

# Relationship between Power Spectrum and Autocorrelation Function

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# AutoCorrelation Function

$N$  Gaussian random variables

## Definition

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) e^{+j\omega t} d\omega = A[R_{XX}(t, t + \tau)]$$

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} E \left[ \frac{1}{2T} \int_{-T}^{+T} X(t_1) e^{+j\omega t_1} dt_1 \int_{-T}^{+T} X(t_2) e^{-j\omega t_2} dt_2 \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} E[X(t_1)X(t_2)] e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

# AutoCorrelation and Expectation

$N$  Gaussian random variables

## Definition

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} E[X(t_1)X(t_2)] e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

$$E[X(t_1)X(t_2)] = R_{XX}(t_1, t_2)$$

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XX}(t_1, t_2) e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

# Inverse Transform

$N$  Gaussian random variables

## Definition

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XX}(t_1, t_2) e^{-j\omega(t_2 - t_1)} dt_2 dt_1$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{XX}(\omega) e^{+j\omega t} d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XX}(t_1, t_2) e^{-j\omega(t_2 - t_1)} dt_2 dt_1 \right\} e^{+j\omega\tau} d\omega$$

$$= \left\{ \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XX}(t_1, t_2) \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{+j\omega(\tau - t_1 - t_2)} d\omega \right\} dt_2 dt_1$$



