\tfrac{1}{2}\right)^2 z^3+\cdots\right] - \left[2^0 z^1+2^1 z^2+2^2 z^3+\cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1}+\left(\tfrac{1}{2}\right)^1 z^{-2}+\left(\tfrac{1}{2}\right)^2 z^{-3}+\cdots\right] - \left[2^0 z^{-1}+2^1 z^{-2}+2^2 z^{-3}+\cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[2^0 z^1+2^{-1} z^2+2^{-2} z^3+\cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1+\left(\tfrac{1}{2}\right)^{-1} z^2+\left(\tfrac{1}{2}\right)^{-2} z^3+\cdots\right]$$

$$\boxed{x_n = +2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)}$$

$f(z)$ $\quad |z| < 0.5$ $\quad |z| > 2$

causal $\quad$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① − Ⓐ $\quad \boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z| < 0.5} \quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad \boxed{(n \geq 0)}$

$$-\left(2 + 2^2 z + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

① − Ⓑ $\quad \boxed{\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z| > 2} \quad \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}} \quad \boxed{(n < 0)}$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② − Ⓐ $\quad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z| > 2} \quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}} \quad \boxed{(n < 1)}$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right) + \left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

② − Ⓑ $\quad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z| < 0.5} \quad \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}} \quad \boxed{(n \geq 1)}$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

$$\frac{3}{2}\;\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① – Ⓐ $\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\quad \boxed{|z| < 0.5}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad \boxed{(n<1)}$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

① – Ⓑ $\boxed{\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\quad \boxed{|z| > 2}$ $\quad \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad \boxed{(n \geq 1)}$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{3}{2}\;\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② – Ⓐ $\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\quad \boxed{|z| > 2}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad \boxed{(n \geq 0)}$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

② – Ⓑ $\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\quad \boxed{|z| < 0.5}$ $\quad \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\quad \boxed{(n < 0)}$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$f(z) \longleftrightarrow a_n$

$X(z) \longleftrightarrow x_n$

1 – A

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

2 – A

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

1 – B

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

2 – B

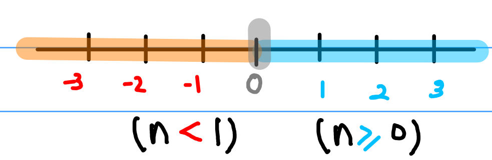
$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

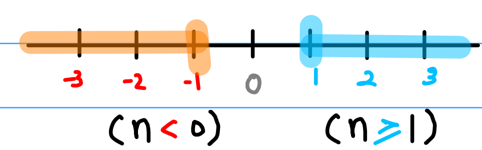
$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = a_{-n}$$

$$a_n = x_{-n}$$





$(n \geq 0) \longleftrightarrow (n < 1)$

$(n \geq 1) \longleftrightarrow (n < 0)$

I $\quad (z^{-1}, R^{-1}) \Longleftrightarrow (a_{-n}, -N)$

1-A $\qquad\qquad$ **2-A**

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5} \quad \xLeftrightarrow{z^{-1},\ R^{-1}} \quad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}\quad (n \geq 0) \quad \xLeftrightarrow{a_{-n},\ -N} \quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}\quad (n<1)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}\quad (n<1) \quad \xLeftrightarrow{a_{-n},\ -N} \quad x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}\quad (n \geq 0)$$

1-B $\qquad\qquad$ **2-B**

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2} \quad \xLeftrightarrow{z^{-1},\ R^{-1}} \quad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0) \quad \xLeftrightarrow{a_{-n},\ -N} \quad a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}\quad (n \geq 1)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}\quad (n \geq 1) \quad \xLeftrightarrow{a_{-n},\ -N} \quad x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0)$$

Ⅱ $\quad (z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

①−Ⓐ

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$\Big\Vert\, z \qquad \Updownarrow R^{-1}$

①−Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

②−Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$\Big\Vert\, z \qquad \Updownarrow R^{-1}$

②−Ⓑ

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

①−Ⓐ

$$a_n=\boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n\geq 0)$$

$\Updownarrow -a_n \qquad \Updownarrow N^c$

①−Ⓑ

$$a_n=\boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②−Ⓐ

$$a_n=\boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

$\Updownarrow -a_n \qquad \Updownarrow N^c$

②−Ⓑ

$$a_n=\boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n\geq 1)$$

①−Ⓐ

$$x_n=\boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

$\Updownarrow -a_n \qquad \Updownarrow N^c$

①−Ⓑ

$$x_n=\boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n\geq 1)$$

②−Ⓐ

$$x_n=\boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n\geq 0)$$

$\Updownarrow -a_n \qquad \Updownarrow N^c$

②−Ⓑ

$$x_n=\boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

III ①

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

III ②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_{-n}, N^c)$$

1 – A

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

2 – B

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$-a_{-n}, (-N)^c$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0) \quad \Longleftrightarrow \quad a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

1 – B

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

2 – A

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$-a_{-n}, (-N)^c$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0) \quad \Longleftrightarrow \quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

1 – A

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

2 – B

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$-a_{-n}, (-N)^c$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1) \quad \Longleftrightarrow \quad x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

1 – B

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

2 – A

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$-a_{-n}, (-N)^c$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1) \quad \Longleftrightarrow \quad x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

Ⅳ $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$\Big\updownarrow a_{-n} \qquad \Big\updownarrow -N$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

ROC's of interests

$R1(z)$ $\quad$ $R1(z^{-1})$

$R2(z)$ $\quad$ $R2(z^{-1})$

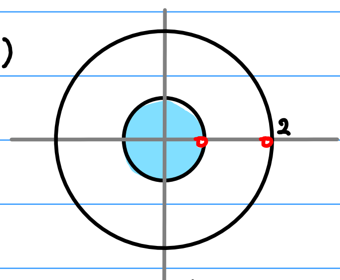
① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$ $\quad \overset{z^{-1}}{\longleftrightarrow} \quad$ ② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$

$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$ $\quad \overset{z^{-1}}{\longleftrightarrow} \quad$ $\boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$

$p_1 = 0.5$ $\quad \overset{z^{-1}}{\longleftrightarrow} \quad$ $p_1^{-1} = 2$

$p_2 = 2$ $\quad\quad\quad\quad\quad$ $p_2^{-1} = 0.5$

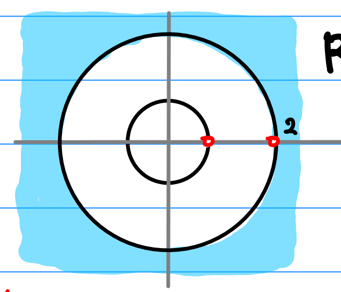
$R1(z)$



$|z| < 0.5$

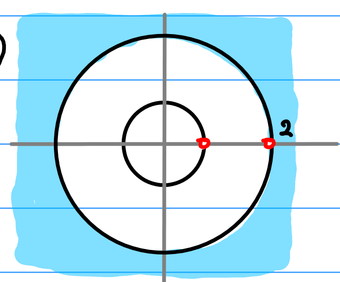
$\overset{z^{-1}}{\longleftrightarrow}$

$R1(z^{-1})$



$|z^{-1}| < 0.5 \quad |z| > 2$

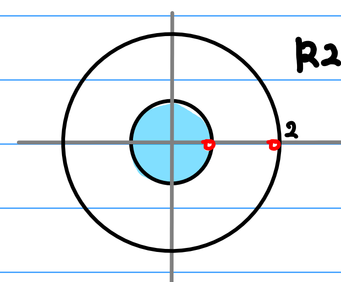
$R2(z)$



$|z| > 2$

$\overset{z^{-1}}{\longleftrightarrow}$

$R2(z^{-1})$



$|z^{-1}| > 2 \quad |z| < 0.5$

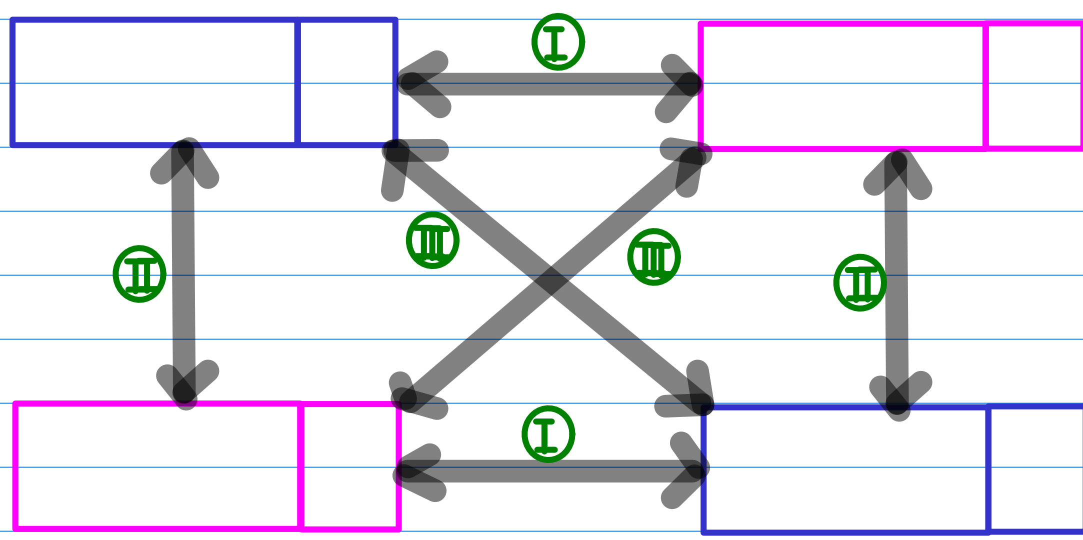
$R1(z^{-1}) = R2(z)$

$R2(z^{-1}) = R1(z)$

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

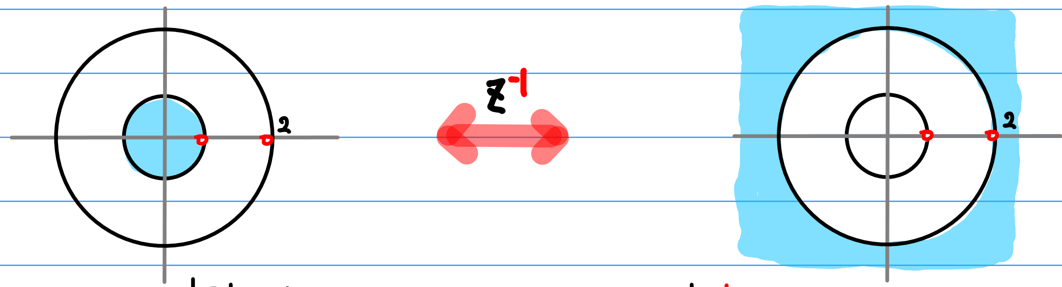
III $(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$(a_n, N) \Leftrightarrow (x_{-n}, -N)$

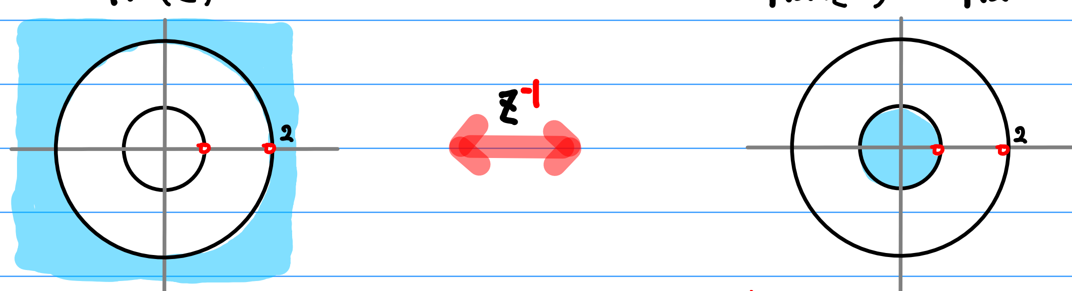
R^{-1}

$R1(z) \triangleq R1$ $\qquad$ $R1(z^{-1}) \triangleq R1^{-1}$



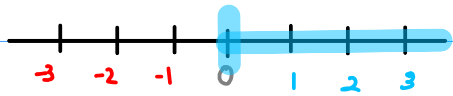
$|z| < 0.5$ $\qquad$ $|z^{-1}| < 0.5$ $\quad$ $|z| > 2$

$R2(z)$ $\qquad$ $R2(z^{-1}) \triangleq R2^{-1}$

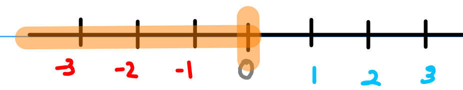


$|z| > 2$ $\qquad$ $|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

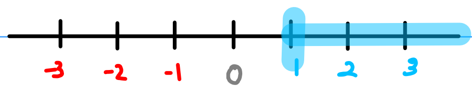
$-N$, N^c



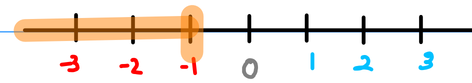
$-N$



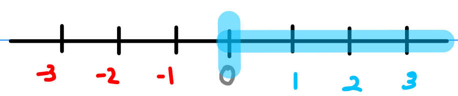
$(n \geq 0)$ $(n < 1)$



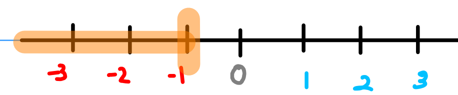
$-N$



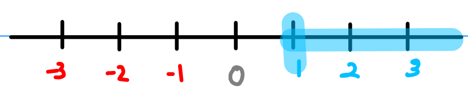
$(n \geq 1)$ $(n < 0)$



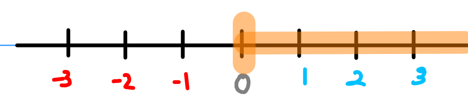
$N^c = \overline{N}$



$(n \geq 0)$ $(n < 0)$



$N^c = \overline{N}$



$(n \geq 1)$ $(n < 1)$

$f(z)$	$ROC(z)$	$\Longleftrightarrow$	a_n	$RNG(n)$
	$\lvert z\rvert < p$			$n \geq 0$

$$(z,\ R) \Leftrightarrow (a_n,\ N)$$

Ⅰ

$f(z^{-1})$	$ROC(z^{-1})$	$\Longleftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$$(z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$$

Ⅱ

$f(z)$	$ROC(z^{-1})$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$

$$(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$$

Ⅲ

$f(z^{-1})$	$ROC(z)$	$\Longleftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$
	$\lvert z\rvert < p$			$n \geq 1$

Ⅰ + Ⅱ

$$(z^{-1},\ R) \Leftrightarrow (-a_{-n},\ (-N)^c) = (-a_{-n},\ -(N^c))$$

Ⅳ

$X(z)$	$ROC(z)$	$\Longleftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert < p$			$n < 1$

$$(a_n,\ N) \Leftrightarrow (x_{-n},\ -N)$$

$$\text{III} = \text{I} + \text{II}$$

III	$f(z^{-1})$	$ROC(z)$, $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$, $n \geq 1$	I + II
	$f(z)$	$ROC(z)$, $\lvert z\rvert < p$	$\Longleftrightarrow$	a_n	$RNG(n)$, $n \geq 0$	
I	$f(z^{-1})$	$ROC(z^{-1})$, $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	a_{-n}	$RNG(-n)$, $n < 1$	
II	$f(z)$	$ROC(z^{-1})$, $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG(n)}$, $n < 0$	
	$f(z^{-1})$	$ROC(z)$, $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{RNG(-n)}$, $n \geq 1$	

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⓘ with ⓘⓥ

	ROC(z)	f(z) $\Longleftrightarrow$ a_n		RNG(n)
	$\lvert z\rvert < p$			$n \geq 0$
I	$\boxed{\text{ROC}(z^{-1})}$	f(z) $\Longleftrightarrow$ $-a_n$		$\overline{\text{RNG}}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$
			$-$ -1	$\overline{\quad}$ complement
IV	ROC(z)	X(z) $\Longleftrightarrow$ a_{-n}		RNG(−n)
	$\lvert z\rvert < p$			$n < 1$
			$-n$	$-n$ symmetrical

Ⅰ

ROC(z^{-1})	$f(z)$	$\Longleftrightarrow$ $-a_n$	$\overline{\text{RNG}}(n)$
$\lvert z\rvert > \frac{1}{p}$			$n < 0$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$\lvert z\rvert < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n<1)$

$\lvert z\rvert > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \geq 1)$

$\{\lvert z\rvert < 0.5\} \cap \{\lvert z\rvert > 2\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$\lvert z\rvert < a$ $\quad X(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Vert$ $\qquad\qquad \Big\downarrow -1\cdot$ $\qquad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

ROC′

$\lvert z\rvert > a^{-1}$ $\quad X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $\qquad -a^{n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad\qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\Vert \qquad\qquad \Vert \qquad\qquad\qquad \Vert \qquad\qquad \Vert$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad\qquad \frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

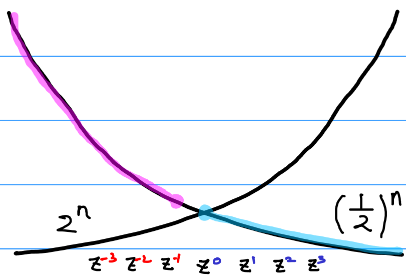
$$\| \qquad\qquad \| \qquad\qquad\qquad \| \qquad\qquad \|$$

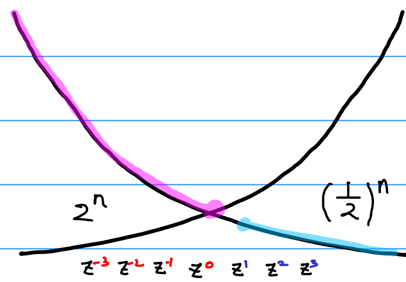
$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

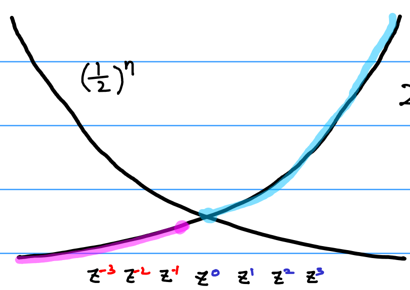
$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k} \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

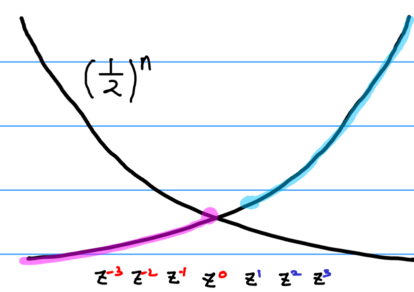
$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots \qquad\qquad z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$$

$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots \qquad\qquad a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$$









Ⅳ

ROC(z)	X(z) ⟺	a_{-n}	RNG(-n)
$\lvert z\rvert < p$			$n \le 0$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

n=0 n=1 n=2 $\qquad$ n=0 n=1 n=2

$|z| < 0.5$ $\qquad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

n=0 n=-1 n=-2 $\qquad$ n=0 n=-1 n=-2

ROC $\quad f(z) = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$

$-n$

ROC $\quad X(z) = \displaystyle\sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\qquad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

II

ROC(z^{-1})	$f(z^{-1})$	a_{-n}	RNG($-n$)
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$ $\quad f(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Big\downarrow z^{-1} \qquad\qquad \Big\downarrow -n$

ROC′

$|z| > a^{-1}$ $\quad f(z^{-1}) = \dfrac{a}{1-az^{-1}} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad a^{-n+1} \qquad n < 1 \quad n < 0 \quad n \geq 0 \quad n \geq 0$

$$= \sum_{k=0}^{-\infty} a^{-k+1} z^{k}$$

$$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$$

$z^{-1}X(z)$ Shifted Sequence

$$\qquad\qquad |z|>1 \qquad\qquad |z|>2$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$\cdot z^{-1} \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z|<0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

$$\cdot z \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z$ $\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z $\quad (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \quad n = 1, 2, 3, \cdots$

$\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1} $\quad (1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \quad n = -1, -2, -3, \cdots$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \quad X(z) \qquad a_n = \boxed{-(\tfrac{1}{2})^{n-1} + 2^{n-1}} \quad (n < 1)$

$|z| > 2 \quad X(z) \qquad b_n = \boxed{+(\tfrac{1}{2})^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \longrightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a \quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\| \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \Big\downarrow -n$

$|z| > a \quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq$

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n+1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$z X(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - 2z^{-1}} \quad (|z| > 2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$\boxed{\begin{array}{lll} & (1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \quad) \quad n & n = 0, 1, 2, \cdots \\ \cdot z^{-1} & \quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots & \\ & (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 & n = 1, 2, 3, \cdots \end{array}}$$

$$z^{-1} X(z) = \frac{z^{-1}}{1 - z^{-1}} - \frac{z^{-1}}{1 - 2z^{-1}} \quad (|z| > 2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$\boxed{\begin{array}{lll} z & (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n & n = 1, 2, 3, \cdots \\ & \quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots & \\ z^{-1} & (1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots) \quad -n & n = -1, -2, -3, \cdots \end{array}}$$

$$z X(z^{-1}) = \frac{z}{1 - z} - \frac{z}{1 - 2z} \quad (|z| < 0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $n=0$ $n=1$ $n=2$

$|z| < 0.5$ $$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $n=0$ $n=-1$ $n=-2$

ROC $$f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

$$\| \qquad \| \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \Big\downarrow -n$$

ROC $$X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC	$f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$	a^{n+1}	$n \geq 0$	$n \geq 1$	$n < 0$	$n < 1$
$\Big\updownarrow z^{-1}$	$\Big\updownarrow z^{-1}$ $\quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n}$	$\Big\downarrow -n$	$\Big\downarrow$	$\Big\downarrow$	$\Big\downarrow$	$\Big\downarrow$
ROC	$X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$	$\left(\frac{1}{a}\right)^{-n+1} = a^{n-1}$	$n < 0$	$n < 1$	$n \geq 0$	$n \geq 1$