

Spectral Analysis of Surface Waves (2A)

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2D FT seismic example (1)

Power Spectra

$$G_{XX} = X^*(f) \cdot X(f)$$

$$G_{YY} = Y^*(f) \cdot Y(f)$$

Cross Spectrum

$$G_{XY} = X^*(f) \cdot Y(f)$$

$$G_{XY} = \frac{1}{N} \sum_{i=1}^N X^*(f) \cdot Y_i(f)$$

$$\Phi(f) = \tan^{-1} \left(\frac{\Im(G_{XY})}{\Re(G_{XY})} \right)$$

Coherence Function

$$\gamma^2(f) = \frac{|G_{XY}(f)|^2}{G_{XX}(f) \cdot G_{YY}(f)}$$

Nazarian & Stokoe 1984

$$V_R(f) = \frac{2\pi f}{\Theta_{12}(f)} \cdot X$$

inter-receiver
distance

Phase Velocity

Phase

cross -power spectrum of the two signals

$$c = \frac{\omega \Delta x}{\Delta \theta}$$

for each angular frequency
the phase velocity c is calculated
from the phase difference and distance

$$\frac{\Delta \theta}{\Delta x} = k$$

$$\frac{\Delta \theta}{\Delta t} = \omega$$

fk analysis

$$V_R(f) = \frac{2\pi f}{k}$$

Phase Velocity

wave number

Using Cross Power Spectrum Phase

The signals of the two receivers in time domain
in frequency domain

$$y_1(t), y_2(t)$$

$$Y_1(\omega), Y_2(\omega)$$

auto-power spectra

$$G_{11}(\omega) = Y_1(\omega)\overline{Y_1(\omega)}$$

$$G_{22}(\omega) = Y_2(\omega)\overline{Y_2(\omega)}$$

cross-power spectra

$$G_{12}(\omega) = Y_1(\omega)\overline{Y_2(\omega)}$$

phase of cross-power spectra

$$\Theta_{12}(\omega) = \tan^{-1} \left[\frac{\Im(G_{12}(\omega))}{\Re(G_{12}(\omega))} \right]$$

time delay between two receivers

$$t(\omega) = \frac{\Theta_{12}(\omega)}{\omega} \quad \frac{2\pi f \cdot t}{2\pi f}$$

phase velocity of the surface wave

$$V_R(\omega) = \frac{D}{t(\omega)} \quad \frac{dist}{time}$$

wavelength

$$\lambda_R(\omega) = \frac{V_R(\omega)}{f}$$

References

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- [3] www.masw.com
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