

# Stationarity

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi

# Outline

- 1 First-Order Stationary Processes
- 2 Higher Order Stationary Processes

# First Order Stationary

$N$  Gaussian random variables

## Definition

if the first order density function does not change with a shift in time origin

$$f_X(x_1; t_1) = f_X(x_1; t_1 + \Delta)$$

must be true for any time  $t_1$  and any real number  $\Delta$  if  $X(t)$  is to be a first-order stationary

# Consequences of stationarity

## $N$ Gaussian random variables

### Definition

$f_X(x, t_1)$  is independent of  $t_1$   
the process mean value is a constant

$$m_X(t) = \bar{X} = \text{constant}$$

the process mean value  
 $N$  Gaussian random variables

## Definition

$$m_X(t) = \bar{X} = \text{constant}$$

$$m_X(t_1) = \int_{-\infty}^{\infty} x f_X(x; t_1) dx$$

$$m_X(t_2) = \int_{-\infty}^{\infty} x f_X(x; t_2) dx$$

$$m_X(t_1) = m_X(t_1 + \Delta)$$

# Second-Order Stationary Process

$N$  Gaussian random variables

## Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, x_2; t_1, t_2) = f_X(x_1, x_2; t_1 + \Delta, t_2 + \Delta)$$

must be true for any time  $t_1, t_2$  and any real number  $\Delta$  if  $X(t)$  is to be a second-order stationary

Auto-correlation function

$$R_{XX}(t, t + \tau) = E[X(t)X(t + \tau)] = R_{XX}(\tau)$$

# $N^{\text{th}}$ -order Stationary Processes

$N$  Gaussian random variables

## Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, \dots, x_N; t_1, \dots, t_N) = f_X(x_1, \dots, x_N; t_1 + \Delta, \dots, t_N + \Delta)$$

must be true for any time  $t_1, \dots, t_N$  and any real number  $\Delta$  if  $X(t)$  is to be a second-order stationary



