System Type (4A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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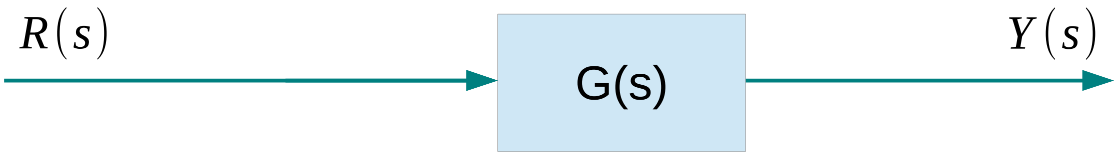
Steady State Response

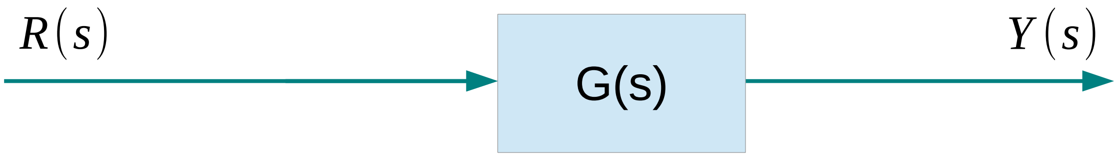


$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^j(s+p_1)\cdots(s+p_n)}$$

$$r(t) = \frac{t^k}{k!}u(t) \qquad \Longleftrightarrow \qquad R(s) = \frac{1}{s^{k+1}} \qquad Y(s) = G(s)R(s) = \frac{G(s)}{s^{k+1}}$$

$$y_{ss}(t) = \lim_{t\to\infty} y(t) \qquad = \qquad \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)R(s) = \lim_{s\to 0} s\frac{G(s)}{s^{k+1}} = \lim_{s\to 0} \frac{G(s)}{s^k}$$

System Types and Input Types

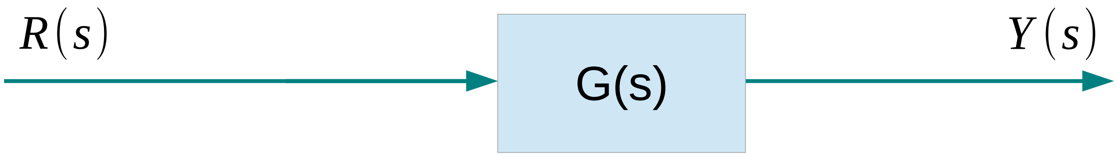


$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^j(s+p_1)\cdots(s+p_n)}$$

$$r(t) = \frac{t^k}{k!}u(t) \qquad y_{ss}(t) = \lim_{s\to 0}\frac{G(s)}{s^k}$$

$u(t)$	$k=0$ *unit step*	$j=0$ *type 0 system*	$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}$
$t\cdot u(t)$	$k=1$ *unit ramp*	$j=1$ *type 1 system*	$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}$
$\frac{t^2}{2}\cdot u(t)$	$k=2$ *unit parabola*	$j=2$ *type 2 system*	$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}$

Type 0 System Responses



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^{0}(s+p_1)\cdots(s+p_n)}$$

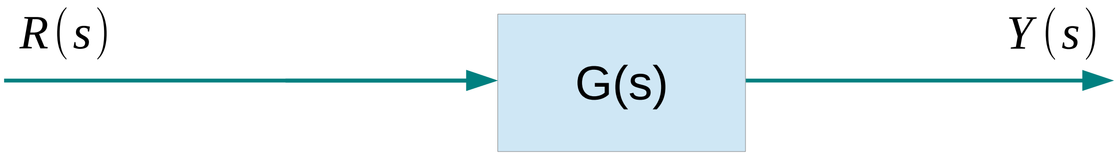
$$r(t) = \frac{t^k}{k!}u(t) \qquad y_{ss}(t) = \lim_{s\to 0}\frac{G(s)}{s^k}$$

$u(t)$ | $k=0$ *unit step* | $\frac{1}{s^1}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{0+0}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \frac{z_1\cdots z_m}{p_1\cdots p_n}$

$t\cdot u(t)$ | $k=1$ *unit ramp* | $\frac{1}{s^2}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{0+1}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

$\frac{t^2}{2}\cdot u(t)$ | $k=2$ *unit parabola* | $\frac{1}{s^3}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{0+2}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

Type 1 System Responses



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^{1}(s+p_1)\cdots(s+p_n)}$$

$$r(t) = \frac{t^k}{k!}u(t)$$

$$y_{ss}(t) = \lim_{s\to 0}\frac{G(s)}{s^k}$$

$u(t)$ — $k=0$ *unit step* — $\frac{1}{s^1}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{1+0}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

$t\cdot u(t)$ — $k=1$ *unit ramp* — $\frac{1}{s^2}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{1+1}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

$\frac{t^2}{2}\cdot u(t)$ — $k=2$ *unit parabola* — $\frac{1}{s^3}$ ➡ $y_{ss}(t) = \lim_{s\to 0}\frac{1}{s^{1+2}}\frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

Type 2 System Responses

$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^2 (s+p_1)\cdots(s+p_n)}$$

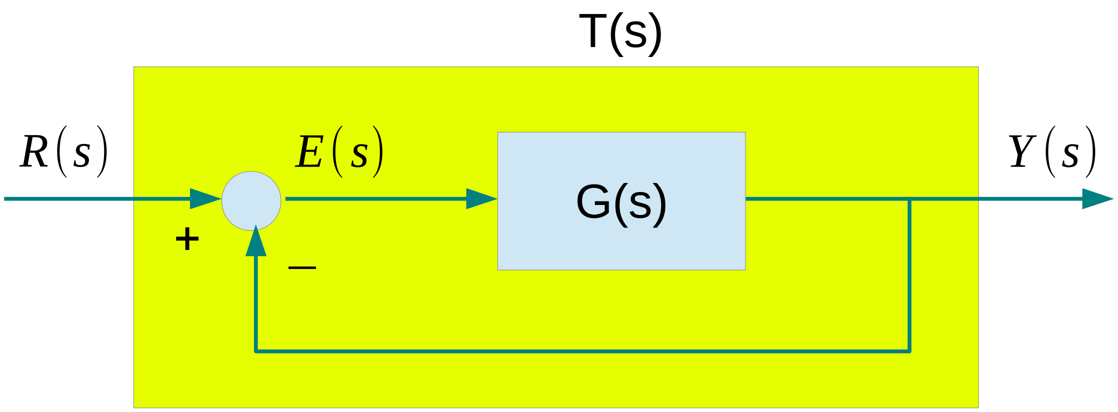
$$r(t) = \frac{t^k}{k!} u(t) \qquad y_{ss}(t) = \lim_{s\to 0} \frac{G(s)}{s^k}$$

$u(t)$ | $k=0$ *unit step* | $\frac{1}{s^1}$ ➡ $y_{ss}(t) = \lim_{s\to 0} \frac{1}{s^{2+0}} \frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

$t \cdot u(t)$ | $k=1$ *unit ramp* | $\frac{1}{s^2}$ ➡ $y_{ss}(t) = \lim_{s\to 0} \frac{1}{s^{2+1}} \frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

$\frac{t^2}{2} \cdot u(t)$ | $k=2$ *unit parabola* | $\frac{1}{s^3}$ ➡ $y_{ss}(t) = \lim_{s\to 0} \frac{1}{s^{2+2}} \frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} = \infty$

Unit Feedback



$$T(s) = \frac{G(s)}{1 + G(s)}$$

$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^j (s+p_1)\cdots(s+p_n)}$$

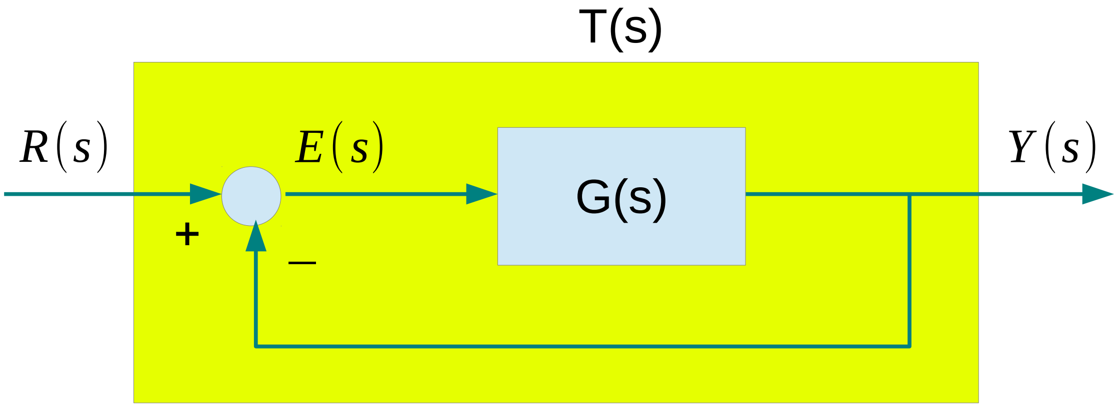
$$E(s) = \frac{1}{1 + G(s)}$$

$$r(t) = \frac{t^k}{k!} u(t) \quad \Longleftrightarrow \quad R(s) = s^{k+1}$$

$$e_{ss}(t) = \lim_{t \to \infty} e(t) \quad = \quad \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{1}{1+G(s)} \frac{1}{s^k}$$

$$= \lim_{s \to 0} \frac{1}{\left\{ 1 + \frac{(s+z_1)\cdots(s+z_m)}{s^j (s+p_1)\cdots(s+p_n)} \right\}} \frac{1}{s^k}$$

Steady State Error : Special Cases



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^{j}(s+p_1)\cdots(s+p_n)}$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^{0}(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^{k}}$$

$j=0$ type 0 system

$k=0$ unit step

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^{1}(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^{k}}$$

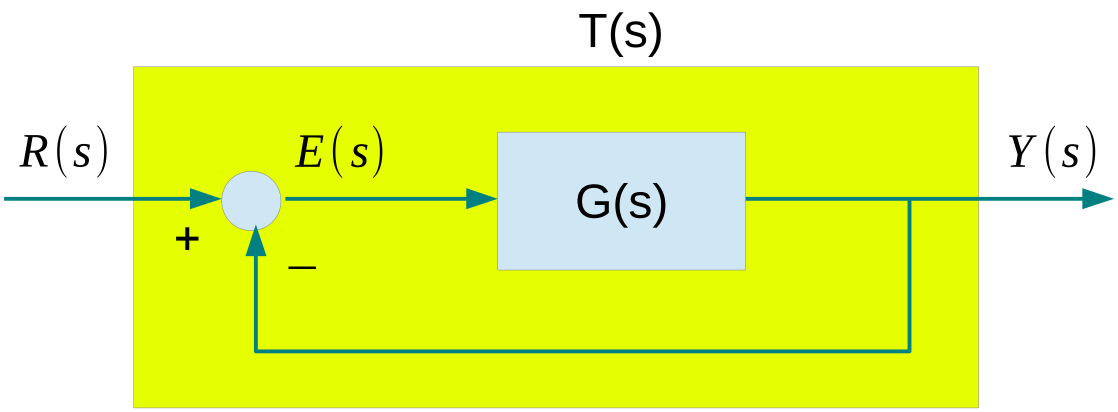
$j=1$ type 1 system

$k=1$ unit ramp

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^{2}(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^{k}}$$

$j=2$ type 2 system

$k=2$ unit parabola



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^j(s+p_1)\cdots(s+p_n)}$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^0}$$

$$\lim_{s\to 0} s^0 G(s) = \lim_{s\to 0} \frac{s^0(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)} = K_p$$

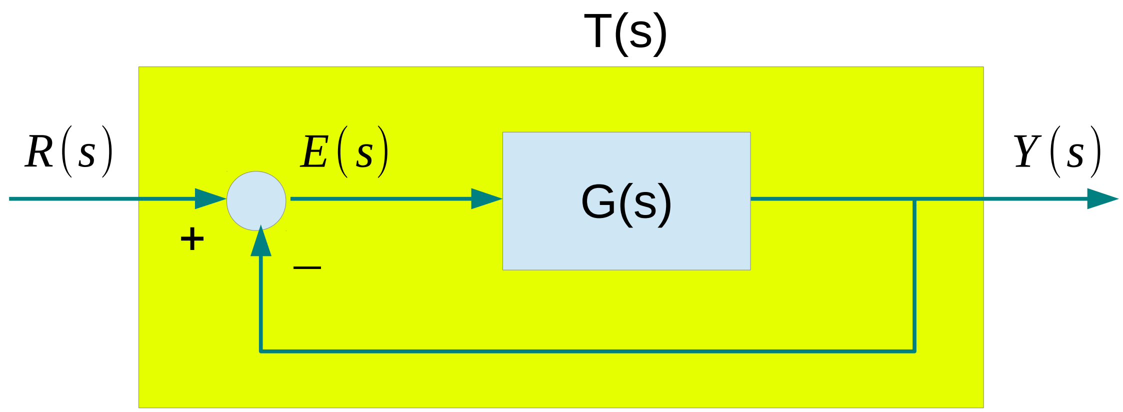
$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^1}$$

$$\lim_{s\to 0} s^1 G(s) = \lim_{s\to 0} \frac{s^1(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)} = K_v$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^2}$$

$$\lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{s^2(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)} = K_a$$

Error Constants (2)



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^j(s+p_1)\cdots(s+p_n)}$$

$j=0 \quad type\ 0\ system$

$$\lim_{s\to 0} s^0 G(s) = \lim_{s\to 0} \frac{s^0(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)} = K_p = \frac{z_1\cdots z_m}{p_1\cdots p_n}$$

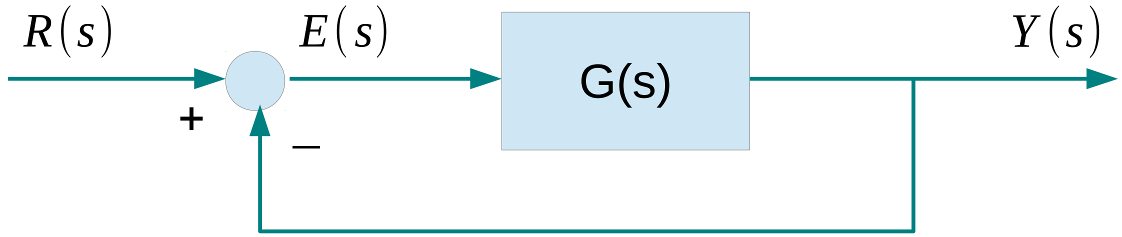
$j=1 \quad type\ 1\ system$

$$\lim_{s\to 0} s^1 G(s) = \lim_{s\to 0} \frac{s^1(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)} = K_v = \frac{z_1\cdots z_m}{p_1\cdots p_n}$$

$j=2 \quad type\ 2\ system$

$$\lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{s^2(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)} = K_a = \frac{z_1\cdots z_m}{p_1\cdots p_n}$$

Type 0 Steady State Error



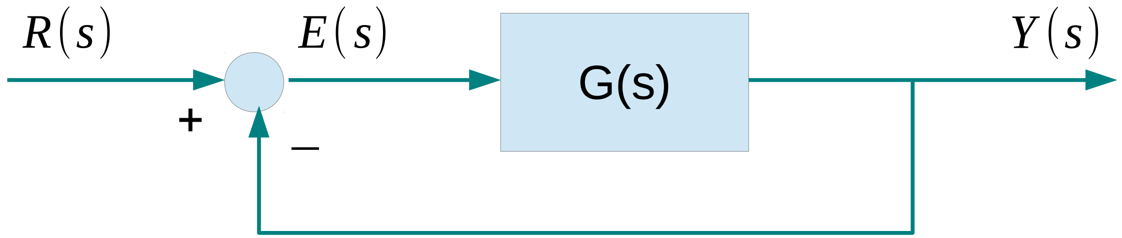
$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}$$

$u(t)$ — $k=0$ *unit step* — $\frac{1}{s^1}$ ➡ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^0} = \frac{1}{1+K_p}$$

$t\cdot u(t)$ — $k=1$ *unit ramp* — $\frac{1}{s^2}$ ➡ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^1} = \infty$$

$\frac{t^2}{2}\cdot u(t)$ — $k=2$ *unit parabola* — $\frac{1}{s^3}$ ➡ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^0(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^2} = \infty$$

Type 1 Steady State Error



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}$$

$u(t)$ | $k=0$ *unit step* | $\frac{1}{s^1}$ ➡

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^0} = 0$$

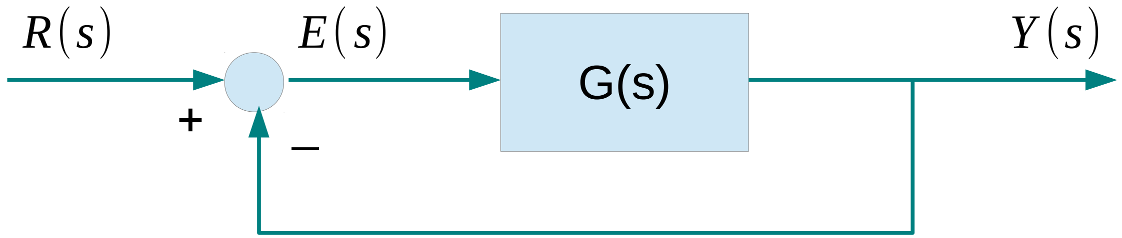
$t \cdot u(t)$ | $k=1$ *unit ramp* | $\frac{1}{s^2}$ ➡

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^1} = \frac{1}{K_v}$$

$\frac{t^2}{2} \cdot u(t)$ | $k=2$ *unit parabola* | $\frac{1}{s^3}$ ➡

$$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^1(s+p_1)\cdots(s+p_n)}\right\}} \frac{1}{s^2} = \infty$$

Type 2 System Responses



$$G(s) = \frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}$$

$u(t)$ | $k=0$ *unit step* | $\frac{1}{s^1}$ ⇒ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}\right\}}\frac{1}{s^0} = 0$$

$t\cdot u(t)$ | $k=1$ *unit ramp* | $\frac{1}{s^2}$ ⇒ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}\right\}}\frac{1}{s^1} = 0$$

$\frac{t^2}{2}\cdot u(t)$ | $k=2$ *unit parabola* | $\frac{1}{s^3}$ ⇒ $$e_{ss}(t) = \lim_{s\to 0} \frac{1}{\left\{1+\frac{(s+z_1)\cdots(s+z_m)}{s^2(s+p_1)\cdots(s+p_n)}\right\}}\frac{1}{s^2} = \frac{1}{K_a}$$

Magnitude & Phase Response : $G(s) = 1 / (s + 3)$

References

[1] http://en.wikipedia.org/
[2] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[3] E. Kreyszig, “Advanced Engineering Mathematics”
[4] D. G. Zill, W. S. Wright, “Advanced Engineering Mathematics”