

Laurent Series and z-Transform Examples case 0.B

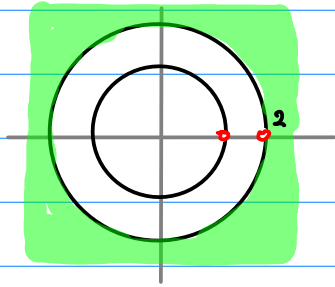
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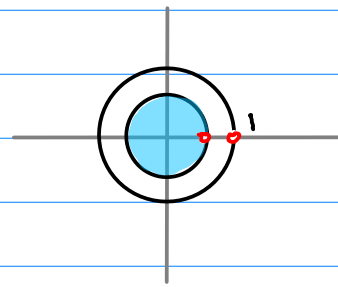
1. B

$$X(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

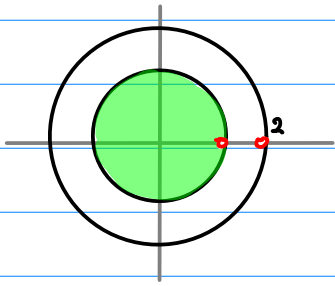


$$\sum_{n=-\infty}^{\infty} [1 - 2^{-n+1}] z^{-n}$$

≡

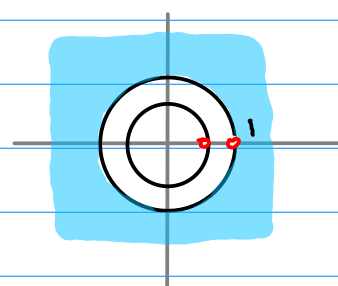


$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$

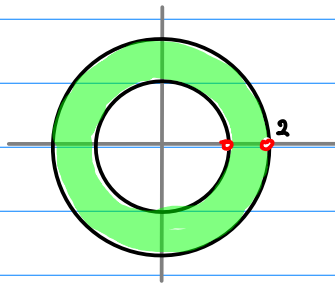


$$\sum_{n=-1}^{-\infty} [-1 + 2^{n+1}] z^{-n}$$

≡

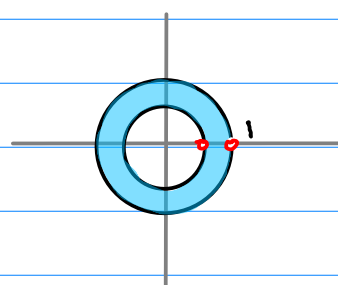


$$\sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$



$$\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n}$$

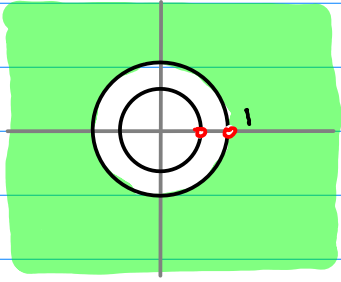
≡



$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$

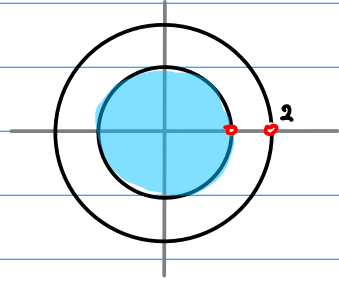
2. B

$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} f(z) = \frac{-1}{(z-1)(z-2)}$$

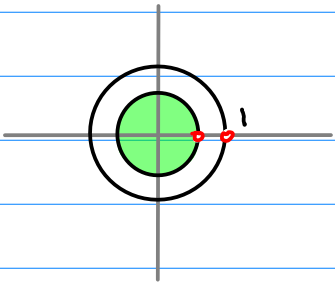


$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$

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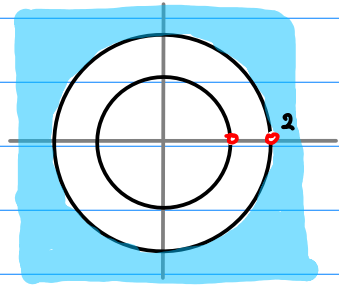


$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$

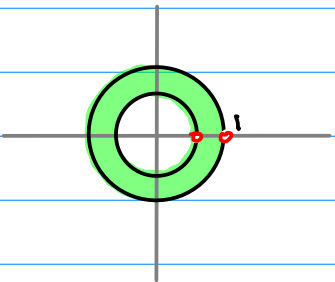


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

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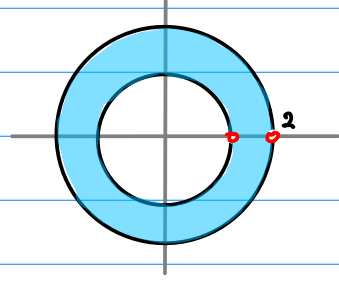


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

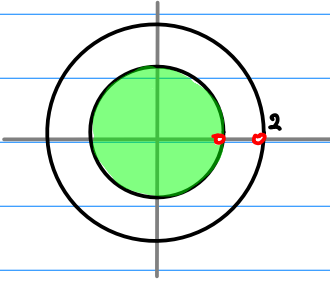
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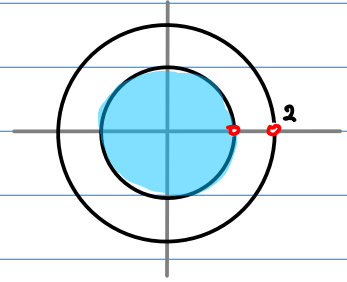
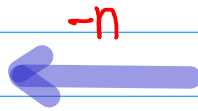
$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3.B

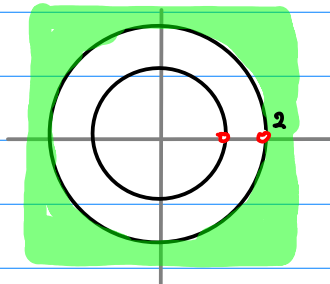
$$X(z) = \frac{-1}{(z-1)(z-2)} = f(z) = \frac{-1}{(z-1)(z-2)}$$



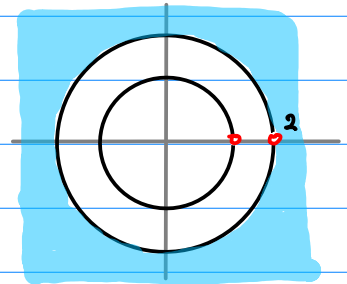
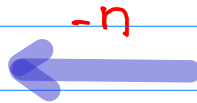
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



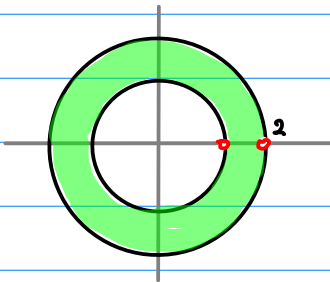
$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$$



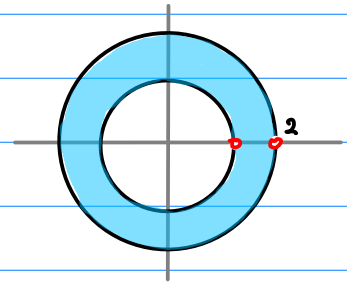
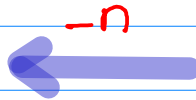
$$\sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^{-n}$$



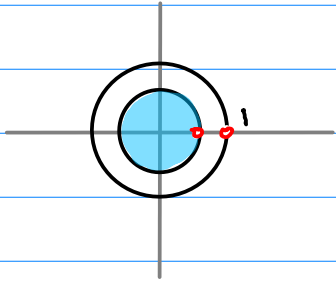
$$-\sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{n-1} z^{-n}$$



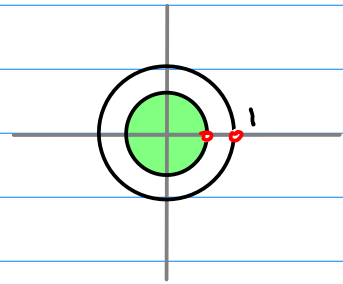
$$+\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}$$

4.B

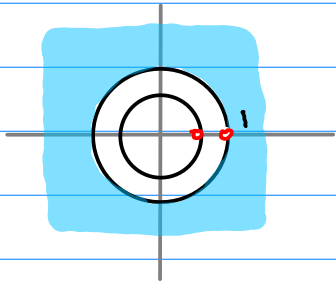
$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



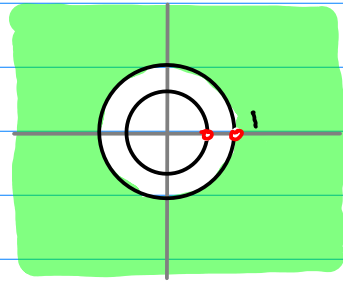
$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n \quad \leftarrow^{-n}$$



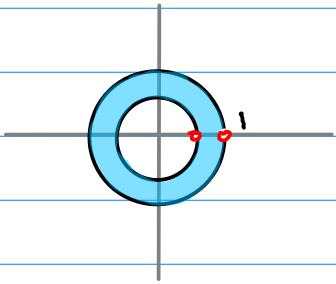
$$\sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$



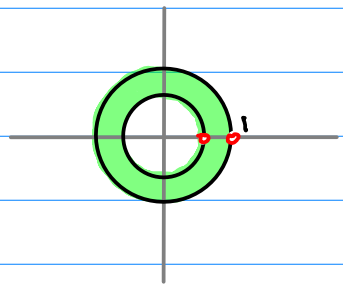
$$\sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n \quad \leftarrow^{-n}$$



$$\sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$



$$+\sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n \quad \leftarrow^{-n}$$



$$+\sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

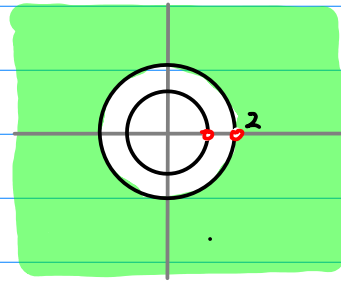
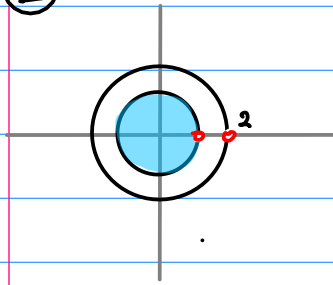
$$x_n = \begin{cases} 1 - 2^{n-1} & (\\ 0 & (\end{cases}$$

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (\\ 2^{n+1} - 1 & (\end{cases}$$

$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z)$$

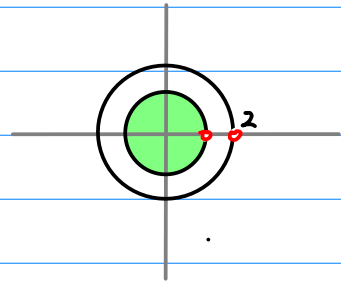
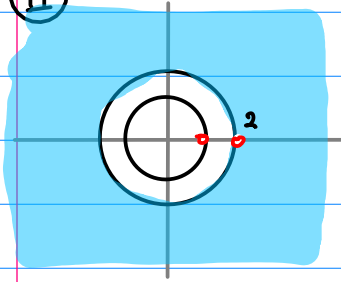
Ⓘ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

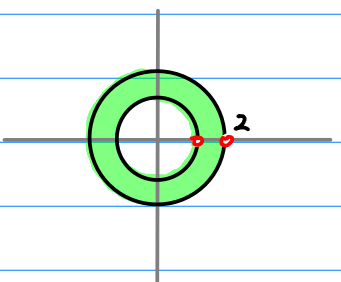
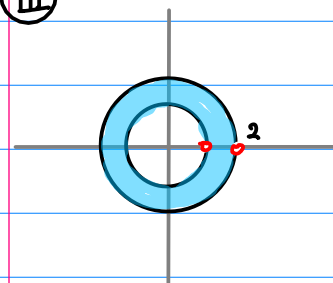
Ⓢ



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⓣ



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{-n+1} - 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$X(z) = \sum_{n=0}^{-\infty} \left(\frac{1}{2}\right)^{-n+1} z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

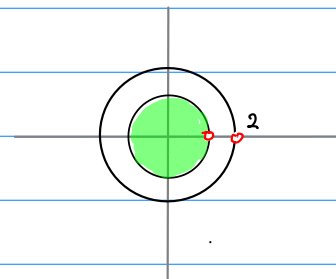
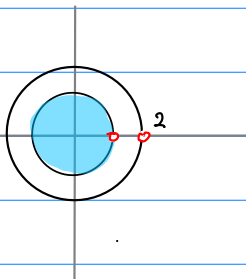
$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{1}{1 - z}$$

$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

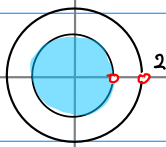
$$= \frac{-z+1+z-2}{(z-1)(z-2)}$$

$$= \frac{-1}{(z-1)(z-2)}$$

$$\left|\frac{z}{2}\right| < 1 \quad \left|\frac{z}{1}\right| < 1$$

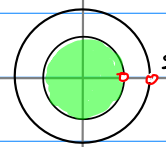


I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

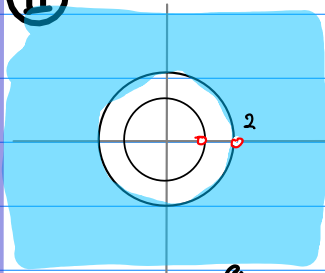
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

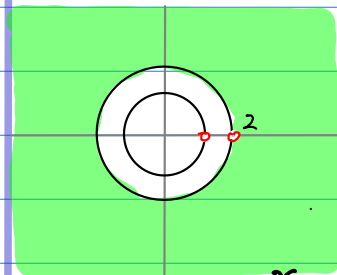
$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

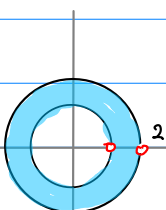
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

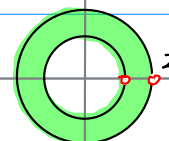
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} \cdot z^{-n}$$

III



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{(1)}{1 - \left(\frac{z}{1}\right)}$$

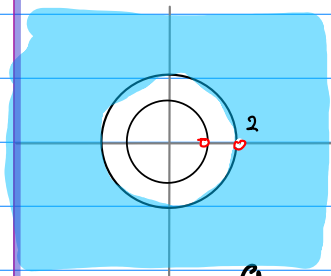
$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

$$= \frac{-z-1 + z-2}{(z-2)(z-1)}$$

$$= \frac{-1}{(z-2)(z-1)}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

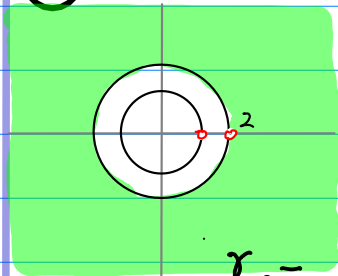
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$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

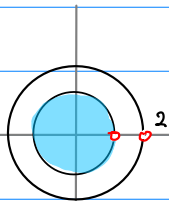
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

Ⓘ



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

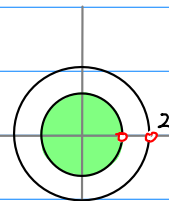
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

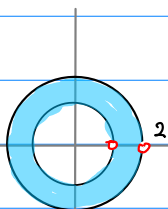
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

Ⓛ



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n+1} - 1 & (n \leq 0) \end{cases}$$

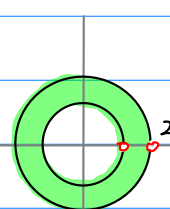
$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

Ⓜ



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n+1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n+1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$



