Spectral Analysis of Surface Waves (2A)

Young Won Lim
3/21/14

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

2D FT seismic example (1)

Power Spectra

$$G_{XX} = X^*(f) \cdot X(f)$$

$$G_{YY} = Y^*(f) \cdot Y(f)$$

Cross Spectrum

$$G_{XY} = X^*(f) \cdot Y(f)$$

$$G_{XY} = \frac{1}{N} \sum_{i=1}^{N} X^*(f) \cdot Y_i(f)$$

$$\Phi(f) = \tan^{-1}\left(\frac{\Im(G_{XY})}{\Re(G_{XY})}\right)$$

Coherence Function

$$\gamma^2(f) = \frac{\left|G_{XY}(f)^2\right|}{G_{XX}(f) \cdot G_{YY}(f)}$$

Nazarian & Stokoe 1984

$$V_R(f) = \frac{2\pi f}{\Theta_{12}(f)} \cdot X$$

inter-receiver
distance

Phase Velocity

Phase

cross -power spectrum of the two signals

$$c = \frac{\omega \Delta x}{\Delta \theta}$$

for each angular frequency
the phase velocity c is calculated
from the phase difference and distance

$$\frac{\Delta \theta}{\Delta x} = k \qquad \frac{\Delta \theta}{\Delta t} = \omega$$

fk analysis

$$V_R(f) = \frac{2\pi f}{k}$$

Phase Velocity wave number

Using Cross Power Spectrum Phase

The signals of the two receivers in time domain	$y_1(t), \quad y_2(t)$
in frequency domain	$Y_1(\omega), \quad Y_2(\omega)$
auto-power spectra	$G_{11}(\omega) = Y_1(\omega)\overline{Y_1(\omega)}$ $G_{22}(\omega) = Y_2(\omega)\overline{Y_2(\omega)}$
cross-power spectra	$G_{12}(\omega) = Y_1(\omega)\overline{Y_2(\omega)}$
phase of cross-power spectra	$\Theta_{12}(\omega) = \tan^{-1}\left[\frac{\Im(G_{12}(\omega))}{\Re(G_{12}(\omega))}\right]$
time delay between two receivers	$t(\omega) = \frac{\Theta_{12}(\omega)}{\omega}$ $\frac{2\pi f \cdot t}{2\pi f}$
phase velocity of the surface wave	$V_R(\omega) = \frac{D}{t(\omega)}$ $\frac{dist}{time}$
wavelength	$\lambda_R(\omega) = \frac{V_R(\omega)}{f}$

References

[1] http://en.wikipedia.org/
[2] S. Foti, et. al., “Application of FK Analysis of Surface Waves for Geotechnical Characterization “
[3] www.masw.com
[4] V. Ganji, S. Nazarian, “Automated Inversion Procedure for Spectral Analysis of Surface Waves”, 1998