Background – Type Classes (1B)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Polymorphism in Haskell

The **polymorphism** features of Haskell

- **Purity** -- side effects
- **higher order functions** -- function passing and returning
- **parameterized algebraic data types**
- **typeclasses**

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Typeclasses

Types in Haskell

- no explicit hierarchy of types
- similar types can *act like* each other
- connect such similar types with the appropriate **typeclasses**

Example:

An Int can *act like* many things

• like an equatable thing,	**Eq**
• like an ordered thing,	**Ord**
• like an enumerable thing, etc.	**Enum**

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Open Typeclasses

Typeclasses are **open**:

- can define our own data type,
- can think about what it can act like
- can connect it with the **typeclasses** that define its behaviors.

action

behavior

typeclasses define behaviors that is very general and abstract

define behaviors

operation of a functions

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Defining behavior

defining behaviors :

the **type declarations** of **functions**

define behaviors

operation of a functions

general and abstract :

A **typeclass defintion** include

the **type declarations** of **functions**,

define behavior

which give a lot of informations

about **functions**

connect

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Examples of defining behavior

Example:

typeclasses that define **operations**

to see if two things are equal

to compare two things by some ordering.

- very **abstract** and elegant behaviors,
- **not** anything very **special**

because these **operations** are most common

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Typeclasses and Instances

typeclasses are like interfaces

defines some behavior
- comparing for *equality*
- comparing for *ordering*
- *enumeration*

instances of that **typeclass**

types possessing such behavior

such *behavior* is defined by
- **function definition**
- function **type declaration** only

a function definition

```
(==) :: a -> a -> Bool
x == y = not (x /= y)
```

- **a type declaration**

a function type declaration

```
(==) :: a -> a -> Bool
```

- **a type declaration**

A function definition can be **overloaded**

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Typeclasses and Type

typeclasses are like interfaces

defines some behavior
- comparing for *equality*
- comparing for *ordering*
- *enumeration*

class AAA bbb where
func1 :: a -> b -> c
func2 :: b -> c -> a

instances of that **typeclass**
types possessing such behavior

instance AAA BBB where
func1 definition
func2 definition

a **type** is an **instance** of a **typeclass** implies

the function types declared by the **typeclass**
are defined (implemented) in the **instance**
so that the functions can be used,
which the **typeclass** defines with that **type**

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Instance Example

the Eq typeclass

defines the functions **==** and **/=**

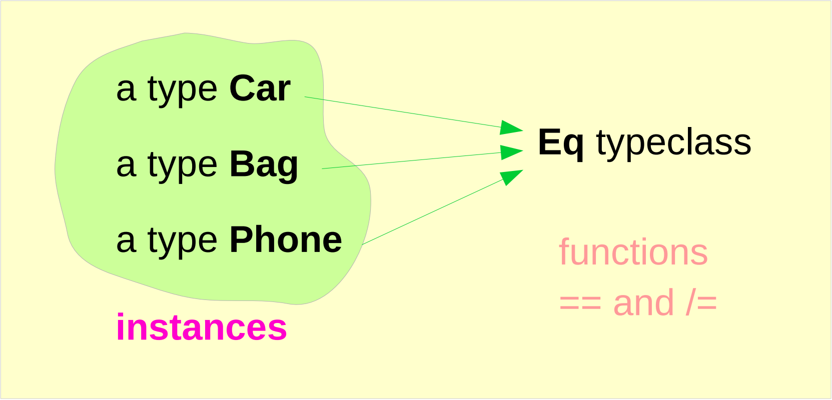
a type Car

comparing two cars c1 and c2 with the equality function ==

The Car type is an **instance** of Eq **typeclass**

Instances : various **types**

Typeclass : a group or a class of these similar types



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Instance of a typeclass (1)

```
data State a = State { runState :: Int -> (a, Int) }


instance Show (State a) where                          not working!



instance (Show a) => Show (State a) where
    show (State f) = show [show i ++ " => " ++ show (f i) | i <- [0..3]]


getState = State (\c -> (c, c))


putState count = State (\_ -> ((), count))
```

(State a) is an instance of Show

a should be an instance of Show

`State { runState = (\c -> (c, c)) }`

`State { runState = (\_ -> ((), c)) }`

https://stackoverflow.com/questions/7966956/instance-show-state-where-doesnt-compile

Instance of a typeclass (2)

```
data State a = State { runState :: Int -> (a, Int) }

instance (Show a) => Show (State a) where
    show (State f) = show [show i ++ " => " ++ show (f i) | i <- [0..3]]

getState = State (\c -> (c, c))
putState count = State (\_ -> ((), count))
```

```
show (State (\c -> (c, c)))  ➡   show (State f)          f ➡  (\c -> (c, c))

0 => show (f 0),        1 => show (f, 1),        2 => show (f, 2),        3 => show (f, 3)
          (0,0),                  (1, 1),                  (2, 2),                  (3, 3)
```

```
*Main> getState
["0 => (0,0)","1 => (1,1)","2 => (2,2)","3 => (3,3)"]

*Main> putState 1
["0 => ((),1)","1 => ((),1)","2 => ((),1)","3 => ((),1)"]
```

https://stackoverflow.com/questions/7966956/instance-show-state-where-doesnt-compile

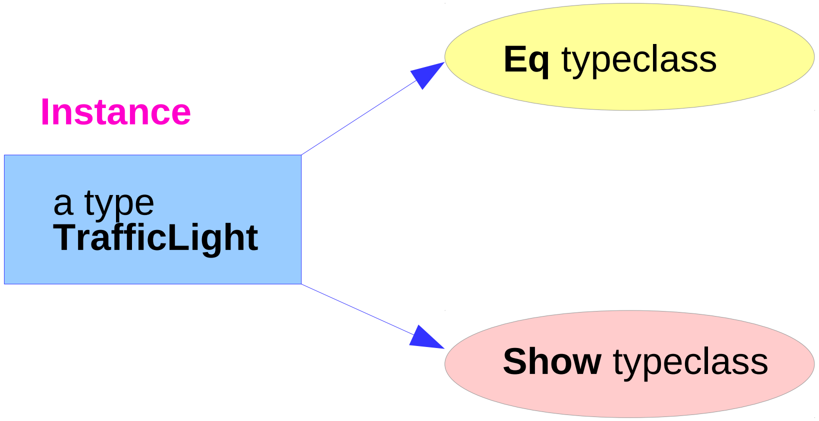
TrafficLight Type Example (2)

```
class Show  a where
    show :: a  -> String
    * * *
```

- a type declaration

```
data TrafficLight = Red | Yellow | Green
```

```
instance Show TrafficLight where
     show Red = "Red light"
     show Yellow = "Yellow light"
     show Green = "Green light"
```



```
ghci> [Red, Yellow, Green]
[Red light,Yellow light,Green light]
```

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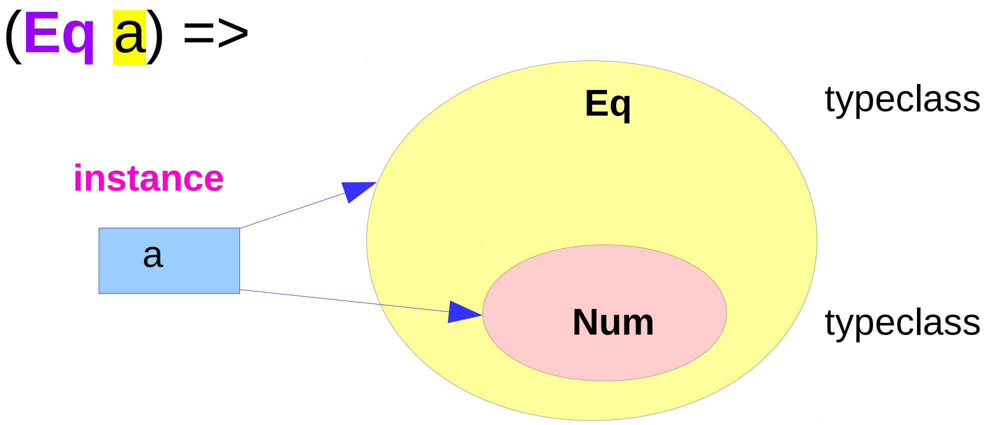
Class Constraints

```
class (Eq a) => Num a where
  ...
```

```
class Num a where
  ...
```

class constraint on a class declaration

an instance of **Eq**
before being an instance of **Num**



Num : a subclass of **Eq**

the required function bodies can be defined in

- the class declaration
- an instance declarations,

we can safely use == because a is a part of **Eq**

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Class Constraints : class & instance declarations

class constraints in **class declarations**

to make a typeclass a subclass of another typeclass

subclass

```
class (Eq a) => Num a where
    …
```

class constraints in **instance declarations**

to express requirements about the contents of some type.

requirements

```
instance (Eq x, Eq y) => Eq (Pair x y) where
   Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
```

http://cmsc-16100.cs.uchicago.edu/2016/Lectures/07-type-classes.php

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Class constraints in instance declaration examples

```
instance (Eq m) => Eq (Maybe m) where
    Just x     ==  Just y          =      x == y   ⇐ Eq m
    Nothing   ==  Nothing          =      True
              _ == _               =      False
```

```
instance (Eq x, Eq y) => Eq (Pair x y) where
    Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
                ↑                  ↑           ↑
           Eq (Pair x y)          Eq x        Eq y
```

Derived instance

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Class constraints and Overloading

```
class Eq a where
  (==)              :: a -> a -> Bool
```

```
instance Eq Integer where
  x == y            =  x `integerEq` y
```

```
instance Eq Float where
  x == y            =  x `floatEq` y
```

```
instance (Eq a) => Eq (Tree a) where
  Leaf a          == Leaf b          =  a == b
  (Branch l1 r1)  == (Branch l2 r2)  =  (l1==l2) && (r1==r2)
  _               == _               =  False
```

== of Eq (Tree a)

== of Eq a

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A Concrete Type and a Type Constructor

a : a concrete type

Maybe : not a concrete type
: a type constructor that takes one parameter
produces a concrete type.

Maybe a : a concrete type

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Instance of **Eq**

```
data TrafficLight = Red | Yellow | Green
```

to define our own type (defining a new data type)

allowed values are Red, Yellow, and Green

no class (type) instances

```
class Eq a where
   (==) :: a -> a -> Bool
   (/=) :: a -> a -> Bool
   x == y = not (x /= y)
   x /= y = not (x == y)

instance Eq TrafficLight where
   Red      == Red      = True
   Green    == Green    = True
   Yellow   == Yellow   = True
          _ == _        = False
```

class :

  defining new **typeclasses**

instance :

  making types instances of a **typeclasses**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Instance of **Show**

```
instance Show TrafficLight where
    show Red      = "Red light"
    show Yellow   = "Yellow light"
    show Green    = "Green light"
```

```
ghci> Red == Red                           ◀   instance Eq TrafficLight
True
ghci> Red == Yellow                        ◀   instance Eq TrafficLight
False
ghci> Red `elem` [Red, Yellow, Green]      ◀   instance Eq TrafficLight
True
ghci> [Red, Yellow, Green]                 ◀   instance Show TrafficLight
[Red light,Yellow light,Green light]
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Instance **Maybe m**

```
instance Eq Maybe where
   ...
```

Maybe is not a concrete type

Maybe m is a concrete type

```
instance Eq (Maybe m) where
  Just x == Just y = x == y
  Nothing == Nothing = True
  _ == _ = False
```

```
instance (Eq m) => Eq (Maybe m) where
   Just x == Just y = x == y
   Nothing == Nothing = True
   _ == _ = False
```

all types of the form **Maybe m**

to be part of the **Eq** typeclass,

but only those types where the **m**
(what's contained inside the **Maybe**)

is also a part of **Eq**.

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Eq, Ord, Show classes

Since equality tests between values are frequently used
most of your own data types should be members of **Eq**.

Prelude classes

- **Eq**
- **Ord**
- **Show**

for the convenience, Haskell has a way to declare
such "obvious" **instance definitions**
using the keyword **deriving**.

https://en.wikibooks.org/wiki/Haskell/Classes_and_types

Deriving instance example

```
data Foo = Foo {x :: Integer, str :: String}

  deriving (Eq, Ord, Show)
```

This makes **Foo** an **instance** of **Eq**

ith an automatically generated

definition of **==**

also an **instance** of **Ord** and **Show**

deriving (Eq, Ord, Show)

```
data Foo = Foo {x :: Integer, str :: String}

instance Eq Foo where
  (Foo x1 str1) == (Foo x2 str2)
        = (x1 == x2) && (str1 == str2)
```

```
*Main> Foo 3 "orange" == Foo 6 "apple"
False
*Main> Foo 3 "orange" /= Foo 6 "apple"
True
```

https://en.wikibooks.org/wiki/Haskell/Classes_and_types

Deriving instance pros and cons

The **types** of **elements** inside the **data** type
must also be **instances of the class** you are deriving.

Deriving instances

- synthesis of functions for a limited set of predefined classes
- against the general Haskell philosophy :
  "built in things are not special",
- induces compact codes
- often reduces errors in coding
  (an example: an instance of Eq such that x == y
  would not be equal to y == x would be flat out wrong).

https://en.wikibooks.org/wiki/Haskell/Classes_and_types

Derivable Classes

Eq

Equality operators **==** and **/=**

Ord

Comparison operators **< <= > >=**; **min**, **max**, and **compare**.

Enum

For enumerations only. Allows the use of list syntax such as [Blue .. Green].

Bounded

Also for enumerations, but can also be used on types that have only one constructor.

Provides **minBound** and **maxBound** as the lowest and highest values that the type can take.

Show

Defines the function **show**, which converts a value into a string, and other related functions.

Read

Defines the function **read**, which parses a string into a value of the type, and other related functions.

https://en.wikibooks.org/wiki/Haskell/Classes_and_types

Functors, Applicatives, Monads

functors: you apply a function to a **wrapped** value
applicatives: you apply a **wrapped** function to a **wrapped** value
monads: you apply a function that returns a **wrapped** value, to a **wrapped** value

functors: using **fmap** or **<$>**
applicatives: using **<*>** or **liftA**
monads: using **>>=** or **liftM**

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Functors

Functors use the **fmap** or <$> functions

```
fmap :: Functor f => (a -> b) -> f a -> f b
<$>   :: Functor f => (a -> b) -> f a -> f b
```

This takes a function and applies to to the wrapped elements

```
fmap (\x -> x + 1) (Just 1)          -- Applies (+1) to the inner value, returning (Just 2)
fmap (\x -> x + 1) Nothing           -- Applies (+1) to an empty wrapper, returning Nothing

fmap (\x -> x + 1) [1, 2, 3]         -- Applies (+1) to all inner values, returning [2, 3, 4]
(\x -> x + 1) <$> [1, 2, 3]          -- Same as above [2, 3, 4]
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Applicatives

Applicatives use the **<*>** function:

```
<*> :: Applicative f => f (a -> b) -> f a -> f b
```

This takes a wrapped function and applies it to the wrapped elements

```
(Just (\x -> x + 1)) <*> (Just 1)        -- Returns (Just 2)
(Just (\x -> x + 1)) <*> Nothing         -- Returns Nothing
Nothing <*> (Just 1)                     -- Returns Nothing
[(*2), (*4)] <*> [1, 2]                  -- Returns [2, 4, 4, 8]
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Monads – return

There are two relevant functions in the **Monad typeclass**:

```
return :: Monad m => a -> m a
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

The return function takes a raw, unwrapped value,
and wraps it up in the desired monadic type.

```
makeJust :: a -> Maybe a
makeJust x = return x

let foo = makeJust 10                -- returns (Just 10)
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Monads – bind

The bind function lets you
temporarily unwrap the inner elements of a **Monad**
and pass them to a function that performs some action
that wraps them back UP in the same monad.

This can be used with the return function in trivial cases:

```
[1, 2, 3, 4] >>= (\x -> return (x + 1))        -- Returns [2, 3, 4, 5]
(Just 1) >>= (\x -> return (x + 1))            -- Returns (Just 2)
Nothing >>= (\x -> return (x + 1))             -- Returns Nothing
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Monads – a binding operand

functions to chain together that don't require to use **return**.

```
getLine :: IO String                  -- return String type value as a result
putStrLn :: String -> IO ()
```

function call examples

```
getLine >>= (\x -> putStrLn x)        -- gets a line from IO and prints it to the console
getLine >>= putStrLn                  -- with currying, this is the same as above
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Monads – a chain of functions

functions to chain together that don't require to use **return**.

```
getLine :: IO String                    -- return String type value as a result
putStrLn :: String -> IO ()
read :: Read a => String -> a
show :: Show a => a -> String
```

-- composite function examples

-- reads a line from IO, converts to a number, adds 10 and prints it

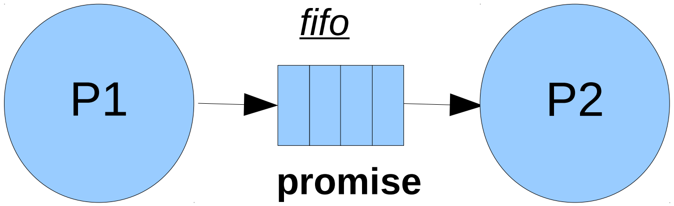
```
getLine >>= (return . read) >>= (return . (+10)) >>= putStrLn . show
```

```
          String               a                    a                String -> ()
getLine     ➡   (return . read)  ➡  (return . (+10))  ➡  putStrLn . show
```

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Promises and Mediators

the concept of **promises** (particularly in Javascript)



A **promise** is an **object** that acts as a placeholder
for the **result value** of an asynchronous, background **computation**,
like fetching some data from a remote service.

it serves as a **mediator**
between the asynchronous computation
and functions that need to **operate** on its anticipated **result**.

Act
Behavior
Operation

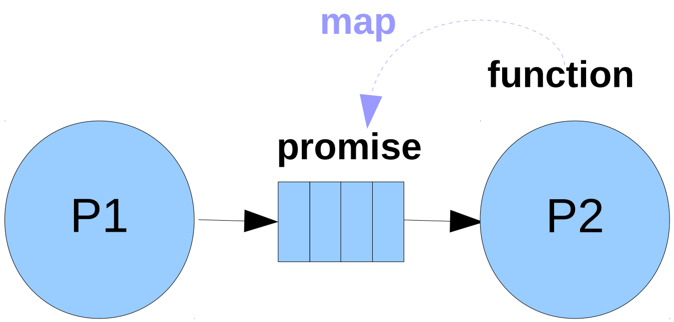
Define
Connect

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Map a function over a promise

A mediator allows us to say
what **function** should apply
to the **result** of a background task,
before that task has completed.

When you **map** a **function** over a **promise**,
the value that your function should apply to
may not have been computed yet
and in fact, if there is an error somewhere
it may never be computed.



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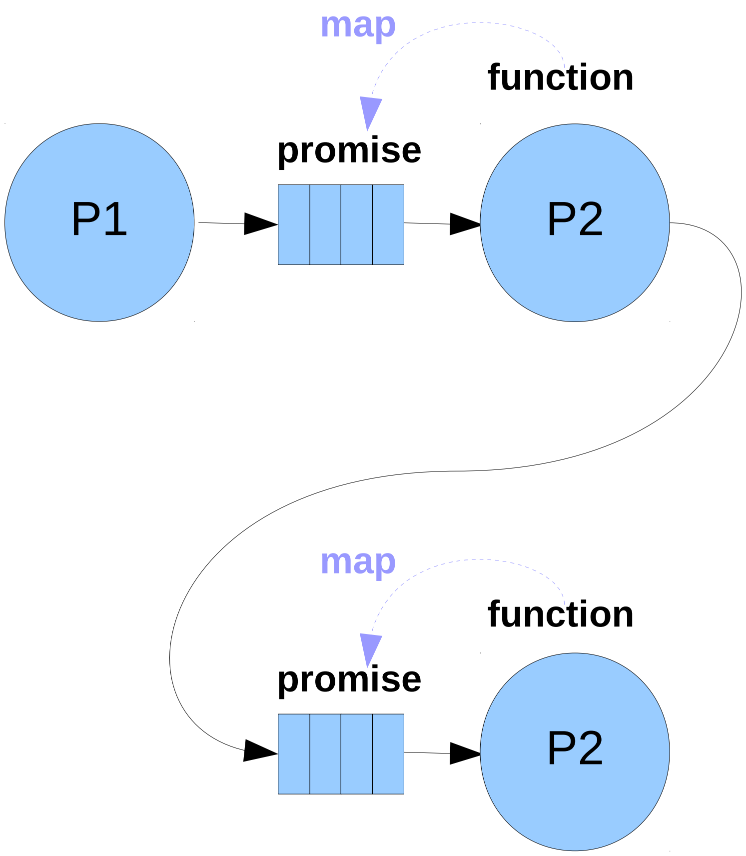
Chaining a function onto a promise

Promise libraries usually support
a **functorial/monadic** API
where you can chain a function onto a **promise**,
which produces another **promise**
that produces the **result** of applying that function
to the original **promise's** result.

the **value** of the **functor**/**monad** interface

Promises allow you to say
what **function** should apply
to the **result** of a background task,
before that task has completed.



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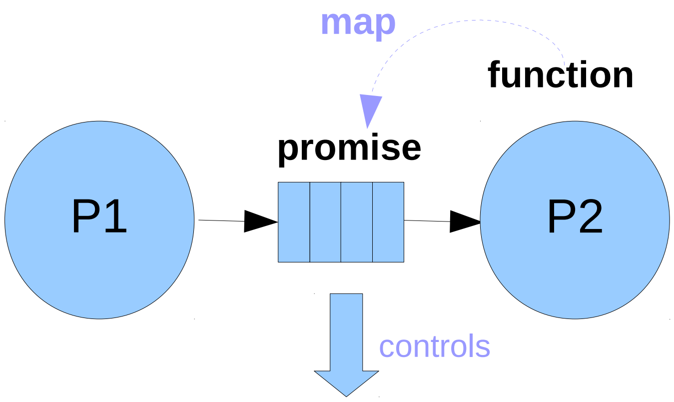
Interfaces

think **functor/applicative/monad**
as **interfaces** for **mediator objects**
that sit in between **functions** and **arguments**,
and <u>connect</u> them indirectly according to some policy.

The <u>simplest</u> way to use a function is
just to <u>call</u> it with some <u>arguments</u>;



<u>when</u> and <u>how</u> <u>many</u> <u>times</u>
the function will be <u>called</u>
what to do with its result.

First-class functions

A **higher-order function** is a function
that <u>takes</u> other functions <u>as arguments</u>
or <u>returns</u> a function <u>as result</u>.

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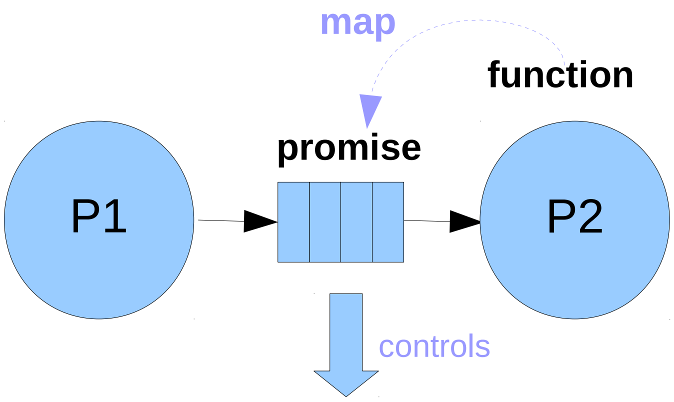
Interfaces with first-class functions

if you have **first-class functions**,
you have other, indirect options—

you can supply the function to a mediator object
that will control when and how many times the
function will be called, and what to do with its result.



when and how many times
the function will be called
what to do with its result.

First-class functions

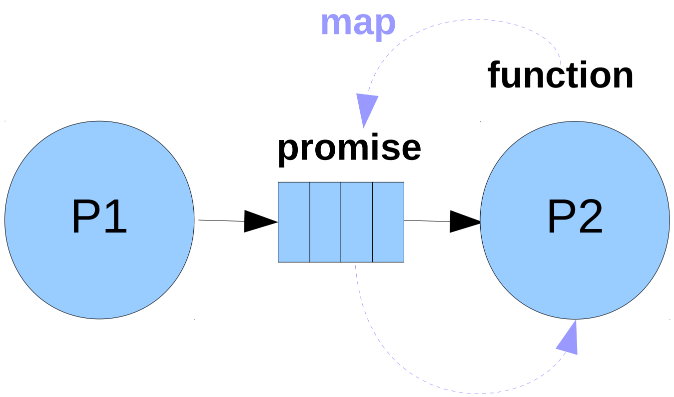
A **higher-order function** is a function
that takes other functions as arguments
or returns a function as result.

https://softwareengineering.stackexchange.com/questions/303472/what-is-the-purpose-of-wrapped-values-in-haskell

Promises and Mediators

Promises call the functions supplied to them
when the result of some background task is completed

The results of those functions are then handed over
to other promises that are waiting for them.



call functions that was received as arguments

hand over the results

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General Monad - MonadPlus

Haskell's **Control.Monad** module defines a typeclass, **MonadPlus**,
that enables abstract the common pattern eliminating **case** expressions.

```
class Monad m => MonadPlus m where
   mzero :: m a
   mplus :: m a -> m a -> m a
```

```
class  (Monad m) => MonadPlus m  where
```

```
instance MonadPlus [] where
   mzero = []
   mplus = (++)
```

```
instance MonadPlus Maybe where
   mzero = nothing
   nothing `mplus` ys  = ys
   xs          `mplus` _    = xs
```

http://book.realworldhaskell.org/read/programming-with-monads.html

General Monad - MonadPlus Laws

The class **MonadPlus** is used for monads that have a zero element and a plus operation:

```
class  (Monad m) => MonadPlus m  where
    mzero           :: m a
    mplus           :: m a -> m a -> m a
```

For lists, the zero value is [], the empty list.
The I/O monad has no zero element and
is not a member of this class.

```
m >>= \x -> mzero      =      mzero
mzero >>= m            =      mzero
```

The zero element laws:

```
m `mplus` mplus        =      m
mplus `mplus` m        =      m
```

The laws governing the mplus operator

The mplus operator is ordinary list concatenation in the list monad.

http://book.realworldhaskell.org/read/programming-with-monads.html

Functional Dependency (fundep)

```
class Mult a b c | a b -> c where
  (*) :: a -> b -> c
```

| a b -> c means

c is <u>uniquely</u> <u>determined</u> from **a** and **b**

fundeps are not standard **Haskell 98**.

(Nor are multi-parameter type classes, for that matter.)

They are, however, supported at least in **GHC** and **Hugs**

and will almost certainly end up in Haskell'.

```
class Mult a b c where
  (*) :: a -> b -> c
```

https://wiki.haskell.org/Functional_dependencies

Functional Dependency – a type inferencer

In a multiparameter typeclass, by default,
the **type variables** are considered independently.

The **type inferencer** has to determine **a** and **b** independently,
then check to see if the **instance** exists.

```
class Foo a b
```

Functional dependencies narrow down possible choices.

effective, useful

```
class Foo a b | a -> b
```

Look, if you determine what **a** is, then there is a unique **b**
so that **Foo a b** exists, so don't bother trying to infer **b**,
just go look up the instance and typecheck that.

https://stackoverflow.com/questions/20040224/functional-dependencies-in-haskell

Functional Dependency – return type polymorphism

Fundep is useful with **return type polymorphism**

```
class Foo a b c where
  bar :: a -> b -> c
```

there's no way to infer

```
  bar (bar "foo" 'c') 1
```

Because we have no way of determining **c** of **a -> b -> c**.

Even if we only wrote one instance for **String** and **Char**,
we have to assume that someone might/will come along
and add another instance later on.

https://stackoverflow.com/questions/20040224/functional-dependencies-in-haskell

Functional Dependency – determining the return type

With **fundeps** we don’t have to specify the **return type**, which is annoying.

And now it's easy to see that the return type **c** of **bar "foo" 'c'** is unique and thus inferable.

```
class Foo a b c | a b -> c where
  bar :: a -> b -> c
```

https://stackoverflow.com/questions/20040224/functional-dependencies-in-haskell

Type Constructors with parameters

Type constructors take other **types** as **parameters**
to eventually produce **concrete types**. – like a **function**

type constructors can be partially applied just like **functions** can

Either String is a **type** that takes one **type**
and produces a **concrete type**,
like **Either String Int**

by using **type declarations**

formally defining how **types** are applied to **type constructors**,
formally defining how **values** are applied to **functions**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Kind of a type

values like

3 – Int
"YEAH" – String
takeWhile – a function value

each have their own type.

types are little **labels** that **values** carry
so that we can reason about the **values**.
types have their own another little **labels**, called **kinds**.

A **kind** can be considered as the **type of a type**.

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Examining the kind of a type

To examine the **kind of a type**

using the **:k** command in GHCI.

```
ghci> :k Int
Int :: *
```

A * means that the type is a **concrete type**.

A **concrete type** is a type that doesn't take any type **parameters**

and **values** can only have types that are **concrete types**.

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Kind of a type constructor

```
ghci> :k Maybe
Maybe :: * -> *
```

The **Maybe type constructor**

takes one **concrete type** (like **Int**)

and returns a **concrete type** (like **Maybe Int**)

Int -> Int represents a **function**

taking an **Int** and returning an **Int**,

*** -> *** represents a **type constructor**

taking an **concrete type** and returning a **concrete type**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Kind of a type constructor applied with a type parameter

apply the **type parameter** to **Maybe**

```
ghci> :k Maybe Int
Maybe Int :: *
```

the **type parameter Int** is applied to **Maybe**

The **kind** of **Maybe Int** is a **concrete type**

```
:t isUpper                        :k isUpper
Char -> Bool                      *

:t isUpper 'A'   (True)           :k isUpper 'A'
Bool                              *
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf