Angle Recording CORDIC
2. Wu

20180331

Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better
the number of iterations
the error performance

$$S_{EAS} = \{ (\sigma \cdot \tan^{-1}(2^{-r})) : \sigma \in \{+1, 0, -1\}, \ r \in \{1, 2, \cdots, n-1\} \}$$

$$S_{EEAS} = \{ (\sigma_1 \cdot \tan^{-1}(2^{-r_1}) + \sigma_2 \cdot \tan^{-1}(2^{-r_2})) : \sigma_1, \sigma_2 \in \{+1, 0, -1\}, \ r_1, r_2 \in \{1, 2, \cdots, n-1\} \}$$

The pse do-rotation
for i-th micro rotations

$$x_{i+1} = x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}]\, y_i$$
$$y_{i+1} = y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}]\, x_i$$

The pseudo-rotated vector $[x_{R_m}, y_{R_m}]$
after R_m (the requred number of micro-rotations)

Needs to be scaled by a factor $K = \prod K_i$

$$K_i = \left[1 + \left(\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}\right)^2\right]^{-\frac{1}{2}}$$

$$\tilde{x}_{i+1} = \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}]\, \tilde{y}_i$$
$$\tilde{y}_{i+1} = \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}]\, \tilde{x}_i$$

$\tilde{x}_0 = x_{R_m}$ $\quad k_1, k_2 \in \{-1, 0, 1\}$

$\tilde{y}_0 = y_{R_m}$ $\quad s_1, s_2 \in \{1, 2, \cdots, n-1\}$

[21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, “A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes,” *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

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A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate θ
with the combination
of selected angle elements
from a pre-defined **EAS**
(Elementary Angle Set)

EAS : all possible values of $\theta(j)$

EAS $\hat{S}_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) \ : \ \alpha^* \in \{-1, 0, +1\},$
$s^* \in \{0, 1, \cdots, N-1\} \}$

EAS $\hat{S}_1$ consists of $\tan^{-1}$(Single **S**igned **p**ower of **t**wo)
$\tan^{-1}$(single **SPT**)
$\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design

to increase the coefficient resolution

→ imploy more SPT terms to represent filter coefficients

[12] H. Samueli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.

[13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS $\hat{S}_1$ consists of $\tan^{-1}$(Single signed power of two)

$\tan^{-1}$(Single SPT)

$\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS $\hat{S}_2$ consists of $\tan^{-1}$(two signed power of two)

$\tan^{-1}$(two SPT)

$\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*})$

Two Signed-Power-of-Two terms

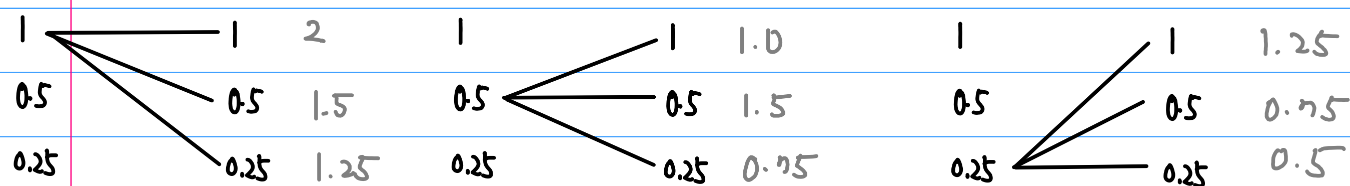
$$S_2 = \{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, \; s_0^*, s_1^* \in \{0, 1, \cdots, w-1\} \}$$

S_1

1	$1 = 2^{-0}$	$\tan^{-1}(2^{-0})$
0.5	$\frac{1}{2} = 2^{-1}$	$\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\tan^{-1}(2^{-2})$

S_2

2	$1+1 = 2^{-0} + 2^{-0}$	$\pm\tan^{-1}(2^{-0} + 2^{-0})$
1.5	$1+\frac{1}{2} = 2^{-0} + 2^{-1}$	$\pm\tan^{-1}(2^{-0} + 2^{-1})$
1.25	$1+\frac{1}{4} = 2^{-0} + 2^{-2}$	$\pm\tan^{-1}(2^{-0} + 2^{-2})$
1.0	$1 = 2^{-0}$	$\pm\tan^{-1}(2^{-0})$
0.75	$\frac{1}{2}+\frac{1}{4} = 2^{-1} + 2^{-2}$	$\pm\tan^{-1}(2^{-1} + 2^{-2})$
0.5	$\frac{1}{2} = 2^{-1}$	$\pm\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\pm\tan^{-1}(2^{-2})$



$2^{-0}, \quad 2^{-1}, \quad 2^{-2}$ $\qquad \{0, 1, 2\} = \{0, 1, w-1\}$

$w=3$

$s_0^*, s_1^* \in \{0, 1, 2\}$

$2^{s_0^*}, 2^{s_1^*} \in \{2^{-0}, 2^{-1}, 2^{-2}\}$

as the word size W increases,
the size of the set S_2 increases exponentially

$$\theta_i = \tan^{-1}\left(\alpha_0(j)\cdot 2^{-S_0(j)} + \alpha_1(j)\cdot 2^{-S_1(j)}\right)$$

R_m : the number of the subangle N_A

$$S_2 = \{ \theta_i \mid i = 0, 1, \cdots R_m \}$$

$$S_2 = \{ \tan^{-1}\left(\alpha_0^* \cdot 2^{-S_0^*} + \alpha_1^* \cdot 2^{-S_1^*}\right) : \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, \; S_0^*, S_1^* \in \{0, 1, \cdots, W-1\} \}$$

the optimization problem of the EEAS-based CORDIC algorithm

given θ and R_m
find $\alpha_0(j)$, $\alpha_1(j)$, $S_0(j)$, and $S_1(j)$
the combination of elementary angles
from EEAS $\hat{S}_2$

minimize the angle quantization error

$$|\xi_{m,\,EEAS}| \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1}\left(\alpha_0(j)\, 2^{-S_0(j)} + \alpha_1(j)\, 2^{-S_1(j)}\right)$$

given $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j)\, 2^{-S_0(j)} + \alpha_1(j)\, 2^{-S_1(j)} \\ \alpha_0(j)\, 2^{-S_0(j)} + \alpha_1(j)\, 2^{-S_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{R_s-1} \sqrt{1 + [\alpha_0(j) \cdot 2^{-S_0(j)} + \alpha_1(j) \cdot 2^{-S_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

micro rotation procedure

the scaling operation

4 additions

increased hardware

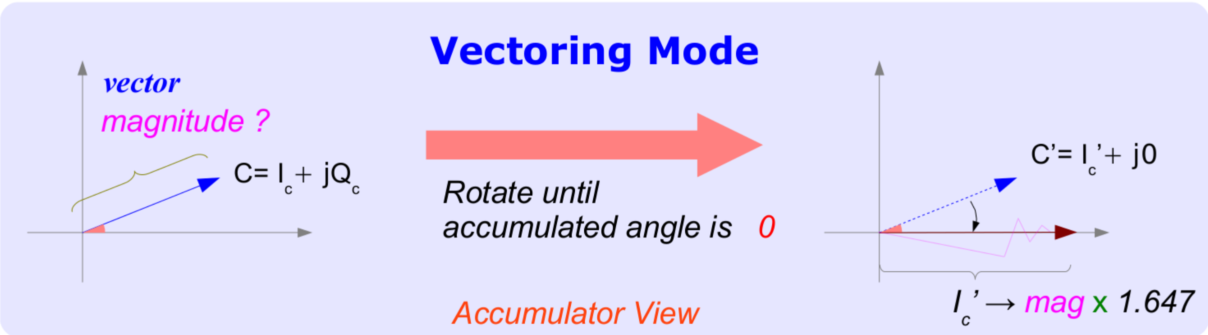
reduced iteration steps

MVR (Modified Vector Rotation)

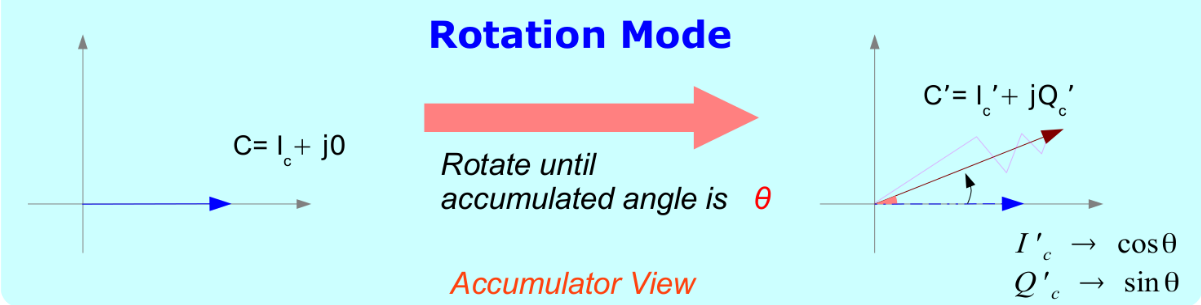
1) Repeat of Elementary Angles θ_i, σ_i

2) fixed total micro-rotation Number R_m

* Vector Rotation Mode
* and the rotation angles are known in advance



vector rotation mode



Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

Three Searching Algorithm

① the selective prerotation
② the selective scaling
③ iteration-tradeoff scheme

Vector Rotation CORDIC Family

0. Conventional CORDIC
1. AR
2. MVR
3. EEAS

⓪ Conventional CORDIC

elementary angle $a(i) = \tan^{-1}(2^{-i})$

the number of elementary angles N

the rotation sequence $\mu(i) = \{-1, +1\}$

+1, -1, -1, +1, +1, ...

the i-the rotation angle $a(i)$

the W-bit word length

the iteration number $N \leq W$

the angle quantization error

$$\xi_{m,\,CORDIC} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$$\mu(i) = 0 \rightarrow \text{skip}$$

$$\xi_{m,\,CORDIC} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad \mu(i) = \{-1, 0, +1\}$$

$$= \theta - \sum_{j=0}^{N'} \tilde{\theta}(j)$$

$$N' \equiv \sum_{i=0}^{N-1} |\mu(i)| \qquad \{|-1|, |0|, |+1|\}$$

the effective iteration number N'

$S(j)$ the rotational sequence

determines the micro-rotation angle in the j-th iteration

er

$i = 0, 1, 2, 3, \cdots, N-1$

$s(j) = 0, 1, 2, 3, \cdots, N-1$ rotational sequence

$\alpha(j) = -1, 0, 0, +1, \cdots, -1$ directional sequence

$j = 0, -, -, 1, \cdots, N'-1$ effective iteration numb

$N' = N-2$

the j-th micro-rotation of $a(s(j))$

elementary angle

$$a(i) = \tan^{-1}(2^{-i})$$
$$a(s(j)) = \tan^{-1}(2^{-s(j)})$$

$$\alpha(j)\, a(s(j)) = \alpha(j) \tan^{-1}(2^{-s(j)})$$

$\Leftrightarrow$ $\mu(i)\, a(i)$

$$\xi_{m,\,\mathrm{CORDIC}} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad \mu(i) = \{-1, 0, +1\}$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)} \right) \right]$$

$$\tilde{\theta}(j) = \alpha(j) \tan^{-1}\left(2^{-s(j)} \right)$$

$$= \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)} \right)$$

$$S_1 = \left\{ \tan^{-1}\left(\boxed{\alpha} \cdot 2^{-\boxed{s}} \right) \quad \boxed{\alpha} \in \{-1, 0, +1\},\ \boxed{s} \in \{0, 1, 2, \cdots N-1\} \right\}$$

② MVR