Laurent Series and z-Transform - Geometric Series Permutations B

20240604 Tue

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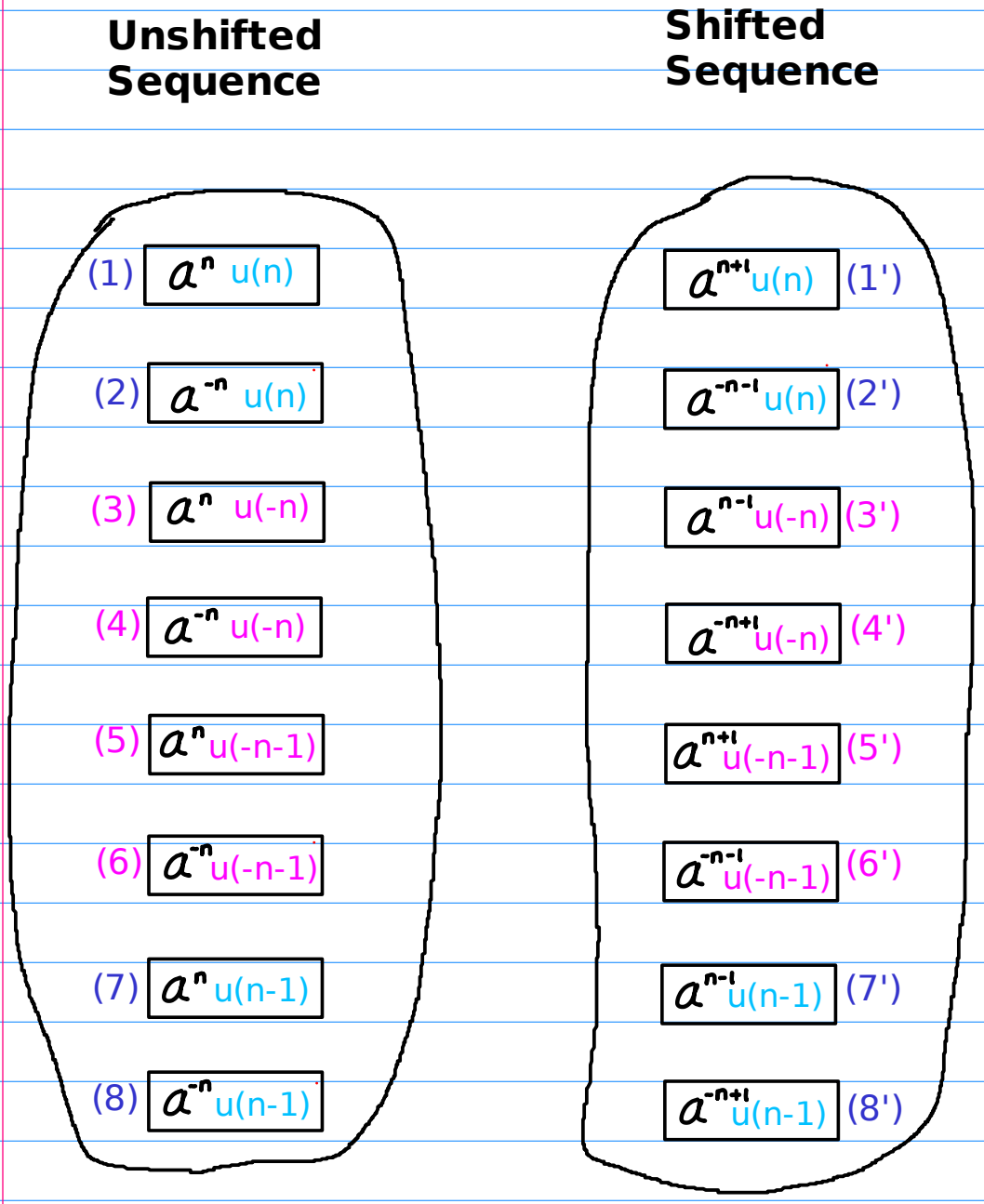


Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Unshifted Sequence	Shifted Sequence	Shifted Sequence
(1) $a^n\,u(n)$	$a^{n+1}u(n)$ (1')	$a^{-n-1}u(n)$ (1'')
(2) $a^{-n}\,u(n)$	$a^{-n-1}u(n)$ (2')	$a^{n+1}u(n)$ (2'')
(3) $a^n\,u(-n)$	$a^{n-1}u(-n)$ (3')	$a^{-n+1}u(-n)$ (3'')
(4) $a^{-n}\,u(-n)$	$a^{-n+1}u(-n)$ (4')	$a^{n-1}u(-n)$ (4'')
(5) $a^n u(-n-1)$	$a^{n+1}u(-n-1)$ (5')	$a^{-n-1}u(-n-1)$ (5'')
(6) $a^{-n}u(-n-1)$	$a^{-n-1}u(-n-1)$ (6')	$a^{n+1}u(-n-1)$ (6'')
(7) $a^n u(n-1)$	$a^{n-1}u(n-1)$ (7')	$a^{-n+1}u(n-1)$ (7'')
(8) $a^{-n}u(n-1)$	$a^{-n+1}u(n-1)$ (8')	$a^{n-1}u(n-1)$ (8'')

many possible permuations are possible
but consider these two first

a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

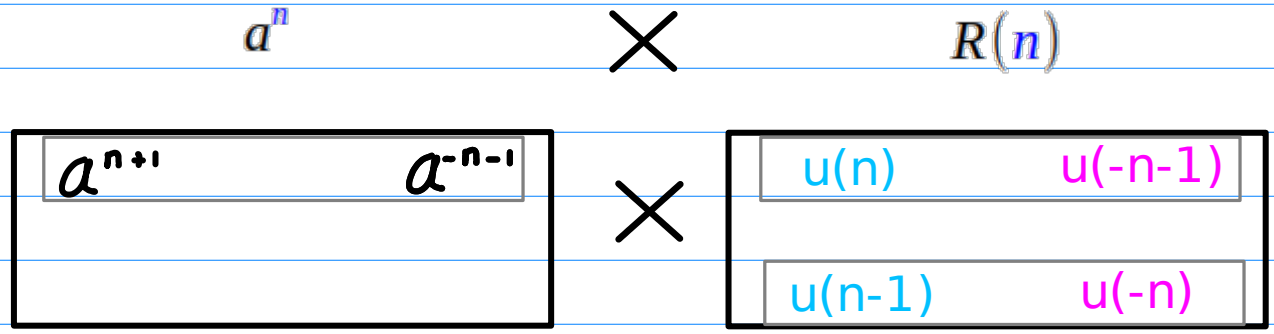
u(n)	u(-n-1)
u(n-1)	u(-n)

(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')
(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')
(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')
(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')

(1'')	a^{n-1}u(n)	a^{-n+1}u(n)	(2'')
(7'')	a^{n+1}u(n-1)	a^{-n-1}u(n-1)	(8'')
(5'')	a^{n-1}u(-n-1)	a^{-n+1}u(-n-1)	(6'')
(3'')	a^{n+1}u(-n)	a^{-n-1}u(-n)	(4'')

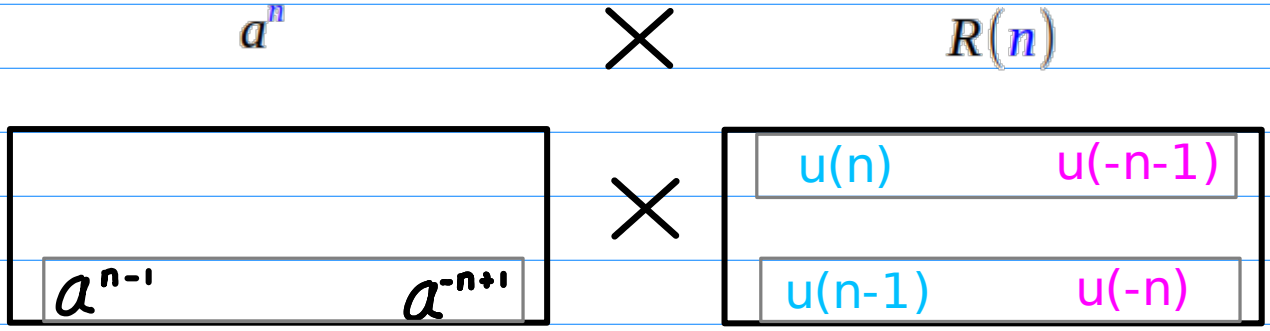
(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')
(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')
(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')
(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')

(1'')	a^{n-1}u(n)	a^{-n+1}u(n)	(2'')
(3'')	a^{n+1}u(-n)	a^{-n-1}u(-n)	(4'')
(5'')	a^{n-1}u(-n-1)	a^{-n+1}u(-n-1)	(6'')
(7'')	a^{n+1}u(n-1)	a^{-n-1}u(n-1)	(8'')



(1')	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	(2')
(7')	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	(8')
(5')	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	(6')
(3')	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	(4')

(1')	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	(2')
(3')	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	(4')
(5')	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	(6')
(7')	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	(8')

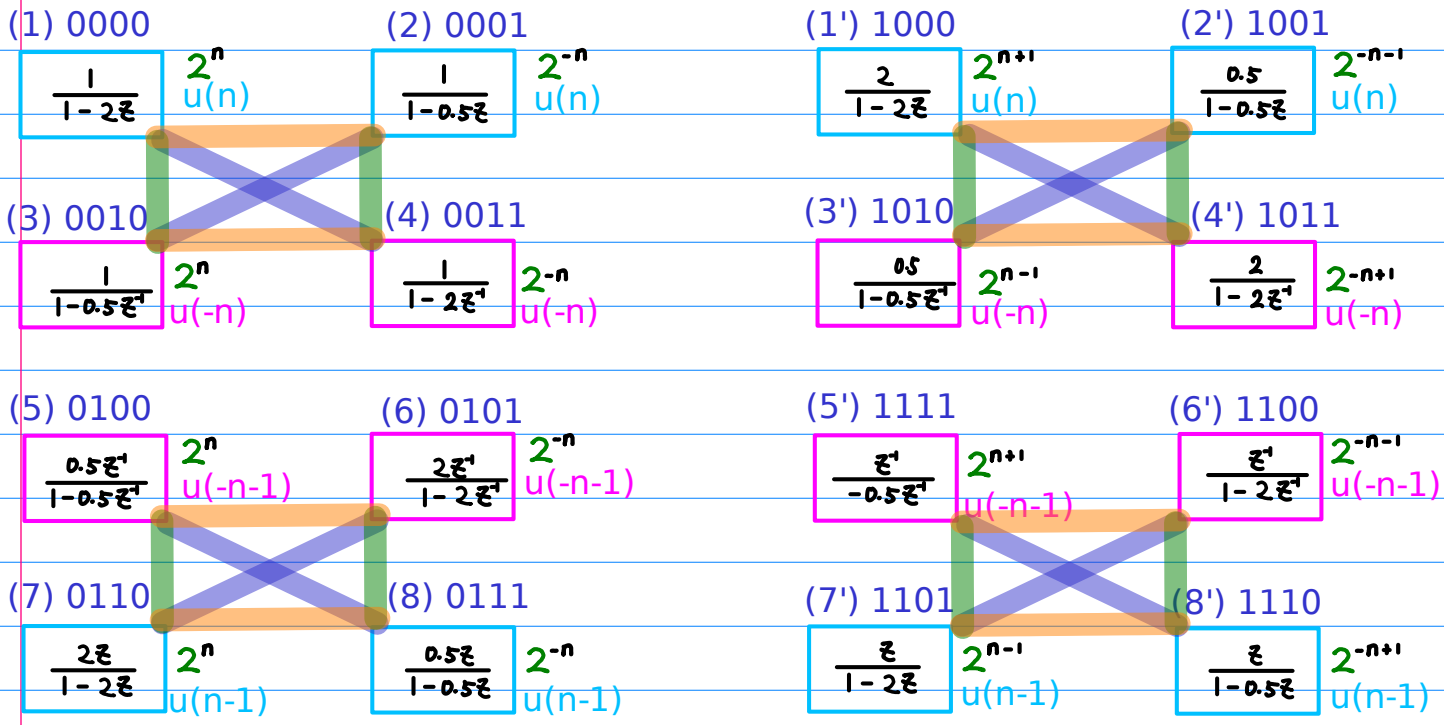


(1'') $a^{n-1}u(n)$		$a^{-n+1}u(n)$	(2'')
(7'') $a^{n+1}u(n-1)$		$a^{-n-1}u(n-1)$	(8'')
(5'') $a^{n-1}u(-n-1)$		$a^{-n+1}u(-n-1)$	(6'')
(3'') $a^{n+1}u(-n)$		$a^{-n-1}u(-n)$	(4'')

(1'') $a^{n-1}u(n)$		$a^{-n+1}u(n)$	(2'')
(3'') $a^{n+1}u(-n)$		$a^{-n-1}u(-n)$	(4'')
(5'') $a^{n-1}u(-n-1)$		$a^{-n+1}u(-n-1)$	(6'')
(7'') $a^{n+1}u(n-1)$		$a^{-n-1}u(n-1)$	(8'')

Inter-permutation (x) --> (x')

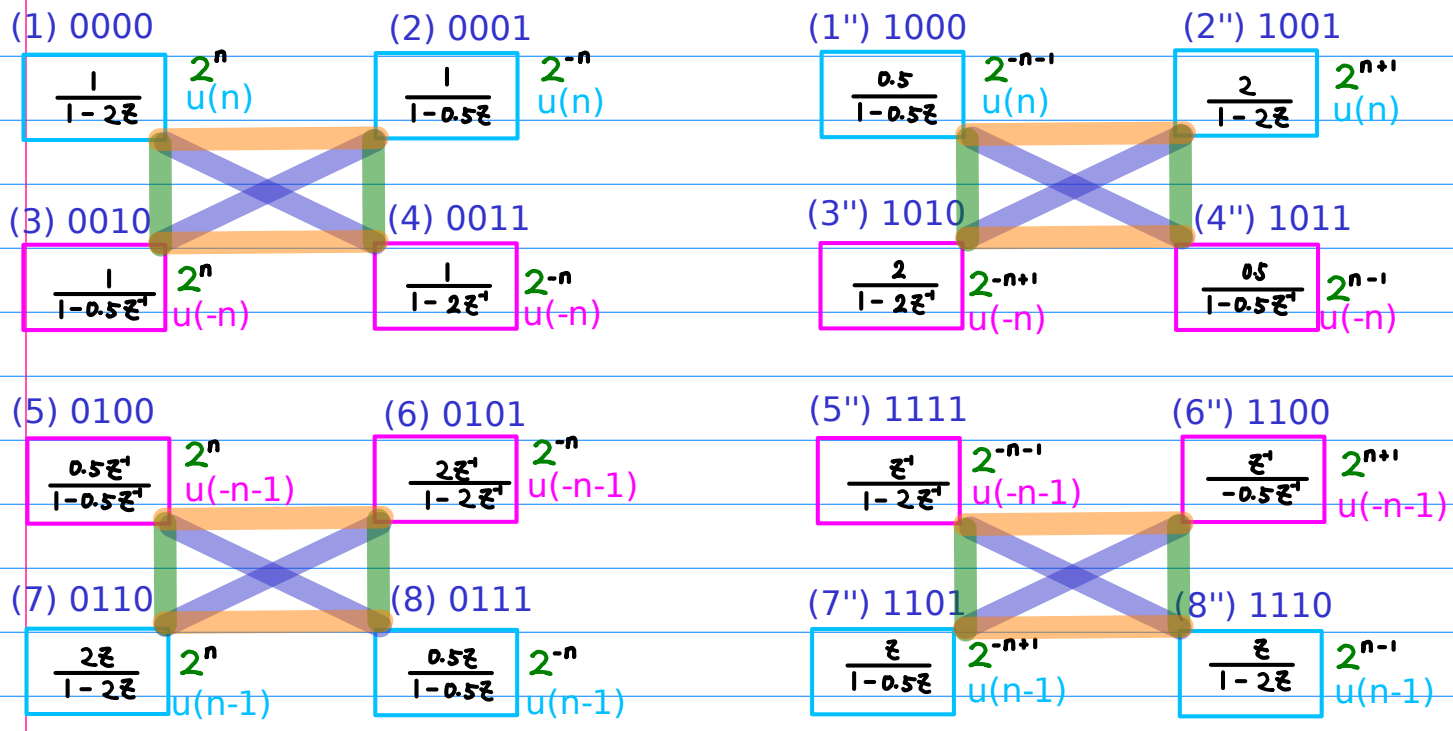
(1)~(8) --> (1')~(8')



(1)	0	0	0	0	→	1	0	0	0	(1′)	n	→	$n+1$	**shl(n)**
(2)	0	0	0	1	→	1	0	0	1	(2′)	n	→	$n+1$	**shl(n)**
(3)	0	0	1	0	→	1	0	1	0	(3′)	n	→	$n-1$	**shr(n)**
(4)	0	0	1	1	→	1	0	1	1	(4′)	n	→	$n-1$	**shr(n)**
(5)	0	1	0	0	→	1	1	0	0	(5′)	n	→	$n+1$	**shl(n)**
(6)	0	1	0	1	→	1	1	0	1	(6′)	n	→	$n+1$	**shl(n)**
(7)	0	1	1	0	→	1	1	1	0	(7′)	n	→	$n-1$	**shr(n)**
(8)	0	1	1	1	→	1	1	1	1	(7′)	n	→	$n-1$	**shr(n)**

Inter-permutation (x) --> (x'')

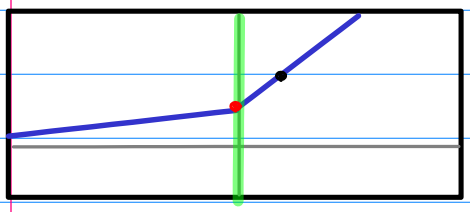
(1)~(8) --> (1'')~(8'')



shl(n) = n + 1
shr(n) = n - 1

(1) -- (1')
(5) -- (5')

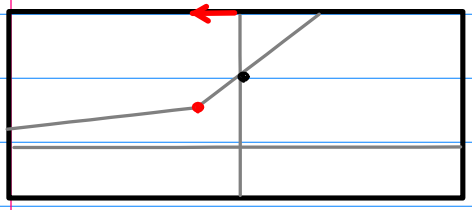
2^n



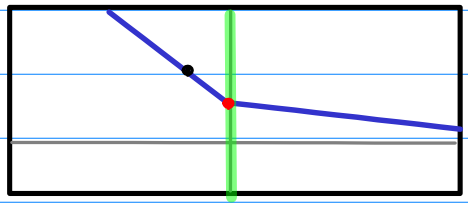
shift left

n ← n+1

2^{n+1}



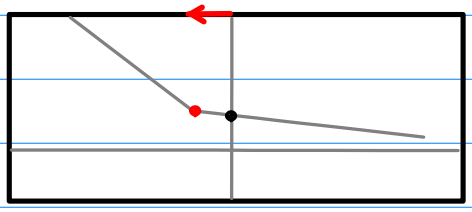
(2) -- (2')
(6) -- (6')



2^{-n}

shift left

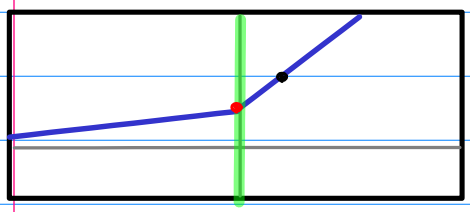
n ← n+1



$2^{-(n+1)}$

(3) -- (3')
(7) -- (7')

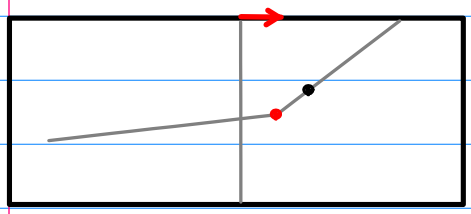
2^n



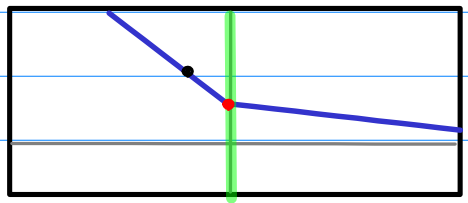
shift right

n ← n-1

2^{n-1}



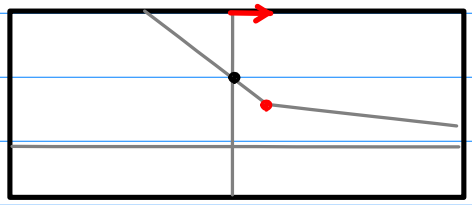
(4) -- (4')
(8) -- (8')



2^{-n}

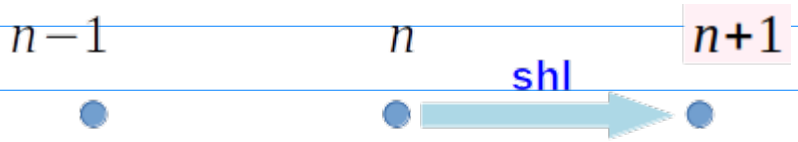
shift right

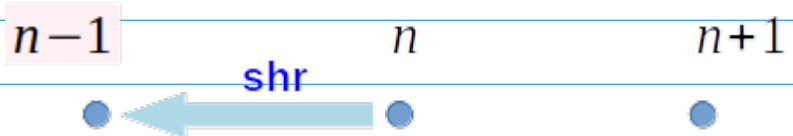
n ← n-1



$2^{-(n-1)}$

$R(n)$			$SHR(n)$
$u(n)$	$n \leftarrow n-1$	shr	$u(n-1)$
$u(n-1)$	$n \leftarrow n+1$	shl	$u(n)$
$u(-n)$	$n \leftarrow n+1$	shl	$u(-n-1)$
$u(-n-1)$	$n \leftarrow n-1$	shr	$u(-n)$



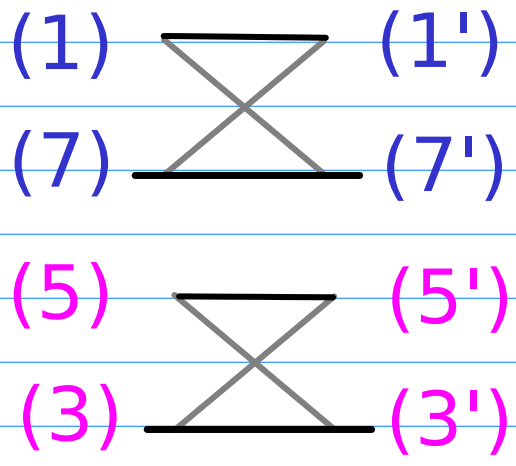


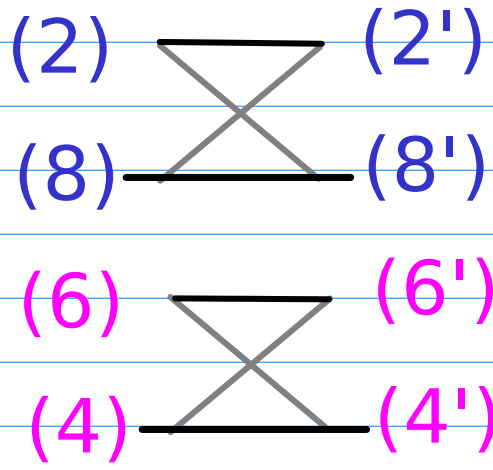
(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

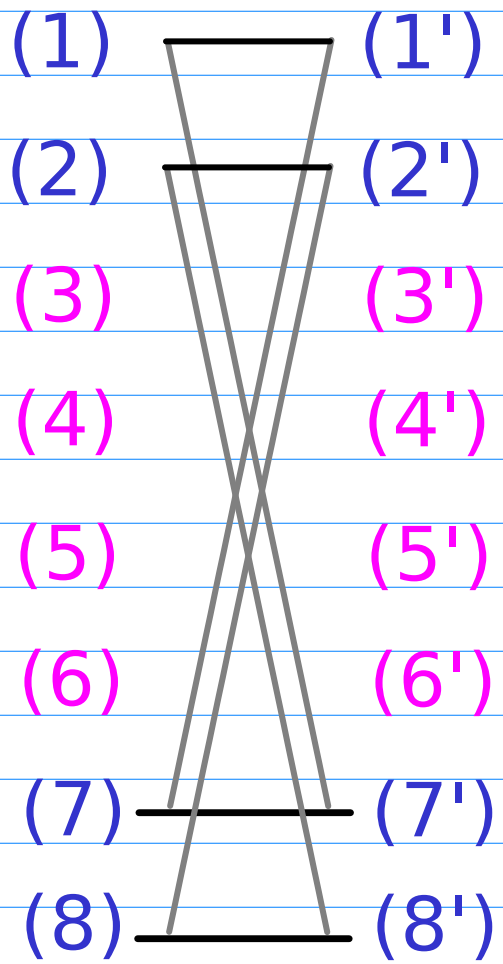
(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

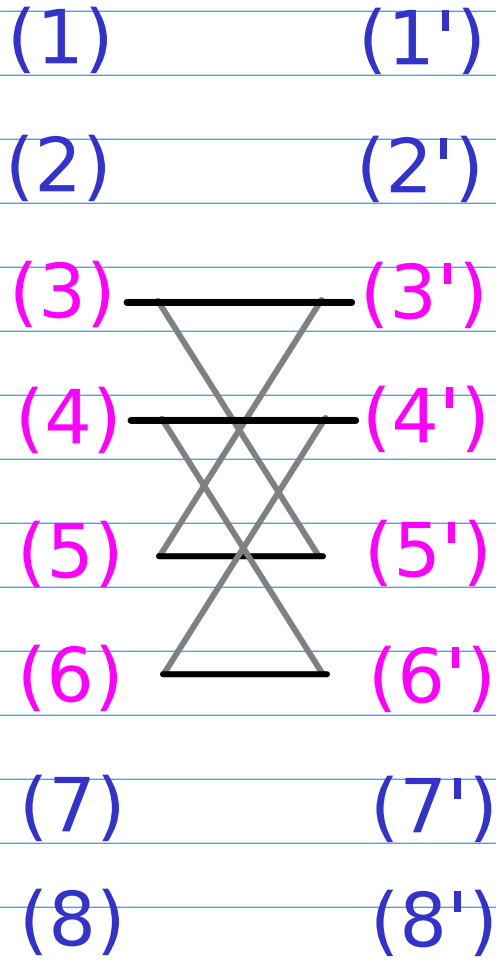
(1) - (1')
(2) - (2')
(3) - (3')
(4) - (4')

(5) - (5')
(6) - (6')
(7) - (7')
(8) - (8')





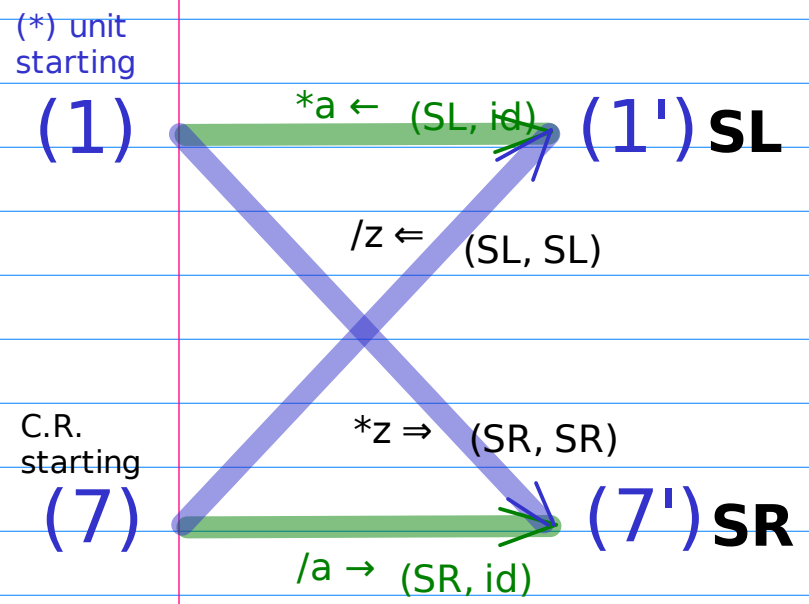


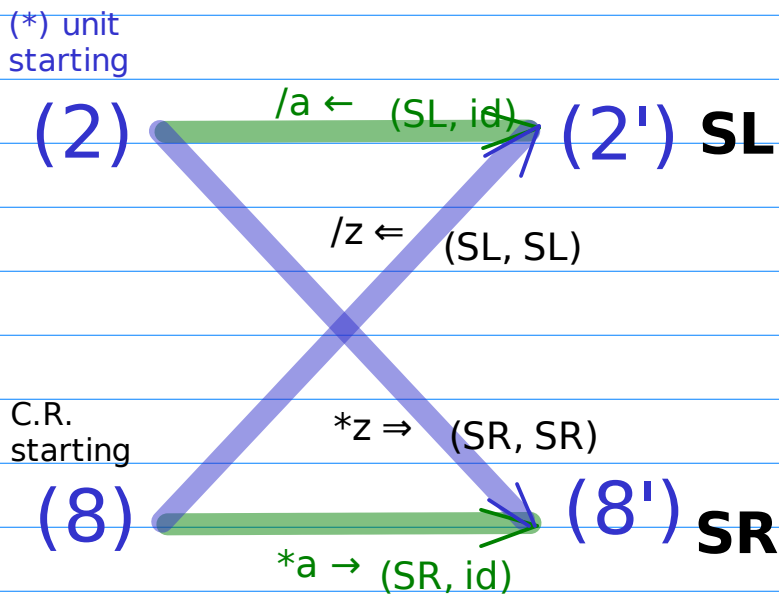


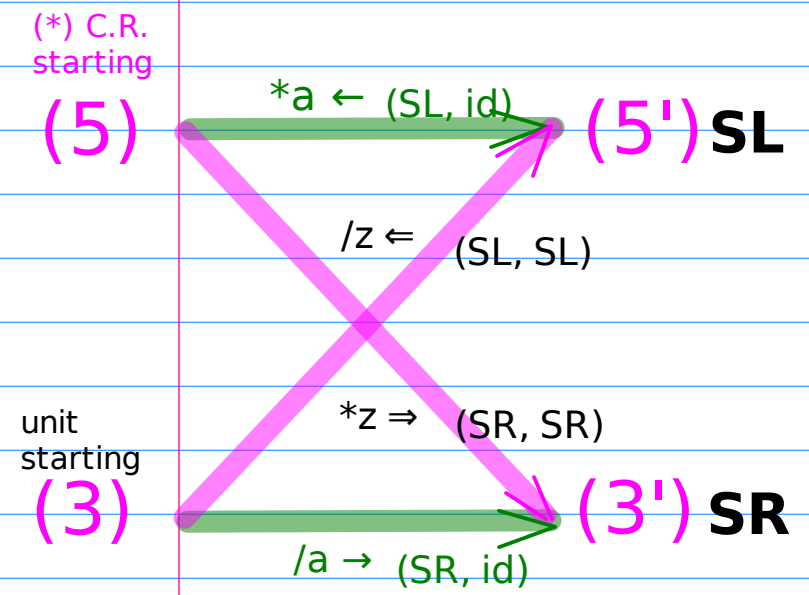
Butterfly Relations

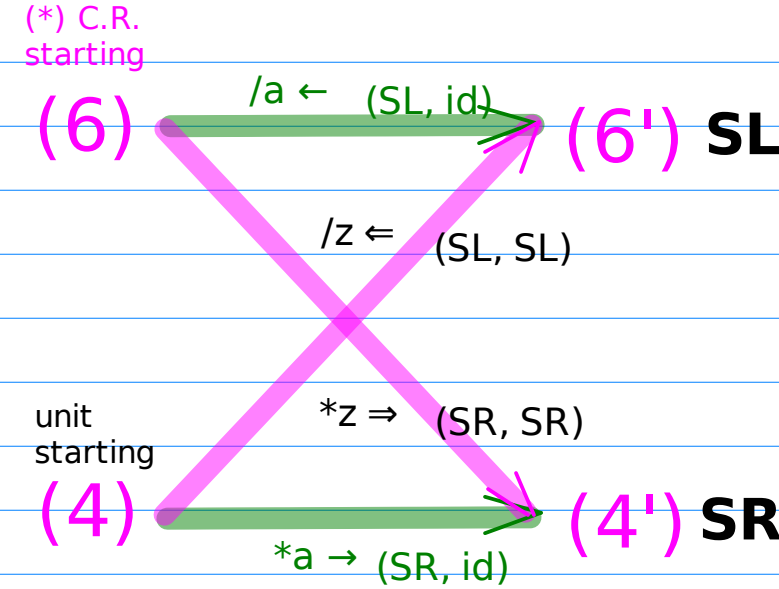
(x) --> (x')

(1)~(8) --> (1')~(8')







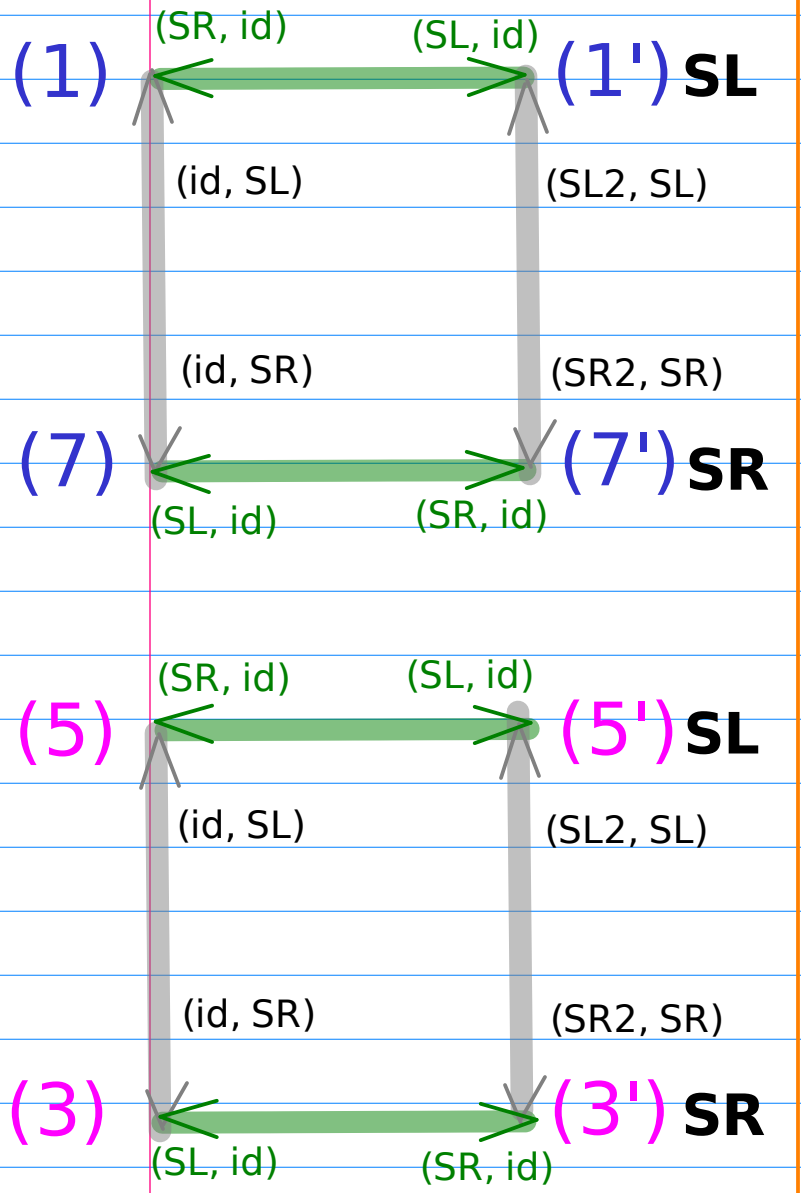


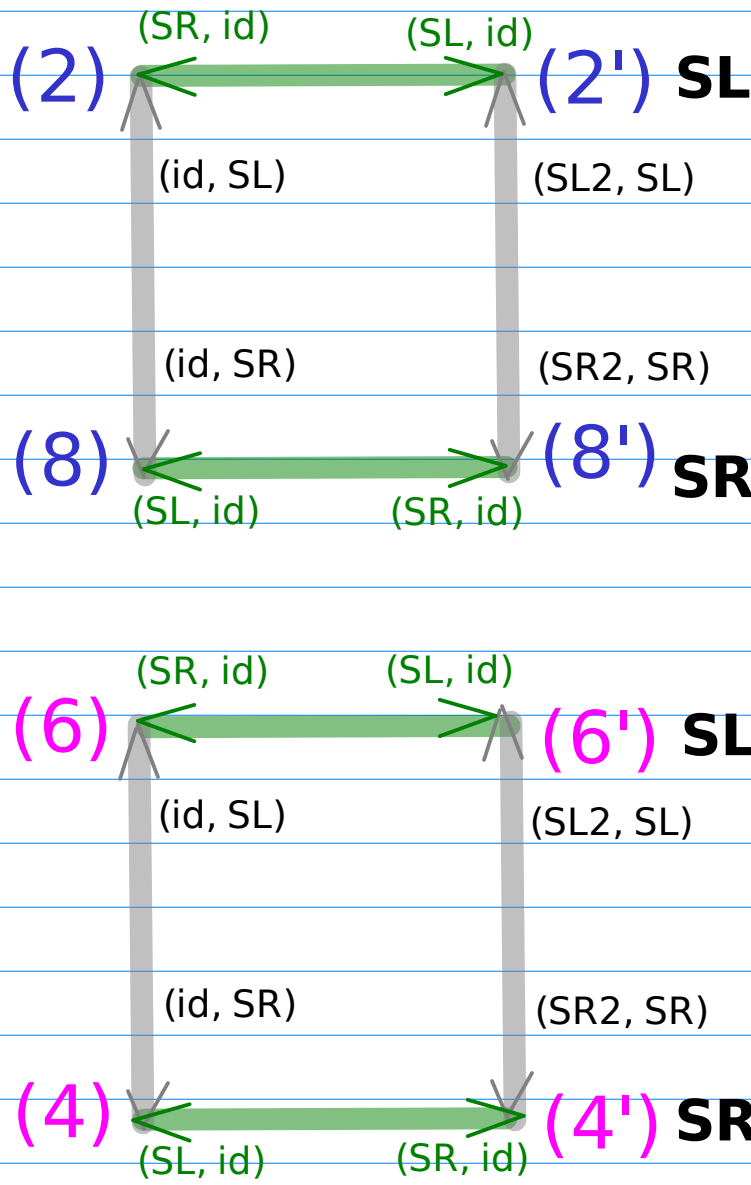
(Exp Shift, Range Shift)

Butterfly Relations

(x) --> (x')

(1)~(8) --> (1')~(8')

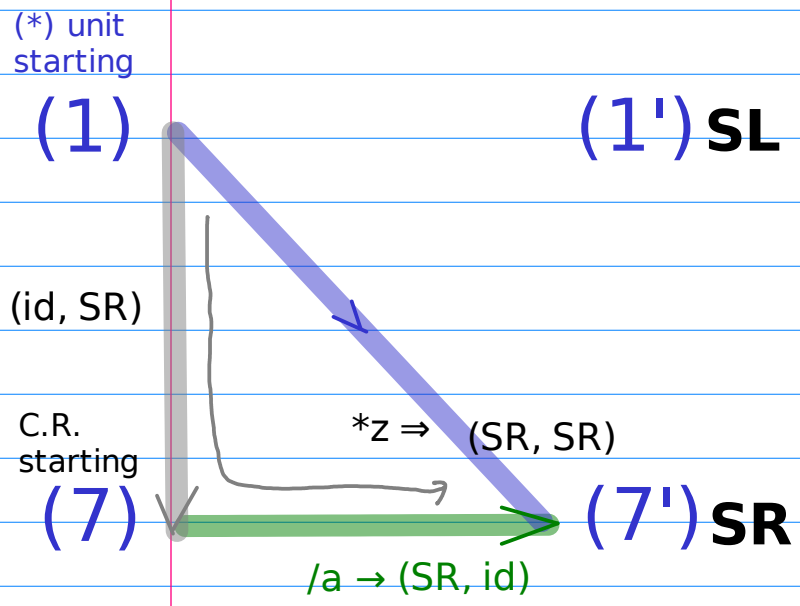


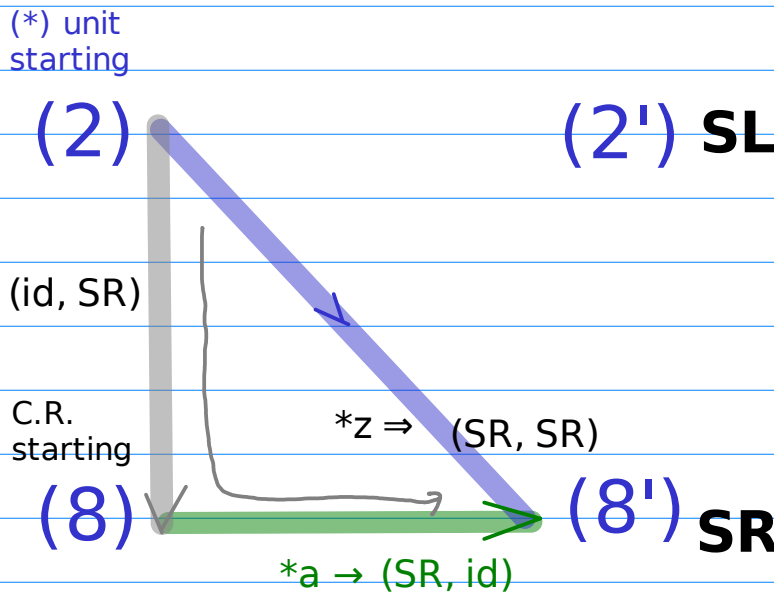


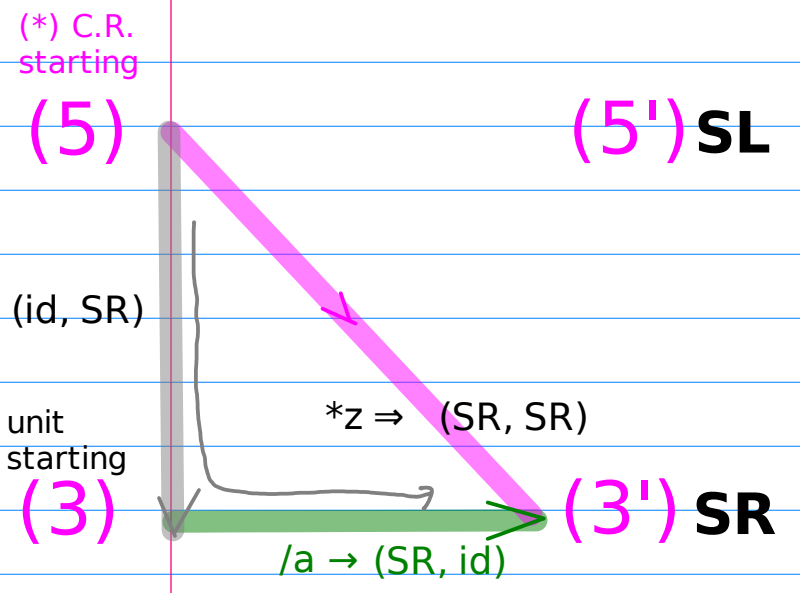
(Exp Shift, Range Shift)

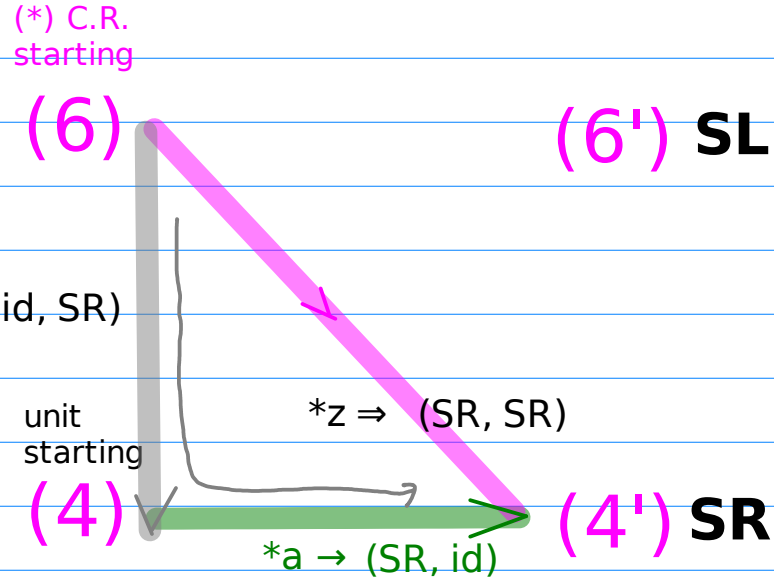
Decomposition of Exp and Rng Shifts (1)

(id, SR) + (SR, id) = (SR, SR)



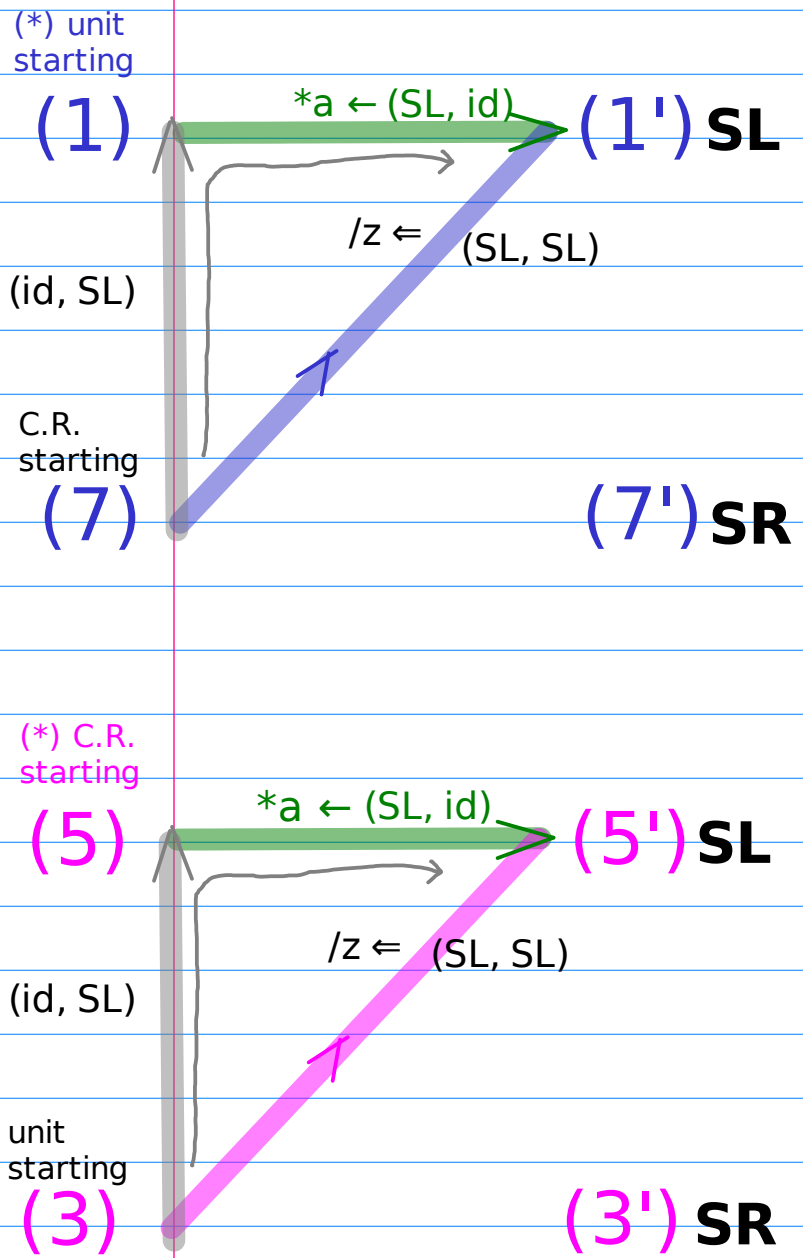


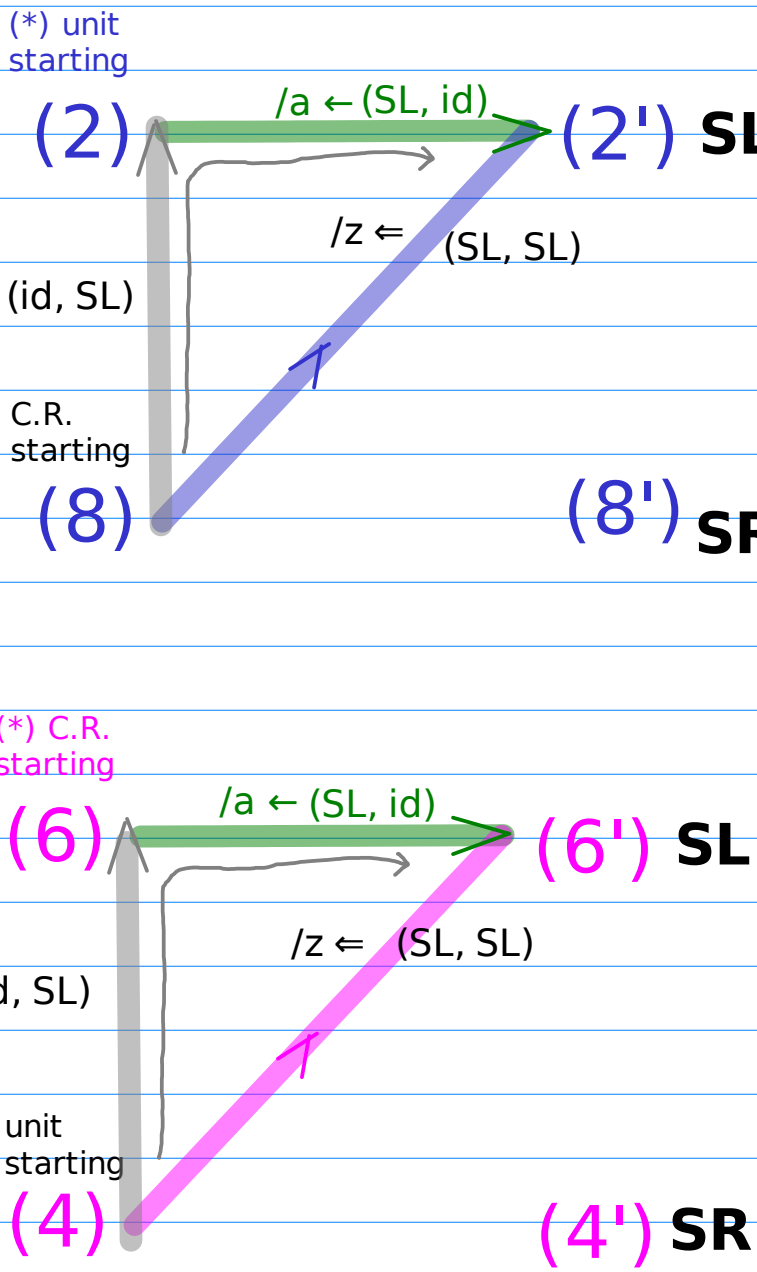




Decomposition of Exp and Rng Shifts (2)

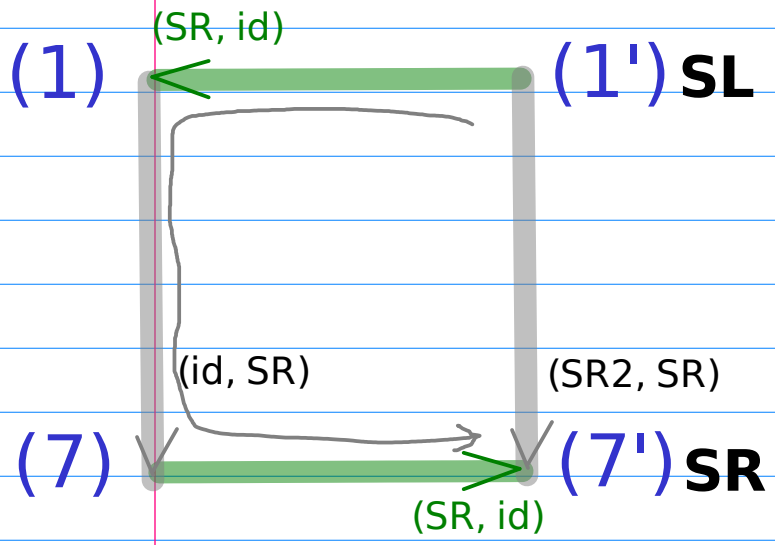
(id, SL) + (SL, id) = (SL, SL)

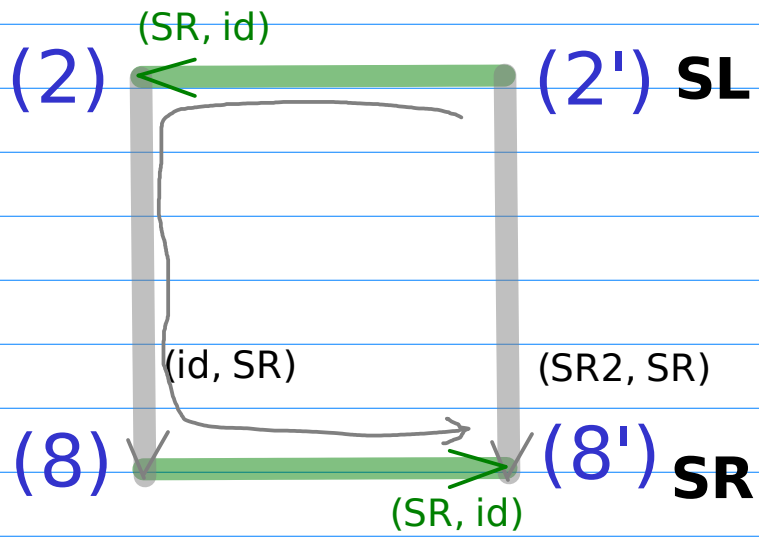


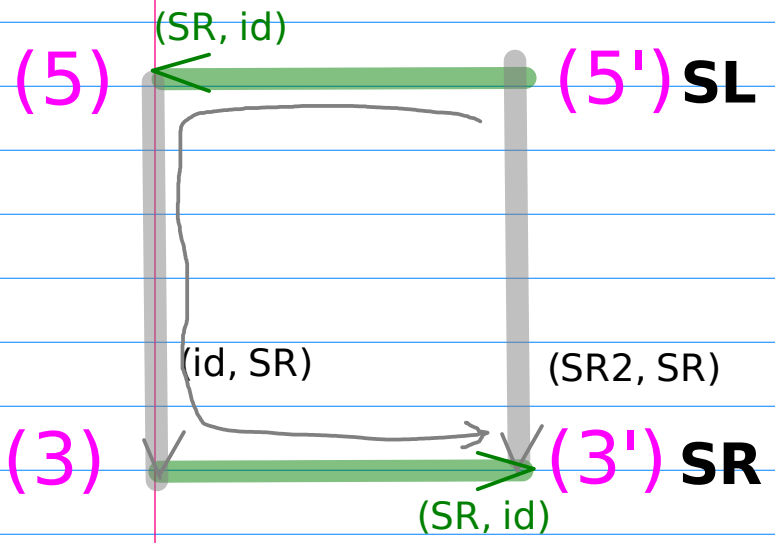


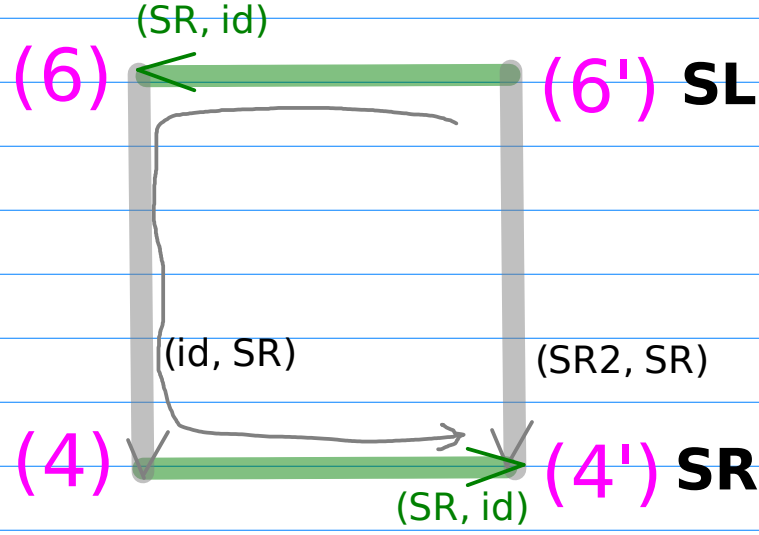
Decomposition of Exp and Rng Shifts (3)

(SR, id) + (id, SR) + (SR, id) = (SR2, SR)







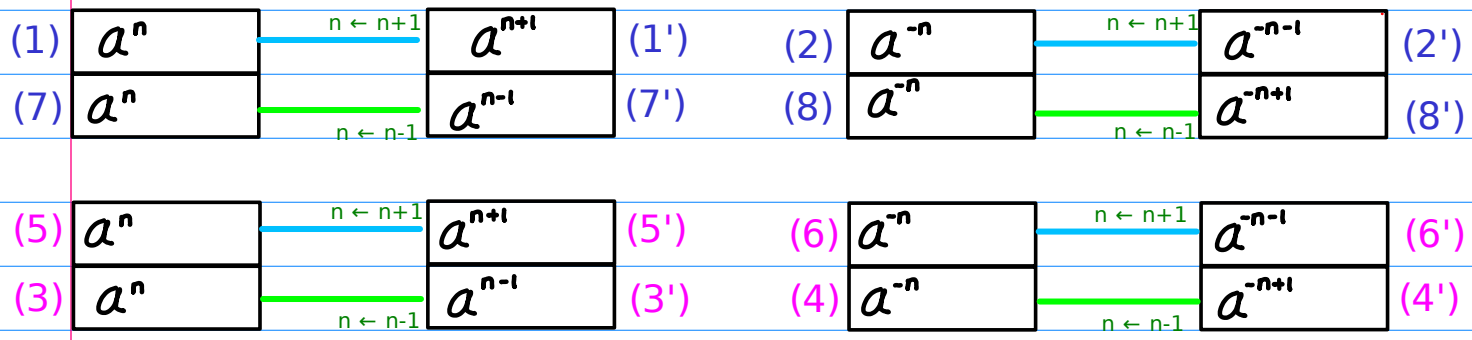


(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SL, SL)

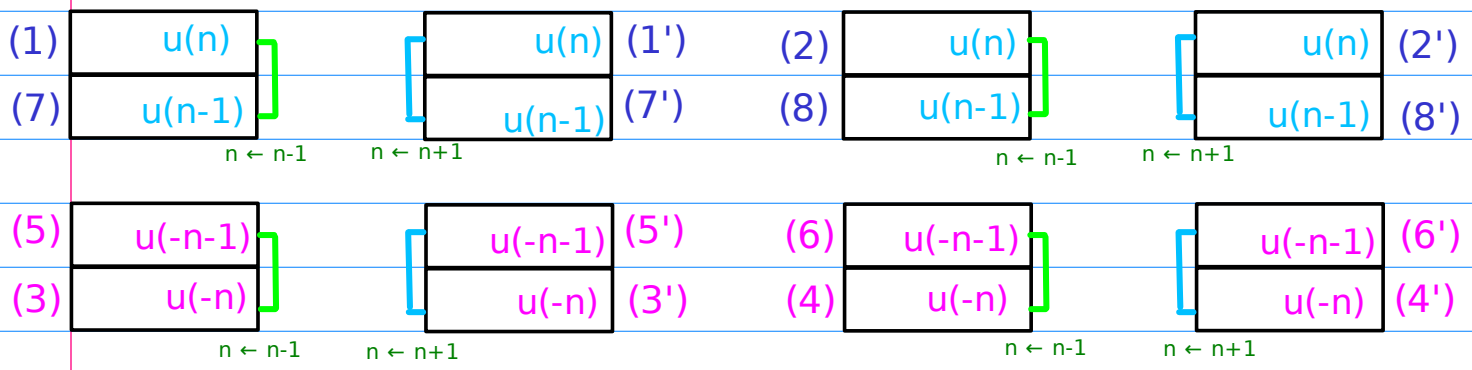
(x) --> (x')

(1)~(8) --> (1')~(8')

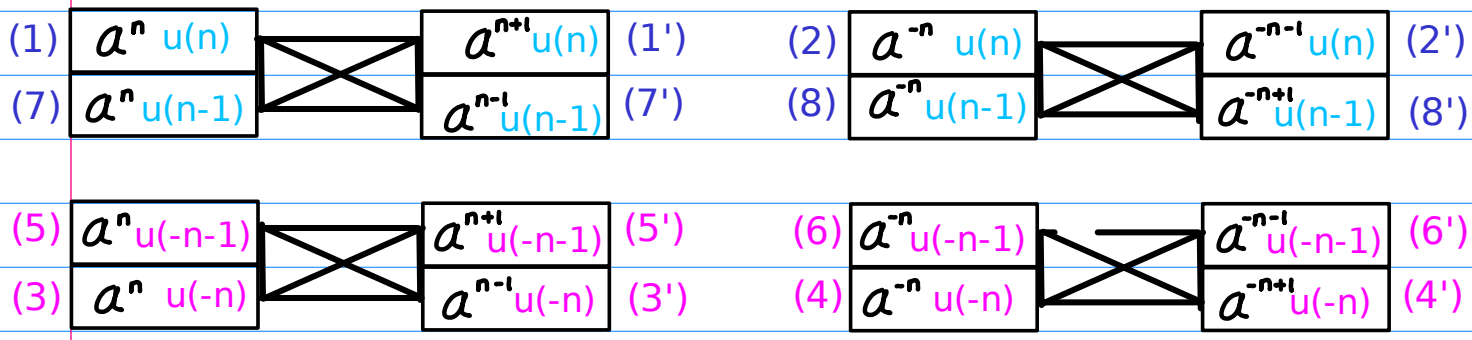
Exponent Shifts



Range Shifts



Exponent & Range Permutations

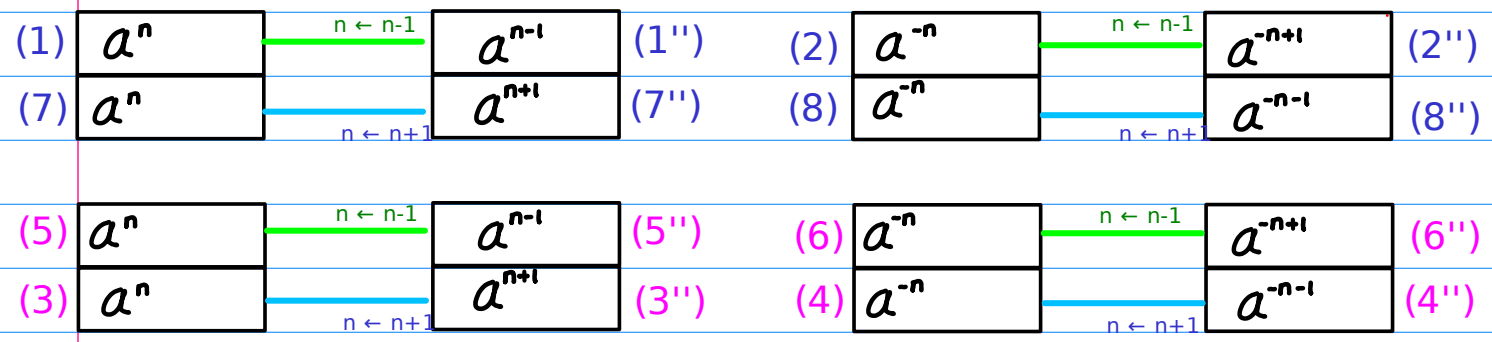


(id, SR) + (SL, id) = (SL, SR)
(id, SL) + (SR, id) = (SR, SL)

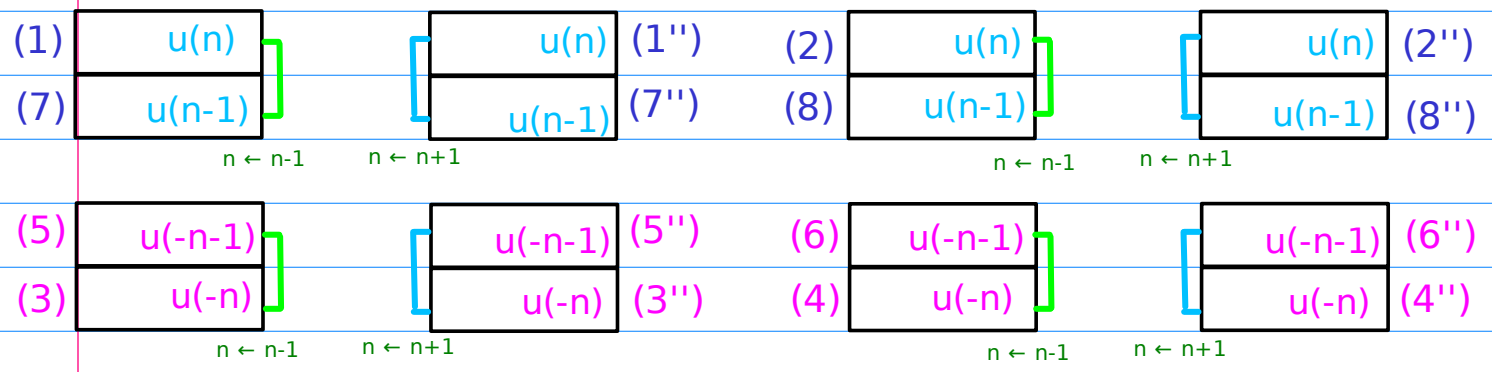
(x) --> (x'')

(1)~(8) --> (1'')~(8'')

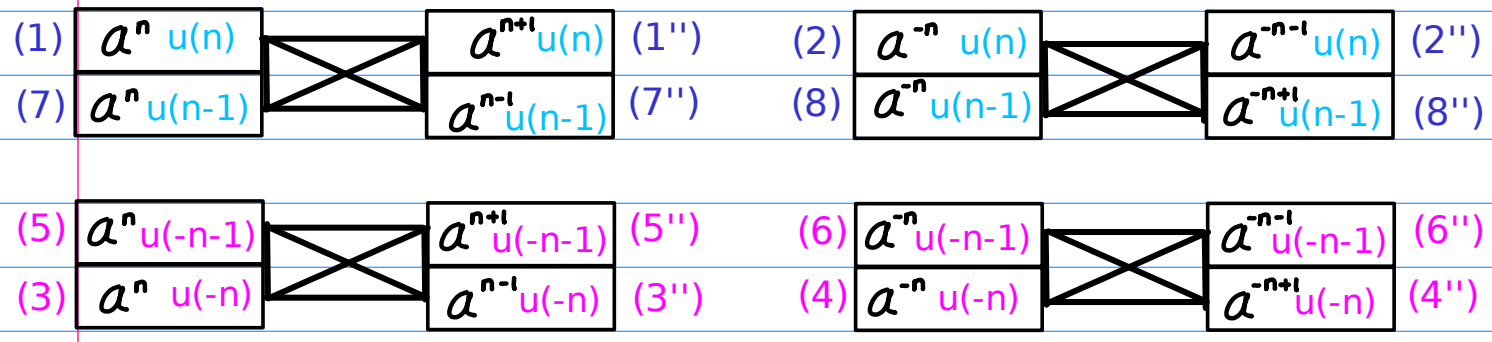
Exponent Shifts



Range Shifts



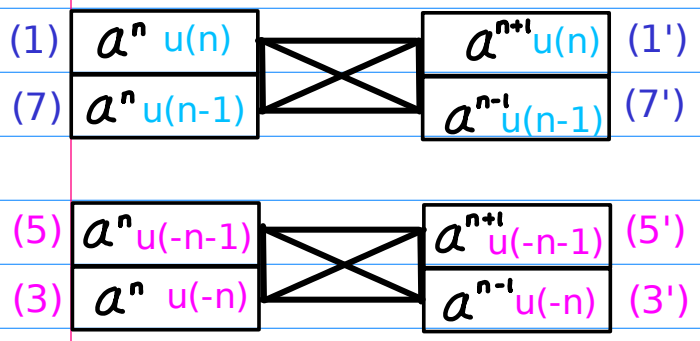
Exponent & Range Permutations

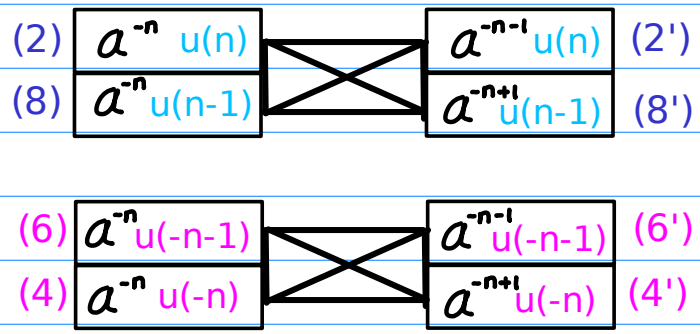


(id, SR) + (SR, id) = (SR, SR)
(id, SL) + (SL, id) = (SL, SL)

(x) --> (x')

(1)~(8) --> (1')~(8')

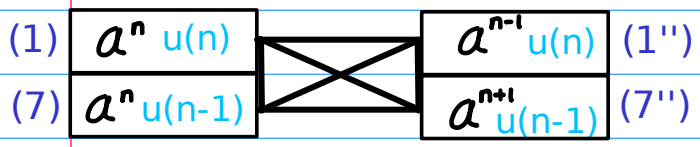


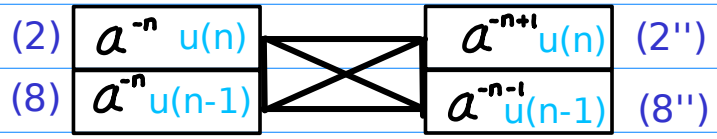


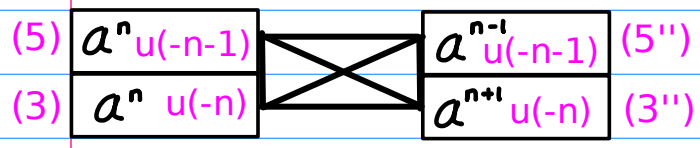
(id, SR) + (SL, id) = (SL, SR)
(id, SL) + (SR, id) = (SR, SL)

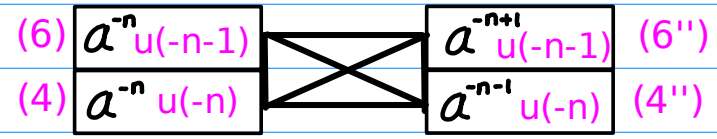
(x) --> (x'')

(1)~(8) --> (1'')~(8'')

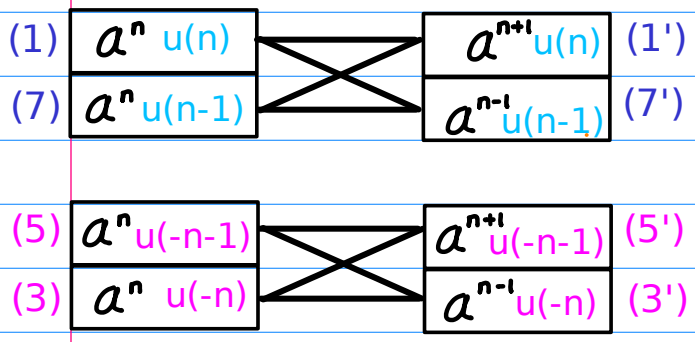


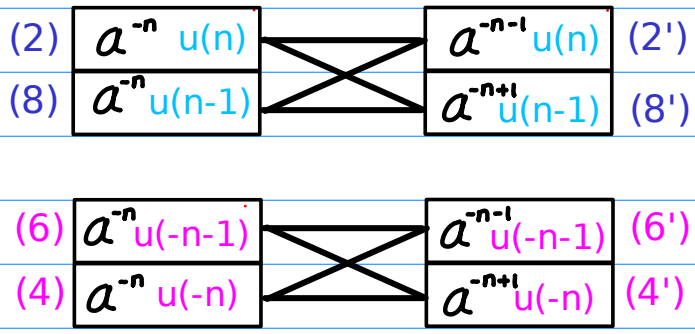


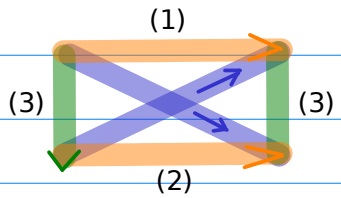


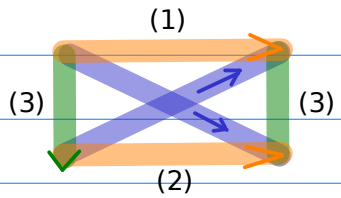


Exponent Permutations









(1) shift left exponent

(2) shift right exponent

(3) shift right range

(1) shift left exponent

(2) shift right exponent

(3) shift right range

(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)

(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)

(SL, id) shift left exponent

(id, SL) shift left range

(SL, SL)

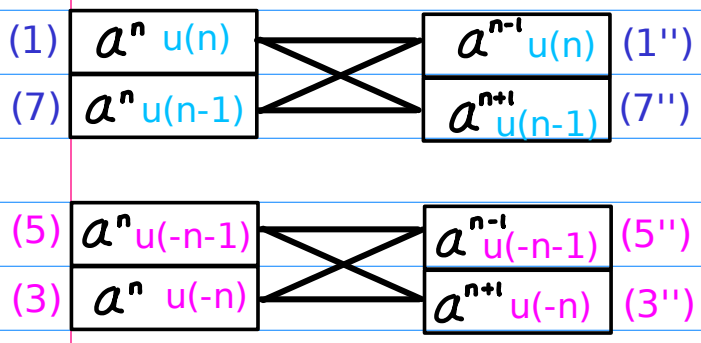
(SL, id) shift left exponent

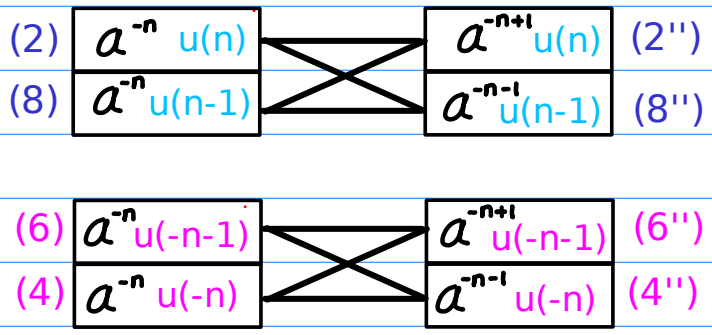
(id, SL) shift left range

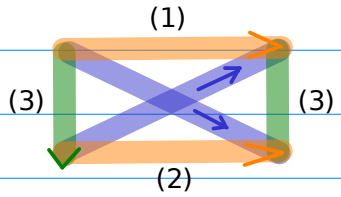
(SL, SL)

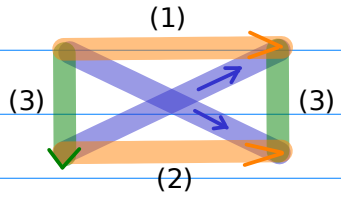
(SR, id) + (id, SR) = (SR, SR)
(SL, id) + (id, SL) = (SL, SL)

(SR, id) + (id, SR) = (SR
(SL, id) + (id, SL) = (SL,









(1) shift right exponent

(2) shift left exponent

(3) shift right range

(1) shift right exponent

(2) shift left exponent

(3) shift right range

(SL, id) shift right exponent

(id, SR) shift right range

(SL SR)

(SL, id) shift right exponent

(id, SR) shift right range

(SL, SR)

(SR, id) shift left exponent

(id, SL) shift left range

(SR, SL)

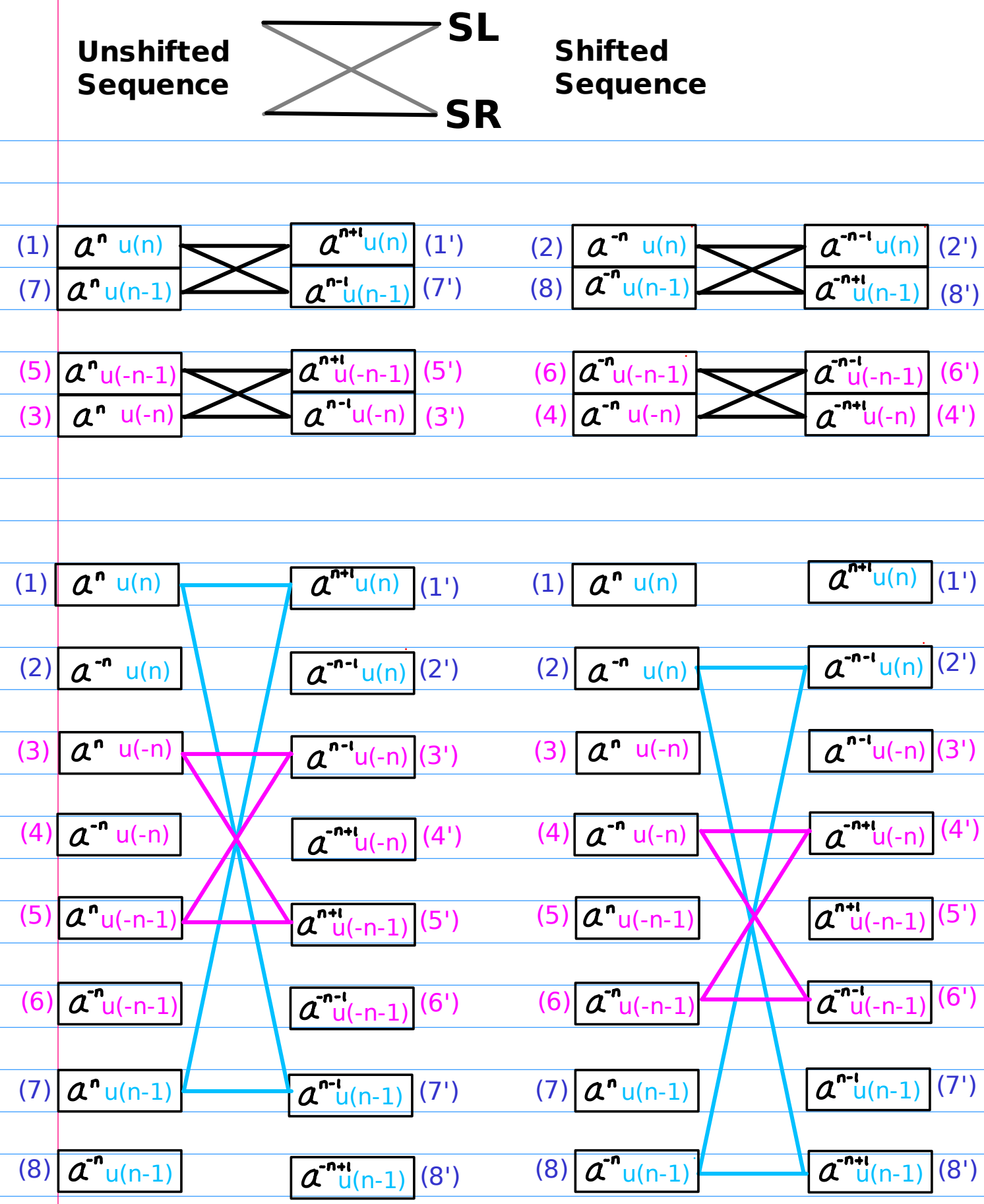
(SR, id) shift left exponent

(id, SL) shift left range

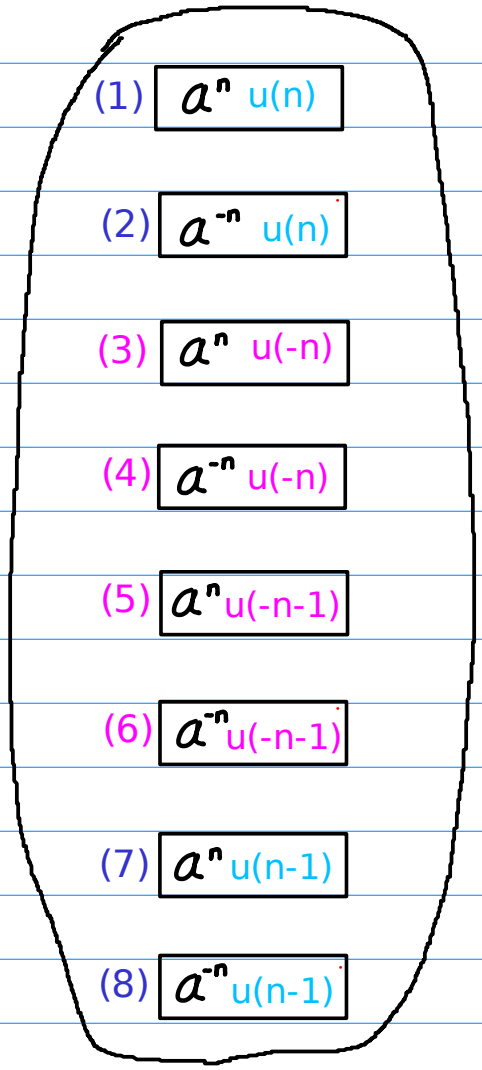
(SR, SL)

(SL, id) + (id, SR) = (SL, SR)
(SR, id) + (id, SL) = (SR, SL)

(SL, id) + (id, SR) = (SL
(SR, id) + (id, SL) = (SL


Unshifted Sequence
SL
SR
Shifted Sequence
(1) a^{n} u(n)
(7) a^{n} u(n-1)
a^{n+l}u(n) (1')
a^{n-l}u(n-1) (7')
(5) a^{n}u(-n-1)
(3) a^{n} u(-n)
a^{n+l}u(-n-1) (5')
a^{n-l}u(-n) (3')
(2) a^{-n} u(n)
(8) a^{-n}u(n-1)
a^{-n-l}u(n) (2')
a^{-n+l}u(n-1) (8')
(6) a^{-n}u(-n-1)
(4) a^{-n} u(-n)
a^{-n-l}u(-n-1) (6')
a^{-n+l}u(-n) (4')
(1) a^{n} u(n)
(2) a^{-n} u(n)
(3) a^{n} u(-n)
(4) a^{-n} u(-n)
(5) a^{n}u(-n-1)
(6) a^{-n}u(-n-1)
(7) a^{n}u(n-1)
(8) a^{-n}u(n-1)
a^{n+l}u(n) (1')
a^{-n-l}u(n) (2')
a^{n-l}u(-n) (3')
a^{-n+l}u(-n) (4')
a^{n+l}u(-n-1) (5')
a^{-n-l}u(-n-1) (6')
a^{n-l}u(n-1) (7')
a^{-n+l}u(n-1) (8')
(1) a^{n} u(n)
(2) a^{-n} u(n)
(3) a^{n} u(-n)
(4) a^{-n} u(-n)
(5) a^{n}u(-n-1)
(6) a^{-n}u(-n-1)
(7) a^{n}u(n-1)
(8) a^{-n}u(n-1)
a^{n+l}u(n) (1')
a^{-n-l}u(n) (2')
a^{n-l}u(-n) (3')
a^{-n+l}u(-n) (4')
a^{n+l}u(-n-1) (5')
a^{-n-l}u(-n-1) (6')
a^{n-l}u(n-1) (7')
a^{-n+l}u(n-1) (8')

Intra-Permuations over (1) ~ (8)



$a^{n+1}u(n)$ (1')

$a^{-n-1}u(n)$ (2')

$a^{n-1}u(-n)$ (3')

$a^{-n+1}u(-n)$ (4')

$a^{n+1}u(-n-1)$ (5')

$a^{-n-1}u(-n-1)$ (6')

$a^{n-1}u(n-1)$ (7')

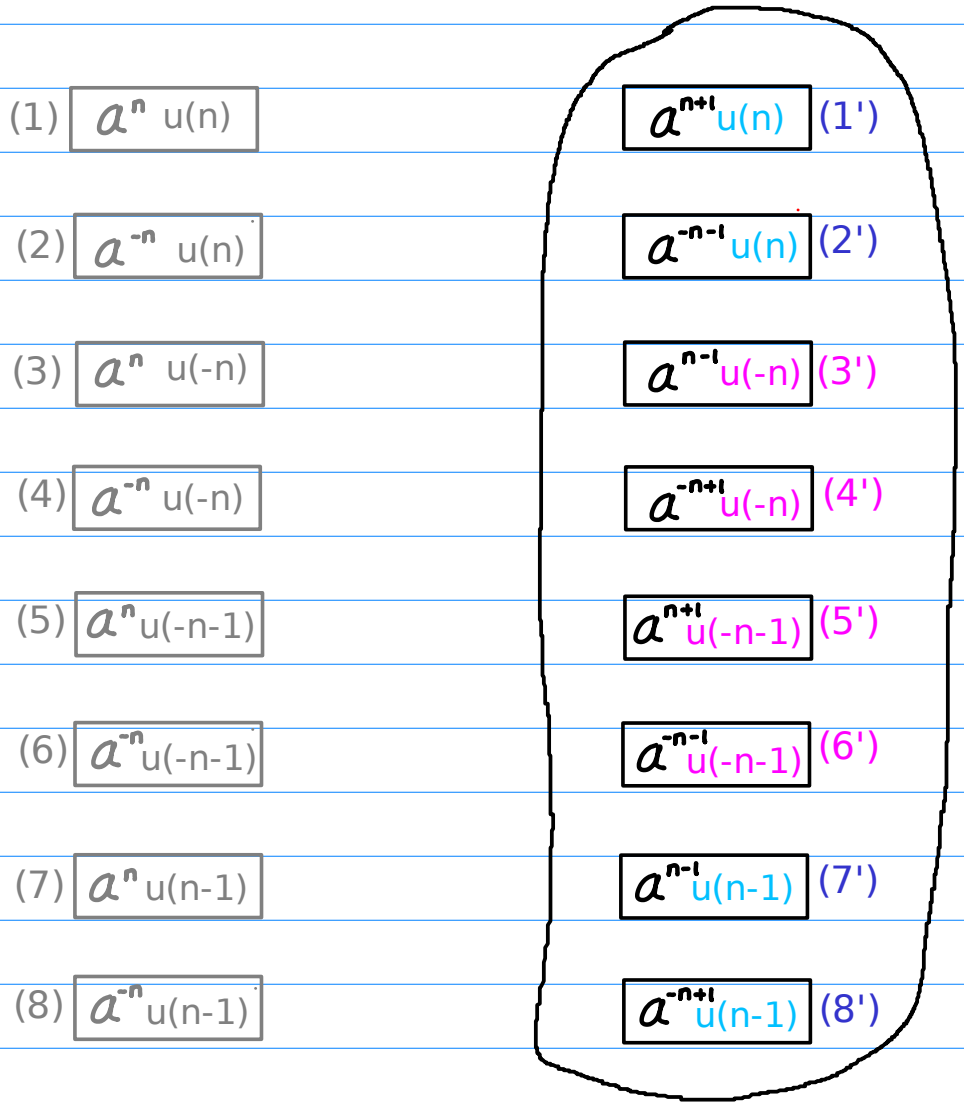
$a^{-n+1}u(n-1)$ (8')

A. Flipping
- **Base Inverting**
- **Range Flipping**

B. Range Shifting = Range Flipping + Range Complementing
- **Range Flipping**
- **Range Complementing**

C. Complementary Inverting
- **Base Inverting**
- **Range Complementing**

Intra-Permuations over (1') ~ (8')



D. Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipppir

E. Shifting2 = Exponent Shifting2 + Range Shifting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Range Complementing

F. Complementary Inverting

Base Inverting

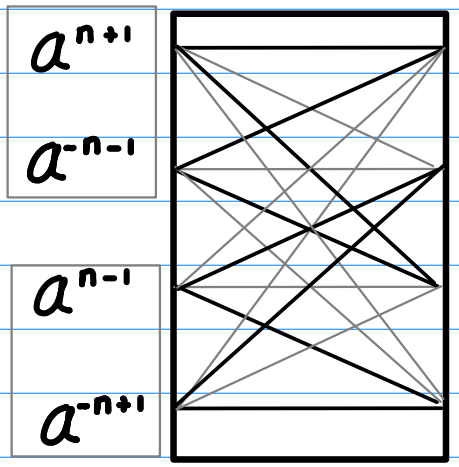
Range Complementing

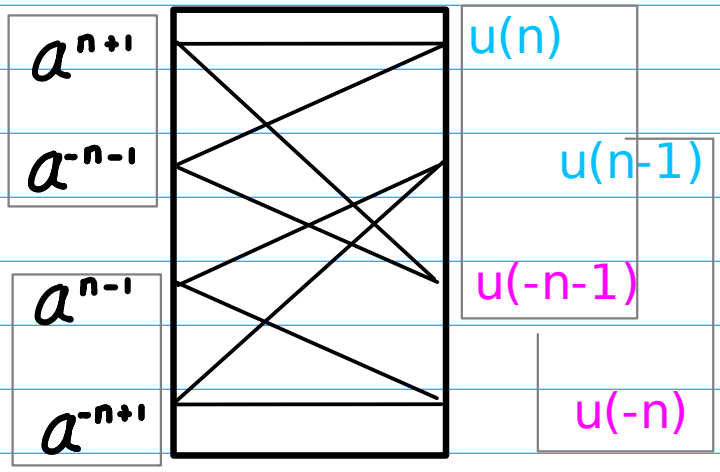
a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(n-1)	u(-n)





n ← n+2 or
n ← n - 2

(1') $a^{n+1}u(n)$		$a^{-n-1}u(n)$	(2')
(3') $a^{n-1}u(-n)$		$a^{-n+1}u(-n)$	(4')
(5') $a^{n+1}u(-n-1)$		$a^{-n-1}u(-n-1)$	(6')
(7') $a^{n-1}u(n-1)$		$a^{-n+1}u(n-1)$	(8')

(1'') $a^{n-1}u(n)$		$a^{-n+1}u(n)$	(2'')
(3'') $a^{n+1}u(-n)$		$a^{-n-1}u(-n)$	(4'')
(5'') $a^{n-1}u(-n-1)$		$a^{-n+1}u(-n-1)$	(6'')
(7'') $a^{n+1}u(n-1)$		$a^{-n-1}u(n-1)$	(8'')

permutation over (1) ~ (8)

A. Flipping
    Base Inverting
    Range Flipping

B. Range Shifting = Range Flipping + Range Complementing
    Range Flipping
    Range Complementing

C. Complementary Inverting
    Base Inverting
    Range Complementing

permutation over (1') ~ (8')

D. Flipping2
    Base Inverting
    Shifted Range Flipping = Exponent Shifting2 + Range Flippping

E. Shifting2 = Exponent Shifting2 + Range Shifting
    Shifted Range Flipping = Exponent Shifting2 + Range Flipping
    Range Complementing

F. Complementary Inverting
    Base Inverting
    Range Complementing

Range Shifting = Range Flipping + Range Complementing
Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
    = Exponent Shifting2 + Range (Flippping+Complementing)
    = Exponent Shfiting2 + Range Shifting

permutation over (1) ~ (8)

A. Flipping
- Base Inverting
- Range Flipping

(1) - (4) (5) - (8)
(2) - (3) (6) - (7)
(3) - (2) (7) - (6)
(4) - (1) (8) - (5)

B. Range Shifting
- Range Flipping
- Range Complementing

(1) - (7) (5) - (3)
(2) - (8) (6) - (4)
(3) - (5) (7) - (1)
(4) - (6) (8) - (2)

C. Complementary Inverting
- Base Inverting
- Range Complementing

(1) - (6) (5) - (2)
(6) - (1) (6) - (1)
(2) - (5) (7) - (4)
(5) - (2) (8) - (3)

permutation over (1') ~ (8')

D. Flipping2
- Base Inverting
- Shifted Range Flipping

(1') - (4') (5') - (8')
(2') - (3') (6') - (7')
(3') - (2') (7') - (6')
(4') - (1') (8') - (5')

E. Shifting2
- Shifted Range Flipping
- Range Complementing

(1') - (7') (5') - (3')
(2') - (8') (6') - (4')
(3') - (5') (7') - (1')
(4') - (6') (8') - (2')

F. Complementary Inverting
- Base Inverting
- Range Complementing

(1') - (6') (5') - (2')
(6') - (1') (6') - (1')
(2') - (5') (7') - (4')
(5') - (2') (8') - (3')

permutation over (1) ~ (8)

	A	B	C
Base Inverting	x		x
Range Flipping	x	x	
Range Complementing		x	x

permutation over (1') ~ (8')

	D	E	F
Base Inverting	x		x
Shifted Range Flipping	x	x	
Range Complementing		x	x

Range Shifting = Range Flipping + Range Complementing
Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flippping+Complementing)
= Exponent Shfiting2 + Range Shifting

Over (1) ~ (8)

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Range Flipping $R(n) \Longleftrightarrow R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

Over (1') ~ (8')

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Shifted Range Flipping $a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

A.I Flipping
Base Inverting
Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

B.I Range Shifting
Range Flipping
Range Complementing

$$R(n) \Longleftrightarrow R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

C.I Complementary Inverting
Base Inverting
Range Complementing

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

D.I Flipping2
Base Inverting
Shifted Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh\,2(n)} R(-n)$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

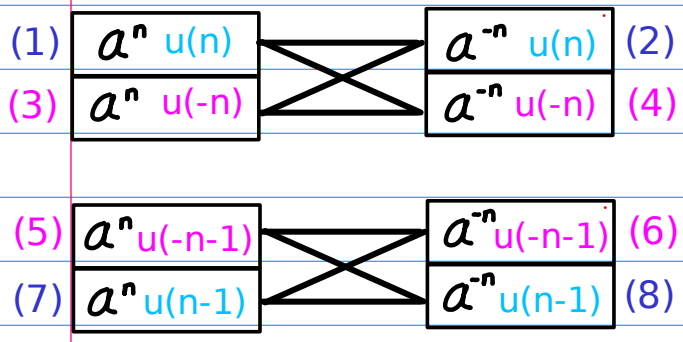
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

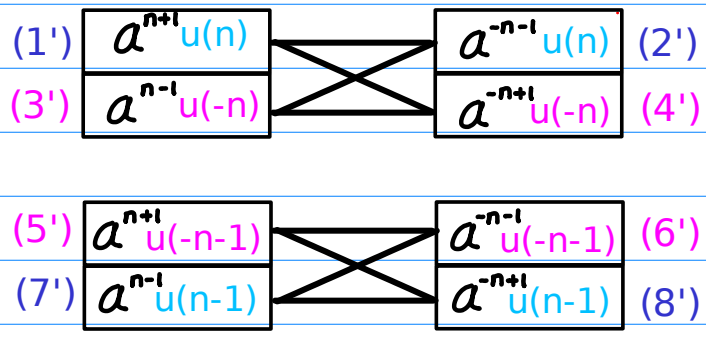
F.I Complementary Inverting
Base Inverting
Range Complementing

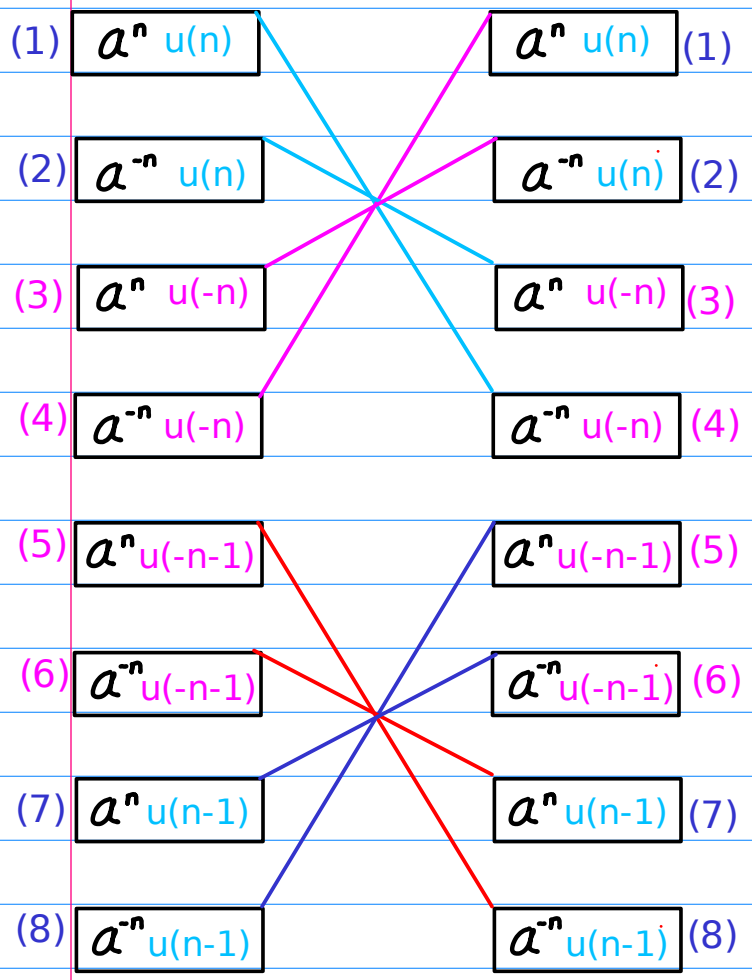
$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

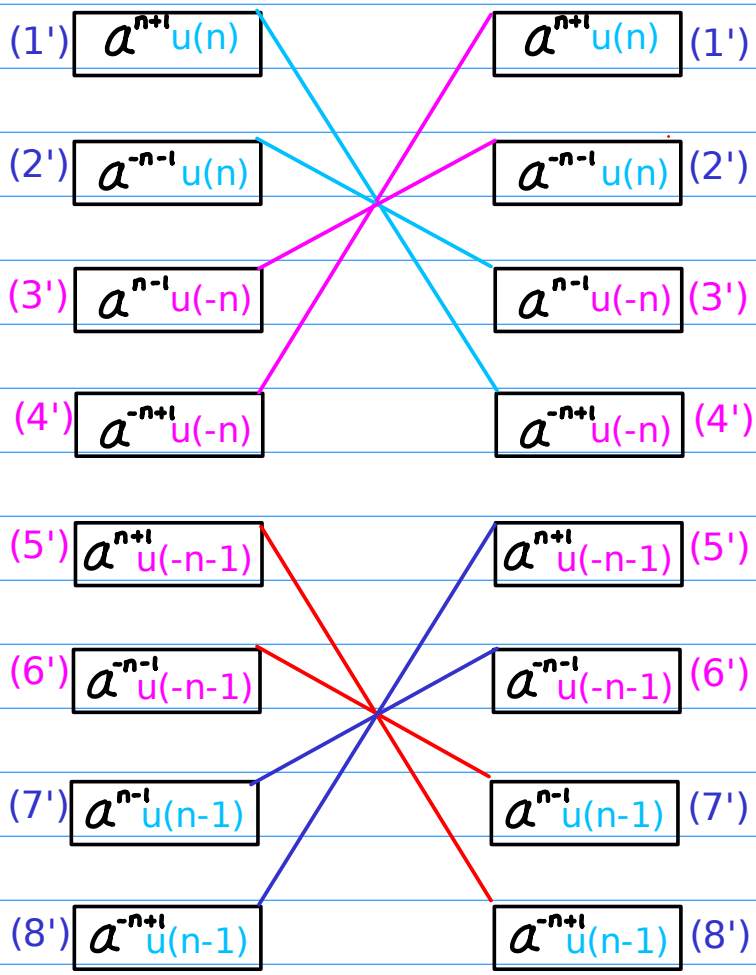
A.I Flipping
Base Inverting
Range Flipping

D.I Flipping2
Base Inverting
Shifted Range Flipping









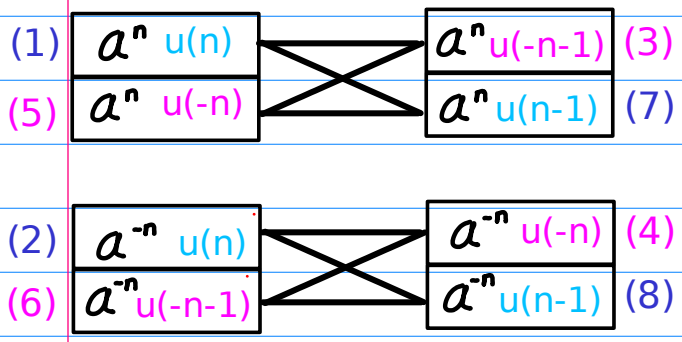
(1) - (4)
(2) - (3)
(3) - (2)
(4) - (1)

(5) - (8)
(6) - (7)
(7) - (6)
(8) - (5)

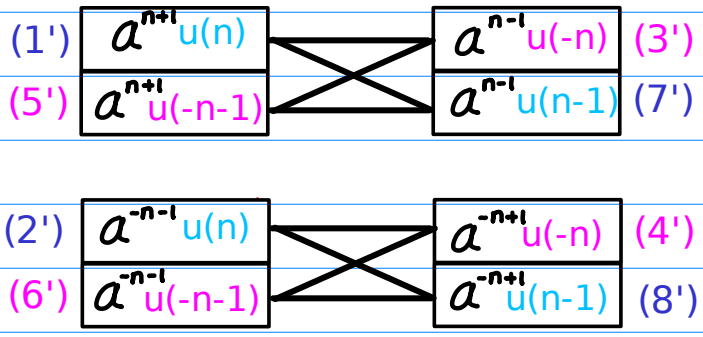
(1') - (4')
(2') - (3')
(3') - (2')
(4') - (1')

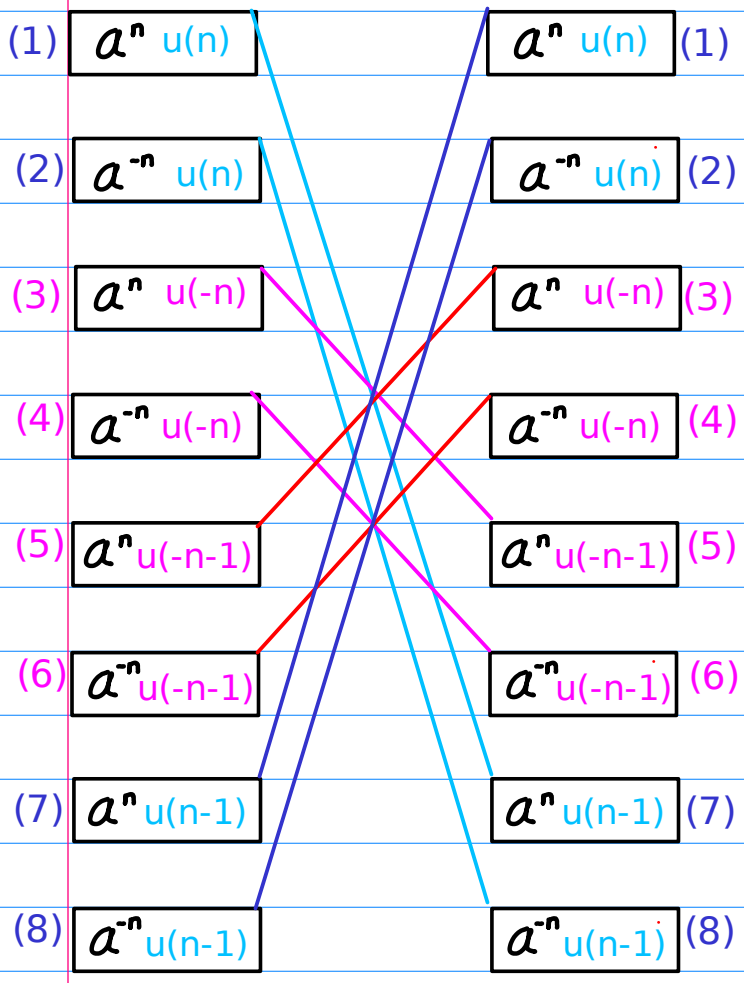
(5') - (8')
(6') - (7')
(7') - (6')
(8') - (5')

B.I Range Shifting
Range Flipping
Range Complementing



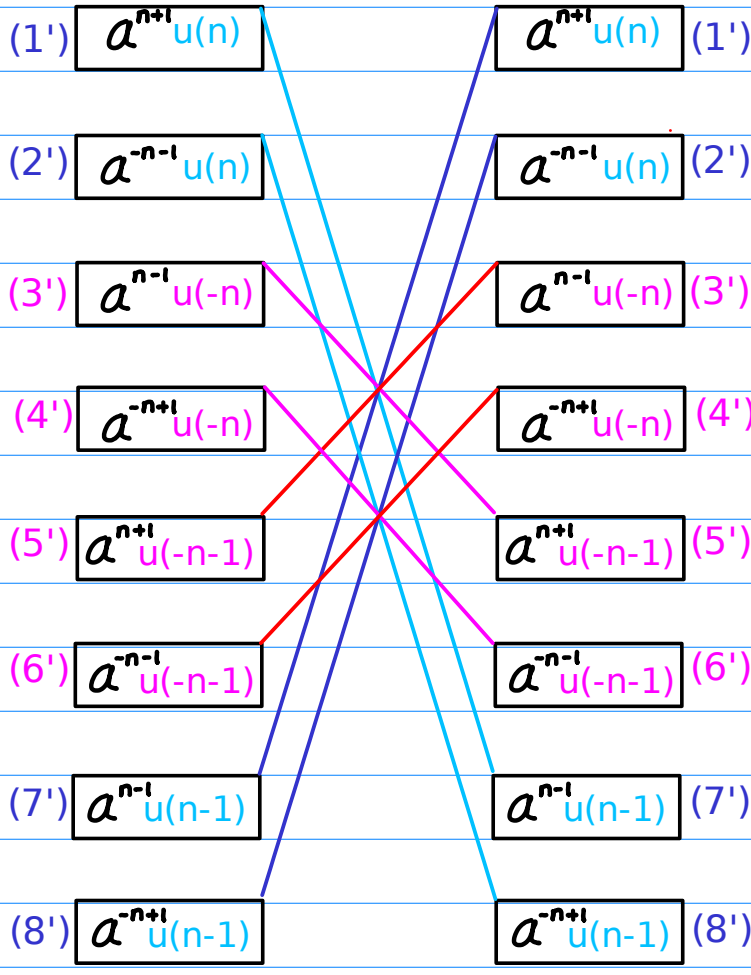
E.I Shifting2
Shifted Range Flipping
Range Complementing





(1) - (7)
(2) - (8)
(3) - (5)
(4) - (6)

(5) - (3)
(6) - (4)
(7) - (1)
(8) - (2)

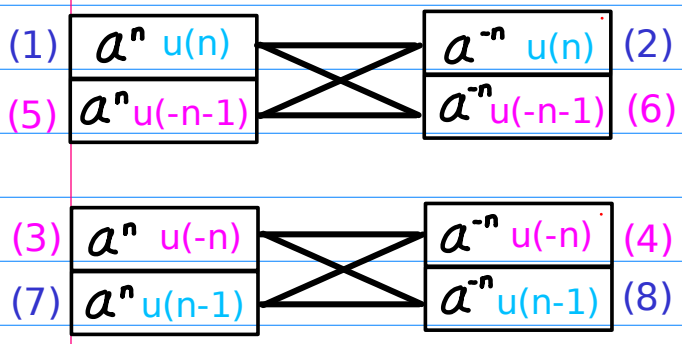


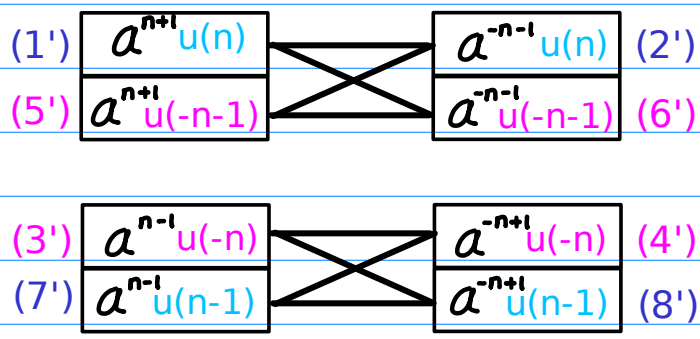
(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')

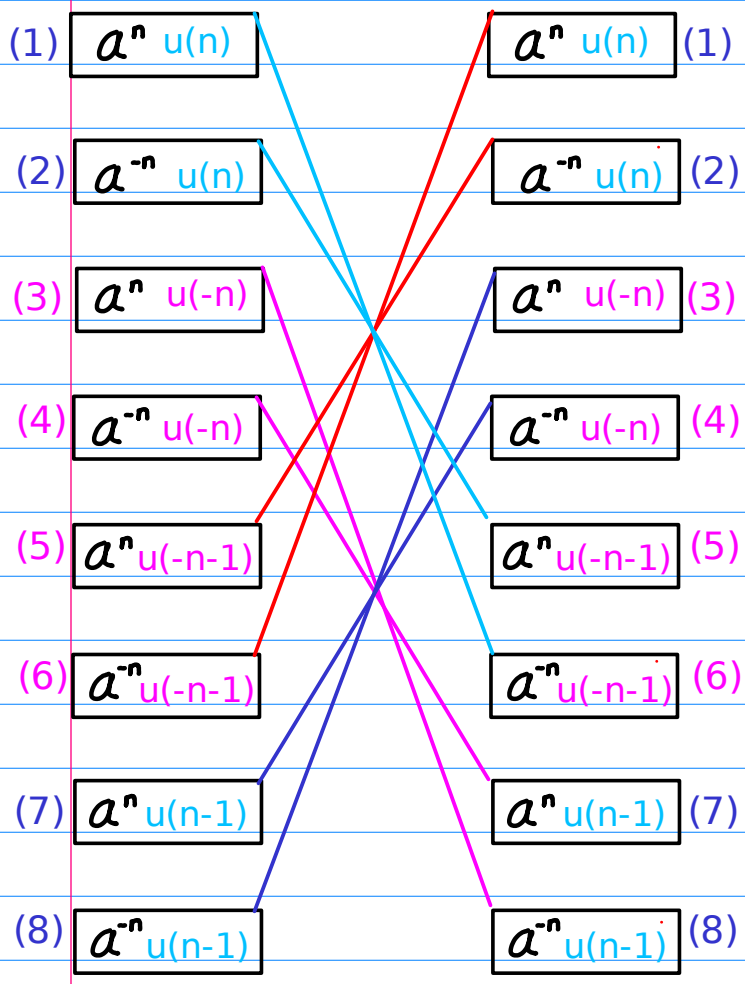
(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

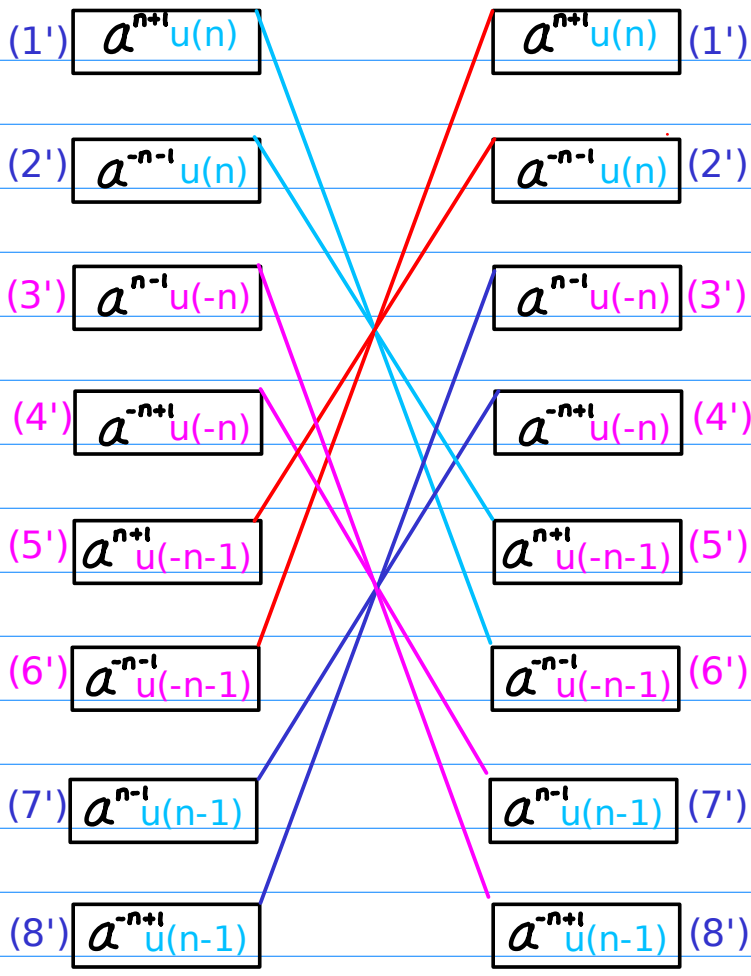
C.I Complementary Inverting
Base Inverting
Range Complementing

F.I Complementary Inverting
Base Inverting
Range Complementing









(1) - (6)
(6) - (1)
(2) - (5)
(5) - (2)

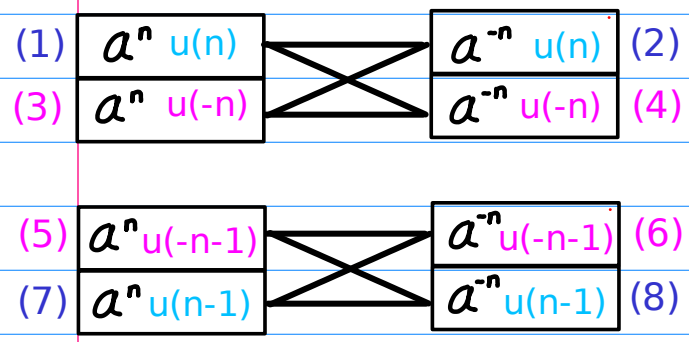
(5) - (2)
(6) - (1)
(7) - (4)
(8) - (3)

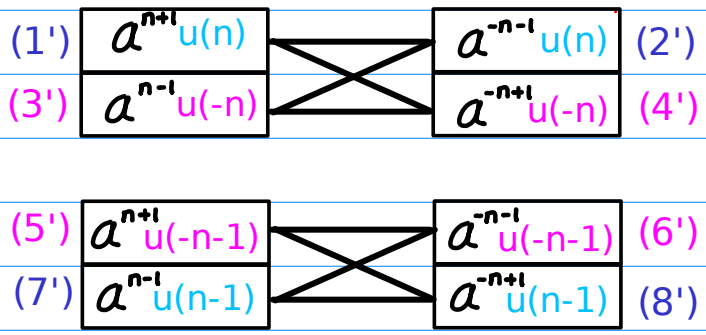
(1') - (6')
(6') - (1')
(2') - (5')
(5') - (2')

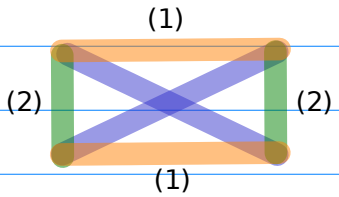
(5') - (2')
(6') - (1')
(7') - (4')
(8') - (3')

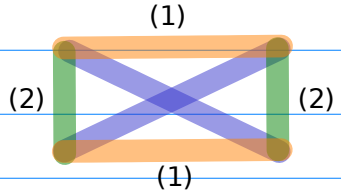
A.I Flipping
(1) Base Inverting
(2) Range Flipping

D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flipping









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

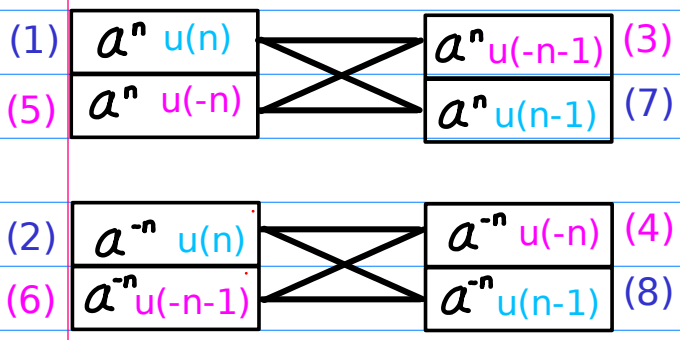
b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

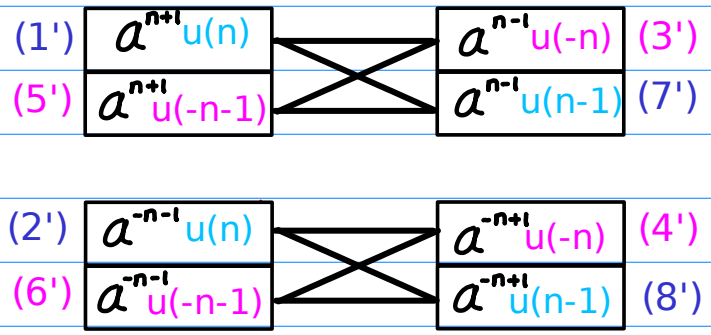
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

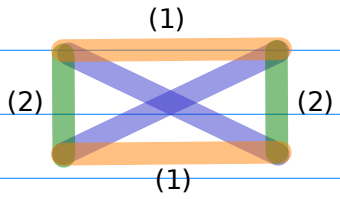
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

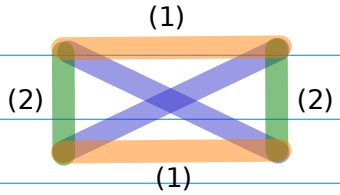
B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing









$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

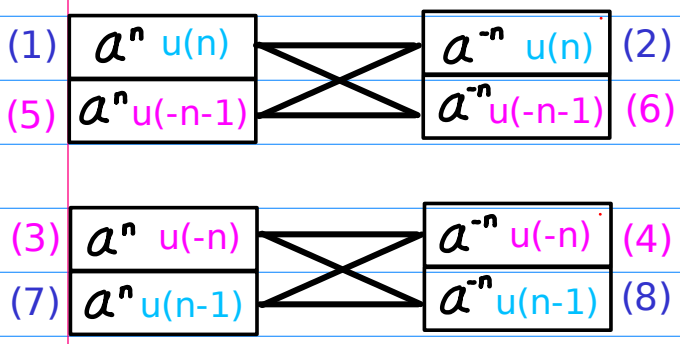
b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

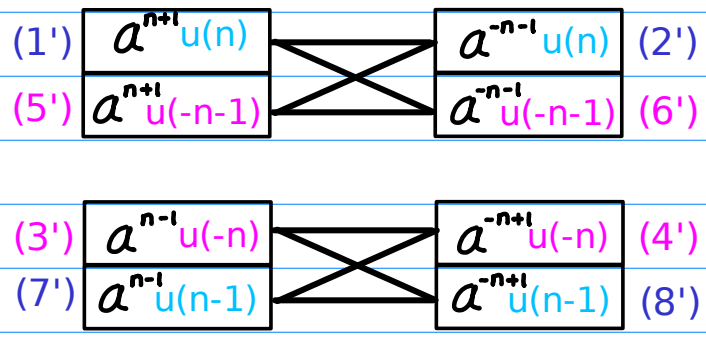
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

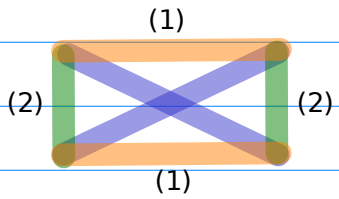
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

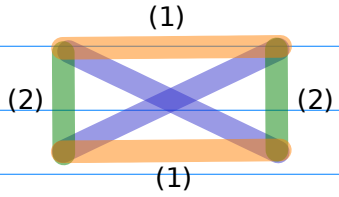
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

b^n	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

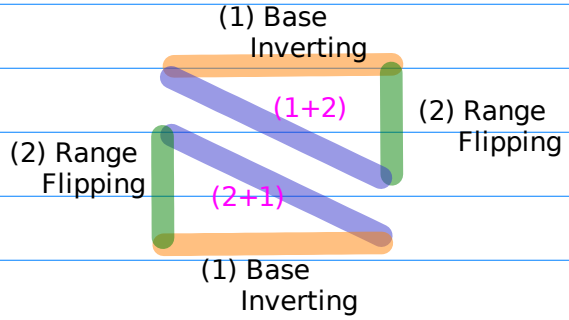
$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

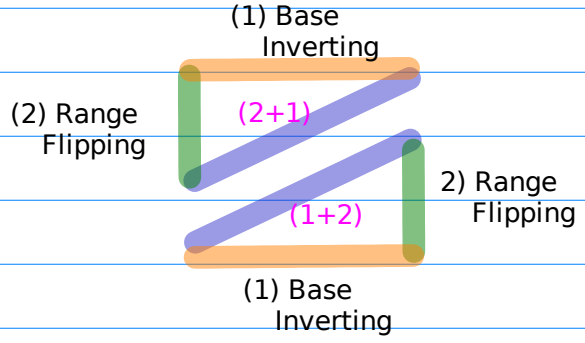
A.I Flipping
(1) Base Inverting
(2) Range Flipping

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^n R(-n)$$
$$\xrightarrow{(1)} a^{-n} R(-n)$$



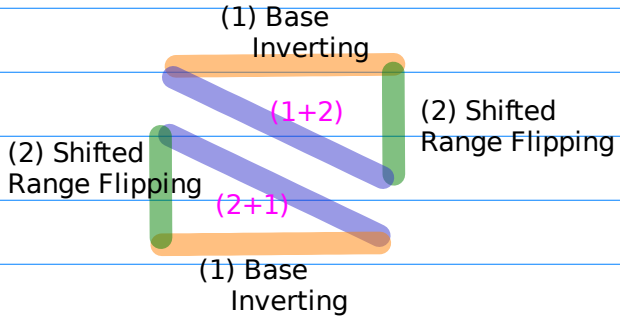


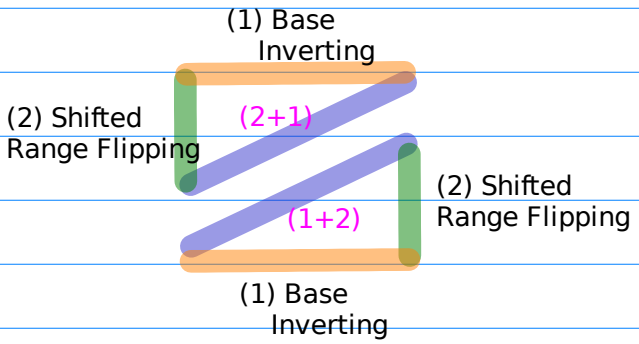
D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flippin

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$
$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$





B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing

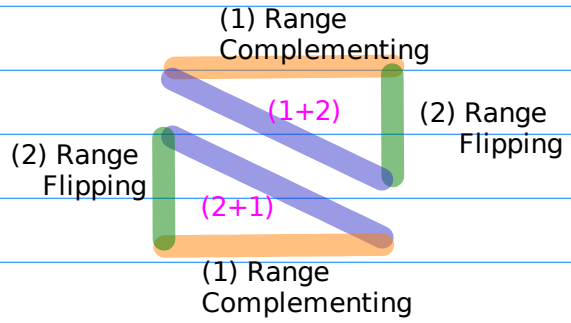
$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

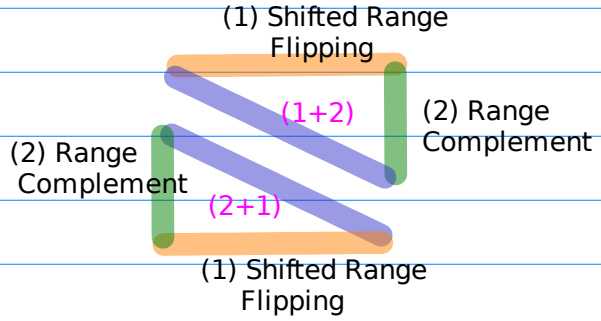
$$a^n R(n) \overset{(1)}{\rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\rightarrow} a^{-n} \overline{R(n)}$$

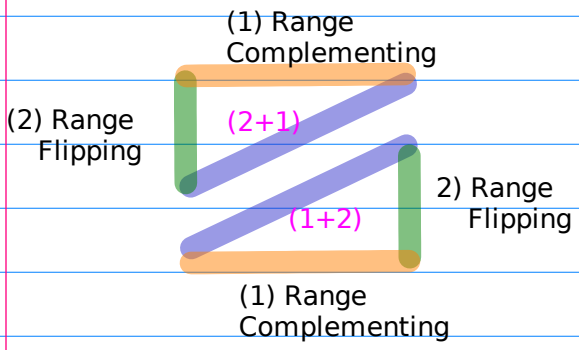
$$a^n R(n) \overset{(1)}{\rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\rightarrow} a^{-sh2(n)} R(-n)$$

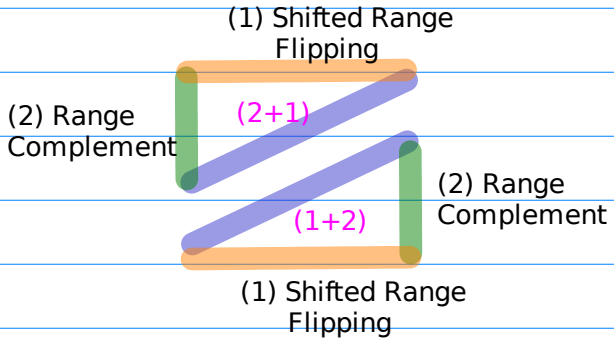
$$a^n rng(n) \overset{(2)}{\rightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\rightarrow} a^{-n} \overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\rightarrow} a^{sh2(n)} R(-n)$$
$$\overset{(1)}{\rightarrow} a^{-sh2(n)} R(-n)$$







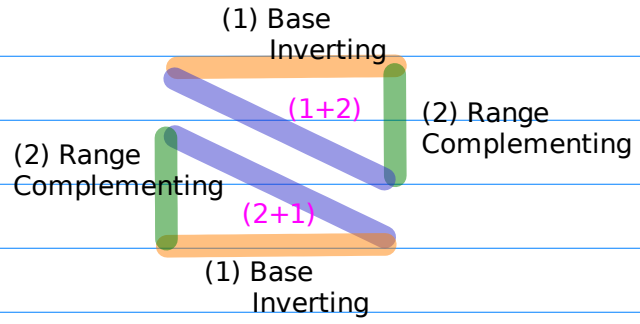


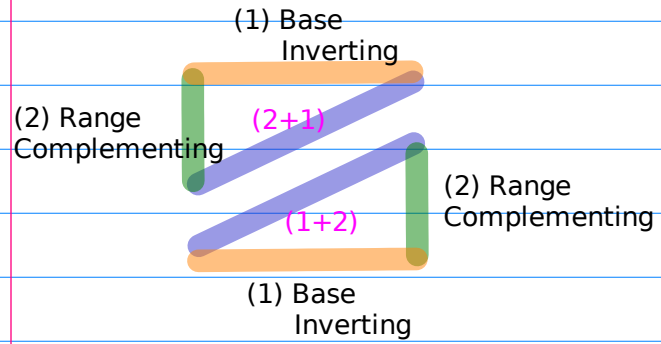
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$



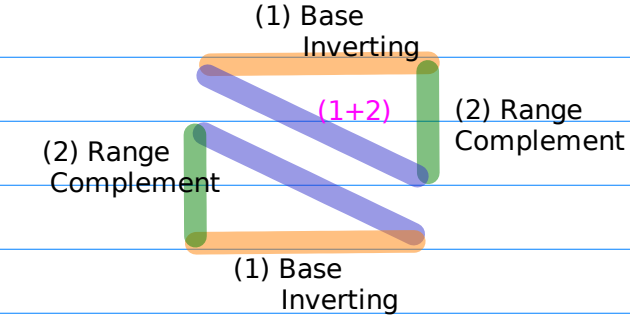


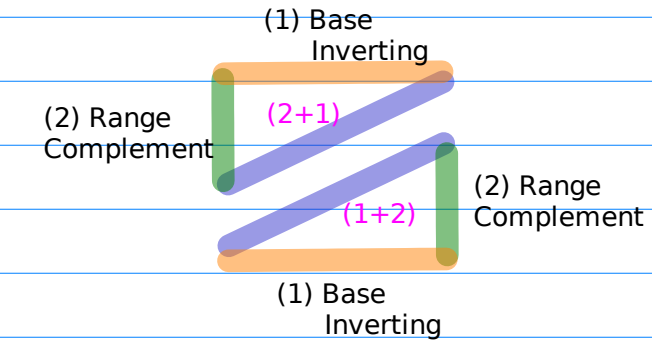
F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

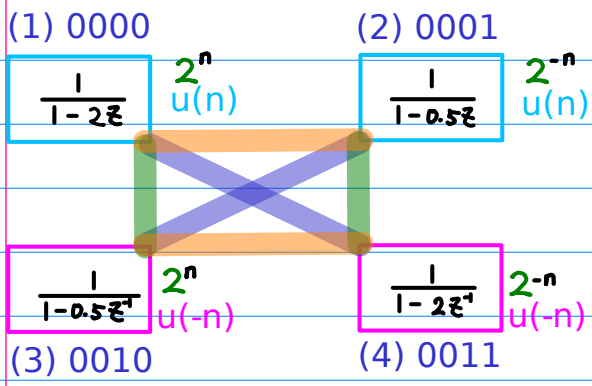
$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

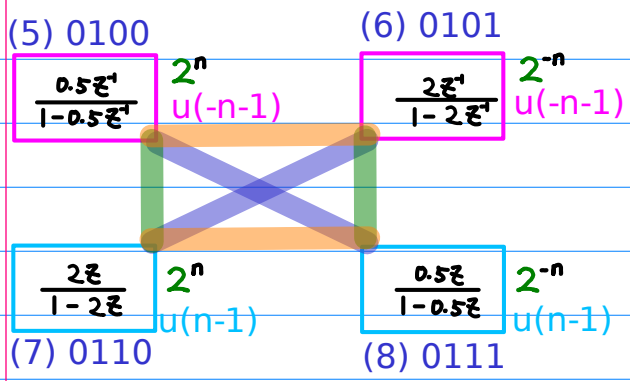
$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$



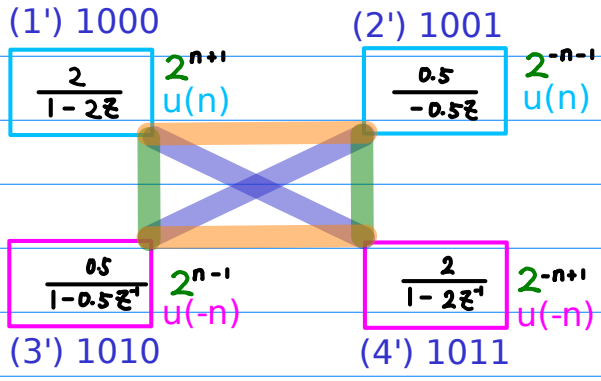


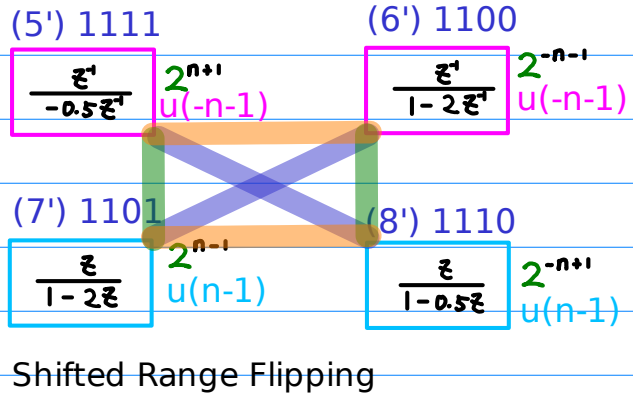
A.I Flipping
Base Inverting
Range Flipping





D.I Flipping2
Base Inverting
Shifted Range Flipping



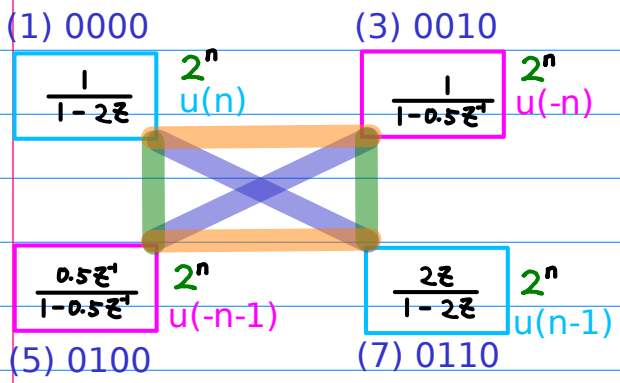


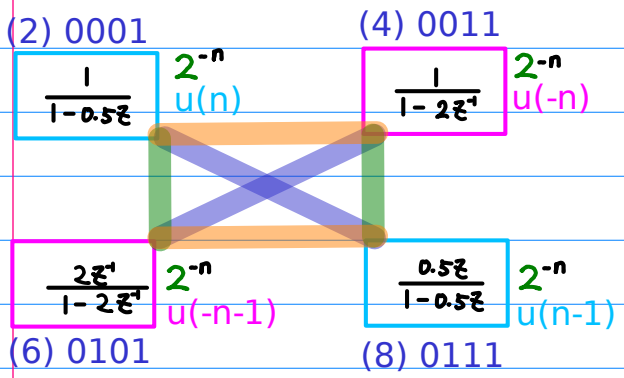
Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$
$$R(n) \quad \Longleftrightarrow \quad R(-n)$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-n} R(-n)$$

$$a^n \quad \Longleftrightarrow \quad a^{-n}$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{sh2(n)} R(-n)$$
$$a^n R(n) \quad \Longleftrightarrow \quad a^{-sh2(n)} R(-n)$$

B.I Range Shifting
Range Flipping
Range Complementing

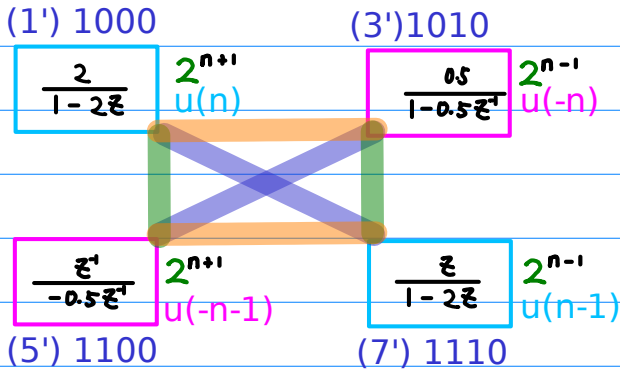


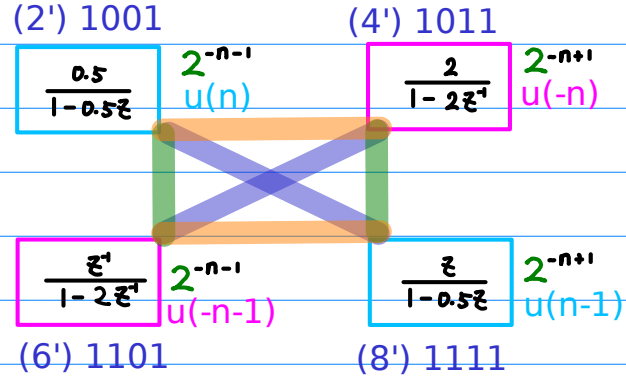


Range Shifting
= Range Flipping
+ Range Complementing

$$R(n) \Longleftrightarrow R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

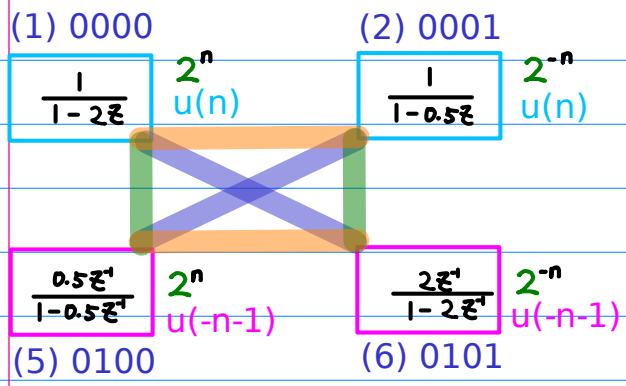


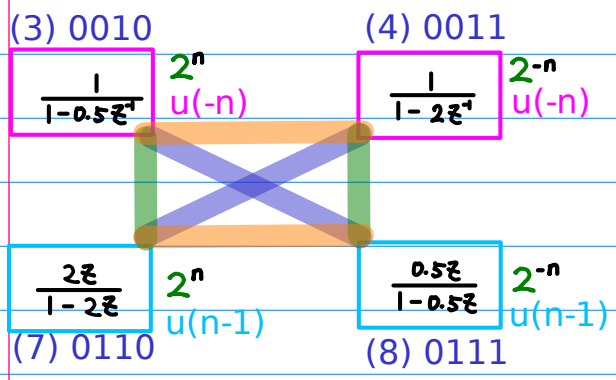


Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

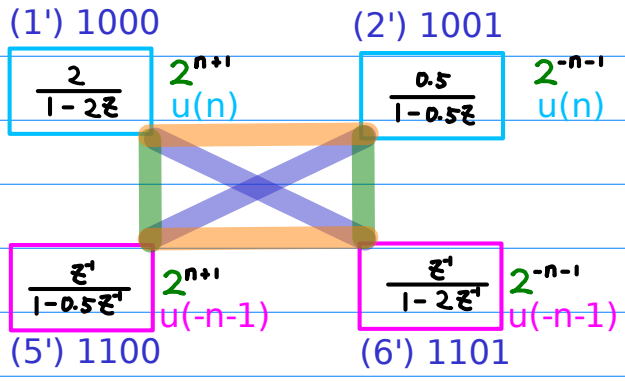
C.I Complementary Inverting
Base Inverting
Range Complementing

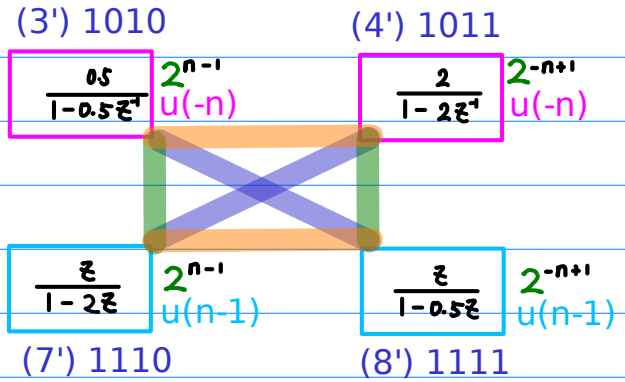




$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

F.I Complementary Inverting
Base Inverting
Range Complementing

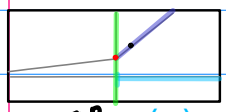




$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

A.II Flipping
Base Inverting
Range Flipping

(1) 0000



a^n u(n)

(4) 0011

a^{-n} u(-n)

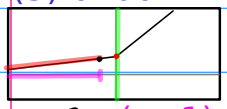
(3) 0010

a^n u(-n)

(2) 0001

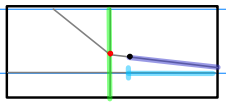
a^{-n} u(n)

(5) 0100



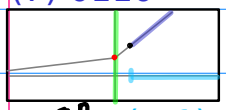
a^n u(-n-1)

(8) 0111



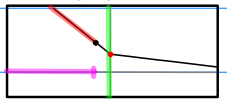
a^{-n} u(n-1)

(7) 0110



a^n u(n-1)

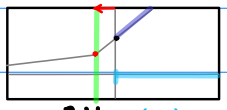
(6) 0101



a^{-n} u(-n-1)

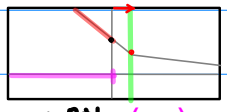
D.II Flipping2
Base Inverting
Shifted Range Flipping

(1') 1000



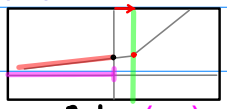
a^{n+1} u(n)

(4') 1011



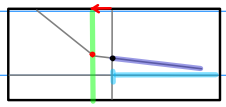
a^{-n+1} u(-n)

(3') 1010



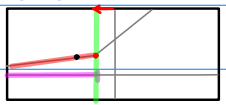
a^{n-1} u(-n)

(2') 1001



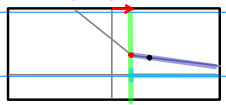
a^{-n-1} u(n)

(5') 1100



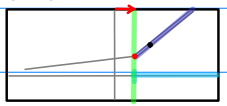
a^{n+1} u(-n-1)

(8') 1111



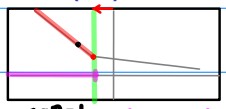
a^{-n+1} u(n-1)

(7') 1110



a^{n-1} u(n-1)

(6') 1101



a^{-n-1} u(-n-1)

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow R(-n)$$

$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

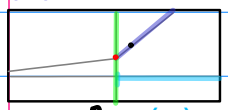
$$a^n \Longleftrightarrow a^{-n}$$

$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$

B.II Range Shifting
Range Flipping
Range Complementing

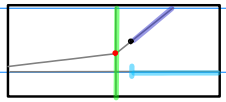
E.II Shifting2
Shifted Range Flipping
Range Complementing

(1) 0000



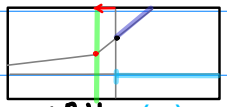
a^n u(n)

(7) 0110



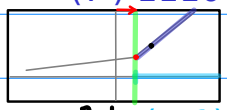
a^n u(n-1)

(1') 1000



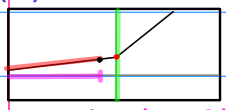
a^{n+1} u(n)

(7') 1110



a^{n-1} u(n-1)

(5) 0100

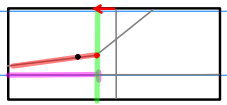


a^n u(-n-1)

(3) 0010

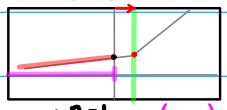
a^n u(-n)

(5') 1100



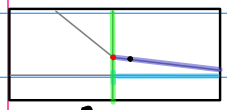
a^{n+1} u(-n-1)

(3') 1010



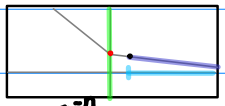
a^{n-1} u(-n)

(2) 0001



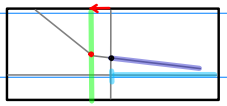
a^{-n} u(n)

(8) 0111



a^{-n} u(n-1)

(2') 1001

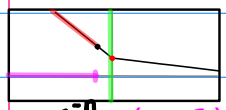


a^{-n-1} u(n)

(8') 1111

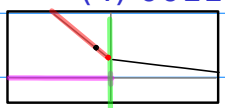
a^{-n+1} u(n-1)

(6) 0101



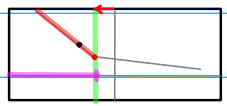
a^{-n} u(-n-1)

(4) 0011



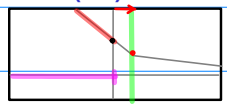
a^{-n} u(-n)

(6') 1101



a^{-n-1} u(-n-1)

(4') 1011



a^{-n+1} u(-n)

Range Shifting
= Range Flipping
+ Range Complementing

$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$R(n) \Longleftrightarrow R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

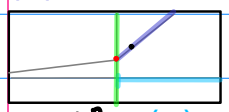
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

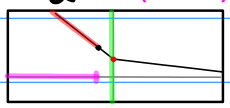
C.II Complementary Inverting
Base Inverting
Range Complementing

(1) 0000



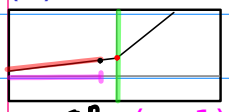
a^n u(n)

a^{-n} u(-n-1)



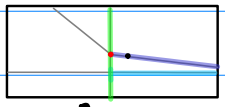
(6) 0101

(5) 0100



a^n u(-n-1)

(2) 0001

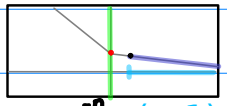


a^{-n} u(n)

(3) 0010

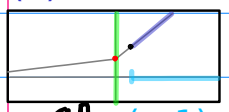
a^n u(-n)

(8) 0111



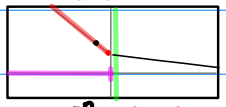
a^{-n} u(n-1)

(7) 0110



a^n u(n-1)

(4) 0011



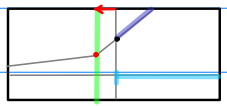
a^{-n} u(-n)

$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$

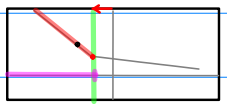
F.II Complementary Inverting
Base Inverting
Range Complementing

(1') 1000



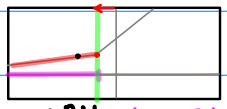
a^{n+1} u(n)

a^{-n-1}u(-n-1)



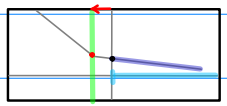
(6') 1101

(5') 1100



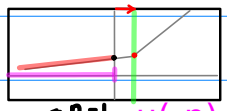
a^{n+1} u(-n-1)

(2') 1001



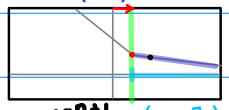
a^{-n-1} u(n)

(3') 1010



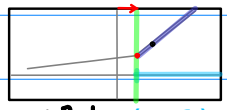
a^{n-1} u(-n)

(8') 1111



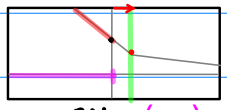
a^{-n+1} u(n-1)

(7') 1110



a^{n-1} u(n-1)

(4') 1011



a^{-n+1} u(-n)

$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$