Sequence (2A)

- Summation Notation (Σ)
- Flow Chart
- Partial Sum of G.P.
- Partial Sum of A.P.

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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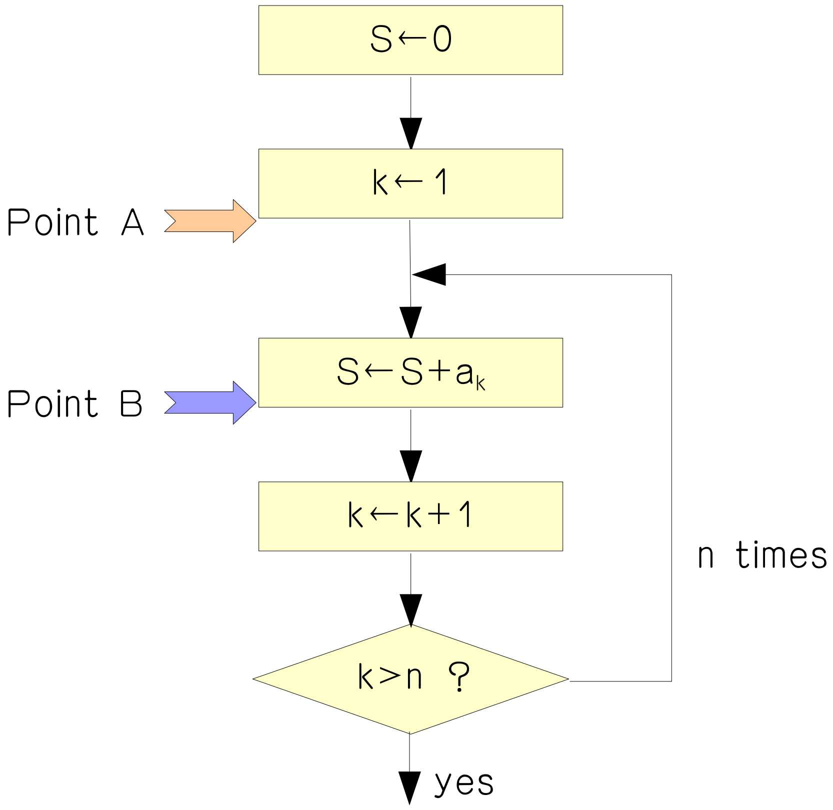
Sigma Notation and Flow Chart (1)

$$S_n = \sum_{k=1}^{n} a_k$$

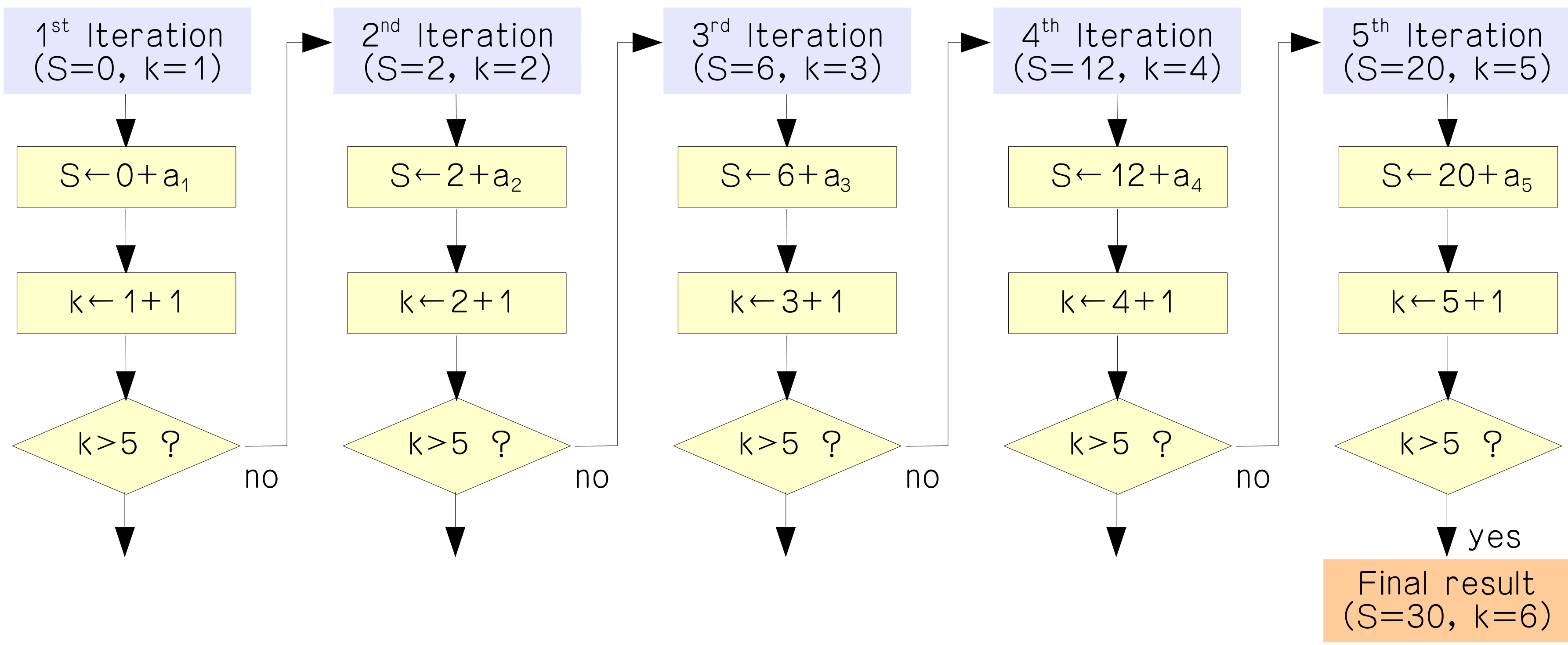
$$= a_1 + a_2 + a_3 + \quad \cdots \quad + a_n$$



	A	B				
K	1	1	2	3	4	5
Ak		2	4	6	8	10
S	0	2	6	12	20	30

$a_1 = 2$,
$a_2 = 4$,
$a_3 = 6$,
$a_4 = 8$,
$a_5 = 10$

Sigma Notation and Flow Chart (2)



$a_1=2$,
$a_2=4$,
$a_3=6$,
$a_4=8$,
$a_5=10$

	A	B				
K	1	1	2	3	4	5
A_k		2	4	6	8	10
S	0	2	6	12	20	30

Sigma Notation and Flow Chart (3)

$S \leftarrow S + a_1 = (0) + a_1$ — After 1st Iteration (S=2, k=2)

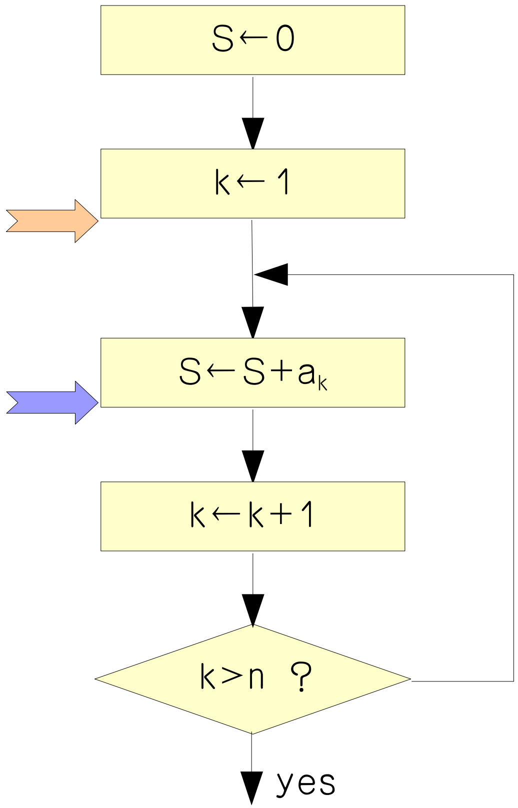
$S \leftarrow S + a_2 = (a_1) + a_2$ — After 2nd Iteration (S=6, k=3)

$S \leftarrow S + a_3 = (a_1 + a_2) + a_3$ — After 3rd Iteration (S=12, k=4)

$S \leftarrow S + a_4 = (a_1 + a_2 + a_3) + a_4$ — After 4th Iteration (S=20, k=5)

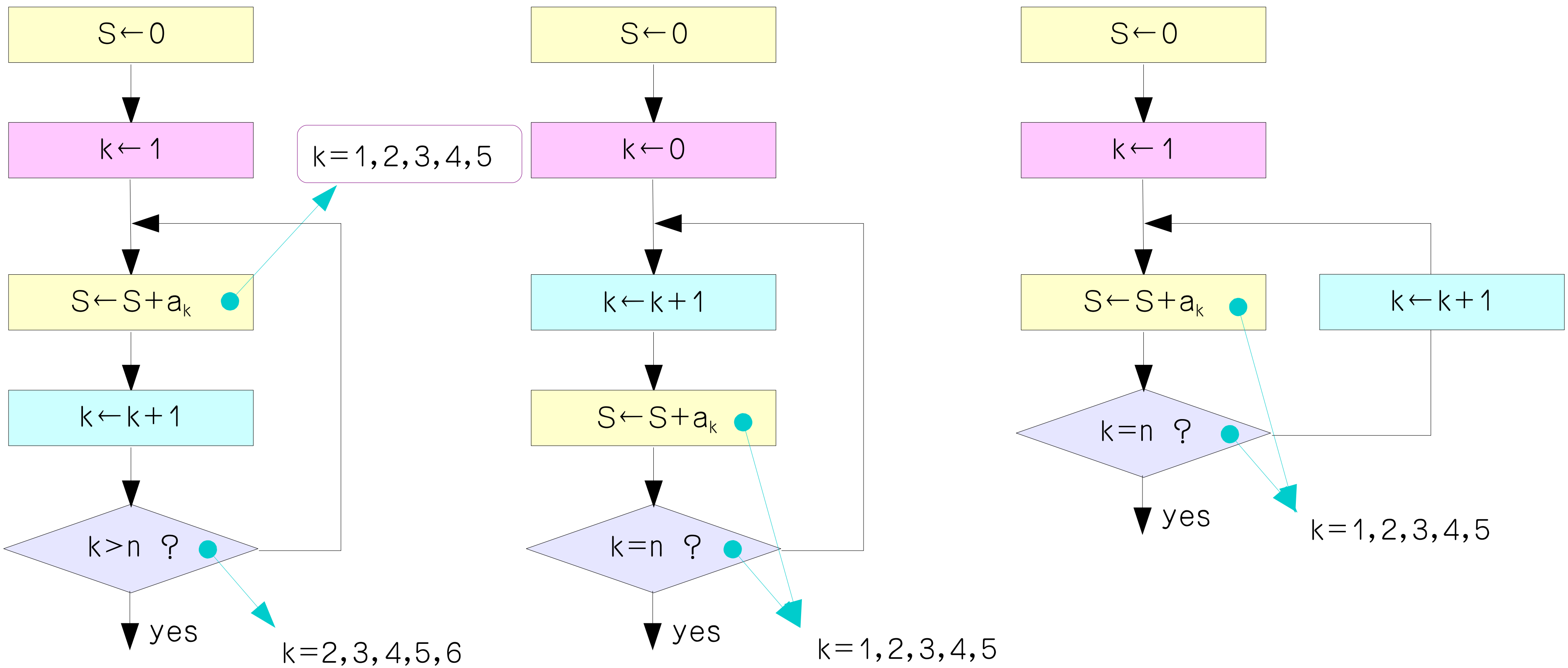
$S \leftarrow S + a_5 = (a_1 + a_2 + a_3 + a_4) + a_5$ — After 5th Iteration (S=30, k=6)

$a_1 = 2$, $a_2 = 4$, $a_3 = 6$, $a_4 = 8$, $a_5 = 10$



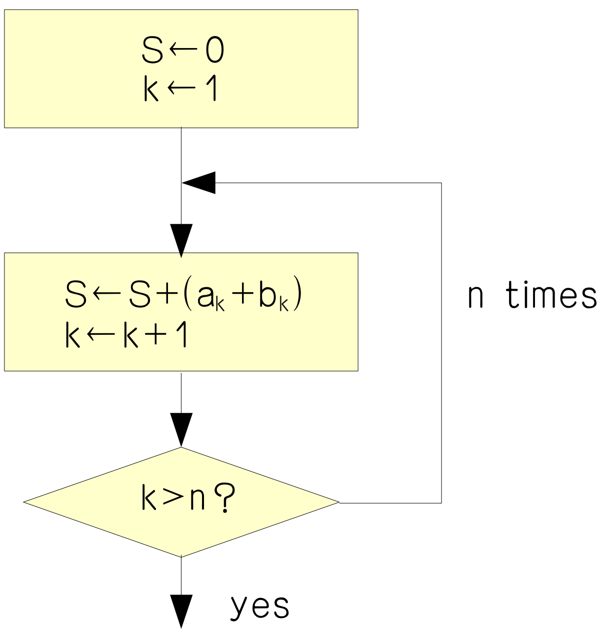
	A	B				
K	1	1	2	3	4	5
Ak		2	4	6	8	10
S	0	2	6	12	20	30

Sigma Notation and Flow Chart (4)

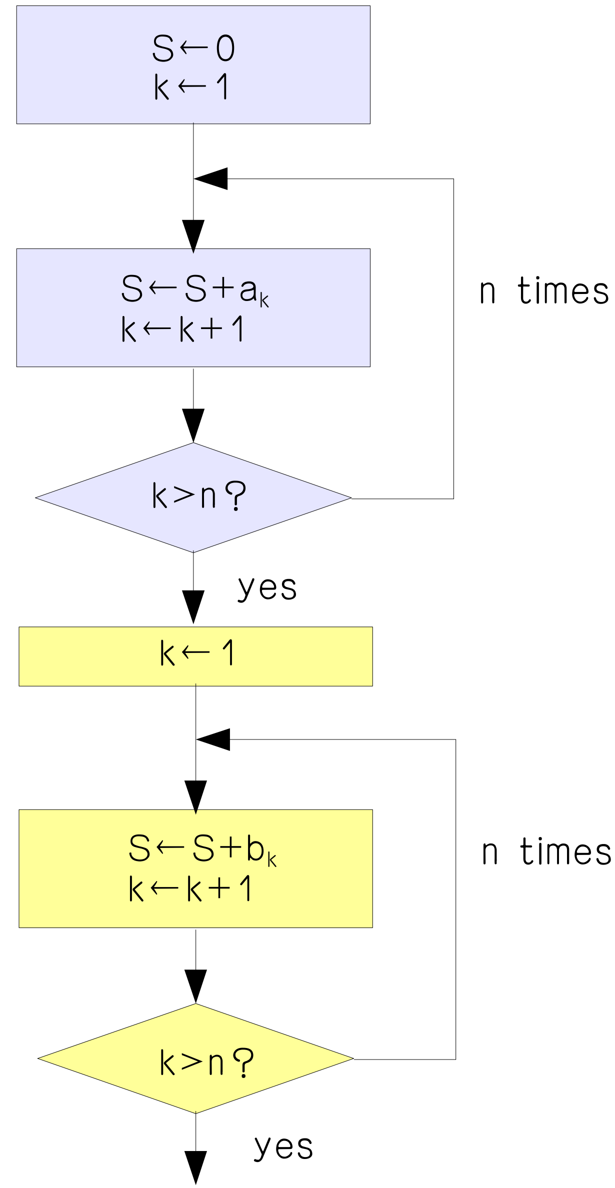


Associativity and Commutativity of +

$$S_n = \sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$



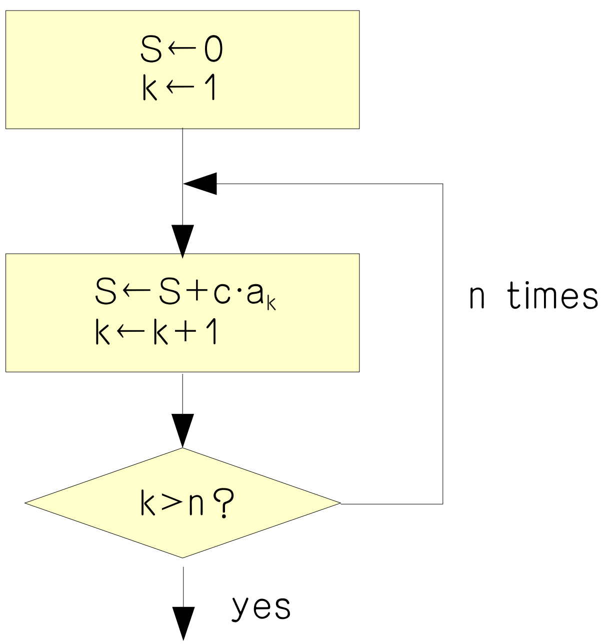
$$\begin{array}{c} +(a_1+b_1) \\ +(a_2+b_2) \\ +(a_3+b_3) \\ \hline +(a_1+a_2+a_3) \\ +(b_1+b_2+b_3) \end{array}$$



Distributivity of X over +

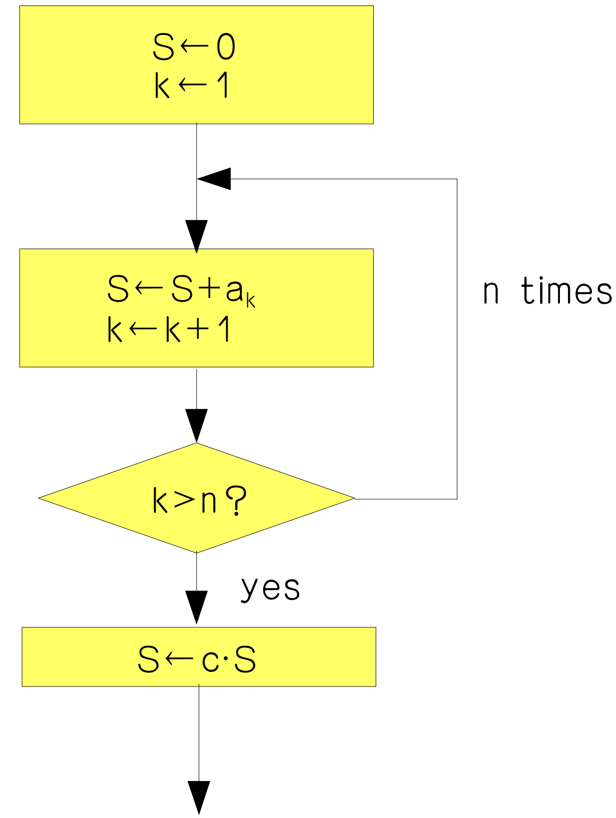
$$S_n = \sum_{k=1}^{n} c \cdot a_k$$

$$= c \cdot \sum_{k=1}^{n} a_k$$



$c \cdot a_1 + c \cdot a_2 + c \cdot a_3$

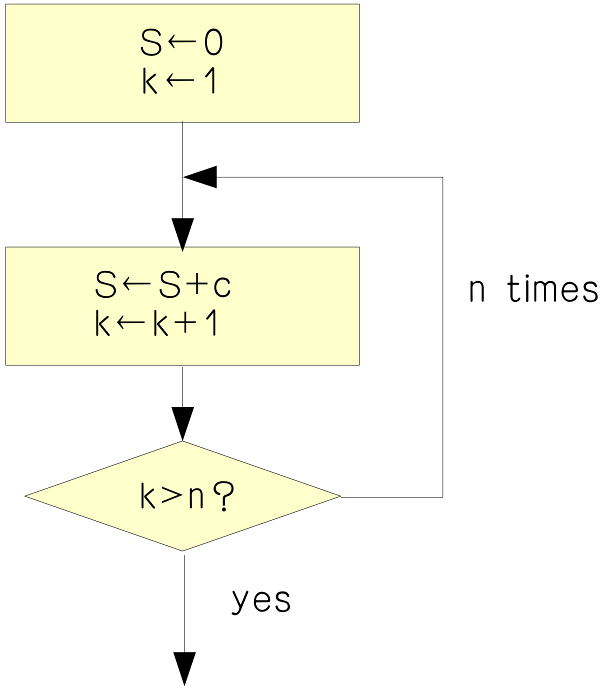
$= c \cdot (a_1 + a_2 + a_3)$



Constant Accumulation

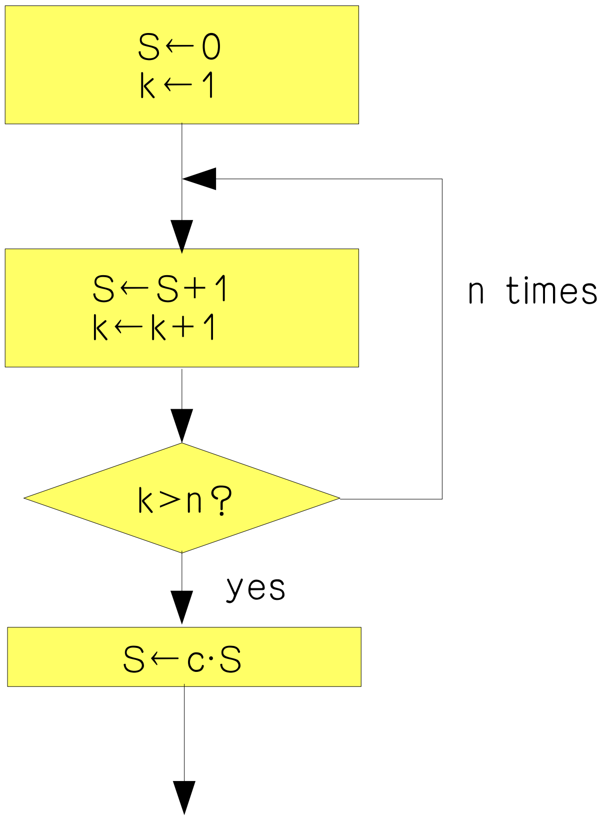
$$S_n = \sum_{k=1}^{n} c$$

$$= c \cdot \sum_{k=1}^{n} 1 = c \cdot n$$



$c \cdot 1 + c \cdot 1 + c \cdot 1$

$= c \cdot (1 + 1 + 1)$



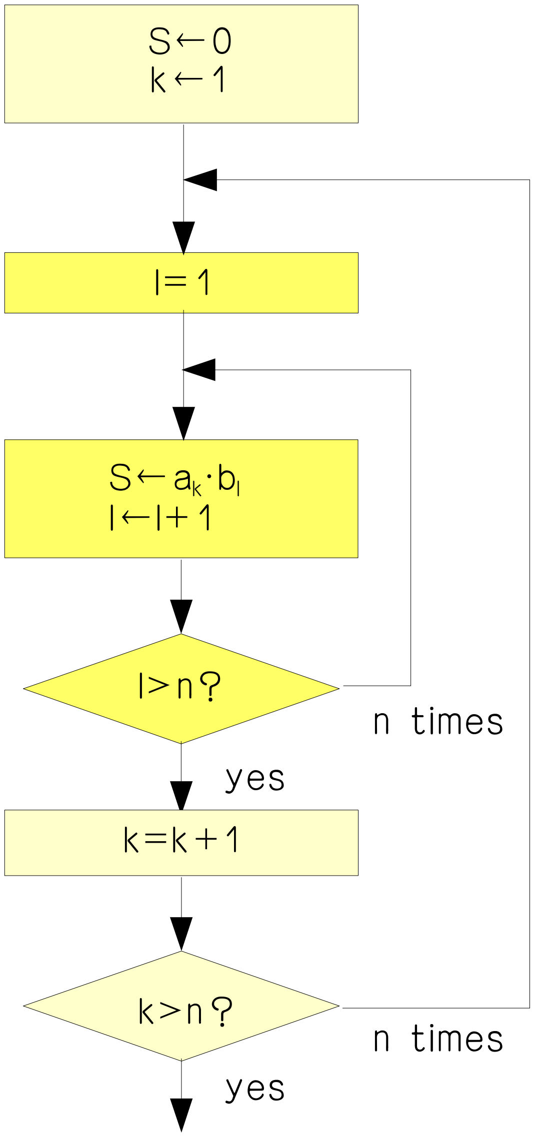
Nested Loop (1)

$$S_n = \sum_{k=1}^{n} \{ \sum_{l=1}^{n} (a_k \cdot b_l) \}$$

$+a_1 \cdot b_1$
$+a_1 \cdot b_2$
$+a_1 \cdot b_3$

$+a_2 \cdot b_1$
$+a_2 \cdot b_2$
$+a_2 \cdot b_3$

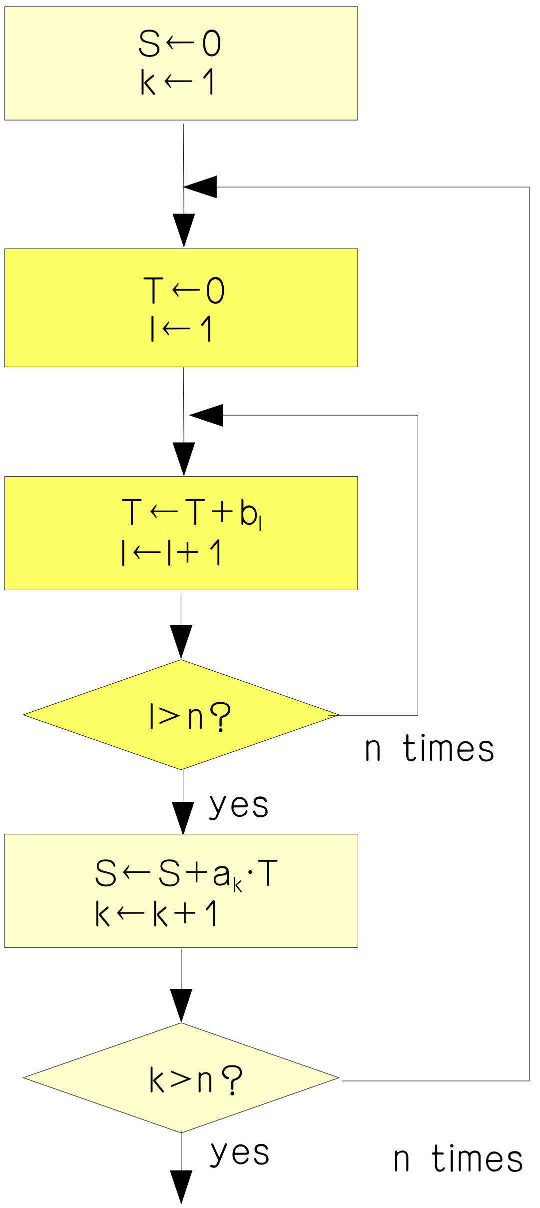
$+a_3 \cdot b_1$
$+a_3 \cdot b_2$
$+a_3 \cdot b_3$



Nested Loop (2)

$$S_n = \sum_{k=1}^{n} \{ a_k \cdot (\sum_{l=1}^{n} b_l) \}$$

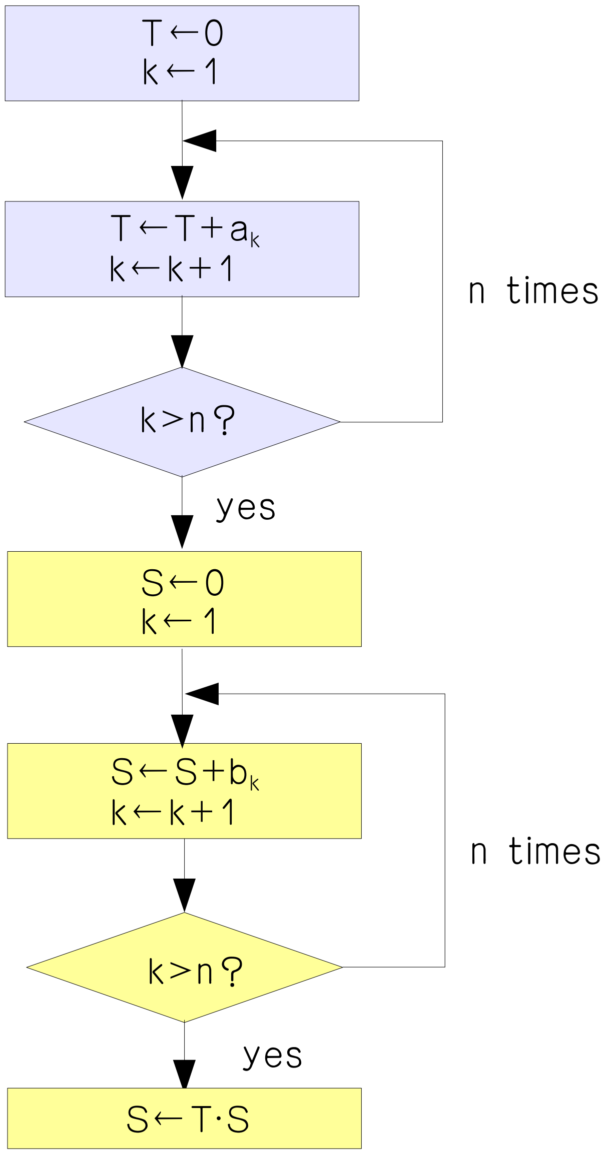
$+a_1 \cdot (b_1+b_2+b_3)$
$+a_2 \cdot (b_1+b_2+b_3)$
$+a_3 \cdot (b_1+b_2+b_3)$



Nested Loop (3)

$$S_n = \left(\sum_{k=1}^{n} a_k\right) \cdot \left(\sum_{l=1}^{n} b_l\right)$$

$$(a_1 + a_2 + a_3) \cdot (b_1 + b_2 + b_3)$$



Nested Loop (4)

$$S_n = \sum_{k=1}^{n} \{ \sum_{l=1}^{n} (a_k \cdot b_l) \}$$

$+a_1 \cdot b_1$ $+a_2 \cdot b_1$ $+a_3 \cdot b_1$
$+a_1 \cdot b_2$ $+a_2 \cdot b_2$ $+a_3 \cdot b_2$
$+a_1 \cdot b_3$ $+a_2 \cdot b_3$ $+a_3 \cdot b_3$

$$S_n = \sum_{k=1}^{n} \{ a_k \cdot (\sum_{l=1}^{n} b_l) \}$$

$+a_1 \cdot (b_1 + b_2 + b_3)$
$+a_2 \cdot (b_1 + b_2 + b_3)$
$+a_3 \cdot (b_1 + b_2 + b_3)$

$$S_n = (\sum_{k=1}^{n} a_k) \cdot (\sum_{l=1}^{n} b_l)$$

$(a_1 + a_2 + a_3) \cdot (b_1 + b_2 + b_3)$

Factorization (1) – 인수분해

$$(1-x^n) = (1-x)(1+x+x^2+ \cdots +x^{n-1})$$

distributivity

$$\begin{array}{r} 1+x+x^2+ \cdots +x^{n-1} \\ -x-x^2-x^3- \cdots -x^n \\ \hline 1 \qquad\qquad\qquad -x^n \end{array}$$

$$(x^n-1) = (x-1)(1+x+x^2+ \cdots +x^{n-1})$$

distributivity

$$\begin{array}{r} x+x^2+x^3+ \cdots +x^n \\ -1-x-x^2- \cdots -x^{n-1} \\ \hline -1 \qquad\qquad\qquad +x^n \end{array}$$

Factorization (2) – 인수분해

$$(1-x^2) = (1-x)(1+x)$$

$$(1-x^3) = (1-x)(1+x+x^2)$$

$$(1-x^4) = (1-x)(1+x+x^2+x^3)$$

$$(1-x^n) = (1-x)(1+x+x^2+ \cdots +x^{n-1})$$

Cf)

$$(1+x)^n = {}_nC_0 + {}_nC_1 \cdot x + {}_nC_2 \cdot x^2 + {}_nC_3 \cdot x^3 + \cdots + {}_nC_n \cdot x^n$$

$$(1-x)^n = {}_nC_0 - {}_nC_1 \cdot x + {}_nC_2 \cdot x^2 - {}_nC_3 \cdot x^3 + \cdots + (-1)^n \cdot {}_nC_n \cdot x^n$$

$$(1-x^n) = (1-x)(1+x+x^2+\cdots+x^{n-1})$$

$$(x^n-1) = (x-1)(1+x+x^2+\cdots+x^{n-1})$$

$$(1+x+x^2+\cdots+x^{n-1}) =$$

$$\sum_{i=1}^{n} x^{i-1} = \begin{cases} \dfrac{(1-x^n)}{(1-x)} = \dfrac{(x^n-1)}{(x-1)} & \text{when } x \neq 1 \\ n & \text{when } x = 1 \end{cases}$$

Partial Sum of G.P.

$$a_n = a \cdot r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a \cdot r^{i-1}$$

$$= (a + ar + ar^2 + \cdots + ar^{n-1})$$

$$= a(1 + r + r^2 + \cdots + r^{n-1})$$

$$= \frac{a \cdot (1 - r^n)}{(1 - r)} = \frac{a \cdot (r^n - 1)}{(r - 1)} \quad \text{when } x \neq 1$$

Partial Sum of A.P.

$$a_n = a + (n-1)\cdot d, \quad a = a_1, \quad l = a_n = a + (n-1)\cdot d$$

$$S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n = \sum_{i=1}^{n} a_i$$

$$S_n = a_n + a_{n-1} + \cdots + a_2 + a_1 = \sum_{i=1}^{n} a_{n-i+1} \quad \text{(Reverse Order)}$$

$$2S_n = \sum_{i=1}^{n} (a_i + a_{n-i+1}) = \sum_{i=1}^{n} \left[\{a + (i-1)\cdot d\} + \{a + (n-i)\cdot d\}\right]$$

$$= \sum_{i=1}^{n} \{2a + (n-1)\cdot d\} = n\cdot\{2a + (n-1)\cdot d\}$$

$$S_n = \frac{n\cdot\{2a + (n-1)\cdot d\}}{2} = \frac{n\cdot(a+l)}{2}$$

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
[5] 홍성대, “기본/실력 수학의 정석,” 성지출판