

Moments (2B)

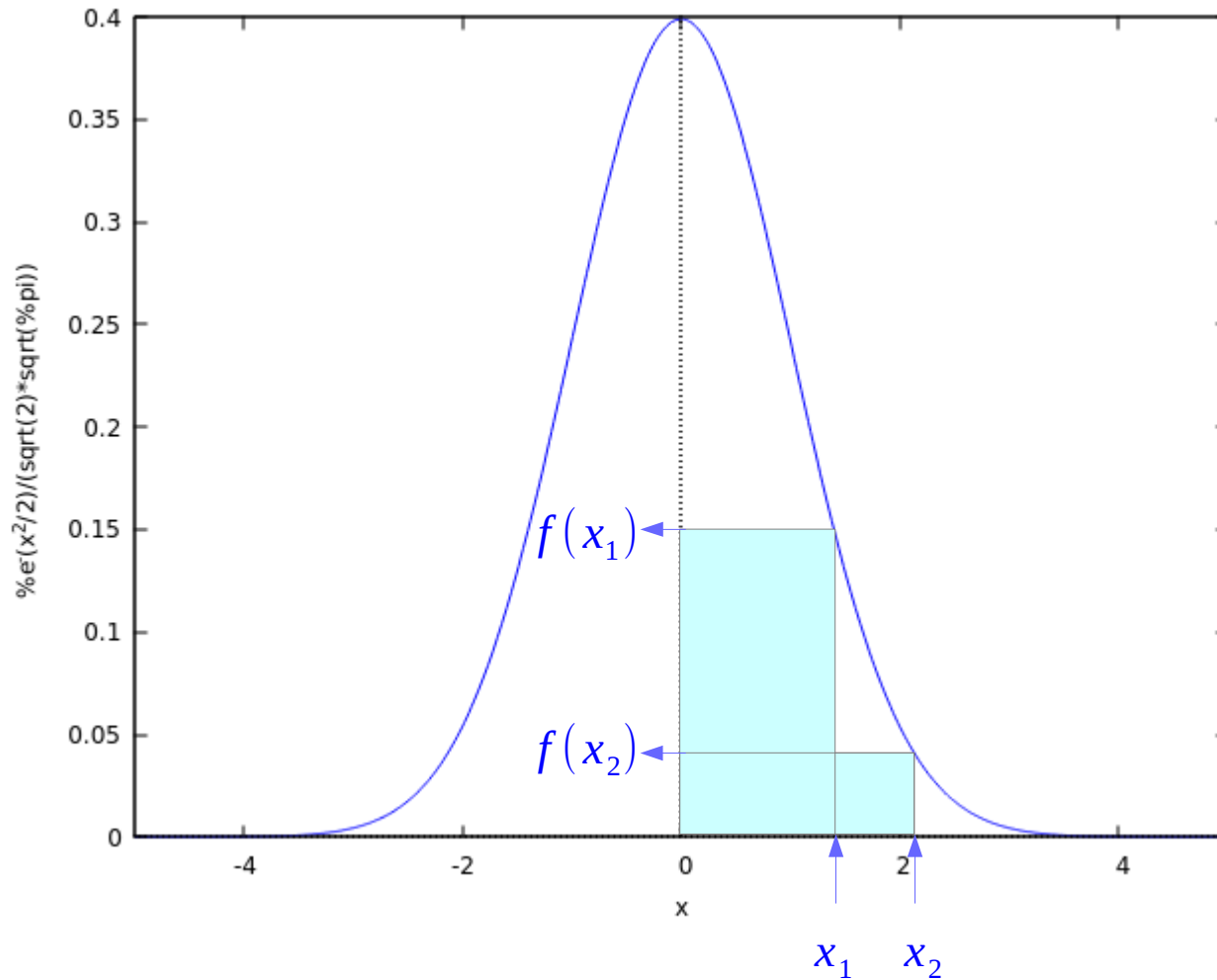
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Gaussian Distribution



$$\int_{-\infty}^{\infty} x f(x) dx$$

$$\sum_{k=-\infty}^{\infty} x_k f(x_k) \Delta_k$$

$$\sum_{k=-\infty}^{\infty} x_k y_k \Delta_k$$

The Probability Density Function (pdf)

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(x_1y_1+x_2y_2+x_3y_3)^2 = x_1^2y_1^2+x_2^2y_2^2+x_3^2y_3^2+2x_1x_2y_1y_2+2x_2x_3y_2y_3+2x_3x_1y_3y_1$$

$$E[X] = \sum x_i y_i = x_1y_1+x_2y_2+x_3y_3$$

$$E[X^2] = \sum x_i^2 y_i = x_1^2y_1+x_2^2y_2+x_3^2y_3$$

$$\begin{aligned} E[X^2]-E^2[X] &= (x_1^2y_1+x_2^2y_2+x_3^2y_3) - (x_1y_1+x_2y_2+x_3y_3)^2 \\ &= (x_1^2y_1+x_2^2y_2+x_3^2y_3) - (x_1^2y_1^2+x_2^2y_2^2+x_3^2y_3^2) \\ &\quad - (2x_1y_1x_2y_2+2x_2y_2x_3y_3+2x_3y_3x_1y_1) \end{aligned}$$

The Probability Density Function (pdf)

$$\begin{aligned}
 E[X^2] - E^2[X] &= (x_1^2 y_1 + x_2^2 y_2 + x_3^2 y_3) - (x_1 y_1 + x_2 y_2 + x_3 y_3)^2 \\
 &= (x_1^2 y_1 + x_2^2 y_2 + x_3^2 y_3) - (x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2) \\
 &\quad - (2x_1 y_1 x_2 y_2 + 2x_2 y_2 x_3 y_3 + 2x_3 y_3 x_1 y_1)
 \end{aligned}$$

$$+ x_1^2 y_1$$

$$- x_1^2 y_1^2$$

$$- x_1 y_1 x_2 y_2$$

$$- x_1 y_1 x_3 y_3$$

$$+ x_2^2 y_2$$

$$- x_2^2 y_2^2$$

$$- x_2 y_2 x_1 y_1$$

$$- x_2 y_2 x_3 y_3$$

$$+ x_3^2 y_3$$

$$- x_3^2 y_3^2$$

$$- x_3 y_3 x_1 y_1$$

$$- x_3 y_3 x_2 y_2$$

$$x_1 y_1 (x_1 - x_1 y_1 - x_2 y_2 - x_3 y_3)$$

$$x_2 y_2 (x_2 - x_1 y_1 - x_2 y_2 - x_3 y_3)$$

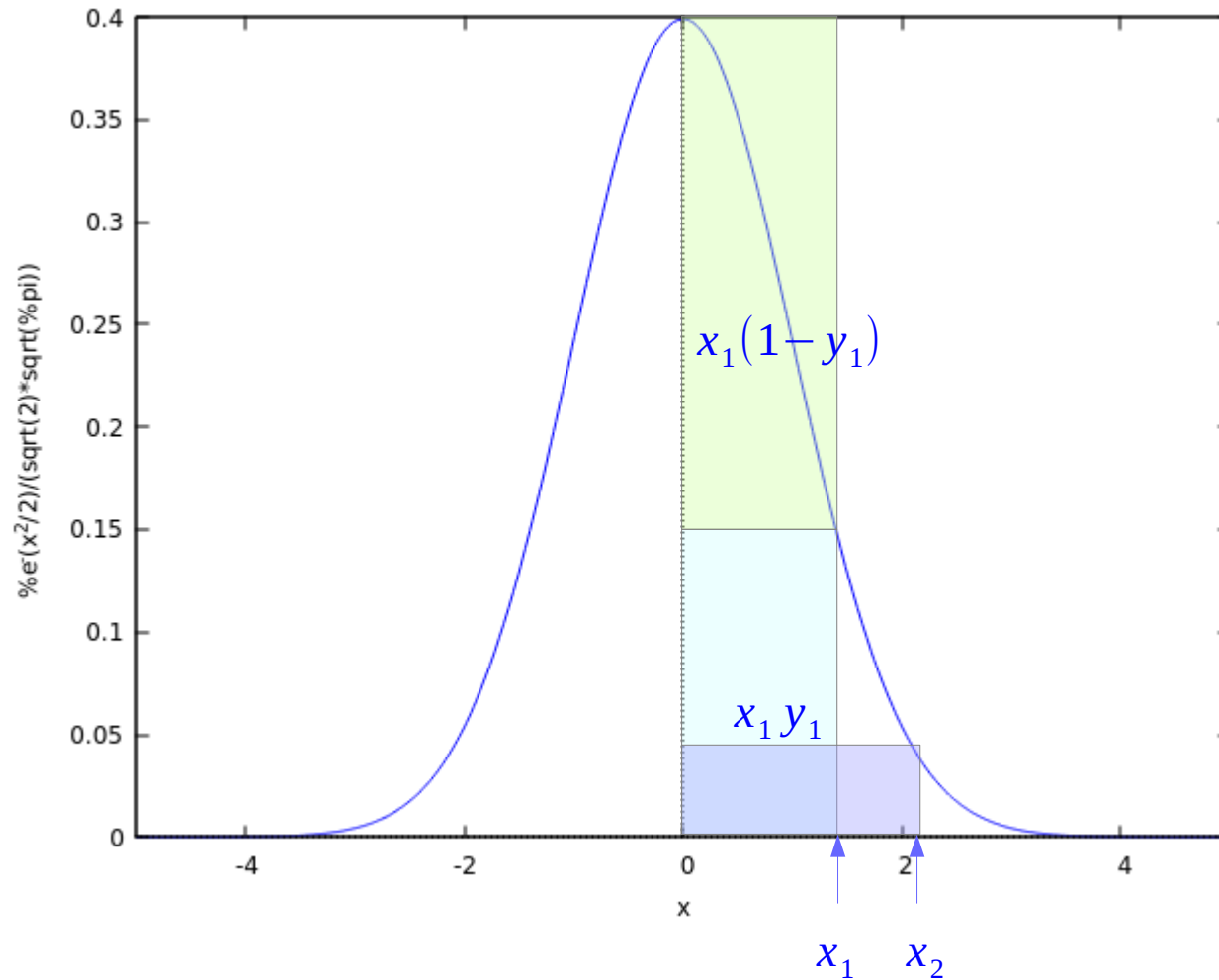
$$x_3 y_3 (x_3 - x_1 y_1 - x_2 y_2 - x_3 y_3)$$

$$= x_1 y_1 (x_1 - E[X])$$

$$= x_2 y_2 (x_2 - E[X])$$

$$= x_3 y_3 (x_3 - E[X])$$

Gaussian Distribution

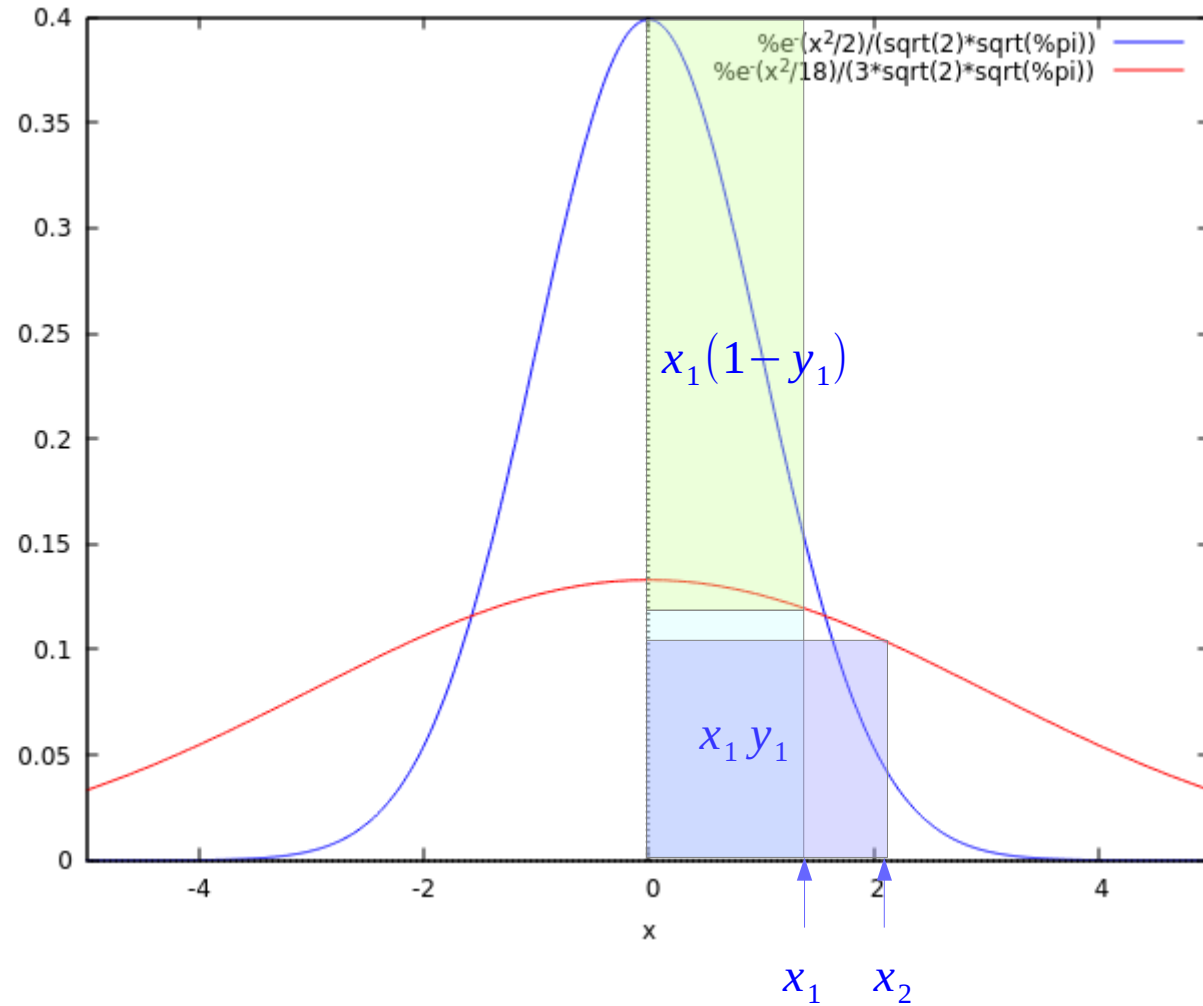


$$\int_{-\infty}^{\infty} x f(x) dx$$

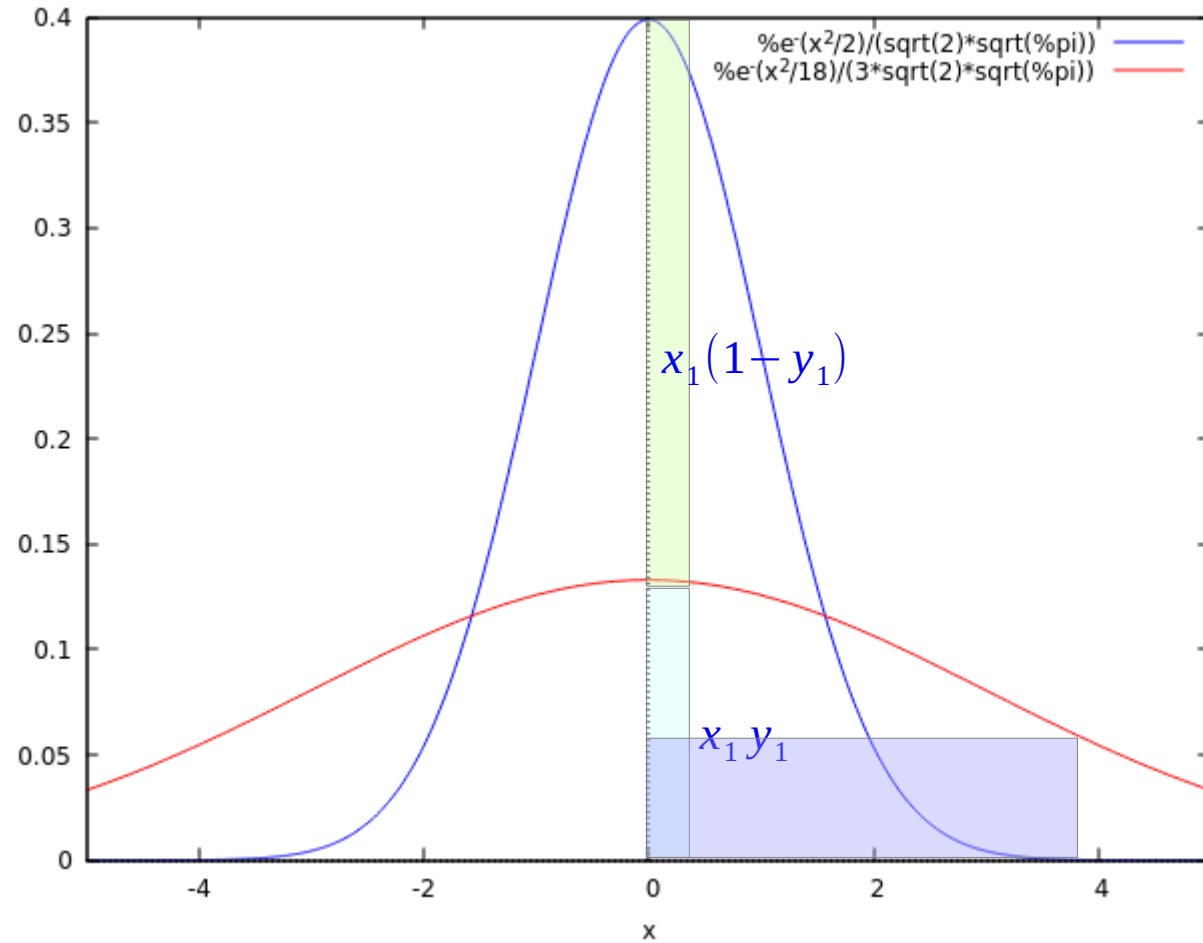
$$\sum_{k=-\infty}^{\infty} x_k f(x_k) \Delta_k$$

$$\sum_{k=-\infty}^{\infty} x_k y_k \Delta_k$$

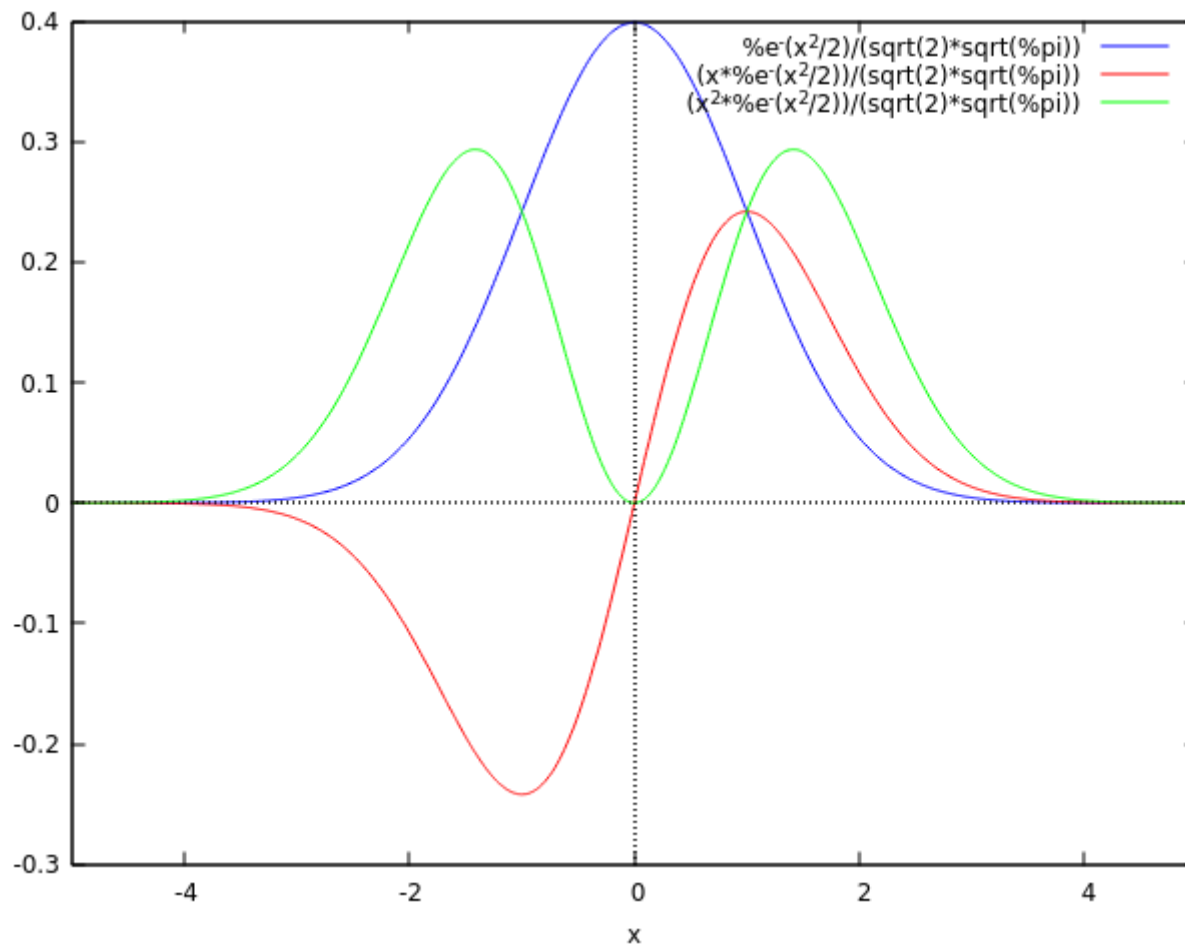
Moment Functions



Moment Functions



Moment Function Plots



Numerical Integration Results

```
(%i16) float(integrate(f(x,0,1), x, -10, +10));  
(%o16) 1.0
```

```
(%i15) float(integrate(g(x,0,1), x, -10, +10));  
(%o15) 0.0
```

```
(%i27) float(integrate(h(x,0,1), x, -10, +10));  
(%o27) 0.9999999999999998
```

The core function test

```
(%i28) m1(x) := %e^(-(x-2)^2);
```

```
(%o28) m1(x) := %e-(x-2)2
```

```
(%i29) m2(x) := m1(x) * x;
```

```
(%o29) m2(x) := m1(x) x
```

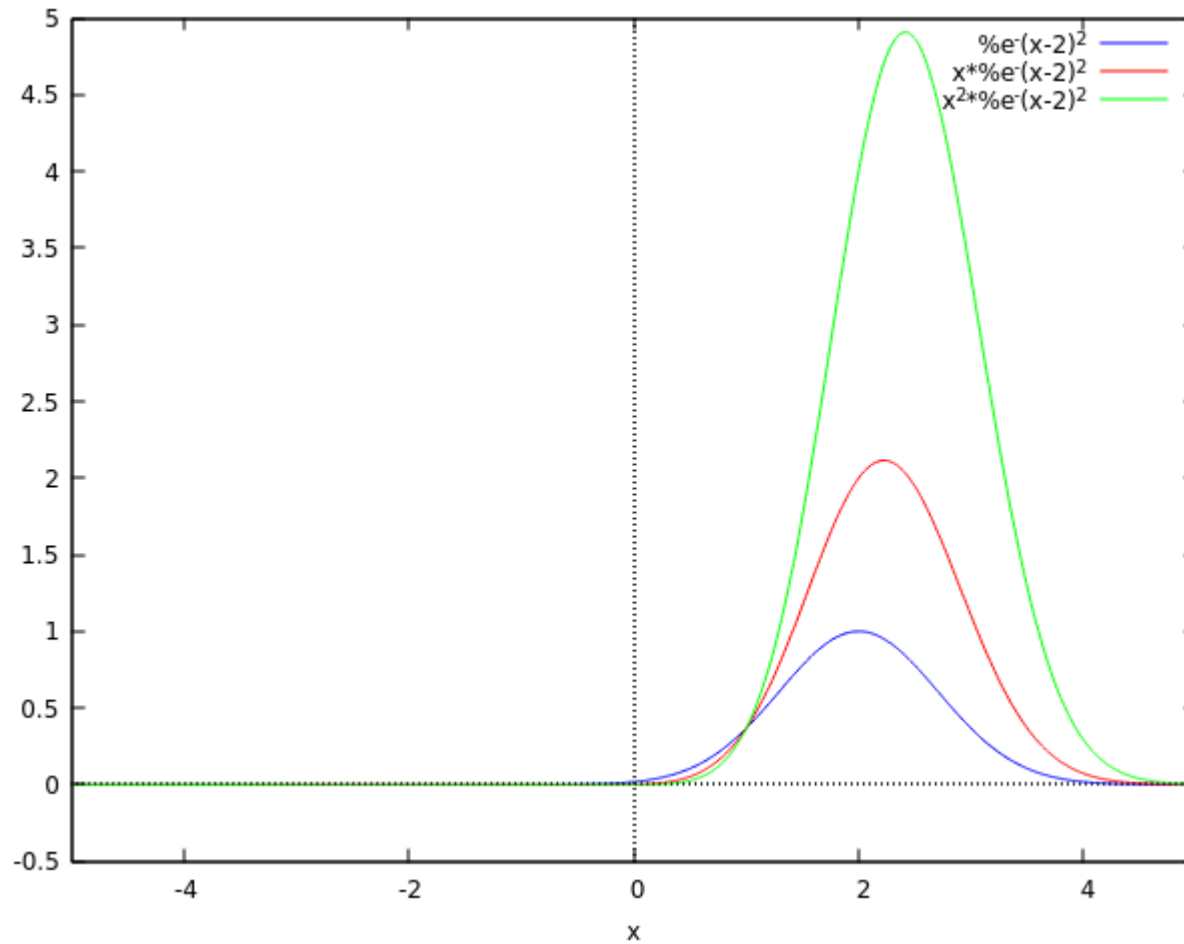
```
(%i30) m3(x) := m1(x) * x^2;
```

```
(%o30) m3(x) := m1(x) x2
```

```
(%i31) plot2d([m1(x), m2(x), m3(x)], [x, -5, 5], [plot_format, gnuplot]);
```

```
(%o31) [/home/young/maxout.gnuplot]
```

The core function plots



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann