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Motional emf (electromotive force)

Wikipedia seems unable to state an adequate formal definition of emf. Wiktionary essentially defines it as a *volt*, i.e., energy (*joules*) divided by charge (*coulombs*.) Physics calls it a *special type of potential difference.*

Physics at the introductory and the most fundamental levels are both works in progress. The goal of all physics is not to understand nature, but to predict it with the greatest possible precision. If you see a voltage you don't fully understand, just call it an *ee-em-eff* and seek the rules that govern its behavior.

Figure 2a illustrates how our first encounter with emf involved whatever chemical process acts to raise the potential of electrons flowing through a battery: by Coulomb's law electrons naturally flow from negative to positive charge. But inside the battery they move in the opposite direction. Figure 2b illustrates our second encounter with Faraday's observation that a time-varying magnetic field can also induce a voltage.

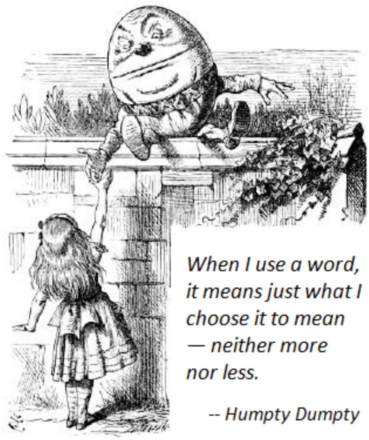

Fig. 1

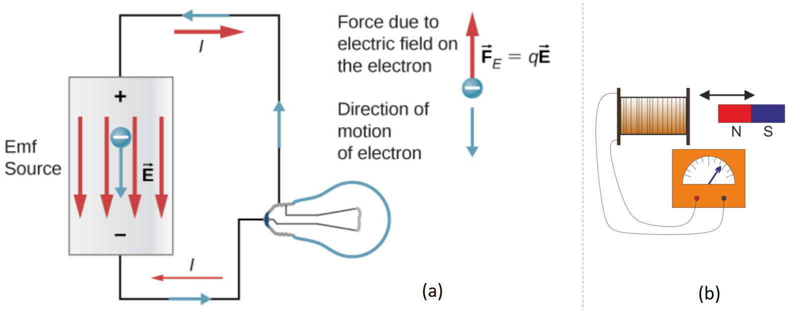

Fig. 2

All of the sciences are based on observations that lead to discovery. But physics is special in that inconsistencies in an established theory can give birth to a new theory before any observations are made. Figure 2a illustrates a mechanism by which electrons gain electric potential energy through a yet to be understood "force" that could rescue the concept of energy conservation by providing the work required to raise the potential energy of the electrons. Figure 2b represents the fact that relative motion between a magnet and a conducting coil also induces an emf.

We begin with a statement of Faraday's law:

$$\varepsilon = -\frac{d\Phi_m}{dt}, \text{ where } \Phi_m = \int \vec{B}\cdot d\vec{A} \to BA\cos\theta \qquad \text{(i)}$$

The simplification in (i) holds when the magnetic field is uniform over a certain area, and permits us to separately discuss the three terms in $BA\cos\theta$:

(ii)

$$\varepsilon = \underbrace{-\frac{dB}{dt}A\cos\theta}_{\text{case 1}} \underbrace{- B\frac{dA}{dt}\cos\theta}_{\text{case 2}} \underbrace{- BA\frac{d}{dt}\cos\theta}_{\text{case 3}}$$

Case 1 is associated with Fig2b and has been previously discussed. It should be noted that complexity beyond the scope of this discussion resides in case 1 and Fig2b. They involve special relativity and radiation effects, but for sufficiently gentle motions it does not matter the magnet or the coil is moving. Case 3 is discussed in the textbook for a rotating loop in the presence of a magnetic field.

Case 2 is interesting for two reasons. First, it is not always true that a change in the enclosed flux generates an emf! Shown below are two situations where the area of a loop changes in a region of uniform magnetic field without generating an emf.

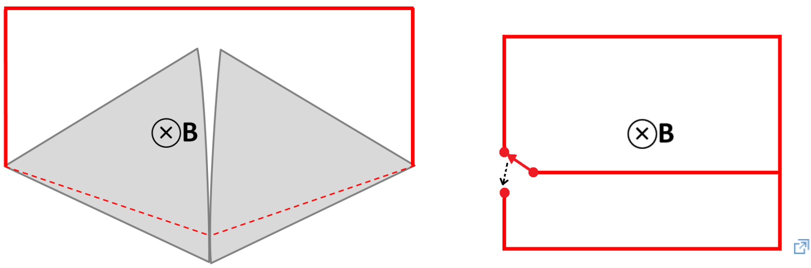

Fig. 3 See Wikipedia: Faraday's law of induction

The other reason case 2 $(dA/dt \neq 0)$ interesting is that the question can be resolved by a thought experiment. The *thought* or *Gedanken* experiment is a device that uses fundamental principles to guess a result of an experiment without actually performing the experiment.

Figure 4 shows a device consisting on two parallel rails situated in an external uniform magnetic field. A conducting can freely slide in the horizontal direction. The rod's velocity is v and the distance between the rails is l. The area varies with time as $dA/dt = vl$. In order to move at constant speed, a force F_a must be exerted at the rails. The case-2 term in (ii) yields a value of $\varepsilon = vBl$ for the emf that, according to Ohm's law, yields a current $I = vBl/R$. If the rod is moving at constant velocity, the net force must be zero, which implies that the applied force F_a must exactly cancels the magnetic force $F_m = BIl$.

As explained in OpenStax University Physics, the power provided by a force F_a acting on the moving object is $P = F_a v$, which must exactly matches the I^2R power dissipated by the resistor. In other words, if the emf generated by this device did not obey Faraday's law, we would be forced to abandon energy conservation because the work done moving the current-carrying rod through the magnetic field would not match the energy delivered to the resistor.

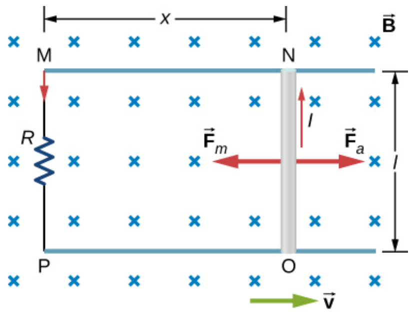

Fig. 4 A conducting rod is pushed to the right at constant velocity. The resulting change in the magnetic flux induces a current in the circuit (Fig. 13.12 in the current textbook.)

Another thought experiment on Figure 4

Though beyond the scope of this course, another thought experiment inspired by Figure 4 can lead to further insights into electromagnetism. Let $R \to \infty$ in the figure, effectively making it an open circuit in which the current $I = 0$. As with the battery of Fig2a, the electric field and the force associated with the must emf cancel to ensure that no current flows. Now put yourself in the reference frame of the moving conducting rod ... *inside* the conductor. Since charged particles are free to move in a conductor, and no current is flowing, we know that the net force on any charged particle must vanish, which is true in the reference frame of the stationary rails due to the v cross B force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \text{(iii)}$$

The problem with (iii) is that $\vec{v} = 0$ in the moving rod's reference frame. This inconsistency is ultimately resolved by recognizing that an observer moving through a "stationary" magnetic field will perceive an electric field. In other words, if we use primes to denote what is perceived in the moving frame of the rod, (iii) looks something like this:

$$\vec{F} = q\left(\vec{E} + \vec{E}'\right) = 0 \qquad \text{(iv)}$$

where $\vec{E}'$ is the external field perceived by an observer moving through the magnetic field. Further discussion of this is usually reserved for a more advanced course on electrodynamics.

Rail Gun

The reader should not confuse the system of Fig. 4 with that of Fig. 5, which is a slightly different linear motor called the rail gun. The two systems differ in that the magnetic field in Fig. 5 is produced by the current in the parallel rails. In our analysis of Fig. 4 we neglected the magnetic field caused by these parallel currents. This is a valid approximation in the limit that the resistance R is sufficiently large.

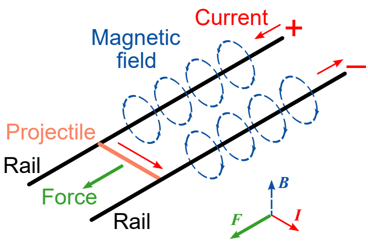

Fig. 5 Rail gun

Identify all true statements:

- [ ] Within the context of this essay, the **predictions** made by an equation are **more important** than knowing the exact definitions of the terms used.
- [ ] The formula, $dA/dt = vl$, used for a rectangle **does not apply** to the rate at which a circle's area expands, where l is the circumfrence and v is the rate at which the radius increases.
- [ ] In Fig. 4 the magnetic field is an **internal** field (i.e., caused by the current flowing through the parallel rails.)
- [ ] In the gedanken experiment using Fig. 4 to postulate that an observer moving through a magnetic field experiences an electric field, the resistance R in the circuit was set to **zero**.
- [ ] In the gedanken experiment using Fig. 4 to postulate that an observer moving through a magnetic field experiences an electric field, the electric field is **parallel** to the rails.
- [ ] The magnetic field in the rail gun of Fig. 5 is **homogeneous**.

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