Functions (4A)

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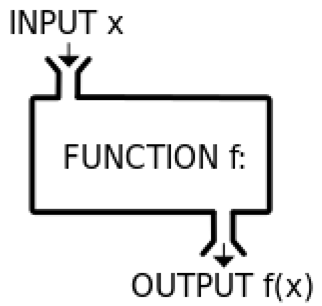
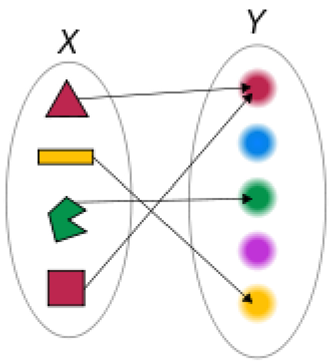
Function



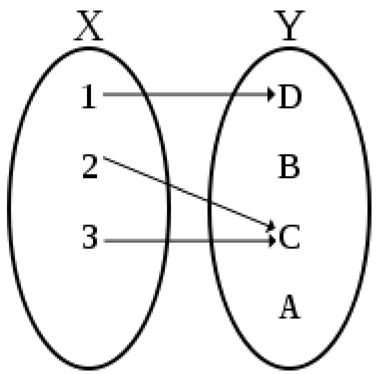
A function f takes an input x, and returns a single output f(x). One metaphor describes the function as a "machine" or "black box" that for each input returns a corresponding output.



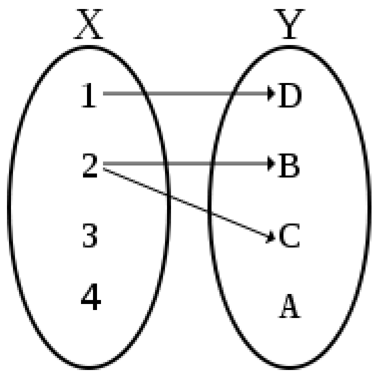
A function that associates any of the four colored shapes to its color.

https://en.wikipedia.org/wiki/Function_(mathematics)

Function



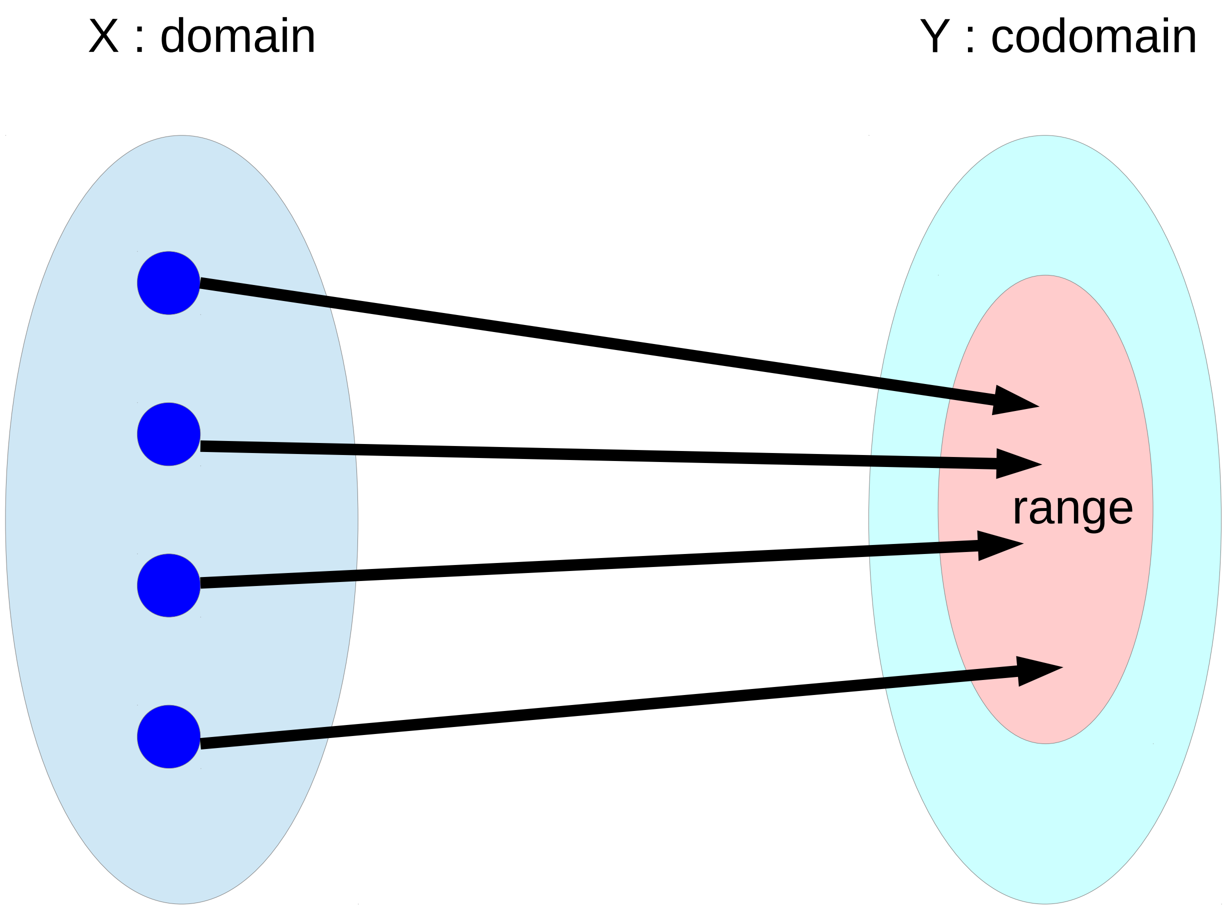
The above diagram represents a function with domain {1, 2, 3}, codomain {A, B, C, D} and set of ordered pairs {(1,D), (2,C), (3,C)}. The image is {C,D}.



However, this second diagram does not represent a function. One reason is that 2 is the first element in more than one ordered pair. In particular, (2, B) and (2, C) are both elements of the set of ordered pairs. Another reason, sufficient by itself, is that 3 is not the first element (input) for any ordered pair. A third reason, likewise, is that 4 is not the first element of any ordered pair.

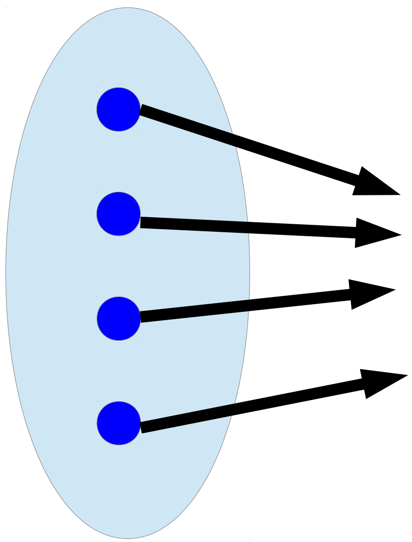
https://en.wikipedia.org/wiki/Function_(mathematics)

Domain, Codomain, and Range



Function condition : One emanating arrow

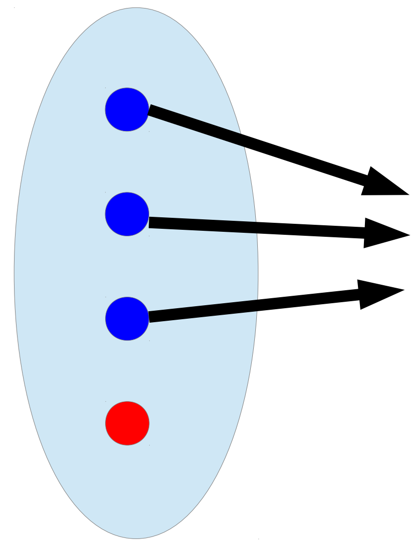
Each, one starting arrow



Function (O)

Relation (O)

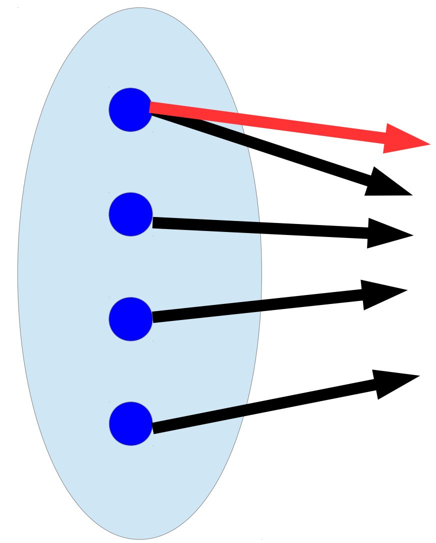
Some, no starting arrow



Function (X)

Relation (O)

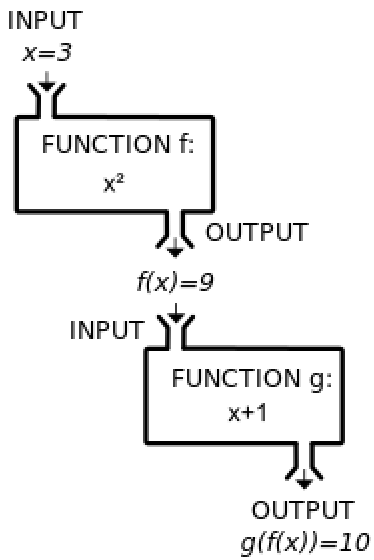
Some, two starting arrows

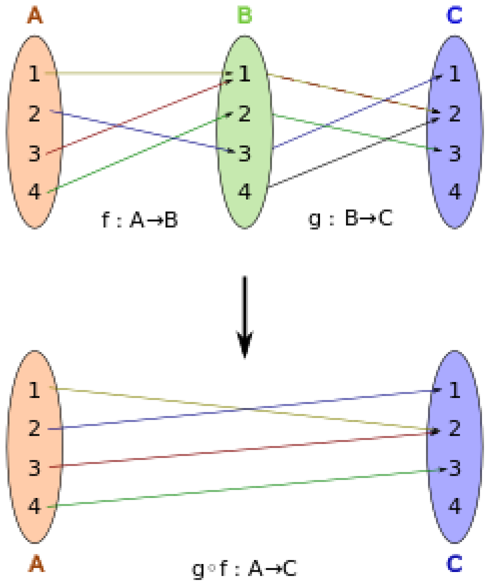


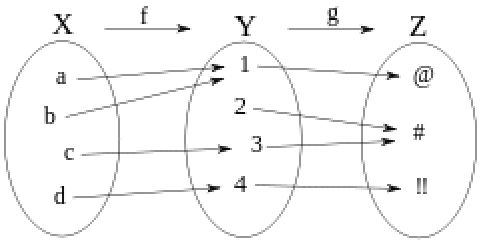
Function (X)

Relation (O)

Composite Function



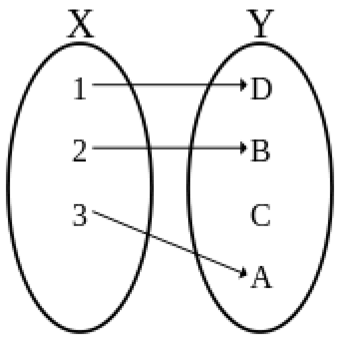




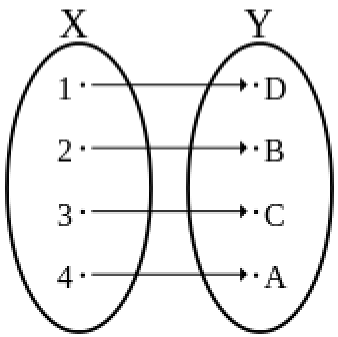
A composite function g(f(x)) can be visualized as the combination of two "machines". The first takes input x and outputs f(x). The second takes as input the value f(x) and outputs g(f(x)).

https://en.wikipedia.org/wiki/Function_(mathematics)

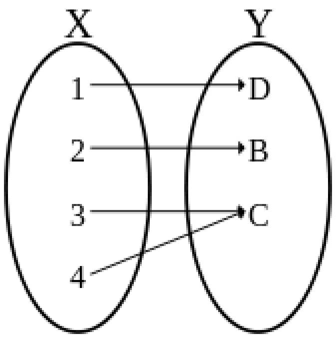
Injective Function



An **injective** non-**surjective** function (injection, not a bijection)



An **injective surjective** function (**bijection**)



A non-**injective surjective** function (surjection, not a bijection)

https://en.wikipedia.org/wiki/Function_(mathematics)

Injective Function

In mathematics, an **injective** function or **injection** or **one-to-one** function is a function that preserves **distinctness**:

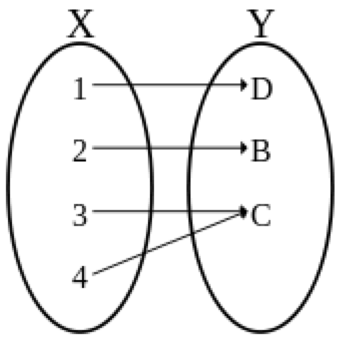
it never maps distinct elements of its **domain** to the same element of its **codomain**.

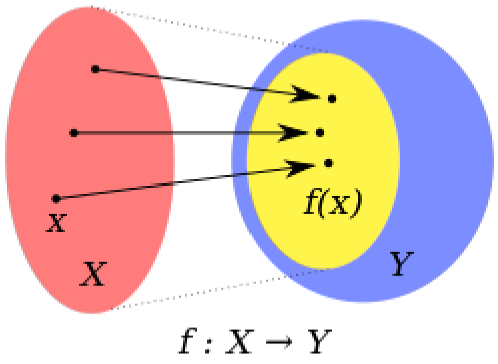
every element of the function's **codomain** is the image of at most one element of its **domain**.

The term one-to-one function must not be confused with one-to-one correspondence (a.k.a. **bijective** function),
which uniquely maps all elements in both **domain** and **codomain** to each other.

https://en.wikipedia.org/wiki/Function_(mathematics)

Surjective Function





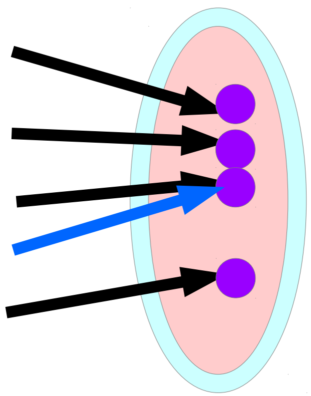
A surjective function from domain X to codomain Y. The function is surjective because every point in the codomain is the value of f(x) for at least one point x in the domain.

A non-surjective function from domain X to codomain Y. The smaller oval inside Y is the image (also called range) of f. This function is not surjective, because the image does not fill the whole codomain. In other words, Y is colored in a two-step process: First, for every x in X, the point f(x) is colored yellow; Second, all the rest of the points in Y, that are not yellow, are colored blue. The function f is surjective only if there are no blue points.

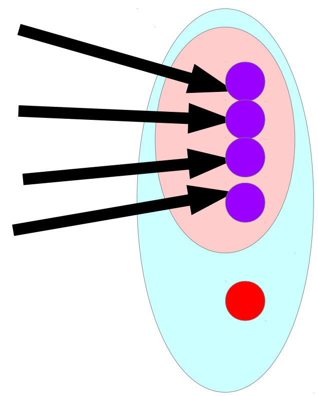
https://en.wikipedia.org/wiki/Function_(mathematics)

Surjective Functions

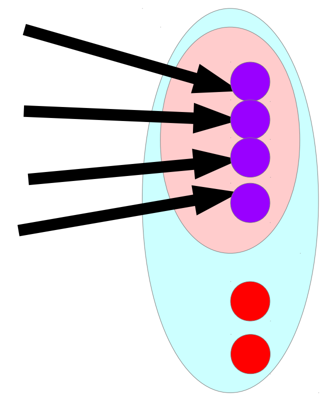
Range = Codomain
Every, arriving arrow(s)



Surjective (O)



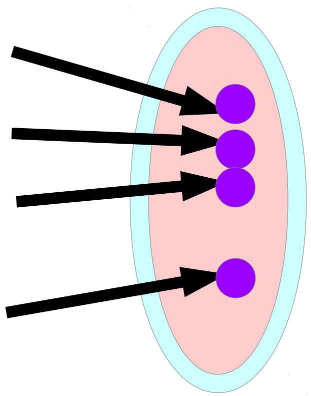
Surjective (X)



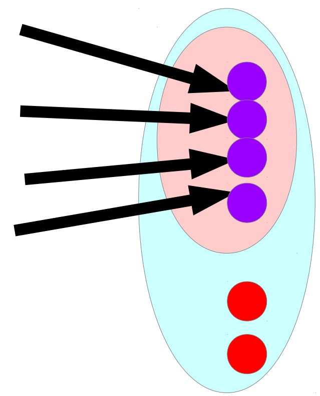
Surjective (X)

Injective Functions

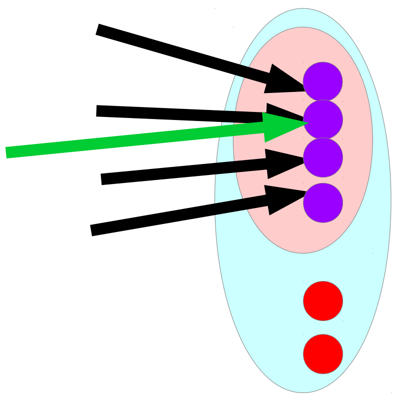
Every, Less than one
arriving arrow



Injective (O)



Injective (O)



Injective (X)

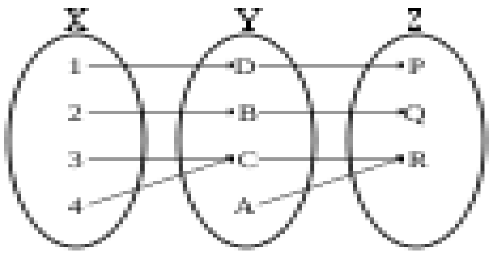
Surjective Function

A **surjective** function is a function
whose **image** is equal to its **codomain**.

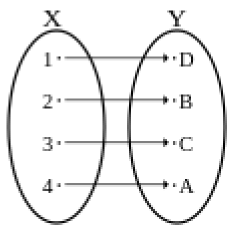
a function f with **domain** X and **codomain** Y is surjective
if for every y in Y there exists at least one x in X with f(x) = y.

https://en.wikipedia.org/wiki/Function_(mathematics)

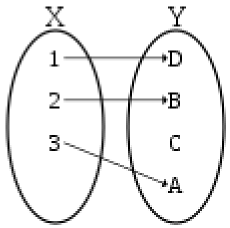
Surjective Functions



Surjective composition: the first function need not be surjective.



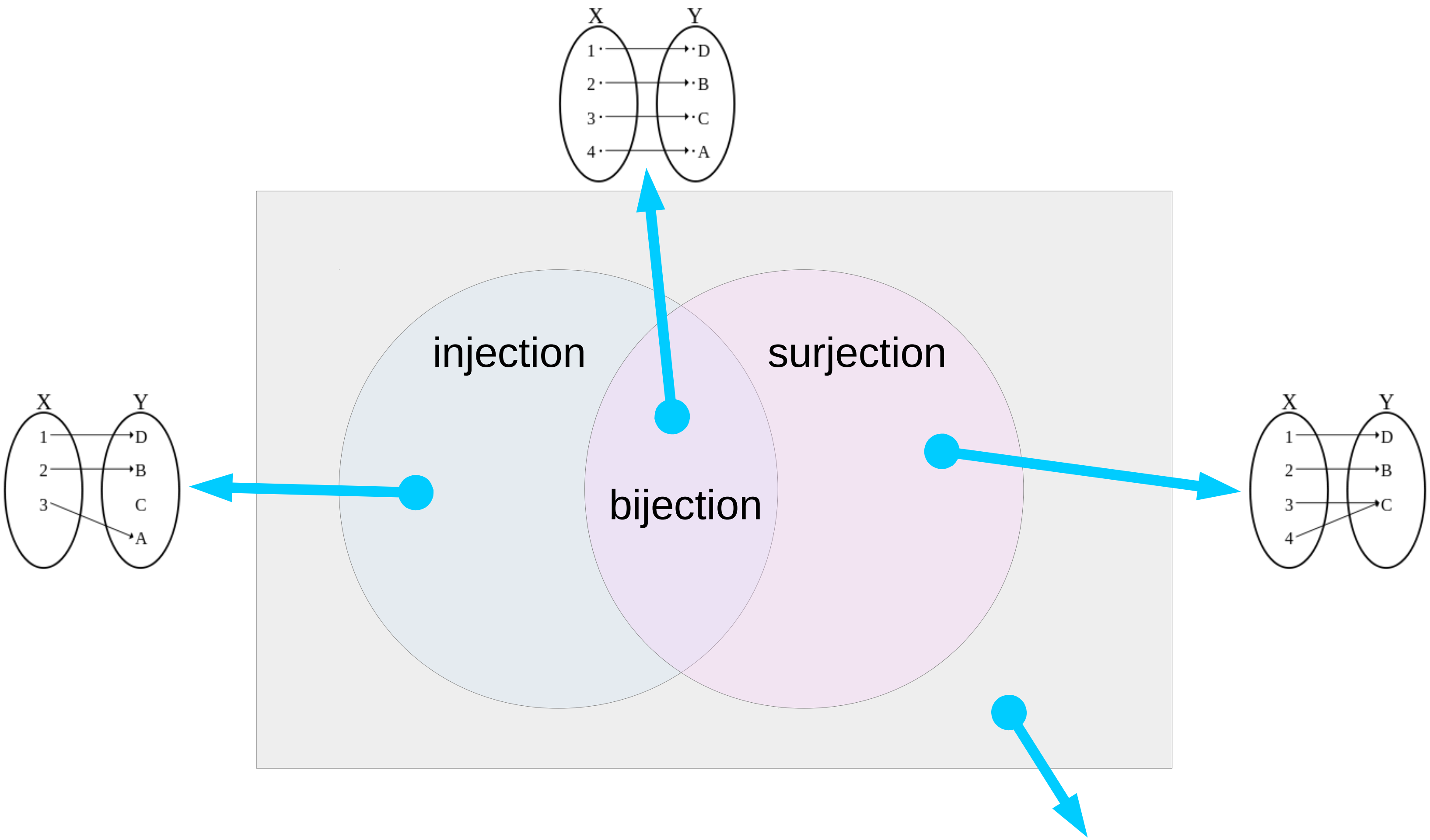
Another surjective function. (This one happens to be a bijection)

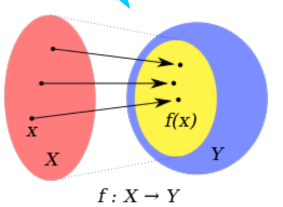


A non-surjective function. (This one happens to be an injection)

https://en.wikipedia.org/wiki/Function_(mathematics)

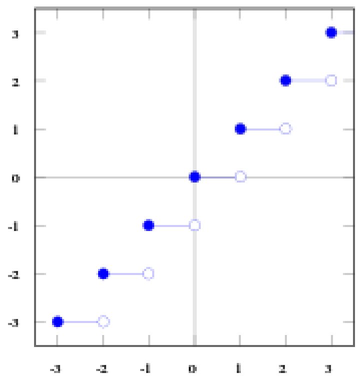
Types of Functions

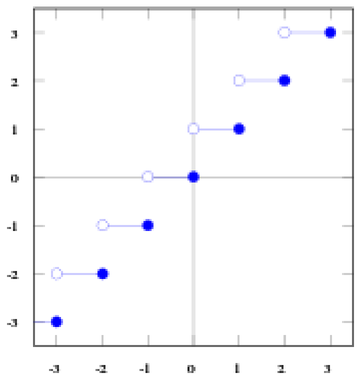




https://en.wikipedia.org/wiki/Function_(mathematics)

Floor and Ceiling Functions





x	Floor $\lfloor x \rfloor$	Ceiling $\lceil x \rceil$	Fractional part $\{x\}$
2	2	2	0
2.4	2	3	0.4
2.9	2	3	0.9
−2.7	−3	−2	0.3
−2	−2	−2	0

https://en.wikipedia.org/wiki/Function_(mathematics)

Floor and Ceiling Functions

In the following formulas, *x* and *y* are real numbers, *k*, *m*, and *n* are integers, and $\mathbb{Z}$ is the set of integers (positive, negative, and zero).

Floor and ceiling may be defined by the set equations

$$\lfloor x \rfloor = \max\{m \in \mathbb{Z} \mid m \le x\},$$
$$\lceil x \rceil = \min\{n \in \mathbb{Z} \mid n \ge x\}.$$

Since there is exactly one integer in a half-open interval of length one, for any real *x* there are unique integers *m* and *n* satisfying

$$x - 1 < m \le x \le n < x + 1.$$

Then $\lfloor x \rfloor = m$ and $\lceil x \rceil = n$ may also be taken as the definition of floor and ceiling.

https://en.wikipedia.org/wiki/Function_(mathematics)

Floor and Ceiling Functions

the floor function is the function that takes as input a real number x and gives as output the **greatest** integer less than or equal to x, denoted **floor(x) = $\lfloor x \rfloor$**.

Similarly, the ceiling function maps x to the **least** integer greater than or equal to x, denoted **ceiling(x) = $\lceil x \rceil$**.

https://en.wikipedia.org/wiki/Function_(mathematics)

Transform

In mathematics, particularly in semigroup theory, a **transformation** is any function *f* mapping a set *X* to itself, i.e. $f:X \to X$.[1][2][3] In other areas of mathematics, a transformation may simply be any function, regardless of domain and codomain.[4] This wider sense shall not be considered in this article; refer instead to the article on function for that sense.

Examples include linear transformations and affine transformations, rotations, reflections and translations. These can be carried out in Euclidean space, particularly in dimensions 2 and 3. They are also operations that can be performed using linear algebra, and described explicitly using matrices.

http://en.wikipedia.org/wiki/Derivative

Linear Map

In mathematics, a **linear map** (also called a **linear mapping**, **linear transformation** or, in some contexts, **linear function**) is a mapping $V \to W$ between two modules (including vector spaces) that preserves (in the sense defined below) the operations of addition and scalar multiplication. Linear maps can generally be represented as matrices, and simple examples include rotation and reflection linear transformations.

An important special case is when $V = W$, in which case the map is called a **linear operator**, or an endomorphism of V. Sometimes the term *linear function* has the same meaning as *linear map*, while in analytic geometry it does not.

A linear map always maps linear subspaces onto linear subspaces (possibly of a lower dimension); for instance it maps a plane through the origin to a plane, straight line or point.

In the language of abstract algebra, a linear map is a module homomorphism. In the language of category theory it is a morphism in the category of modules over a given ring.

http://en.wikipedia.org/wiki/Derivative

Linear Transform Matrices

- rotation by 90 degrees counterclockwise:

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- rotation by angle θ counterclockwise:

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

- reflection against the x axis:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- reflection against the y axis:

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

- scaling by 2 in all directions:

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

- horizontal shear mapping:

$$\mathbf{A} = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$$

- squeeze mapping:

$$\mathbf{A} = \begin{pmatrix} k & 0 \\ 0 & 1/k \end{pmatrix}$$

- projection onto the y axis:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

http://en.wikipedia.org/wiki/Derivative

References

[1] http://en.wikipedia.org/
[2]