

Laurent Series and z-Transform - Geometric Series Applications

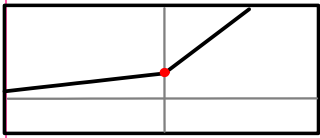


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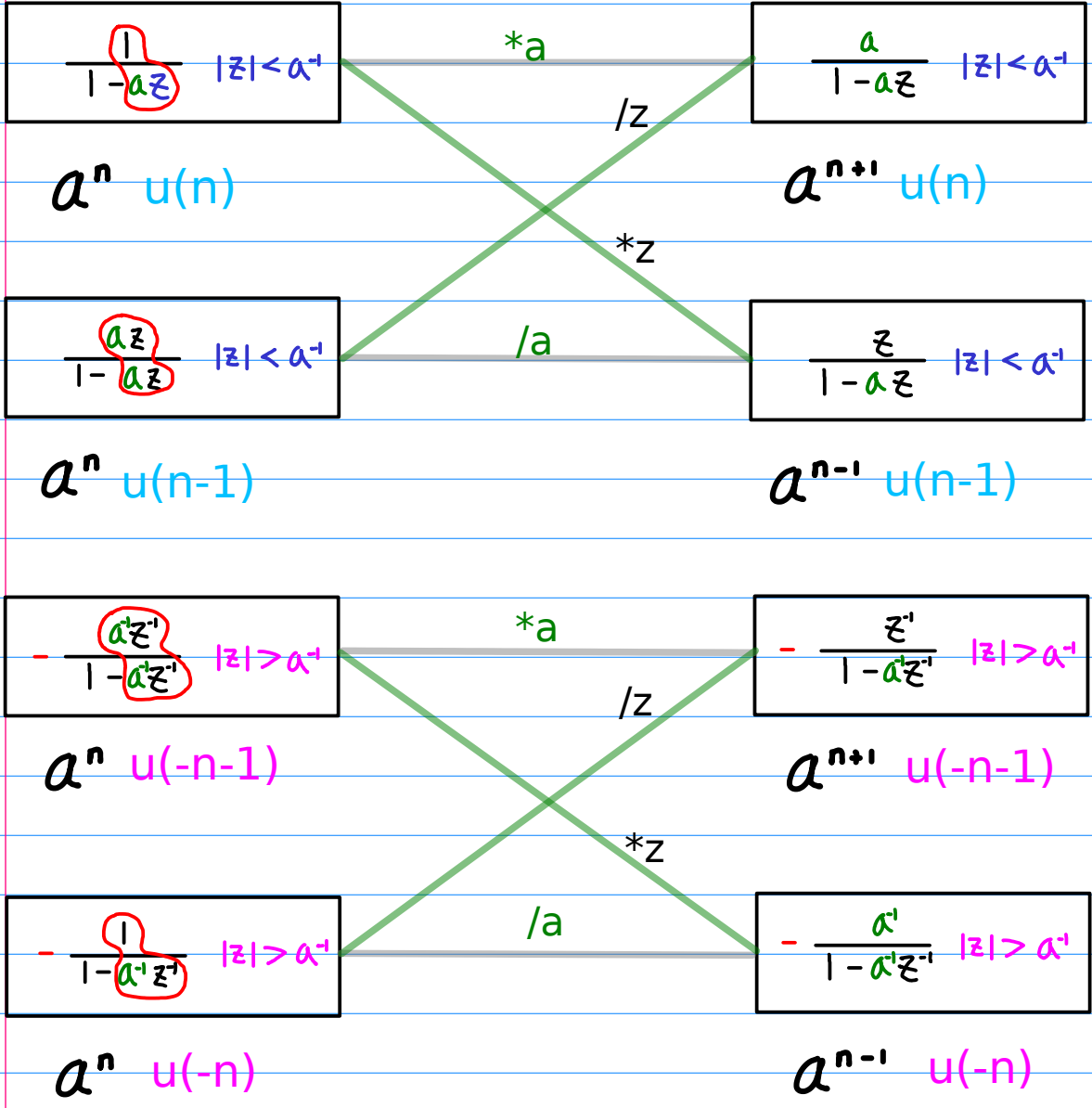
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a^n

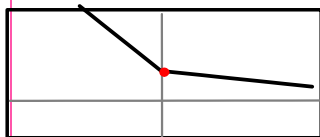


$*a$
 $/a$

Shifting a sequence



$$a^{-n}$$



/a

*a

Shifting a sequence

$$\frac{1}{1 - a^{-1}z} \quad |z| < a$$

$$a^{-n} u(n)$$

/a

/z

$$\frac{a^{-1}}{1 - a^{-1}z} \quad |z| < a$$

$$a^{-n-1} u(n)$$

*z

$$\frac{a^{-1}z}{1 - a^{-1}z} \quad |z| < a$$

$$a^{-n} u(n-1)$$

*a

$$\frac{z}{1 - a^{-1}z} \quad |z| < a$$

$$a^{-n+1} u(n-1)$$

$$-\frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a$$

$$a^{-n} u(-n-1)$$

/a

/z

$$-\frac{z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a$$

$$a^{-n-1} u(-n-1)$$

*z

$$-\frac{1}{1 - a^{-1}z^{-1}} \quad |z| > a$$

$$a^{-n} u(-n)$$

*a

$$-\frac{a}{1 - a^{-1}z^{-1}} \quad |z| > a$$

$$a^{-n+1} u(-n)$$

2 formulas

Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form

$$\begin{array}{l} \frac{1}{z - p} \begin{cases} \text{causal} \\ \text{anti-causal} \end{cases} \begin{cases} \frac{p^{-1}}{1 - p^{-1}z} \triangleq f(z) = \chi(z^{-1}) \\ \frac{z^{-1}}{1 - pz^{-1}} \triangleq \gamma(z) = g(z^{-1}) \end{cases} \end{array}$$

$$\begin{array}{l} \frac{1}{z^{-1} - p} \begin{cases} \text{causal} \\ \text{anti-causal} \end{cases} \begin{cases} -\frac{p^{-1}}{1 - p^{-1}z^{-1}} \triangleq \chi(z) = f(z^{-1}) \\ \frac{z}{1 - pz} \triangleq g(z) = \gamma(z^{-1}) \end{cases} \end{array}$$

Simple Pole Form

Geometric Series Form

Geometric Series (1)

2 formulas

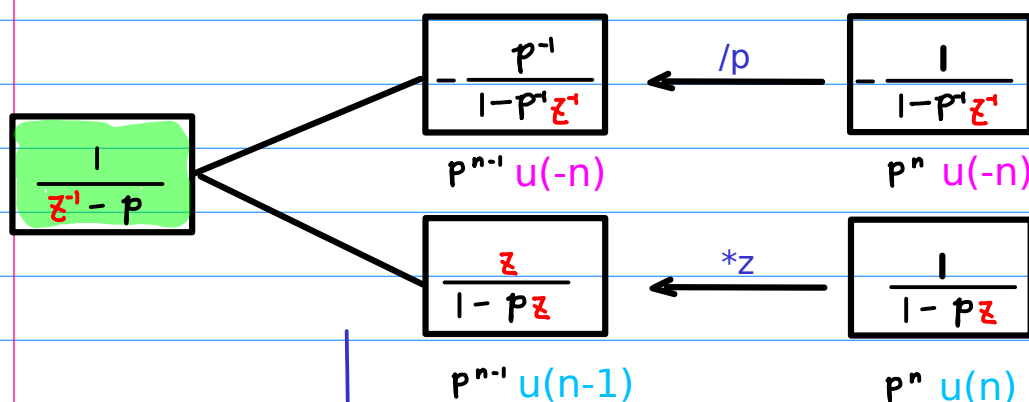
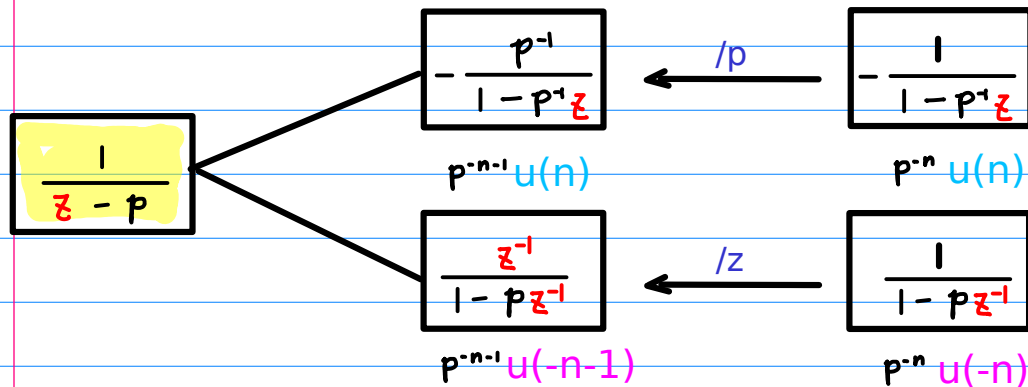
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series (2)

2 formulas

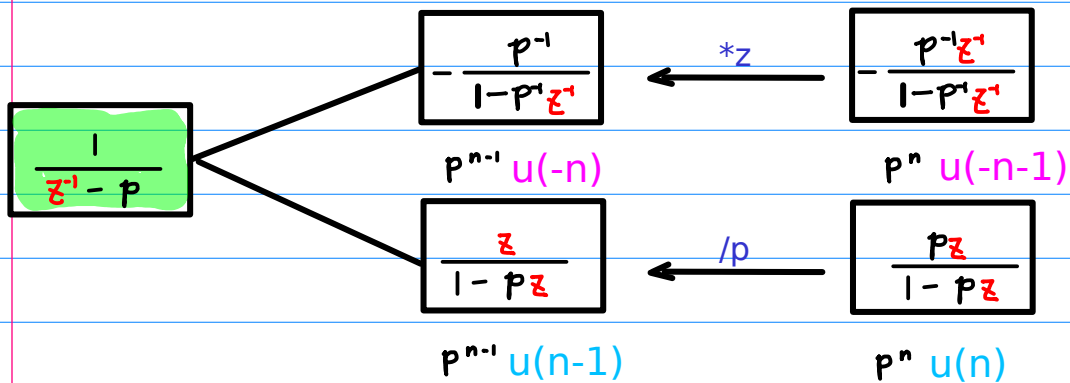
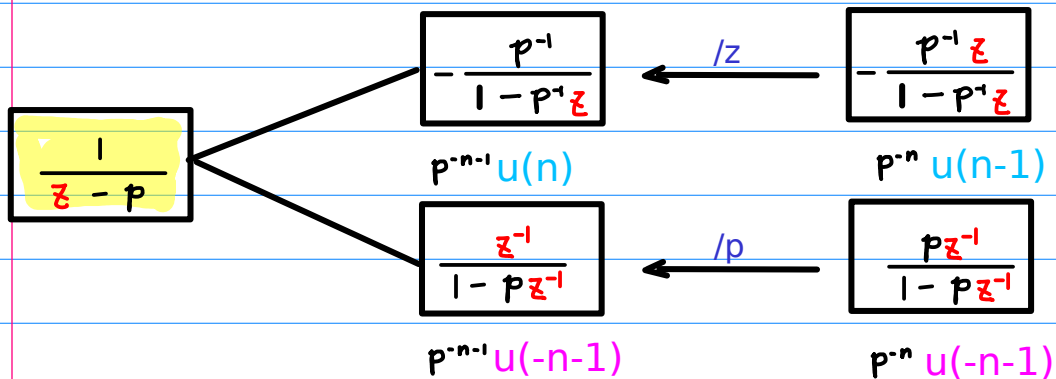
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series Form Combinations with a unit start term

$$-\frac{1}{1 - az} \quad -a^n u(n)$$

$$-\frac{1}{1 - az^{-1}} \quad -a^{-n} u(-n)$$

$$+\frac{1}{1 - a^{-1}z^{-1}} \quad a^n u(-n)$$

$$+\frac{1}{1 - a^{-1}z} \quad a^{-n} u(n)$$

$$-\frac{1}{1 - a^{-1}z} \quad -a^{-n} u(n)$$

$$-\frac{1}{1 - a^{-1}z^{-1}} \quad -a^n u(-n)$$

$$+\frac{1}{1 - az^{-1}} \quad a^{-n} u(-n)$$

$$+\frac{1}{1 - az} \quad a^n u(n)$$

Geometric Series Form Combinations with a common-ratio start term

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$- \frac{az}{1 - az}$$

$a^n u(n-1)$

$$- \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$+ \frac{az}{1 - az}$$

$a^n u(n-1)$

$$- \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

Geometric Series - a unit start term

Laurent Series

Geometric Series - a unit start term

z-Transform

Geometric Series - a unit start term

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

$$-\frac{1}{1-\underline{az}}$$

$$-(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$a_n = -a^n u(n) \quad (n \geq 0)$$

$$|z| < a^{-1}$$

$$-\frac{1}{1-\underline{az^{-1}}}$$

$$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$-(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \dots)$$

$$a_n = -\left(\frac{1}{a}\right)^n u(-n) \quad (n < 1)$$

$$|z| > a$$

$$+\frac{1}{1-\underline{a^{-1}z^{-1}}}$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$a_n = a^n u(-n) \quad (n < 1)$$

$$|z| > a^{-1}$$

$$+\frac{1}{1-\underline{a^{-1}z}}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \dots)$$

$$a_n = \left(\frac{1}{a}\right)^n u(n) \quad (n \geq 0)$$

$$|z| < a$$

$$-\frac{1}{1-\underline{a^{-1}z}}$$

$$-(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$-(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \dots)$$

$$a_n = -\left(\frac{1}{a}\right)^n u(n) \quad (n \geq 0)$$

$$|z| < a$$

$$-\frac{1}{1-\underline{a^{-1}z^{-1}}}$$

$$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$a_n = -a^n u(-n) \quad (n < 1)$$

$$|z| > a^{-1}$$

$$+\frac{1}{1-\underline{az^{-1}}}$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \dots)$$

$$a_n = \left(\frac{1}{a}\right)^n u(-n) \quad (n < 1)$$

$$|z| > a$$

$$+\frac{1}{1-\underline{az}}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$a_n = a^n u(n) \quad (n \geq 0)$$

$$|z| < a^{-1}$$

Geometric Series - a unit start term

z-Transform

$$-\frac{1}{1-\underline{a}z}$$

$$|z| < a^{-1}$$

$$- (a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = -\underline{a}^n u(-n) \quad (n \geq 0)$$

$$a_n = -(\frac{1}{a})^n u(-n) \quad (n < 1)$$

$$-\frac{1}{1-\underline{a}z^{-1}}$$

$$|z| > a$$

$$- (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -(\frac{1}{a})^n u(-(-n)) \quad (n < 1)$$

$$a_n = -\underline{a}^n u(n) \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}^{-1}z}$$

$$|z| > a^{-1}$$

$$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = \underline{a}^n u(-(-n)) \quad (n < 1)$$

$$a_n = (\frac{1}{a})^n u(n) \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}^{-1}z^{-1}}$$

$$|z| < a$$

$$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = (\frac{1}{a})^{-n} u(-n) \quad (n \geq 0)$$

$$a_n = \underline{a}^n u(-n) \quad (n < 1)$$

$$-\frac{1}{1-\underline{a}^{-1}z}$$

$$|z| < a$$

$$- (a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -(\frac{1}{a})^{-n} u(-n) \quad (n \geq 0)$$

$$a_n = -\underline{a}^n u(-n) \quad (n < 1)$$

$$-\frac{1}{1-\underline{a}^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$- (a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -\underline{a}^n u(-(-n)) \quad (n < 1)$$

$$a_n = -(\frac{1}{a})^n u(n) \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}z^{-1}}$$

$$|z| > a$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = (\frac{1}{a})^{-n} u(-(-n)) \quad (n < 1)$$

$$a_n = \underline{a}^n u(n) \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}z}$$

$$|z| < a^{-1}$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = \underline{a}^n u(-n) \quad (n \geq 0)$$

$$a_n = (\frac{1}{a})^n u(-n) \quad (n < 1)$$

Geometric Series - a unit start term

Laurent Series vs. z-Transform

$$-\frac{1}{1-\underline{a}z}$$

$$|z| < a^{-1}$$

$$-\frac{1}{1-\underline{a}z^{-1}}$$

$$|z| > a$$

$$- (a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$- (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

Laurent

$$a_n = -a^n \quad (n \geq 0)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 1)$$

z-Trans

$$a_n = -(\frac{1}{a})^n \quad (n < 1)$$

$$a_n = -a^n \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$+\frac{1}{1-\underline{a}^{-1}z}$$

$$|z| < a$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

Laurent

$$a_n = a^n \quad (n < 1)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 0)$$

z-Trans

$$a_n = (\frac{1}{a})^n \quad (n \geq 0)$$

$$a_n = a^n \quad (n < 1)$$

$$-\frac{1}{1-\underline{a}^{-1}z}$$

$$|z| < a$$

$$-\frac{1}{1-\underline{a}^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$- (a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$- (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

Laurent

$$a_n = -(\frac{1}{a})^n \quad (n \geq 0)$$

$$a_n = -a^n \quad (n < 1)$$

z-Trans

$$a_n = -a^n \quad (n < 1)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 0)$$

$$+\frac{1}{1-\underline{a}z^{-1}}$$

$$|z| > a$$

$$+\frac{1}{1-\underline{a}z}$$

$$|z| < a^{-1}$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

Laurent

$$a_n = (\frac{1}{a})^n \quad (n < 1)$$

$$a_n = a^n \quad (n \geq 0)$$

z-Trans

$$a_n = a^n \quad (n \geq 0)$$

$$a_n = (\frac{1}{a})^n \quad (n < 1)$$

Geometric Series - a non-unit start term
Laurent Series

Geometric Series - a non-unit start term
z-Transform

Geometric Series - a non-unit start term
Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$|z| > a^{-1}$$

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$|z| < a$$

$$((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$a_n = a^n u(-n-1) \quad (n < 0)$$

$$a_n = (\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

$$- \frac{az}{1 - az}$$

$$- (a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$|z| < a^{-1}$$

$$- \frac{az^{-1}}{1 - az^{-1}}$$

$$- (a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$|z| > a$$

$$- ((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$a_n = -a^n u(n-1) \quad (n \geq 1)$$

$$a_n = -(\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$$(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$|z| > a$$

$$((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$+ \frac{az}{1 - az}$$

$$(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$|z| < a^{-1}$$

$$a_n = (\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

$$a_n = a^n u(n-1) \quad (n \geq 1)$$

$$- \frac{a^{-1}z}{1 - a^{-1}z}$$

$$- (a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$|z| < a$$

$$- ((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$- (a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$|z| > a^{-1}$$

$$a_n = -(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

$$a_n = -a^n u(-n-1) \quad (n < 0)$$

Geometric Series - a non-unit start term

z-Transform

$$+ \frac{a^{-1} z^{-1}}{1 - a^{-1} z^{-1}}$$

$$|z| > a^{-1}$$

$$(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \dots)$$

$$a_n = a^{-n} u(-n-1) \quad (n < 0)$$

$$a_n = (\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

$$+ \frac{a^{-1} z}{1 - a^{-1} z}$$

$$|z| < a$$

$$(a^{-1} z^1 + a^{-2} z^2 + a^{-3} z^3 + \dots)$$

$$((\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + (\frac{1}{a})^{-3} z^3 + \dots)$$

$$a_n = (\frac{1}{a})^{-n} u(-n-1) \quad (n \geq 1)$$

$$a_n = a^n u(-n-1) \quad (n < 0)$$

$$- \frac{a z}{1 - a z}$$

$$|z| < a^{-1}$$

$$- (a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$- ((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$a_n = -a^n u(-n-1) \quad (n \geq 1)$$

$$a_n = -(\frac{1}{a})^n u(n-1) \quad (n < 0)$$

$$- \frac{a z^{-1}}{1 - a z^{-1}}$$

$$|z| > a$$

$$- (a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$- ((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$a_n = -(\frac{1}{a})^{-n} u(-n-1) \quad (n < 0)$$

$$a_n = -a^n u(n-1) \quad (n \geq 1)$$

$$+ \frac{a z^{-1}}{1 - a z^{-1}}$$

$$|z| > a$$

$$(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \dots)$$

$$a_n = (\frac{1}{a})^{-n} u(-n-1) \quad (n < 0)$$

$$a_n = a^n u(n-1) \quad (n \geq 1)$$

$$+ \frac{a z}{1 - a z}$$

$$|z| < a^{-1}$$

$$(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$((\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + (\frac{1}{a})^{-3} z^3 + \dots)$$

$$a_n = a^{-n} u(-n-1) \quad (n \geq 1)$$

$$a_n = (\frac{1}{a})^n u(-n-1) \quad (n < 0)$$

$$- \frac{a^{-1} z}{1 - a^{-1} z}$$

$$|z| < a$$

$$- (a^{-1} z^1 + a^{-2} z^2 + a^{-3} z^3 + \dots)$$

$$- ((\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + (\frac{1}{a})^{-3} z^3 + \dots)$$

$$a_n = -(\frac{1}{a})^{-n} u(-n-1) \quad (n \geq 1)$$

$$a_n = -a^n u(n-1) \quad (n < 0)$$

$$- \frac{a^{-1} z^{-1}}{1 - a^{-1} z^{-1}}$$

$$|z| > a^{-1}$$

$$- (a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \dots)$$

$$- ((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \dots)$$

$$a_n = -a^{-n} u(-n-1) \quad (n < 0)$$

$$a_n = -(\frac{1}{a})^n u(n-1) \quad (n \geq 1)$$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$	$ z > a^{-1}$	$+ \frac{a^{-1}z}{1 - a^{-1}z}$	$ z < a$
-------------------------------------------	----------------	---------------------------------	-----------

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = a^n \quad (n < 0)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 1)$$

z-Trans

$$a_n = (\frac{1}{a})^n \quad (n \geq 1)$$

$$a_n = a^n \quad (n < 0)$$

$- \frac{az}{1 - az}$	$ z < a^{-1}$	$- \frac{az^{-1}}{1 - az^{-1}}$	$ z > a$
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$$- (a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$- ((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$$- (a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$- ((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

Laurent

$$a_n = -a^n \quad (n \geq 1)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

z-Trans

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$a_n = -a^n \quad (n \geq 1)$$

$+ \frac{az^{-1}}{1 - az^{-1}}$	$ z > a$	$+ \frac{az}{1 - az}$	$ z < a^{-1}$
---------------------------------	-----------	-----------------------	----------------

$$(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

$$(a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$$a_n = a^n \quad (n \geq 1)$$

z-Trans

$$a_n = a^n \quad (n \geq 1)$$

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$- \frac{a^{-1}z}{1 - a^{-1}z}$	$ z < a$	$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$	$ z > a^{-1}$
---------------------------------	-----------	-------------------------------------------	----------------

$$- (a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$- ((\frac{1}{a})^{-1}z^1 + (\frac{1}{a})^{-2}z^2 + (\frac{1}{a})^{-3}z^3 + \dots)$$

$$- (a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$- ((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

Laurent

$$a_n = -(\frac{1}{a})^n \quad (n \geq 1)$$

$$a_n = -a^n \quad (n < 0)$$

z-Trans

$$a_n = -a^n \quad (n < 0)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 1)$$

Complement ROC Pairs - Original Geometric Series Form Combinations

unit	$-\frac{1}{1-a^*z} \quad z < a^*$	$-a^n \quad (n \geq 0)$	$-\frac{1}{1-a^*z^*} \quad z > a$	$-(\frac{1}{a})^n \quad (n < 1)$
non-unit	$\frac{a^*z^*}{1-a^*z^*} \quad z > a^*$	$a^n \quad (n < 0)$	$\frac{a^*z}{1-a^*z} \quad z < a$	$(\frac{1}{a})^n \quad (n \geq 1)$

unit	$\frac{1}{1-a^*z^*} \quad z > a^*$	$a^n \quad (n < 1)$	$\frac{1}{1-a^*z} \quad z < a$	$(\frac{1}{a})^n \quad (n \geq 0)$
non-unit	$-\frac{az}{1-az} \quad z < a^*$	$-a^n \quad (n \geq 1)$	$\frac{az^*}{1-az^*} \quad z > a$	$-(\frac{1}{a})^n \quad (n < 0)$

unit	$-\frac{1}{1-a^*z} \quad z < a$	$-(\frac{1}{a})^n \quad (n \geq 0)$	$-\frac{1}{1-a^*z^*} \quad z > a^*$	$-a^n \quad (n < 1)$
non-unit	$\frac{az^*}{1-az^*} \quad z > a$	$(\frac{1}{a})^n \quad (n < 0)$	$\frac{az}{1-az} \quad z < a^*$	$a^n \quad (n \geq 1)$

unit	$\frac{1}{1-az^*} \quad z > a$	$(\frac{1}{a})^n \quad (n < 1)$	$\frac{1}{1-az} \quad z < a^*$	$a^n \quad (n \geq 0)$
non-unit	$-\frac{a^*z}{1-a^*z} \quad z < a$	$-(\frac{1}{a})^n \quad (n \geq 1)$	$-\frac{a^*z^*}{1-a^*z^*} \quad z > a^*$	$-a^n \quad (n < 0)$

start term

Complement ROC Pairs - Shifted Geometric Series Form Combinations

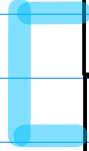
$-\frac{a}{1-a^2} \quad z < a^{-1}$	$-a^{n+1} \quad (n \geq 0)$	$-\frac{a}{1-a^2} \quad z > a$	$-(\frac{1}{a})^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$a^{n+1} \quad (n < 0)$	$\frac{z}{1-a^2 z} \quad z < a$	$(\frac{1}{a})^{n-1} \quad (n \geq 1)$

$\frac{z^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$a^{n+1} \quad (n < 0)$	$\frac{z}{1-a^2 z} \quad z < a$	$(\frac{1}{a})^{n-1} \quad (n \geq 1)$
$-\frac{a}{1-a^2} \quad z < a^{-1}$	$-a^{n+1} \quad (n \geq 0)$	$\frac{a}{1-a^2} \quad z > a$	$-(\frac{1}{a})^{n-1} \quad (n < 1)$

$-\frac{a^{-1}}{1-a^2 z} \quad z < a$	$-(\frac{1}{a})^{n+1} \quad (n \geq 0)$	$-\frac{a^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$-a^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$(\frac{1}{a})^{n+1} \quad (n < 0)$	$\frac{z}{1-a^2 z} \quad z < a$	$a^{n-1} \quad (n \geq 1)$

$\frac{z^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$(\frac{1}{a})^{n+1} \quad (n < 0)$	$\frac{z}{1-a^2 z} \quad z < a$	$a^{n-1} \quad (n \geq 1)$
$-\frac{a^{-1}}{1-a^2 z} \quad z < a$	$-(\frac{1}{a})^{n+1} \quad (n \geq 0)$	$-\frac{a^{-1}}{1-a^2 z^{-1}} \quad z > a^{-1}$	$-a^{n-1} \quad (n < 1)$

Complement ROC Pairs - Reduced Shifted Geometric Series Form Combinations



$-\frac{a}{1-az} \quad z < a^{-1}$	$-a^{n+1} \quad (n \geq 0)$	$-\frac{a}{1-az^{-1}} \quad z > a$	$-(\frac{1}{a})^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^{n+1} \quad (n < 0)$	$\frac{z}{1-a^{-1}z} \quad z < a$	$(\frac{1}{a})^{n-1} \quad (n \geq 1)$



$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-(\frac{1}{a})^{n+1} \quad (n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$(\frac{1}{a})^{n+1} \quad (n < 0)$	$\frac{z}{1-az} \quad z < a^{-1}$	$a^{n-1} \quad (n \geq 1)$

$$2z$$

$$2z^{-1}$$

$$2^{-1}z^{-1}$$

$$2^{-1}z$$

$$|z| < 0.5$$

$$|z| > 2$$

$$|z| > 0.5$$

$$|z| < 2$$

$$- \frac{2}{1 - 2z} \xleftrightarrow{z^{-1}} - \frac{2}{1 - 2z^{-1}}$$

$$\cdot \frac{(2z)^{-1}}{(2z)^{-1}} \cdot \frac{(2z)}{(2z)}$$

$$\cdot \frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \cdot \frac{(2z^{-1})}{(2z^{-1})}$$

$$+ \frac{z^{-1}}{1 - 0.5z^{-1}} \xleftrightarrow{z^{-1}} + \frac{z}{1 - 0.5z}$$

	scale(a)		scale(a)	
	$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\frac{a}{1-az^{-1}} \quad z > a$
Comp.ROC	$\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{a^1 z}{1-a^1 z} \quad z < a$	$\frac{z}{1-a^1 z} \quad z < a$
	scale(1/z)		scale(z)	
	$\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^1 z} \quad z < a$	$\frac{z}{1-a^1 z} \quad z < a$
Comp.ROC	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{a}{1-az^{-1}} \quad z > a$
	scale(1/a)		scale(1/a)	
	$-\frac{1}{1-a^1 z} \quad z < a$	$-\frac{a^1}{1-a^1 z} \quad z < a$	$-\frac{1}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-\frac{a^1}{1-a^1 z^{-1}} \quad z > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
	scale(1/z)		scale(z)	
	$\frac{1}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
Comp.ROC	$-\frac{a^1 z}{1-a^1 z} \quad z < a$	$-\frac{a^1}{1-a^1 z} \quad z < a$	$-\frac{a^1 z^{-1}}{1-a^1 z^{-1}} \quad z > a^{-1}$	$-\frac{a^1}{1-a^1 z^{-1}} \quad z > a^{-1}$

scale(a)

scale(a)

Comp.ROC

$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$-\frac{a}{1-a^{-1}z^{-1}} \quad z > a$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{a}{1-a^{-1}z^{-1}} \quad z > a$

scale(1/a)

scale(1/a)

Comp.ROC

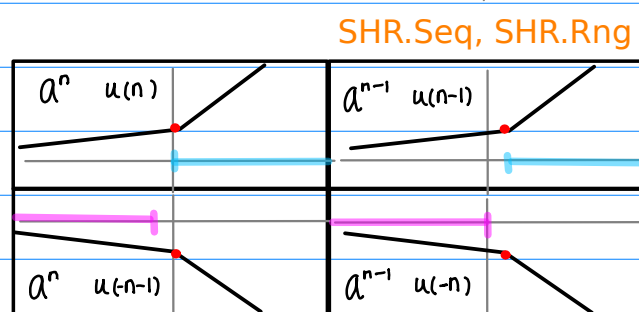
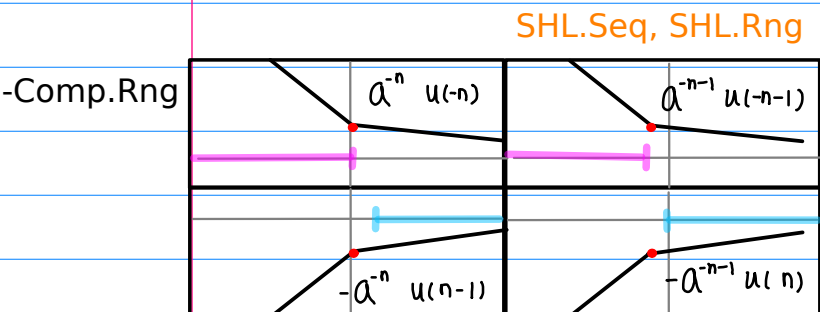
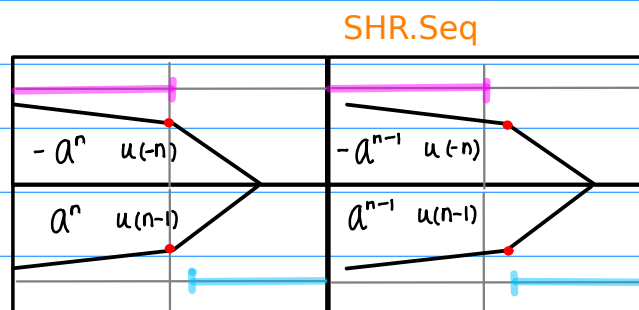
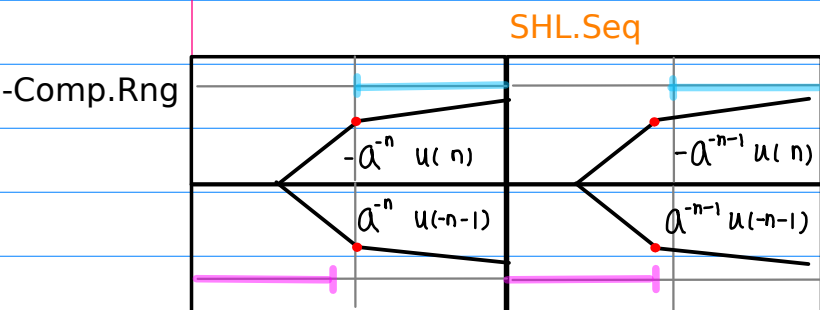
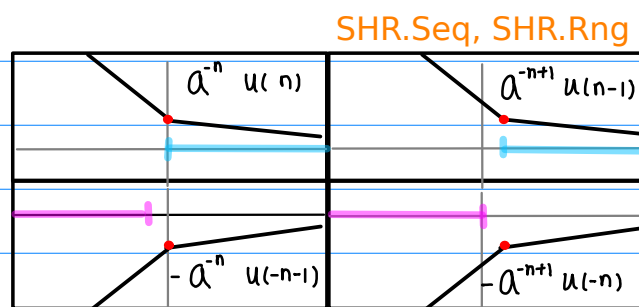
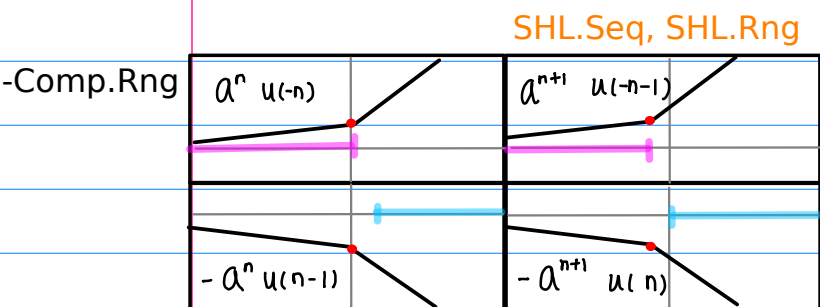
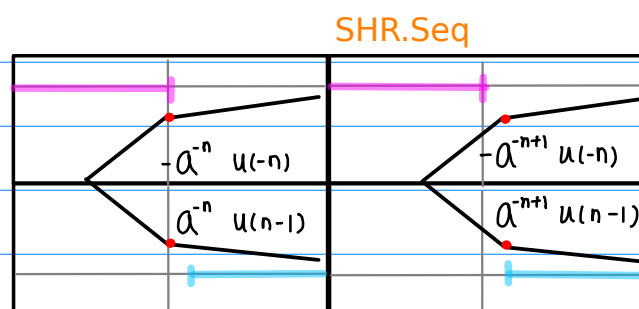
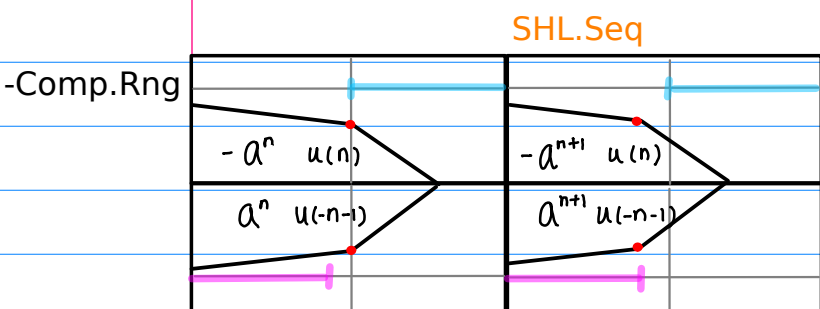
$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a^{-1}$	$\frac{z}{1-a^{-1}z} \quad z < a^{-1}$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{1}{1-a^{-1}z} \quad z < a^{-1}$	$\frac{z}{1-a^{-1}z} \quad z < a^{-1}$
$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$



Left Shifted
Sequence

Right Shifted
Sequence

	scale(a)		scale(a)	
Comp.ROC	$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\frac{a}{1-az^{-1}} \quad z > a$
	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{a}{1-az^{-1}} \quad z > a$
	scale(1/a)		scale(1/a)	
Comp.ROC	$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
	scale(1/z)		scale(z)	
Comp.ROC	$\frac{1}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
	$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$

	SHL.Seq		SHR.Seq	
-Comp.Rng	$-a^n \ (n \geq 0)$ $-(a^0, a^1, a^2, \dots)$	$-a^{n+1} \ (n \geq 0)$ $-(a^1, a^2, a^3, \dots)$	$-(\frac{1}{a})^n \ (n < 1)$ $-(\dots, a^0, a^1, a^0)$	$-(\frac{1}{a})^{n-1} \ (n < 1)$ $-(\dots, a^3, a^2, a^1)$
	$a^n \ (n < 0)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$a^{n+1} \ (n < 0)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$(\frac{1}{a})^n \ (n \geq 1)$ $(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$

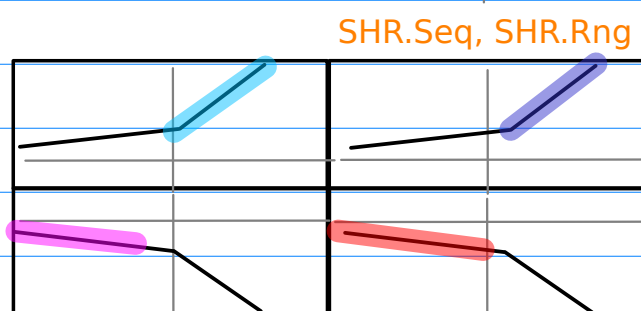
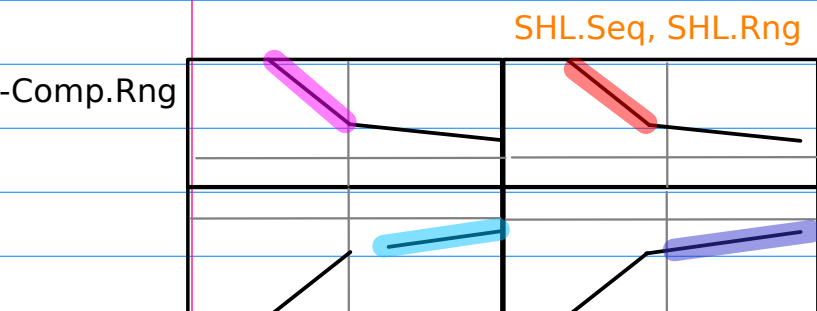
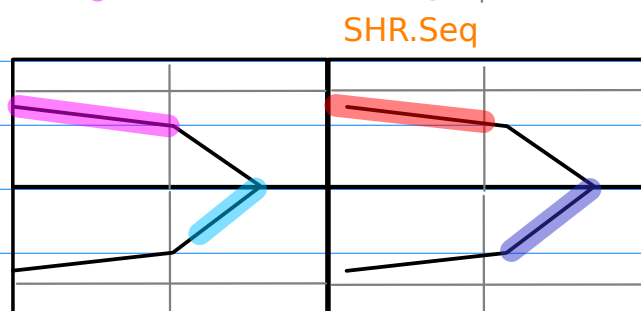
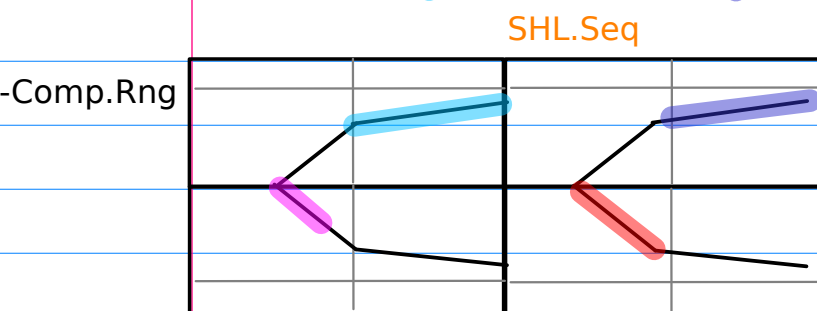
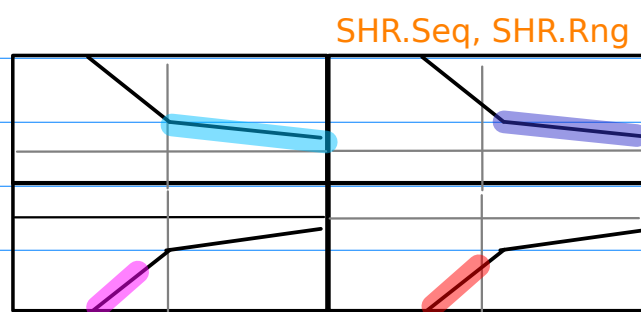
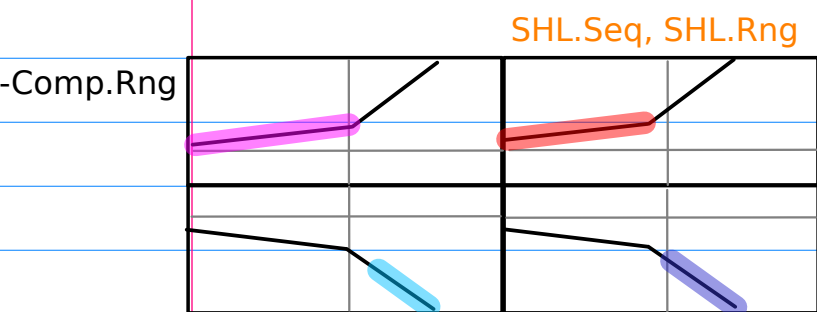
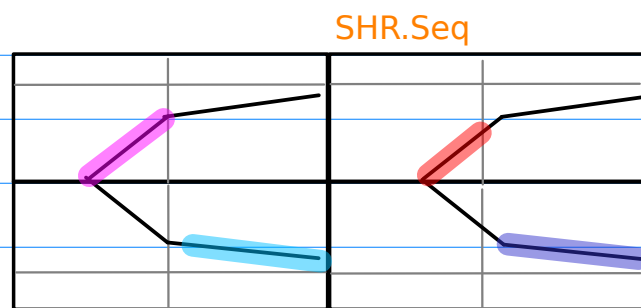
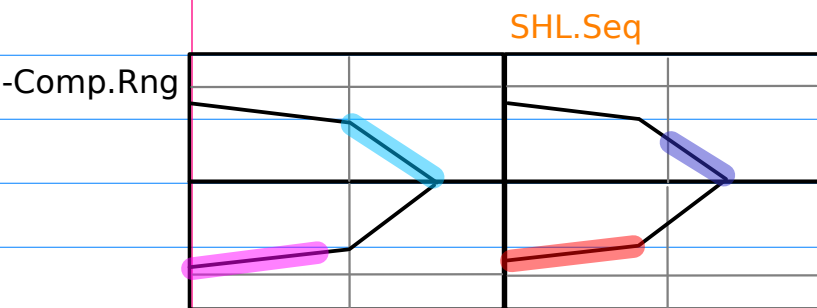
	SHL.Seq, SHL.Rng		SHR.Seq, SHR.Rng	
-Comp.Rng	$a^n \ (n < 1)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$a^{n+1} \ (n < 0)$ $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$(\frac{1}{a})^n \ (n \geq 0)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$ $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$
	$-a^n \ (n \geq 1)$ $-(a^1, a^2, a^3, \dots)$	$-a^{n+1} \ (n \geq 0)$ $-(a^1, a^2, a^3, \dots)$	$-(\frac{1}{a})^n \ (n < 0)$ (\dots, a^3, a^2, a^1)	$-(\frac{1}{a})^{n-1} \ (n < 1)$ (\dots, a^3, a^2, a^1)

	SHL.Seq		SHR.Seq	
-Comp.Rng	$-(\frac{1}{a})^n \ (n \geq 0)$ $-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$-a^n \ (n < 1)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1} \ (n < 1)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$
	$(\frac{1}{a})^n \ (n < 0)$ (\dots, a^3, a^2, a^1)	$(\frac{1}{a})^{n+1} \ (n < 0)$ (\dots, a^2, a^1, a^0)	$a^n \ (n \geq 1)$ (a^1, a^2, a^3, \dots)	$a^{n-1} \ (n \geq 1)$ (a^0, a^1, a^2, \dots)

	SHL.Seq, SHL.Rng		SHR.Seq, SHR.Rng	
-Comp.Rng	$(\frac{1}{a})^n \ (n < 1)$ (\dots, a^2, a^1, a^0)	$(\frac{1}{a})^{n+1} \ (n < 0)$ (\dots, a^2, a^1, a^0)	$a^n \ (n \geq 0)$ (a^0, a^1, a^2, \dots)	$a^{n-1} \ (n \geq 1)$ (a^0, a^1, a^2, \dots)
	$-(\frac{1}{a})^n \ (n \geq 1)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$ $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$-a^n \ (n < 0)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1} \ (n < 1)$ $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$

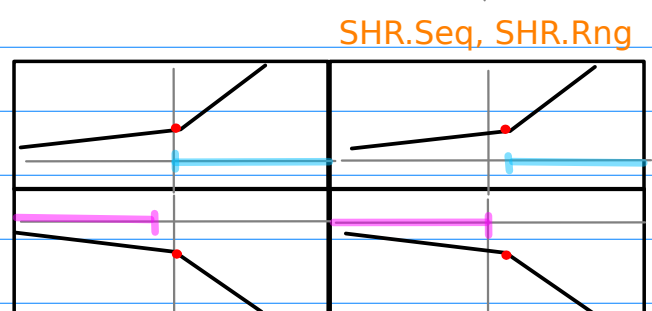
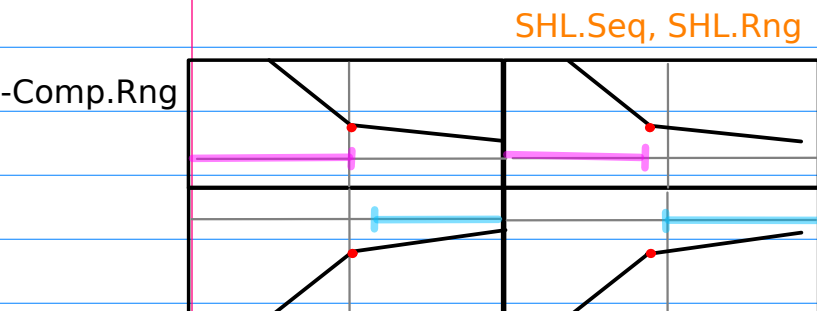
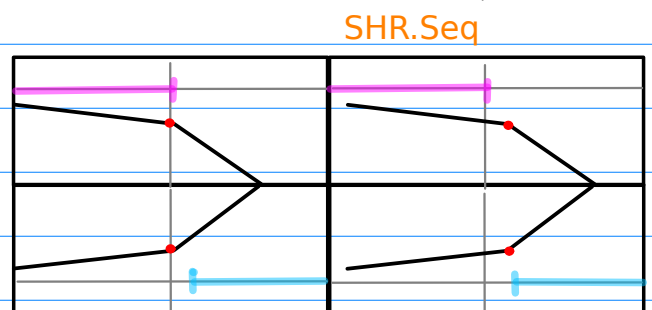
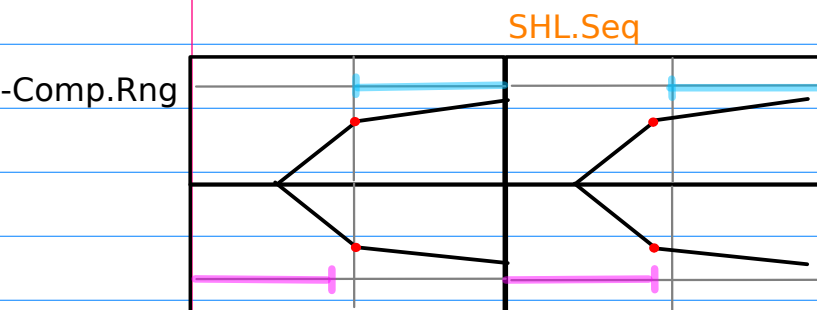
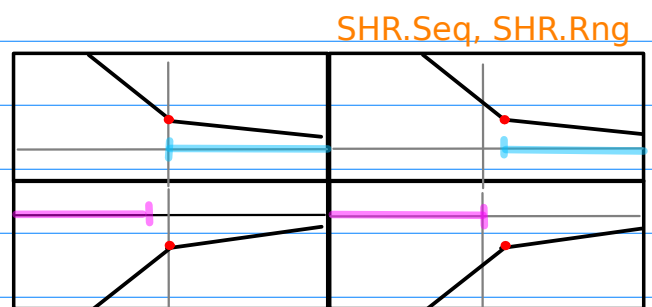
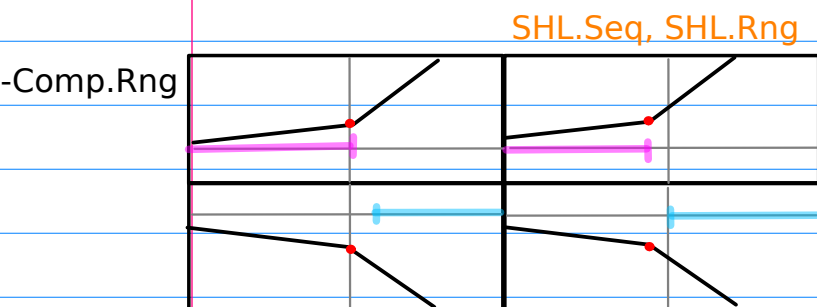
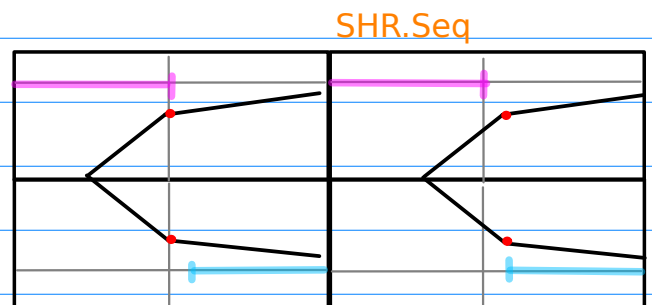
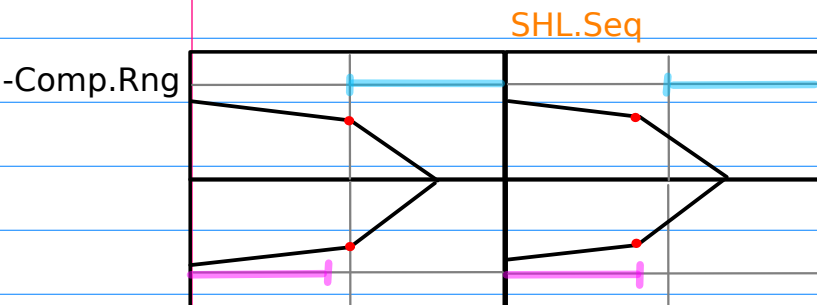
Left Shifted
Sequence

Right Shifted
Sequence



Left Shifted
Sequence

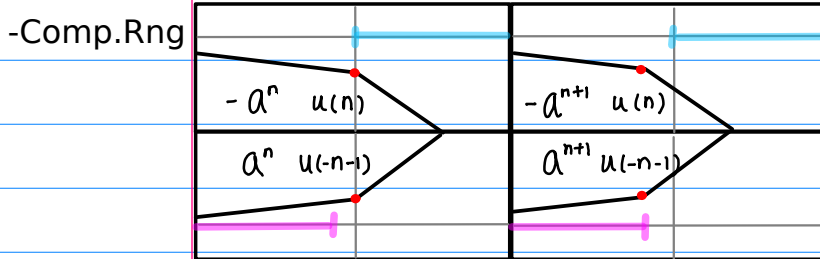
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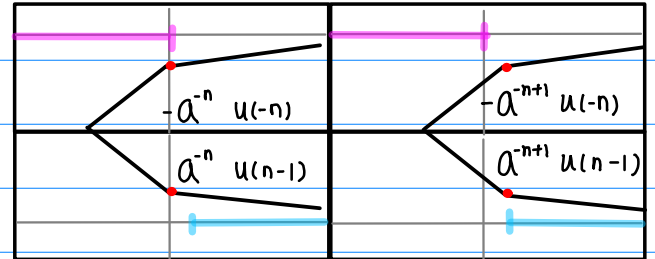
Left Shifted
Sequence

Right Shifted
Sequence

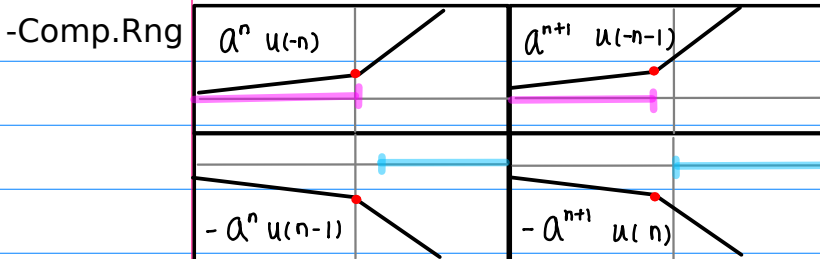
SHL.Seq



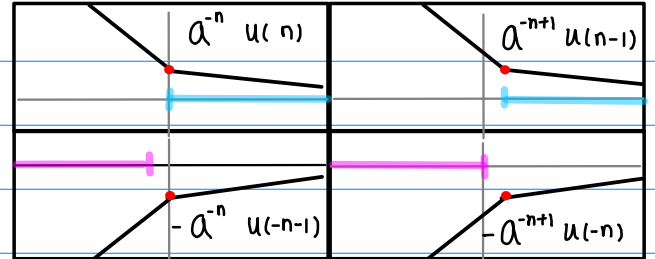
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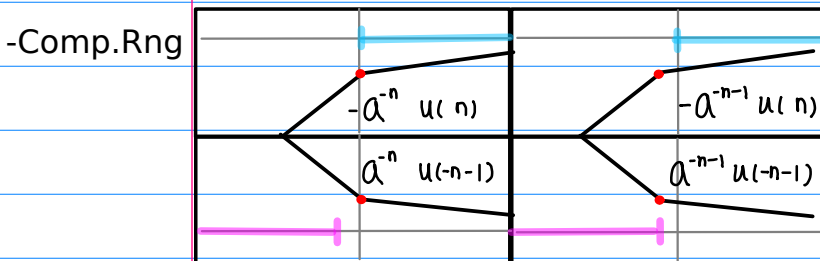
SHL.Seq, SHL.Rng



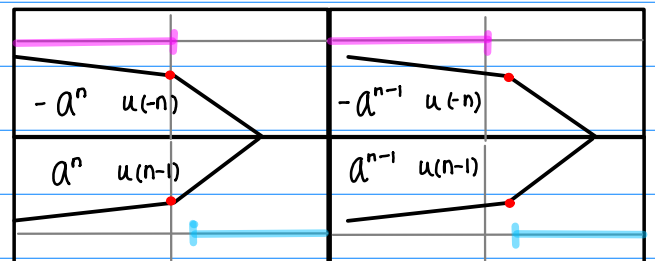
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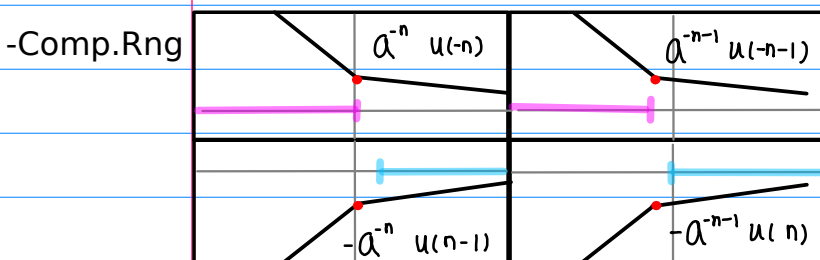
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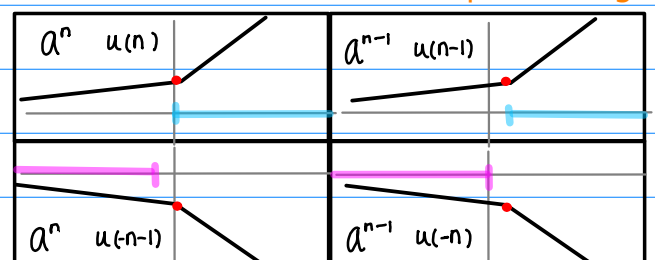
SHR.Seq



SHL.Seq, SHL.Rng



SHR.Seq, SHR.Rng



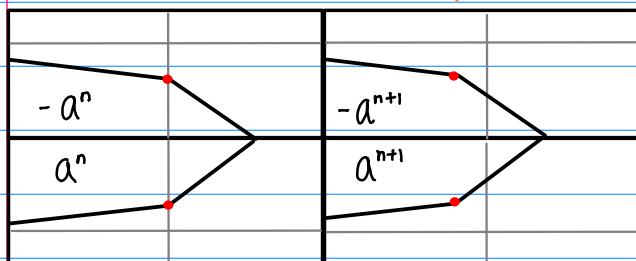
Left Shifted
Sequence

Right Shifted
Sequence

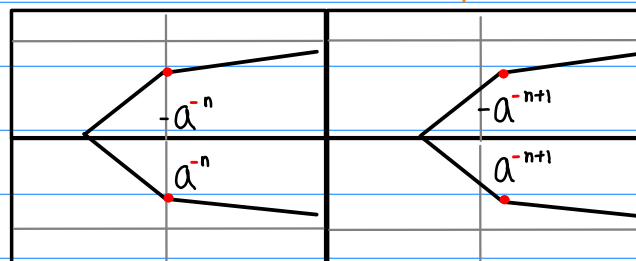
a Sequence Function

SHL.Seq

-Comp.Rng

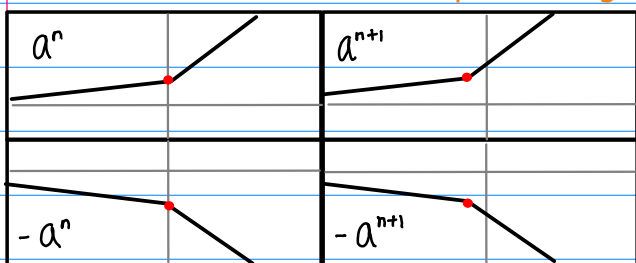


SHR.Seq

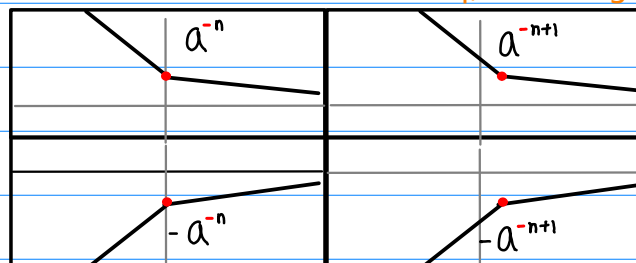


SHL.Seq, SHL.Rng

-Comp.Rng

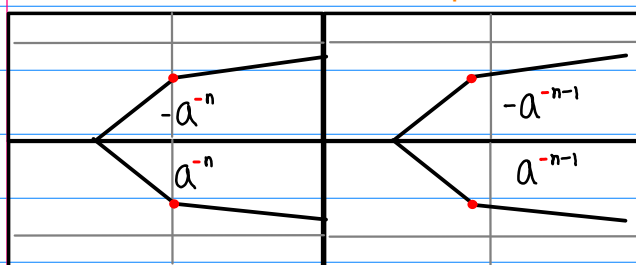


SHR.Seq, SHR.Rng

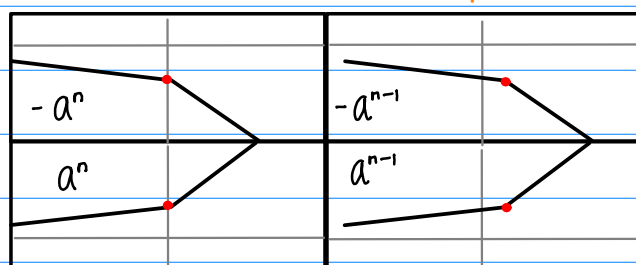


SHL.Seq

-Comp.Rng

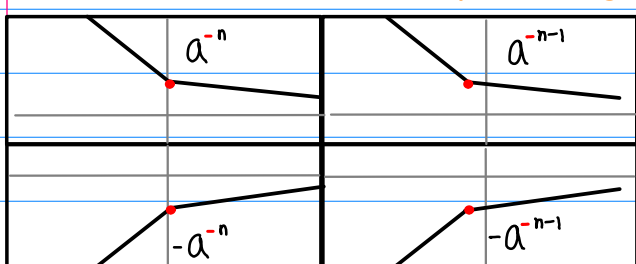


SHR.Seq

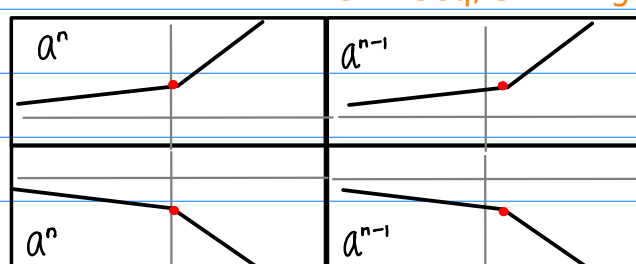


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



Left Shifted
Sequence

Right Shifted
Sequence

Range of a Sequence

