Laurent Series and z-Transform
- Geometric Series
Double Pole Properties A

20190226 Tue

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

z^{-1}

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$ ② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$$+\frac{1}{z-0.5} - \frac{1}{z-2} \qquad -\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $|z| < 0.5$

causal $f_1(z)$

anti-causal $X_1(z)$

$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $|z| > 2$

anti-causal $g_1(z)$

causal $Y_1(z)$

$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $|z| > 2$

anti-causal $f_2(z)$

causal $X_2(z)$

$+\frac{z}{1-0.5z} - \frac{z}{1-2z}$ $|z| < 0.5$

causal $g_2(z)$

anti-causal $Y_2(z)$

$$① \quad \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \qquad\qquad ② \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$+\frac{1}{z-0.5} - \frac{1}{z-2} \qquad\qquad -\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\downarrow \cdot 2 \qquad \downarrow \cdot \frac{1}{2} \qquad\qquad \downarrow \cdot \frac{1}{z} \qquad \downarrow \cdot \frac{1}{z}$$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5 \qquad\qquad -\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2 \qquad\qquad +\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$\uparrow \cdot \frac{1}{z} \qquad \uparrow \cdot \frac{1}{z} \qquad\qquad \uparrow \cdot \frac{1}{2} \qquad \uparrow \cdot 2$$

$$+\frac{1}{z-0.5} - \frac{1}{z-2} \qquad\qquad -\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\boxed{① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$|2z| < 1 \qquad |0.5z| < 1 \qquad |2z^{-1}| < 1 \qquad |0.5z^{-1}| < 1$

$|z| < 0.5 \qquad |z| < 2 \qquad 2 < |z| \qquad 0.5 < |z|$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5} \qquad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2} \qquad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$|0.5z^{-1}| < 1 \qquad |2z^{-1}| < 1 \qquad |0.5z| < 1 \qquad |2z| < 1$

$0.5 < |z| \qquad 2 < |z| \qquad |z| < 2 \qquad |z| < 0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad \boxed{|z|<0.5}$$

$$\downarrow \cdot\frac{1}{2z} \qquad \downarrow \cdot\frac{2}{z}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{|z|>2}$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad \boxed{|z|>2}$$

$$\downarrow \cdot\frac{z}{2} \qquad \downarrow \cdot 2z$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad \boxed{|z|<0.5}$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad \boxed{|z|<0.5}$$

$$\uparrow \cdot 2z \qquad \uparrow \cdot\frac{z}{2}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{|z|>2}$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad \boxed{|z|>2}$$

$$\uparrow \cdot\frac{2}{z} \qquad \uparrow \cdot\frac{1}{2z}$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad \boxed{|z|<0.5}$$

Causal sequence a_n & x_n

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

causal $f_1(z) =$

$$-\left[2+2^2z^1+2^3z^2+\cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

causal $Y_1(z) =$

$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}\ \cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

causal $X_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \qquad -2^{n-1}$$

1 2 3

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \qquad -2^{n-1}$$

1 2 3

Anti-causal sequence a_n & x_n

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

anti-causal $X_1(z)$

$$2=\left(\tfrac{1}{2}\right)^{-1}$$
$$\left(\tfrac{1}{2}\right)=2^{-1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^{1}+\left(\tfrac{1}{2}\right)^{-3}z^{2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}+2^{-2}z^{1}+2^{-3}z^{2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $g_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^{0}+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$2=\left(\tfrac{1}{2}\right)^{-1}$$
$$\left(\tfrac{1}{2}\right)=2^{-1}$$

$$+\left[2^{0}z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

anti-causal $Y_2(z)$

$$+\left[2^{0}z^{1}+2^{-1}z^{2}+2^{-2}z^{3}+\cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{1}+\left(\tfrac{1}{2}\right)^{-1}z^{2}+\left(\tfrac{1}{2}\right)^{-2}z^{3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

causal $f_1(z) =$

$$-\left[2 + 2^2 z^1 + 2^3 z^2 + \cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

anti-causal $X_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right] \qquad +2^{n-1}$$

0 -1 -2

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

anti-causal $g_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] \qquad +2^{n-1}$$

0 -1 -2

causal $Y_1(z) =$

$$-\left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$+\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] \qquad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1 -2 -3

causal $X_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \qquad -2^{n-1}$$

1 2 3

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \qquad -2^{n-1}$$

1 2 3

anti-causal $Y_2(z)$

$$+\left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right] \qquad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$f(z) = +\left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

1-A

2-A

1-B

2-B

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① – Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad |z| < 0.5 \qquad -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

$$-\left(2 + 2^2 z + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

n=0 n=1 n=2 $\qquad$ n=0 n=1 n=2

① – Ⓑ

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z| > 2 \qquad +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$$

n=-1 n=-2 n=-3 $\qquad$ n=-1 n=-2 n=-3

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② – Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z| > 2 \qquad -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)$$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right) + \left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right)$$

n=0 n=-1 n=-2 $\qquad$ n=0 n=-1 n=-2

② – Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \qquad |z| < 0.5 \qquad +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

n=1 n=2 n=3 $\qquad$ n=1 n=2 n=3

$X(z)$ $|z| < 0.5$ $|z| > 2$

anticausal causal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① − Ⓐ $\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<0.5}$ $\boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\boxed{(n<1)}$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $n=0$ $n=-1$ $n=-2$

① − Ⓑ $\boxed{\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\boxed{(n\geq 1)}$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1$ $n=2$ $n=3$ $n=1$ $n=2$ $n=3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② − Ⓐ $\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n\geq 0)}$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $n=0$ $n=1$ $n=2$

② − Ⓑ $\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n<0)}$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1$ $n=-2$ $n=-3$ $n=-1$ $n=-2$ $n=-3$

$$f(z) \longleftrightarrow a_n$$

$$X(z) \longleftrightarrow x_n$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

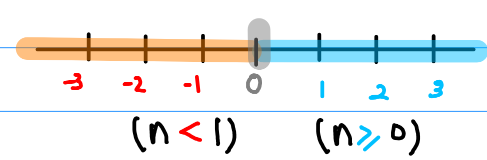
$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

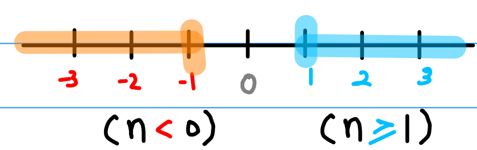
$$x_n = a_{-n}$$

$$a_n = x_{-n}$$

$$(n \geq 0) \longleftrightarrow (n < 1)$$

$$(n \geq 1) \longleftrightarrow (n < 0)$$

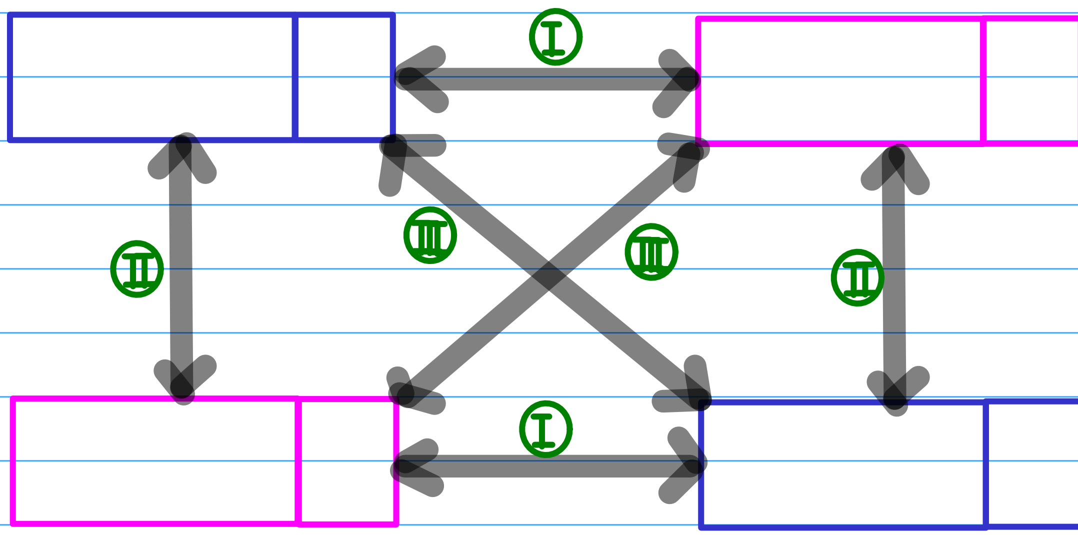




I $\quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $\quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

III $\quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



IV $\quad (a_n, N) \Leftrightarrow (x_{-n}, -N)$

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

①-Ⓐ		②-Ⓐ
$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{\lvert z\rvert<0.5}$	$\xleftrightarrow{z^{-1},\ R^{-1}}$	$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{\lvert z\rvert>2}$
$a_n = \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$	$\xleftrightarrow{a_{-n},\ -N}$	$a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n < 1)$
$x_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n < 1)$	$\xleftrightarrow{a_{-n},\ -N}$	$x_n = \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

①-Ⓑ		②-Ⓑ
$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{\lvert z\rvert>2}$	$\xleftrightarrow{z^{-1},\ R^{-1}}$	$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{\lvert z\rvert<0.5}$
$a_n = \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $(n < 0)$	$\xleftrightarrow{a_{-n},\ -N}$	$a_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq 1)$
$x_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq 1)$	$\xleftrightarrow{a_{-n},\ -N}$	$x_n = \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $(n < 0)$

Ⅱ $(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

$$\Big\|\ z \qquad\qquad \Updownarrow R^{-1}$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$\Big\|\ z \qquad\qquad \Updownarrow R^{-1}$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

①-Ⓐ

$$a_n=\boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}}\quad (n\geq 0)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

①-Ⓑ

$$a_n=\boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0)$$

②-Ⓐ

$$a_n=\boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}}\quad (n<1)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

②-Ⓑ

$$a_n=\boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}}\quad (n\geq 1)$$

①-Ⓐ

$$x_n=\boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}}\quad (n<1)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

①-Ⓑ

$$x_n=\boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}}\quad (n\geq 1)$$

②-Ⓐ

$$x_n=\boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}}\quad (n\geq 0)$$

$$\Updownarrow -a_n \qquad \Updownarrow N^c$$

②-Ⓑ

$$x_n=\boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}}\quad (n<0)$$

III ①

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

①-(A)

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

②-(A)

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

①-(B)

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②-(B)

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

①-(A)

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

②-(A)

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$x_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-(B)

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

②-(B)

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$x_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

Ⅲ ②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \to (z, R^{-1}) \qquad (a_{-n}, -N) \to (-a_n, N^c)$$

① − Ⓐ $\qquad$ ② − Ⓑ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5} \Longleftrightarrow \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0) \xleftrightarrow{-a_{-n},\,(-N)^c} a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① − Ⓑ $\qquad$ ② − Ⓐ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2} \Longleftrightarrow \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0) \xleftrightarrow{-a_{-n},\,(-N)^c} a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

① − Ⓐ $\qquad$ ② − Ⓑ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5} \Longleftrightarrow \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1) \xleftrightarrow{-a_{-n},\,(-N)^c} x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

① − Ⓑ $\qquad$ ② − Ⓐ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2} \Longleftrightarrow \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1) \xleftrightarrow{-a_{-n},\,(-N)^c} x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

IV $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

1-A

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

a_{-n} $\Updownarrow$ $-N$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

2-A

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

a_{-n} $\Updownarrow$ $-N$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

1-B

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

a_{-n} $\Updownarrow$ $-N$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

2-B

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

a_{-n} $\Updownarrow$ $-N$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

Some notations

$R1(z)$ $\quad$ $R1(z^{-1})$

$R2(z)$ $\quad$ $R2(z^{-1})$

R^{-1}

$-N$, $\quad$ N^{c}

ROC's of interests

R1(z) R1(z^{-1})
R2(z) R2(z^{-1})

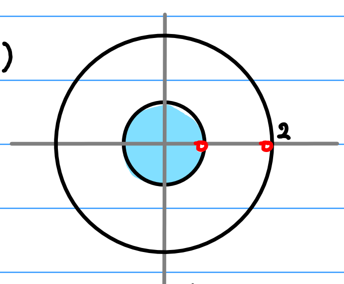
① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$ $\xleftrightarrow{z^{-1}}$ $\boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$

$p_1 = 0.5$ $\xleftrightarrow{z^{-1}}$ $p_1^{-1} = 2$

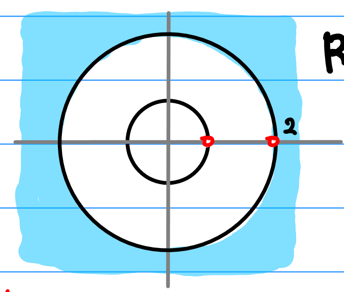
$p_2 = 2$ $\qquad$ $p_2^{-1} = 0.5$

R1(z)



$|z| < 0.5$

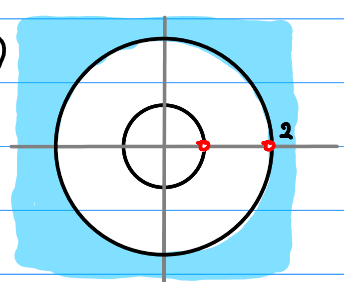
$\xleftrightarrow{z^{-1}}$



R1(z^{-1})

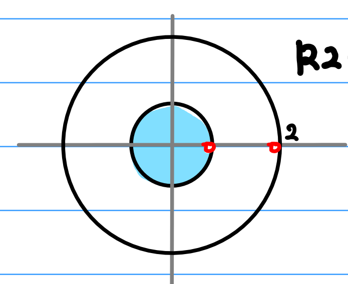
$|z^{-1}| < 0.5$ $\quad$ $|z| > 2$

R2(z)



$|z| > 2$

$\xleftrightarrow{z^{-1}}$



R2(z^{-1})

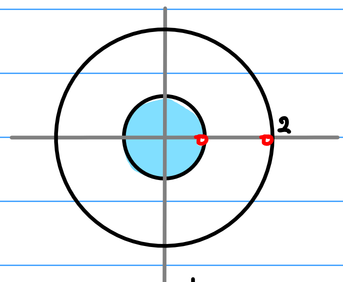
$|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

R1(z^{-1}) = R2(z)

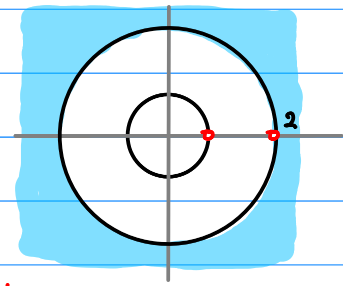
R2(z^{-1}) = R1(z)

R^{-1}

$R1(z) \triangleq R1$ $\qquad$ $R1(z^{-1}) \triangleq R1^{-1}$

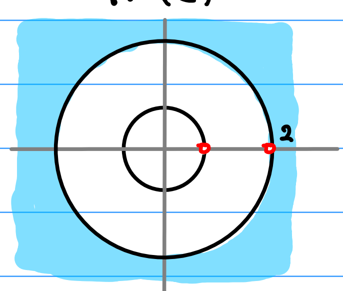


$\xleftrightarrow{z^{-1}}$

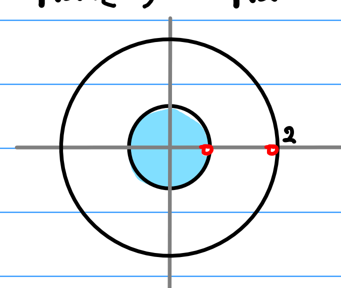


$|z| < 0.5$ $\qquad$ $|z^{-1}| < 0.5$ $\quad$ $|z| > 2$

$R2(z)$ $\qquad$ $R2(z^{-1}) \triangleq R2^{-1}$



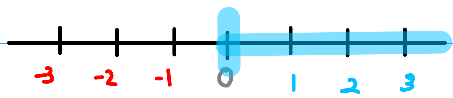
$\xleftrightarrow{z^{-1}}$



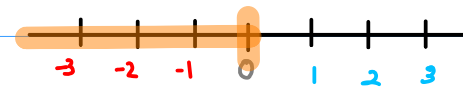
$|z| > 2$ $\qquad$ $|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

$-N$, N^c

$-N$

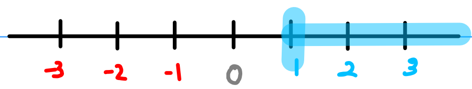


$(n \geq 0)$

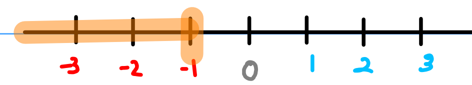


$(n < 1)$

$-N$

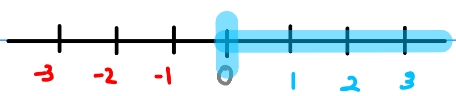


$(n \geq 1)$

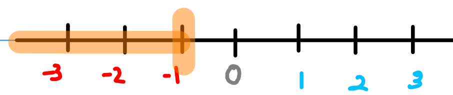


$(n < 0)$

$N^c = \overline{N}$

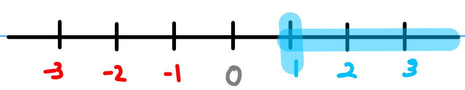


$(n \geq 0)$

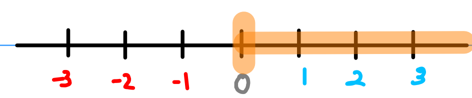


$(n < 0)$

$N^c = \overline{N}$



$(n \geq 1)$



$(n < 1)$

$-$	$\overline{}$	« »
-1	complement	
$-N \triangleq RNG(-n)$	$N^c \triangleq \overline{RNG(n)}$	$(-N)^c = -(N^c)$

$$(z, R) \Leftrightarrow (a_n, N)$$

$f(z)$	$ROC(z)$	$\Leftrightarrow$	a_n	$RNG(n)$
	$\lvert z\rvert < p$			$n \geq 0$

Ⓘ $$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$f(z^{-1})$	$ROC(z^{-1})$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⓘⓘ $$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$f(z)$	$ROC(z^{-1})$	$\Leftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$

Ⓘⓘⓘ $$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

$f(z^{-1})$	$ROC(z)$	$\Leftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$
	$\lvert z\rvert < p$			$n \geq 1$

Ⓘ + Ⓘⓘ

Ⓘⓥ $$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$X(z)$	$ROC(z)$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert < p$			$n < 1$

$$\text{III} = \text{I} + \text{II}$$

III	$f(z^{-1})$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$ $n \geqslant 1$	I + II
	$f(z)$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	a_n	$RNG(n)$ $n \geqslant 0$	
I	$f(z^{-1})$	$ROC(z^{-1})$ $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	a_{-n}	$RNG(-n)$ $n < 1$	
II	$f(z)$	$ROC(z^{-1})$ $\lvert z\rvert > \frac{1}{p}$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG(n)}$ $n < 0$	
	$f(z^{-1})$	$ROC(z)$ $\lvert z\rvert < p$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{RNG(-n)}$ $n \geqslant 1$	

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⓘ with Ⓘⓥ

ROC(z) $\quad f(z) \Longleftrightarrow a_n \quad$ RNG(n)

$|z| < p \qquad\qquad n \geq 0$

Ⓘ $\quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

$$\boxed{\begin{array}{cccc} f(z^{-1}) & \text{ROC}(z^{-1}) \Longleftrightarrow & a_{-n} & \text{RNG}(-n) \\ & |z| > \frac{1}{p} & & n < 1 \end{array}}$$

Ⓘⓥ $\quad (a_n, N) \Leftrightarrow (x_{-n}, -N)$

$(x_n, N) \Leftrightarrow (a_{-n}, -N)$

$$\boxed{\begin{array}{cccc} X(z) & \text{ROC}(z) \Longleftrightarrow & a_{-n} & \text{RNG}(-n) \\ & |z| < p & & n < 1 \end{array}}$$

$-n \qquad -n$

symmetrical

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)}$$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)}$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad |z|<a$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad |z|>a^{-1}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n<1$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad |z|>a^{-1}$$

$$-a^{n+1} \qquad n<0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad |z|<a$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

II $(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$f(z) = -\left[2+2^2z+2^3z^2+\cdots\right] + \left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2 z+\left(\tfrac{1}{2}\right)^3 z^2+\cdots\right]$$

$$\boxed{a_n = -2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}z^{0}+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] + \left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[2^{0}z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] - \left[\left(\tfrac{1}{2}\right)^{0}z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right]$$

$$\boxed{a_n = +2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \qquad (n<0)}$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^{0}z^{1}+\left(\tfrac{1}{2}\right)^{1}z^{2}+\left(\tfrac{1}{2}\right)^{2}z^{3}+\cdots\right] - \left[2^{0}z^{1}+2^{1}z^{2}+2^{2}z^{3}+\cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n \geq 1)}$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad |z|<a$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{n} \qquad |z|>a^{-1}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n<1$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{n} \qquad |z|>a^{-1}$$

$$-a^{n+1} \qquad n<0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{n} \qquad |z|<a$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

Ⅲ $(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$f(z) = -[2 + 2^2 z + 2^3 z^2 + \cdots] + [(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z + (\tfrac{1}{2})^3 z^2 + \cdots]$$

$$\boxed{a_n = -2^{n+1} + (\tfrac{1}{2})^{n+1} \quad (n \geq 0)}$$

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$f(z) = -[(\tfrac{1}{2})^{-1} z^0 + (\tfrac{1}{2})^{-2} z^{-1} + (\tfrac{1}{2})^{-3} z^{-2} + \cdots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$$

$$\boxed{a_n = -(\tfrac{1}{2})^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$f(z) = +[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots] - [(\tfrac{1}{2})^0 z^{-1} + (\tfrac{1}{2})^{-1} z^{-2} + (\tfrac{1}{2})^{-2} z^{-3} + \cdots]$$

$$\boxed{a_n = +2^{n+1} - (\tfrac{1}{2})^{n+1} \quad (n < 0)}$$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$f(z) = +[(\tfrac{1}{2})^0 z^1 + (\tfrac{1}{2})^1 z^2 + (\tfrac{1}{2})^2 z^3 + \cdots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots]$$

$$\boxed{a_n = +(\tfrac{1}{2})^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$\sum_{n=0}^{\infty} a^{n+1} z^n \qquad |z| < a$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^n \qquad |z| > a^{-1}$$

$$(\tfrac{1}{a})^{n-1} \qquad n < 1$$

$$f(z^{-1}) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad |z| > a^{-1}$$

$$-a^{n+1} \qquad n < 0$$

$$f(z) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^n \qquad |z| < a$$

$$-(\tfrac{1}{a})^{n-1} \qquad n \geq 1$$

Ⅳ $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[2 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

Ⅳ $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

$$f(z) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad \boxed{|z| < a}$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{n} \qquad \boxed{|z| > a^{-1}}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n < 1$$

$$X(z^{-1}) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad n < 1$$

$$X(z) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$a^{n+1} \qquad n \geq 0$$

$$f(z^{-1}) = \frac{-z^{-1}}{1 - a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{n} \qquad \boxed{|z| > a^{-1}}$$

$$-a^{n+1} \qquad n < 0$$

$$f(z) = \frac{-z}{1 - a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{n} \qquad \boxed{|z| < a}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

$$X(z) = \frac{-z^{-1}}{1 - a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad n \geq 1$$

$$X(z^{-1}) = \frac{-z}{1 - a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$-a^{n+1} \qquad n < 0$$

$f(z)$	$f(z^{-1})$	a_n	a_{-n}
$f(z^{-1})$	$f(z)$	$-a_n$	$-a_{-n}$

$$f(z) = \frac{a}{1 - az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

$\sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad \boxed{|z| < a}$

a^{n+1} $\quad \boxed{n \geq 0}$

$$f(z^{-1}) = \frac{a}{1 - az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$\sum_{n=0}^{-\infty} a^{-n+1} z^n$ $\quad \boxed{|z| > a^{-1}}$

$\left(\frac{1}{a}\right)^{n-1}$ $\quad \boxed{n < 1}$

$$f(z^{-1}) = \frac{-z^{-1}}{1 - a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$-\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $\quad \boxed{|z| > a^{-1}}$

$-a^{n+1}$ $\quad \boxed{n < 0}$

$$f(z) = \frac{-z}{1 - a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$-\sum_{n=1}^{\infty} a^{-n+1} z^n$ $\quad \boxed{|z| < a}$

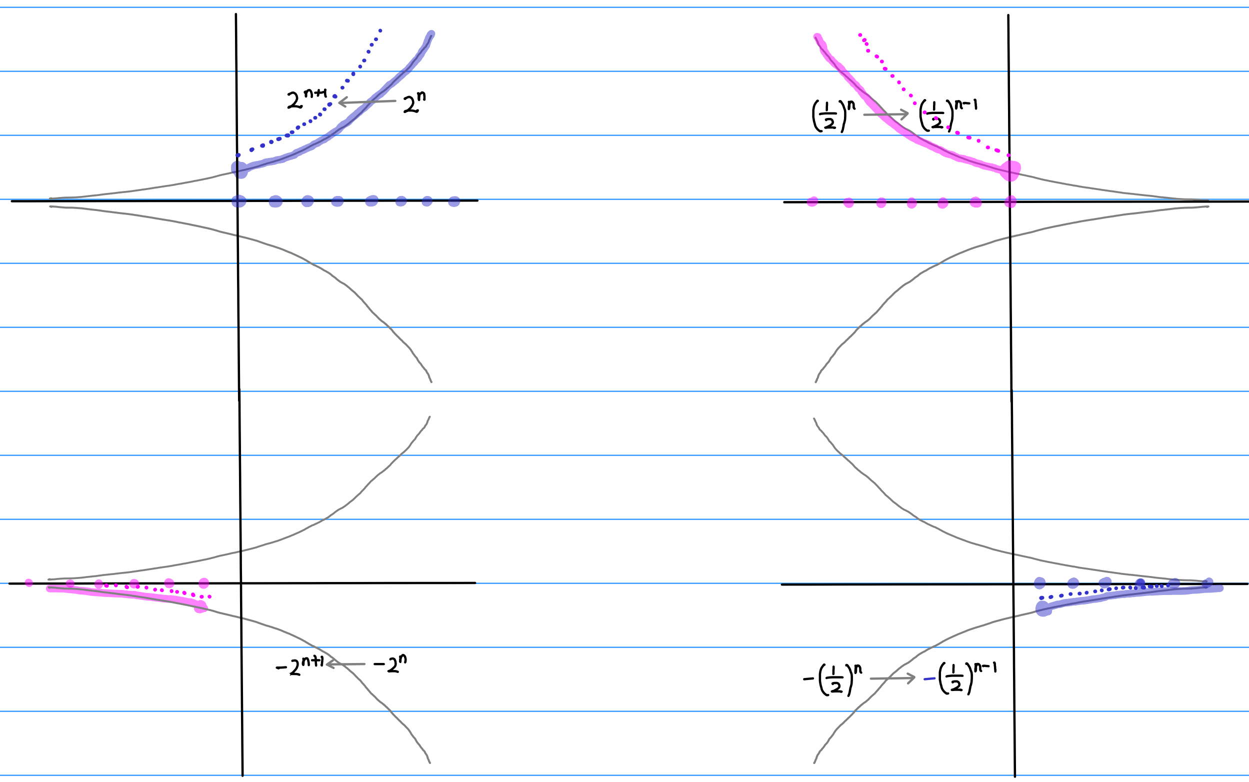
$-\left(\frac{1}{a}\right)^{n-1}$ $\quad \boxed{n \geq 1}$

$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$

$-z^{-1} - a^{-1} z^{-2} - a^{-2} z^{-3} - a^{-3} z^{-4} - \cdots$

$a + a^2 z^{-1} + a^3 z^{-2} + a^4 z^{-3} + \cdots$

$-z^1 - a^{-1} z^2 - a^{-2} z^3 - a^{-3} z^4 - \cdots$



$X(z^{-1})$	$X(z)$	x_n	x_{-n}
$X(z)$	$X(z^{-1})$	$-x_n$	$-x_{-n}$

$$X(z^{-1}) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\sum_{n=0}^{-\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| < a}$$

$$\left(\frac{1}{a}\right)^{n-1} \qquad \boxed{n < 1}$$

$$X(z) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n}$$

$$\sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$a^{n+1} \qquad \boxed{n \geq 0}$$

$$X(z) = \frac{-z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$-\sum_{n=1}^{\infty} a^{-n+1} z^{-n} \qquad \boxed{|z| > a^{-1}}$$

$$-\left(\frac{1}{a}\right)^{n-1} \qquad \boxed{n \geq 1}$$

$$X(z^{-1}) = \frac{-z}{1-a^{-1}z} = -\sum_{n=0}^{\infty} a^{-n} z^{n+1}$$

$$-\sum_{n=-1}^{-\infty} a^{n+1} z^{-n} \qquad \boxed{|z| < a}$$

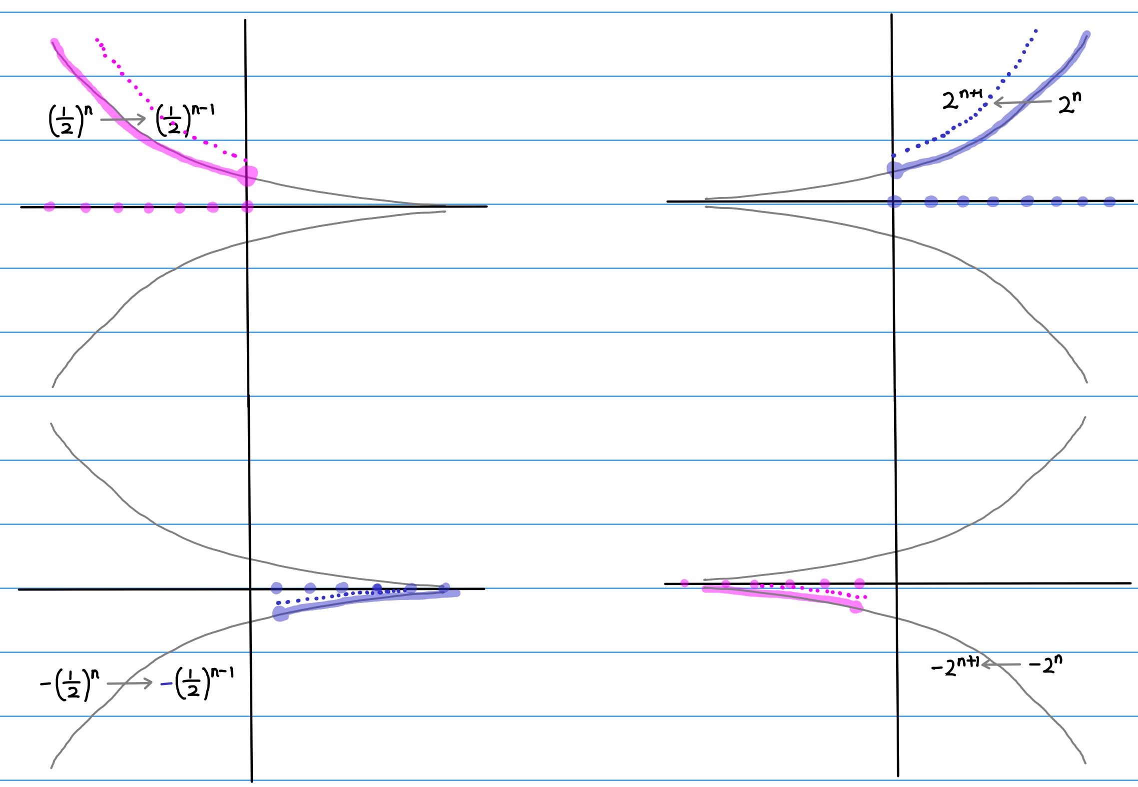
$$-a^{n+1} \qquad \boxed{n < 0}$$

$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$$

$$-z^{-1} - a^{-1} z^{-2} - a^{-2} z^{-3} - a^{-3} z^{-4} - \cdots$$

$$a + a^2 z^{-1} + a^3 z^{-2} + a^4 z^{-3} + \cdots$$

$$-z^{1} - a^{-1} z^{2} - a^{-2} z^{3} - a^{-3} z^{4} - \cdots$$



$$
\begin{tikzcd}
\boxed{f(z):\ -\frac{2}{1-2z}+\frac{0.5}{1-0.5z}\quad |z|<0.5} \arrow[r] \arrow[d] & \boxed{f(z^{-1}):\ -\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}\quad |z|>2} \arrow[d] \\
\boxed{g(z^{-1}):\ +\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}\quad |z|>2} & \boxed{g(z):\ +\frac{z}{1-0.5z}-\frac{z}{1-2z}\quad |z|<0.5} \arrow[l]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\boxed{f(z):\ -\frac{2}{1-2z}+\frac{0.5}{1-0.5z}\quad |z|<0.5} \arrow[r] & \boxed{f(z^{-1}):\ -\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}\quad |z|>2} \\
\boxed{g(z^{-1}):\ +\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}\quad |z|>2} \arrow[u] & \boxed{g(z):\ +\frac{z}{1-0.5z}-\frac{z}{1-2z}\quad |z|<0.5} \arrow[l] \arrow[u]
\end{tikzcd}
$$