

# Sequence (2A)

---

- Summation Notation ( $\Sigma$ )
- Flow Chart
- Partial Sum of G.P.
- Partial Sum of A.P.

Copyright (c) 2008 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

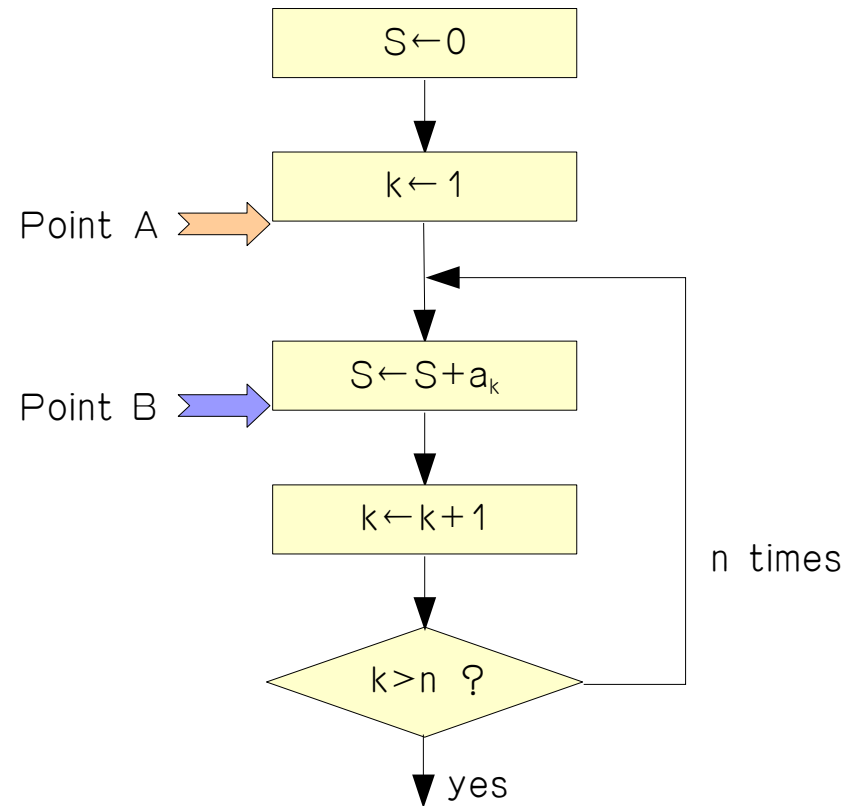
This document was produced by using OpenOffice and Octave.

# Sigma Notation and Flow Chart (1)

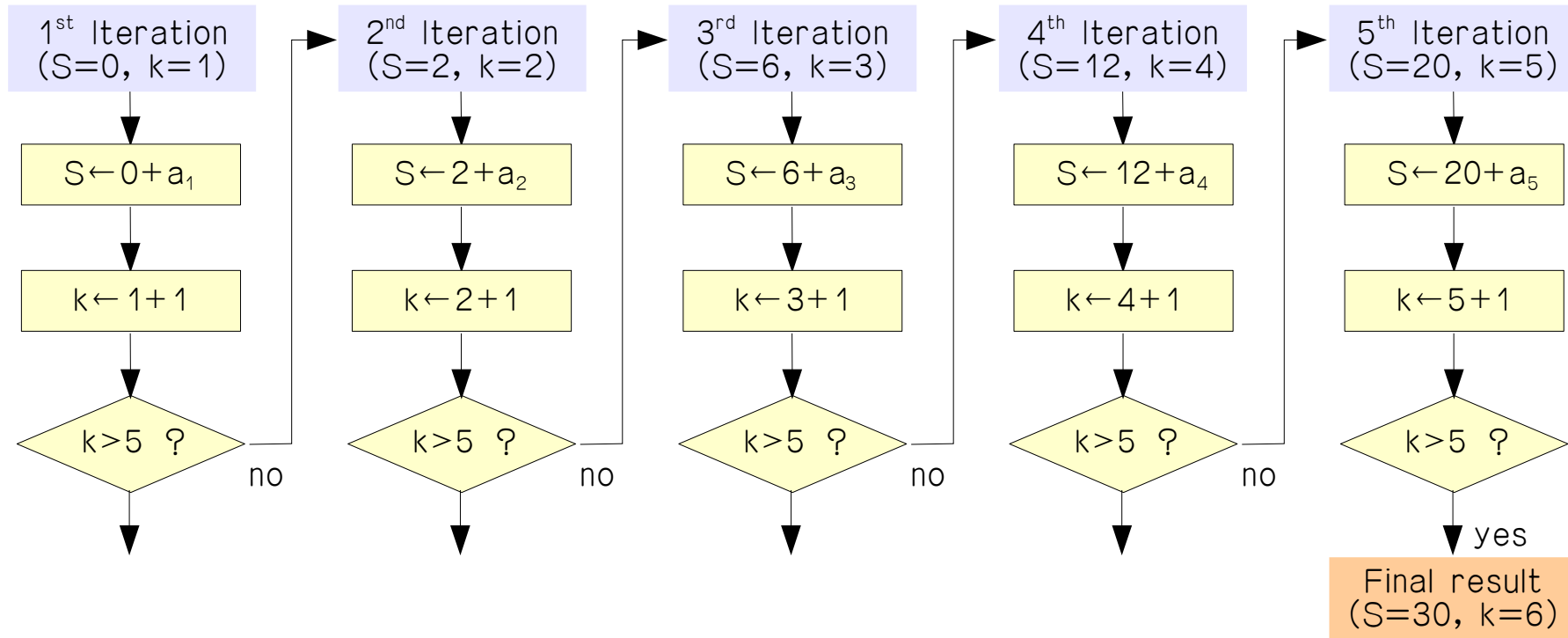
$$S_n = \sum_{k=1}^n a_k$$
$$= a_1 + a_2 + a_3 + \dots + a_n$$

|                | A | B |   |    |    |    |
|----------------|---|---|---|----|----|----|
| K              | 1 | 1 | 2 | 3  | 4  | 5  |
| A <sub>k</sub> |   | 2 | 4 | 6  | 8  | 10 |
| S              | 0 | 2 | 6 | 12 | 20 | 30 |

$a_1=2,$   
 $a_2=4,$   
 $a_3=6,$   
 $a_4=8,$   
 $a_5=10$



# Sigma Notation and Flow Chart (2)



$a_1=2,$   
 $a_2=4,$   
 $a_3=6,$   
 $a_4=8,$   
 $a_5=10$

|       | A | B |   |    |    |    |
|-------|---|---|---|----|----|----|
| K     | 1 | 1 | 2 | 3  | 4  | 5  |
| $A_k$ |   | 2 | 4 | 6  | 8  | 10 |
| S     | 0 | 2 | 6 | 12 | 20 | 30 |

# Sigma Notation and Flow Chart (3)

$$S \leftarrow S + a_1 = (0) + a_1 \quad \text{After 1}^{\text{st}} \text{ Iteration} \quad (S=2, k=2)$$

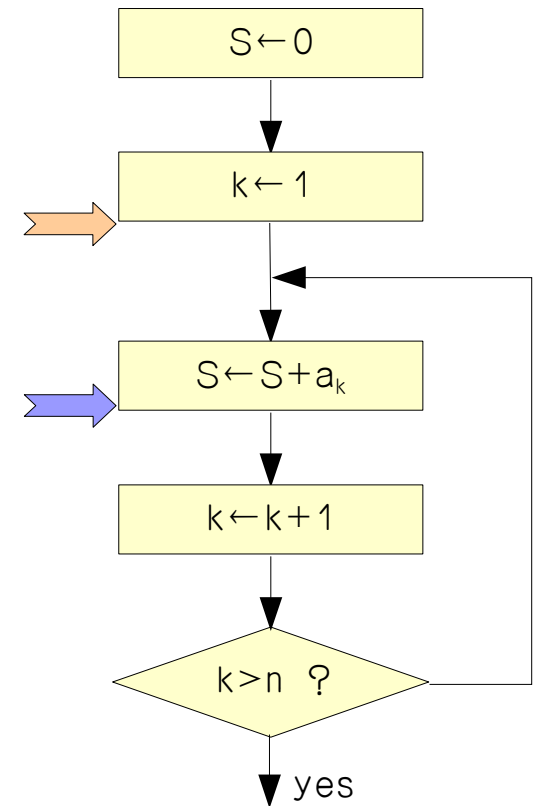
$$S \leftarrow S + a_2 = (a_1) + a_2 \quad \text{After 2}^{\text{nd}} \text{ Iteration} \quad (S=6, k=3)$$

$$S \leftarrow S + a_3 = (a_1 + a_2) + a_3 \quad \text{After 3}^{\text{rd}} \text{ Iteration} \quad (S=12, k=4)$$

$$S \leftarrow S + a_4 = (a_1 + a_2 + a_3) + a_4 \quad \text{After 4}^{\text{th}} \text{ Iteration} \quad (S=20, k=5)$$

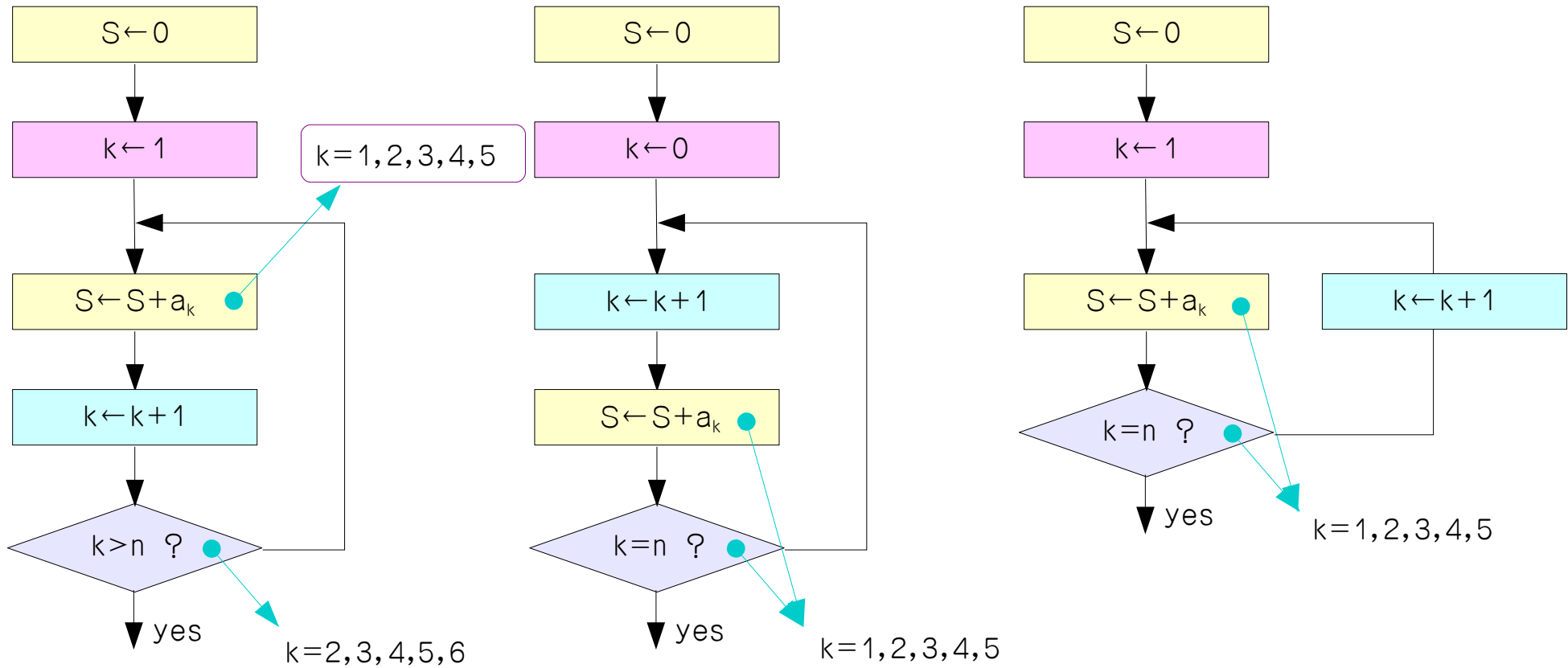
$$S \leftarrow S + a_5 = (a_1 + a_2 + a_3 + a_4) + a_5 \quad \text{After 5}^{\text{th}} \text{ Iteration} \quad (S=30, k=6)$$

$a_1=2,$   
 $a_2=4,$   
 $a_3=6,$   
 $a_4=8,$   
 $a_5=10$



|                | A | B |   |    |    |    |
|----------------|---|---|---|----|----|----|
| K              | 1 | 1 | 2 | 3  | 4  | 5  |
| A <sub>k</sub> |   | 2 | 4 | 6  | 8  | 10 |
| S              | 0 | 2 | 6 | 12 | 20 | 30 |

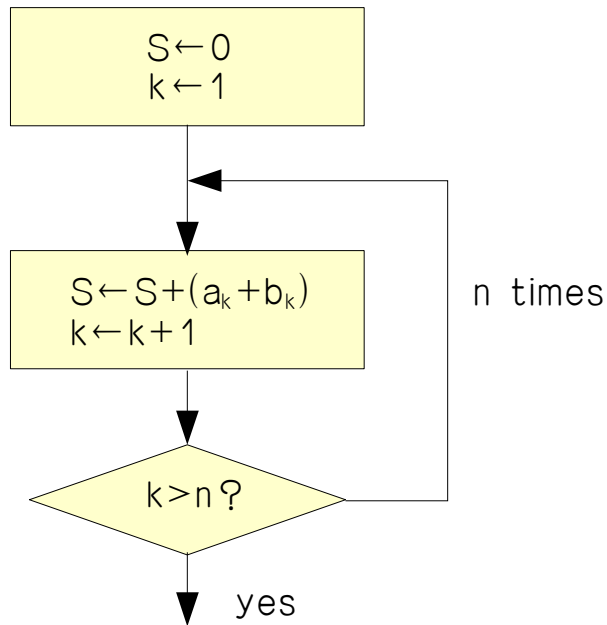
# Sigma Notation and Flow Chart (4)



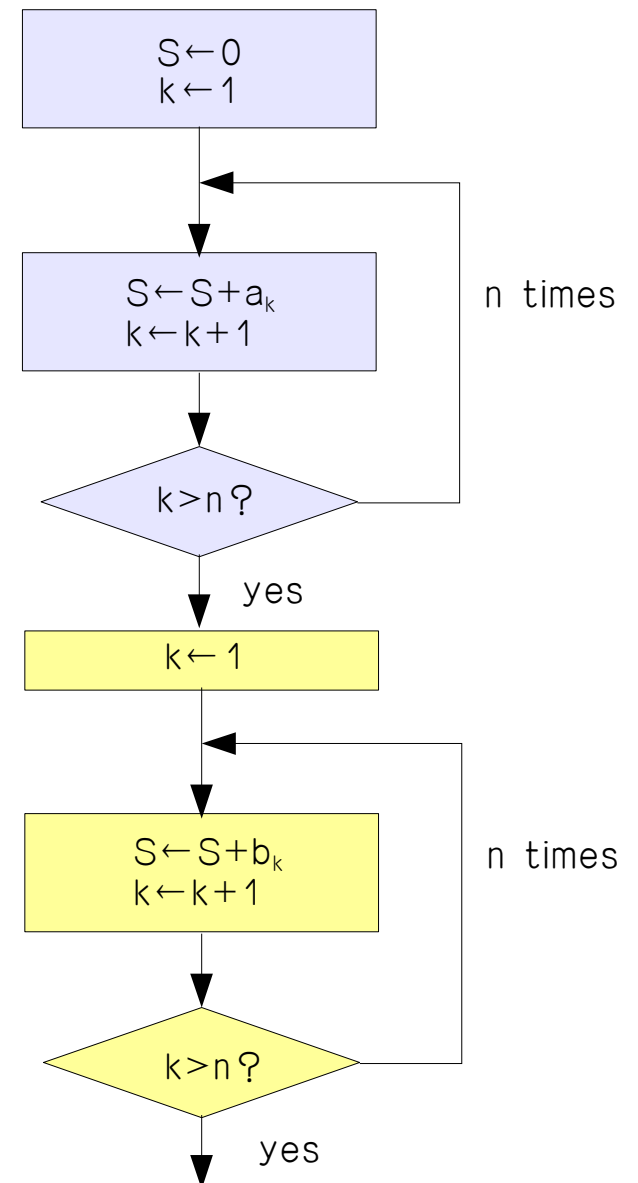
# Associativity and Commutativity of +

$$S_n = \sum_{k=1}^n (a_k + b_k)$$

$$= \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

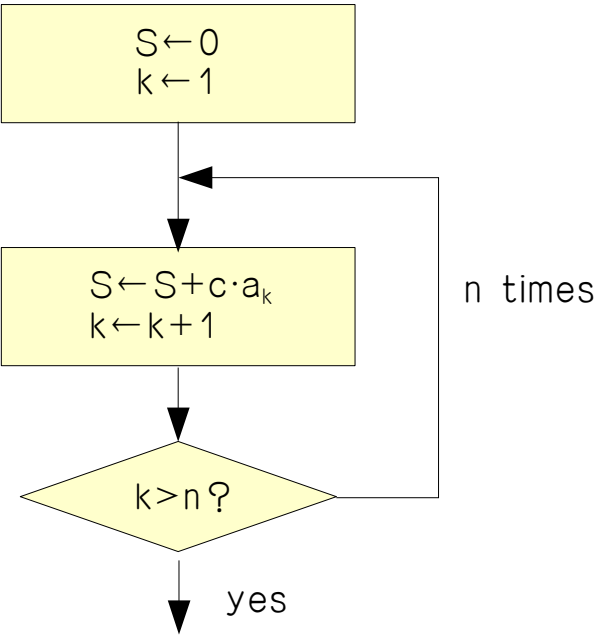


$$\begin{array}{l} +(a_1 + b_1) \\ +(a_2 + b_2) \\ +(a_3 + b_3) \\ \hline +(a_1 + a_2 + a_3) \\ +(b_1 + b_2 + b_3) \end{array}$$

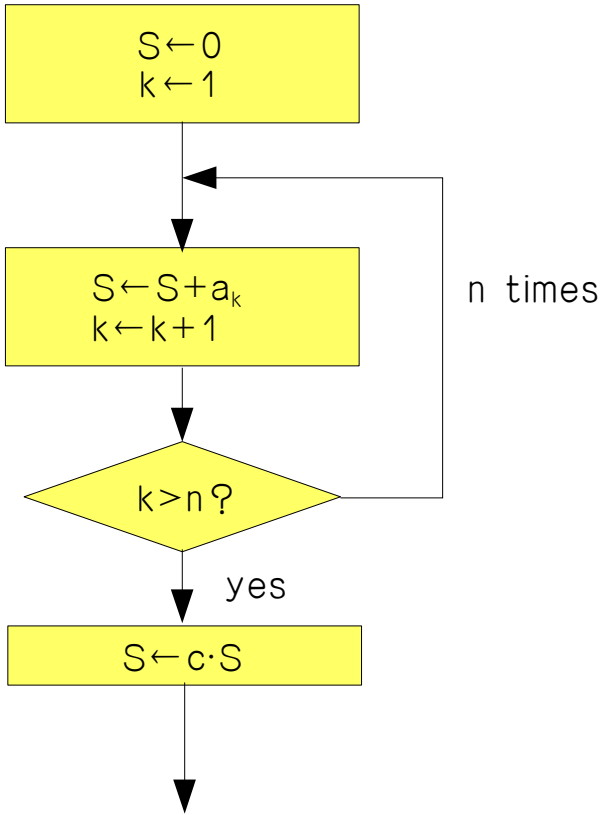


# Distributivity of X over +

$$S_n = \sum_{k=1}^n c \cdot a_k$$
$$= c \cdot \sum_{k=1}^n a_k$$



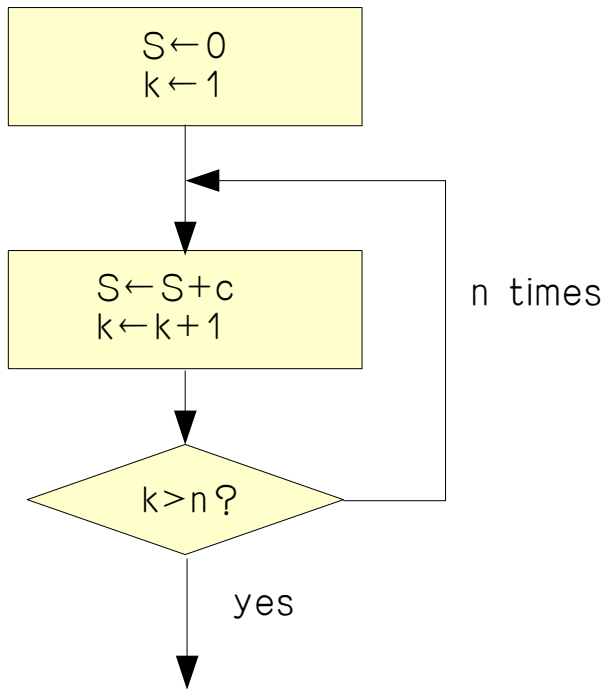
$$c \cdot a_1 + c \cdot a_2 + c \cdot a_3$$
$$= c \cdot (a_1 + a_2 + a_3)$$



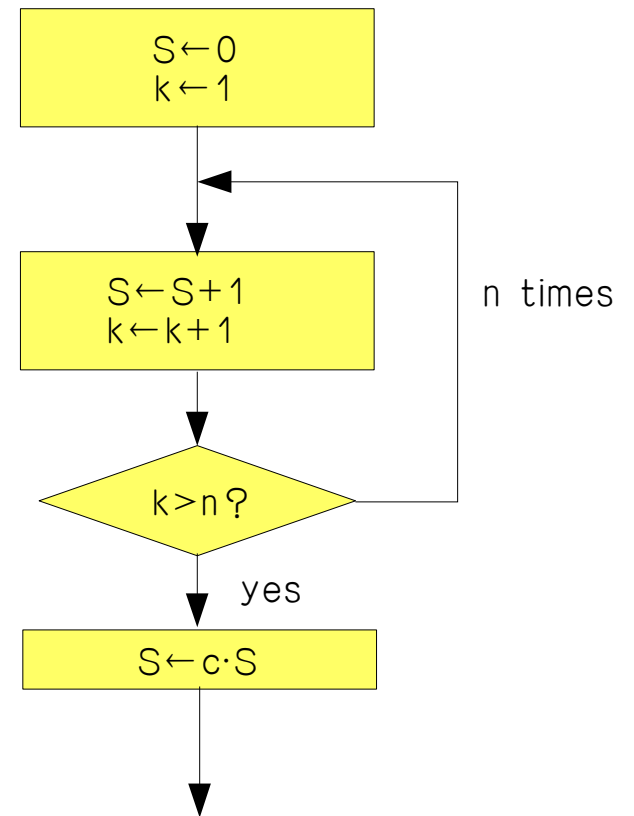


# Constant Accumulation

$$S_n = \sum_{k=1}^n c$$
$$= c \cdot \sum_{k=1}^n 1 = c \cdot n$$



$$c \cdot 1 + c \cdot 1 + c \cdot 1$$
$$= c \cdot (1 + 1 + 1)$$



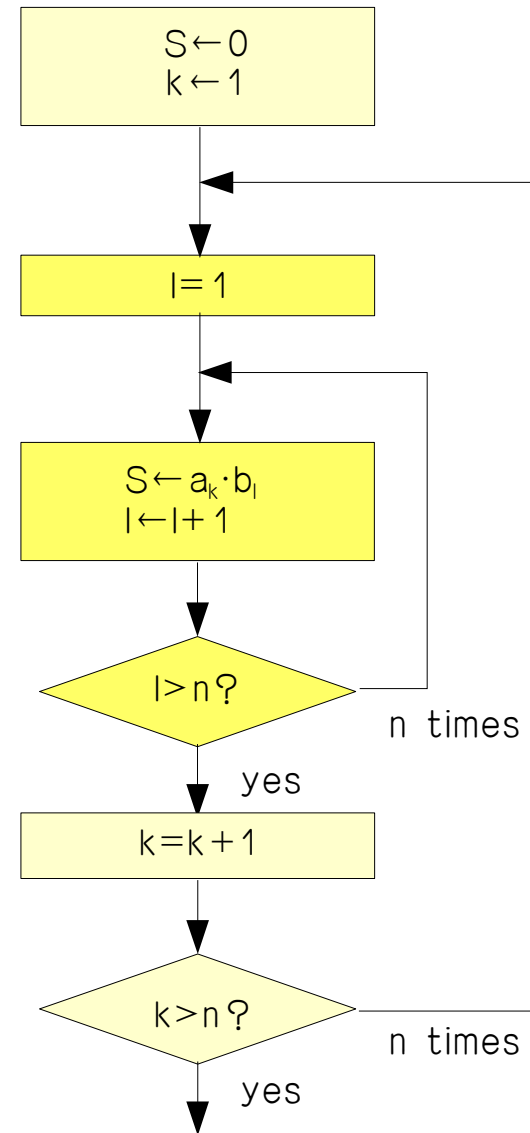
# Nested Loop (1)

$$S_n = \sum_{k=1}^n \left\{ \sum_{l=1}^n (a_k \cdot b_l) \right\}$$

+a<sub>1</sub>·b<sub>1</sub>  
+a<sub>1</sub>·b<sub>2</sub>  
+a<sub>1</sub>·b<sub>3</sub>

+a<sub>2</sub>·b<sub>1</sub>  
+a<sub>2</sub>·b<sub>2</sub>  
+a<sub>2</sub>·b<sub>3</sub>

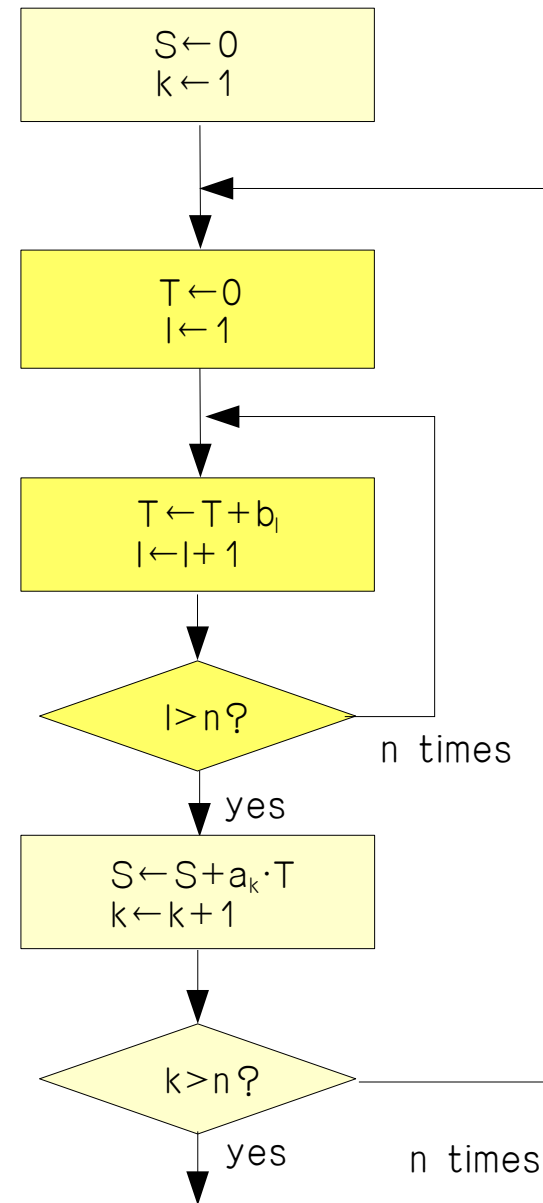
+a<sub>3</sub>·b<sub>1</sub>  
+a<sub>3</sub>·b<sub>2</sub>  
+a<sub>3</sub>·b<sub>3</sub>



# Nested Loop (2)

$$S_n = \sum_{k=1}^n \left\{ a_k \cdot \left( \sum_{l=1}^n b_l \right) \right\}$$

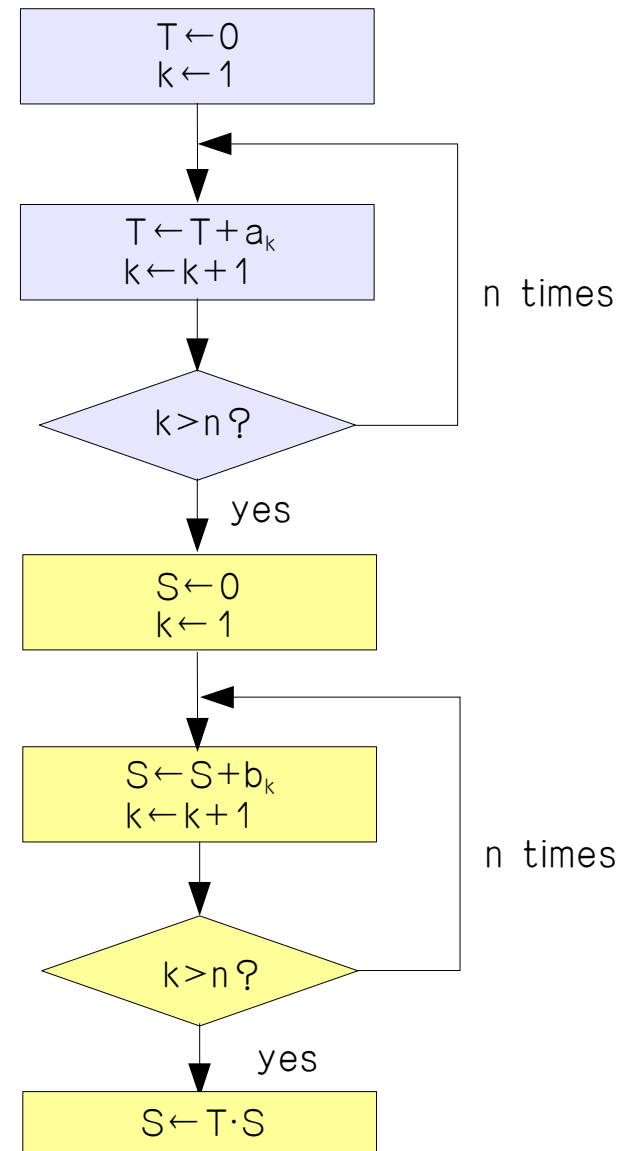
$$\begin{aligned} &+ a_1 \cdot (b_1 + b_2 + b_3) \\ &+ a_2 \cdot (b_1 + b_2 + b_3) \\ &+ a_3 \cdot (b_1 + b_2 + b_3) \end{aligned}$$



# Nested Loop (3)

$$S_n = \left( \sum_{k=1}^n a_k \right) \cdot \left( \sum_{l=1}^n b_l \right)$$

$$(a_1 + a_2 + a_3) \cdot (b_1 + b_2 + b_3)$$



# Nested Loop (4)

$$S_n = \sum_{k=1}^n \left\{ \sum_{l=1}^n (a_k \cdot b_l) \right\}$$

$$\begin{array}{lll} +a_1 \cdot b_1 & +a_2 \cdot b_1 & +a_3 \cdot b_1 \\ +a_1 \cdot b_2 & +a_2 \cdot b_2 & +a_3 \cdot b_2 \\ +a_1 \cdot b_3 & +a_2 \cdot b_3 & +a_3 \cdot b_3 \end{array}$$

$$S_n = \sum_{k=1}^n \left\{ a_k \cdot \left( \sum_{l=1}^n b_l \right) \right\}$$

$$\begin{array}{l} +a_1 \cdot (b_1 + b_2 + b_3) \\ +a_2 \cdot (b_1 + b_2 + b_3) \\ +a_3 \cdot (b_1 + b_2 + b_3) \end{array}$$

$$S_n = \left( \sum_{k=1}^n a_k \right) \cdot \left( \sum_{l=1}^n b_l \right)$$

$$(a_1 + a_2 + a_3) \cdot (b_1 + b_2 + b_3)$$

# Factorization (1) – 인수분해

$$(1-x^n) = (1-x)(1+x+x^2+\dots+x^{n-1})$$

distributivity

$$\begin{array}{r} 1+x+x^2+\dots+x^{n-1} \\ -x-x^2-x^3-\dots-x^n \\ \hline 1 \qquad \qquad \qquad -x^n \end{array}$$

$$(x^n-1) = (x-1)(1+x+x^2+\dots+x^{n-1})$$

distributivity

$$\begin{array}{r} x+x^2+x^3+\dots+x^n \\ -1-x-x^2-\dots-x^{n-1} \\ \hline -1 \qquad \qquad \qquad +x^n \end{array}$$

## Factorization (2) – 인수분해

$$(1-x^2) = (1-x)(1+x)$$

$$(1-x^3) = (1-x)(1+x+x^2)$$

$$(1-x^4) = (1-x)(1+x+x^2+x^3)$$

$$(1-x^n) = (1-x)(1+x+x^2+\dots+x^{n-1})$$

Cf)

$$(1+x)^n = {}_n C_0 + {}_n C_1 \cdot x + {}_n C_2 \cdot x^2 + {}_n C_3 \cdot x^3 + \dots + {}_n C_n \cdot x^n$$

$$(1-x)^n = {}_n C_0 - {}_n C_1 \cdot x + {}_n C_2 \cdot x^2 - {}_n C_3 \cdot x^3 + \dots + (-1)^n \cdot {}_n C_n \cdot x^n$$

# Summation Notation

$$(1 - x^n) = (1 - x)(1 + x + x^2 + \dots + x^{n-1})$$

$$(x^n - 1) = (x - 1)(1 + x + x^2 + \dots + x^{n-1})$$

$$(1 + x + x^2 + \dots + x^{n-1}) =$$

$$\sum_{i=1}^n x^{i-1} = \begin{cases} \frac{(1-x^n)}{(1-x)} = \frac{(x^n-1)}{(x-1)} & \text{when } x \neq 1 \\ n & \text{when } x = 1 \end{cases}$$



# Partial Sum of G.P.

$$a_n = a \cdot r^{n-1}$$

$$\begin{aligned} S_n &= \sum_{i=1}^n a_i = \sum_{i=1}^n a \cdot r^{i-1} \\ &= (a + ar + ar^2 + \dots + ar^{n-1}) \\ &= a(1 + r + r^2 + \dots + r^{n-1}) \end{aligned}$$

$$= \frac{a \cdot (1 - r^n)}{(1 - r)} = \frac{a \cdot (r^n - 1)}{(r - 1)} \quad \text{when } r \neq 1$$

# Partial Sum of A.P.

$$a_n = a + (n-1) \cdot d, \quad a = a_1, \quad l = a_n = a + (n-1) \cdot d$$

$$S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n = \sum_{i=1}^n a_i$$

$$S_n = a_n + a_{n-1} + \cdots + a_2 + a_1 = \sum_{i=1}^n a_{n-i+1} \quad (\text{Reverse Order})$$

$$\begin{aligned} 2S_n &= \sum_{i=1}^n (a_i + a_{n-i+1}) = \sum_{i=1}^n [\{a + (i-1) \cdot d\} + \{a + (n-i) \cdot d\}] \\ &= \sum_{i=1}^n \{2a + (n-1) \cdot d\} = n \cdot \{2a + (n-1) \cdot d\} \end{aligned}$$

$$S_n = \frac{n \cdot \{2a + (n-1) \cdot d\}}{2} = \frac{n \cdot (a+l)}{2}$$

## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
- [4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
- [5] 홍성대, “기본/실력 수학의 정석,” 성지출판