# Applications of Arrays (1A)

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- Viewing an **array** as a **pointer**

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- Viewing an **array** as a **pointer**

int **a**[4] ;

an array **a**

generalization

int (*a)

view **a** as a pointer

virtual pointer
- no real memory location
- constraints :
  value(&**a**) = value(**a**)

- Viewing a **pointer** as an **array**

int (***a**) ;

a pointer **a**

a specific instance

int **a**[N]

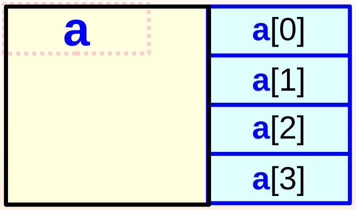
view **a** as an array

N is not fixed

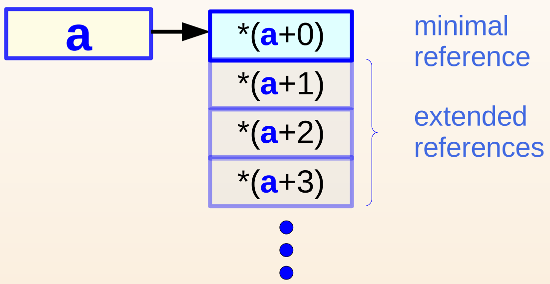
sizeof(**a**) is
not the size of the array
but of a pointer variable

# Array **a** and pointer **a**

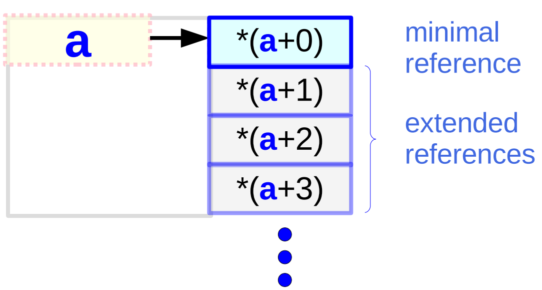
int **a**[4] ;    an array **a**



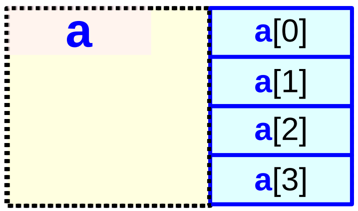

int (*a) ;    a pointer **a**




int (***a**)    **a** as a pointer




int **a**[N]    **a** as an array



# Array **a** and pointer **a**

int **a**[4] ; — an array **a**

- type(**a**) = int [4]
- sizeof(**a**) = an array size (16 bytes)
- value(&**a**) = value(**a**)
- fixed number of elements

int (***a**) ; — a pointer **a**

- type(**a**) = int (*)
- sizeof(**a**) = a pointer size (4 bytes)
- value(&**a**) ≠ value(**a**)
- variable number of elements

int (***a**) — **a** as a pointer

**a** is not a real pointer

- sizeof(**a**) = an array size
- value(&**a**) = value(**a**)

int **a**[N] — **a** as an array

**a** is not a real array

- sizeof(**a**) ≠ an array size<br>= a pointer size
- value(&**a**) ≠ value(**a**)<br>= assigned address

# Relationship between array and array pointer types

| | | | |
|---|---|---|---|
| int b[4][2] ; | declare a **2-d** array b | int (*b)[2] ; | declare a **1-d** array pointer **b** |
| ↓ | generalization | ↓ | a specific instance |
| int (*b) [2] | **b** as a **1-d** array pointer | int b[N][2] | **b** as a **2-d** array |
| int a[4] ; | declare a **1-d** array **a** | int (*a) ; | declare a **0-d** array pointer **a** |
| ↓ | generalization | ↓ | a specific instance |
| int (*a) | **a** as a **0-d** array pointer | int a[N] | **a** as a **1-d** array |

# Array **b** and array pointer **b**

int **b**[4][2] ;

**2-d** array **b**

- type(**b**) = int [4]
- sizeof(**b**) = an array size (32 bytes)
- value(&**b**) = value(**b**)
- fixed number of elements

int (*) [2]

**b** as a **1-d** array pointer

**b** is not a real pointer

- sizeof(**b**) = an array size
- value(&**b**) = value(**b**)

int (***b**) [2] ;

**1-d** array pointer **b**

- type(**b**) = int (*)
- sizeof(**b**) = a pointer size (4 bytes)
- value(&**b**) ≠ value(**b**)
- variable number of elements

int [N][2]

**b** as a **2-d** array

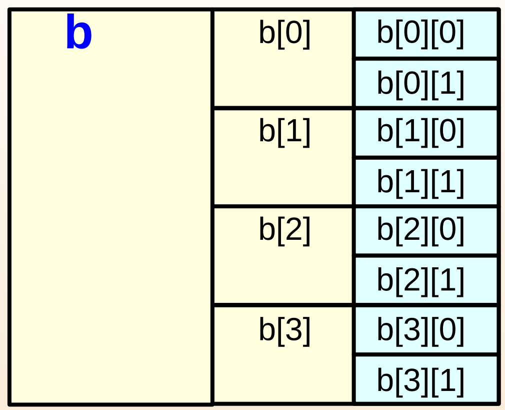
**b** is not a real array

- sizeof(**b**) ≠ an array size
  = a pointer size
- value(&**b**) ≠ value(**b**)
  = assigned address

# Array **b** and array pointer **b**

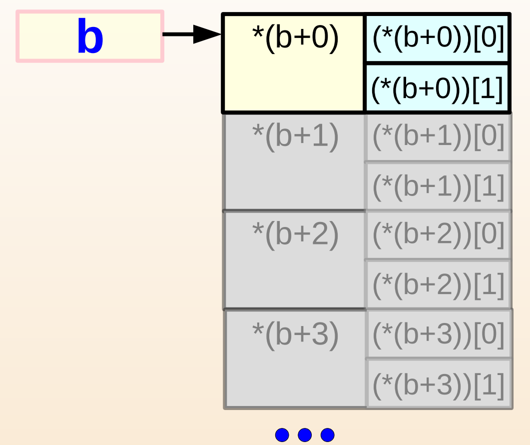
int **b**[4][2] ;

**2-d** array **b**




int (*b) [2] ;

**1-d** array pointer **b**

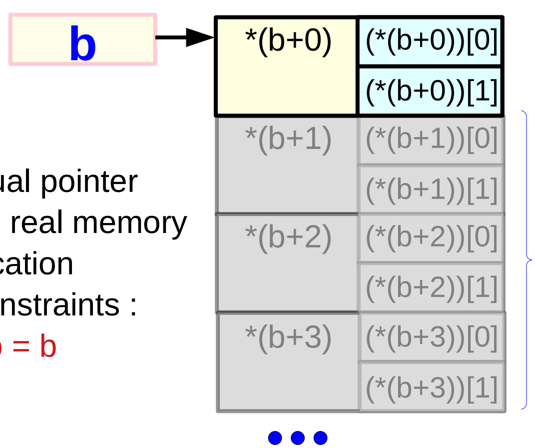



minimal reference

extended references

int (*) [2]

**b** as a **1-d** array pointer



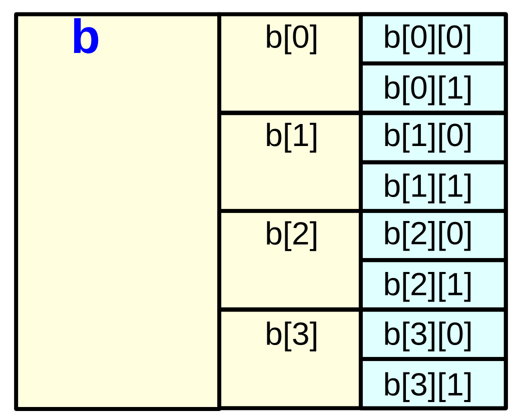

minimal reference

extended references

virtual pointer
- no real memory location
- constraints :
  &b = b

int [N][2]

**b** as a **2-d** array




N is not fixed to 4

sizeof(b) is not the size of the array but the size of a pointer variable

# Dual type - relaxing the 1st dimension of an array

int [4][2] — **2-d** array
more constrained type

relaxing the 1st dimension
generalization
↓ ↑
a specific instance

int (*)[2] — **1-d** array pointer
more general type

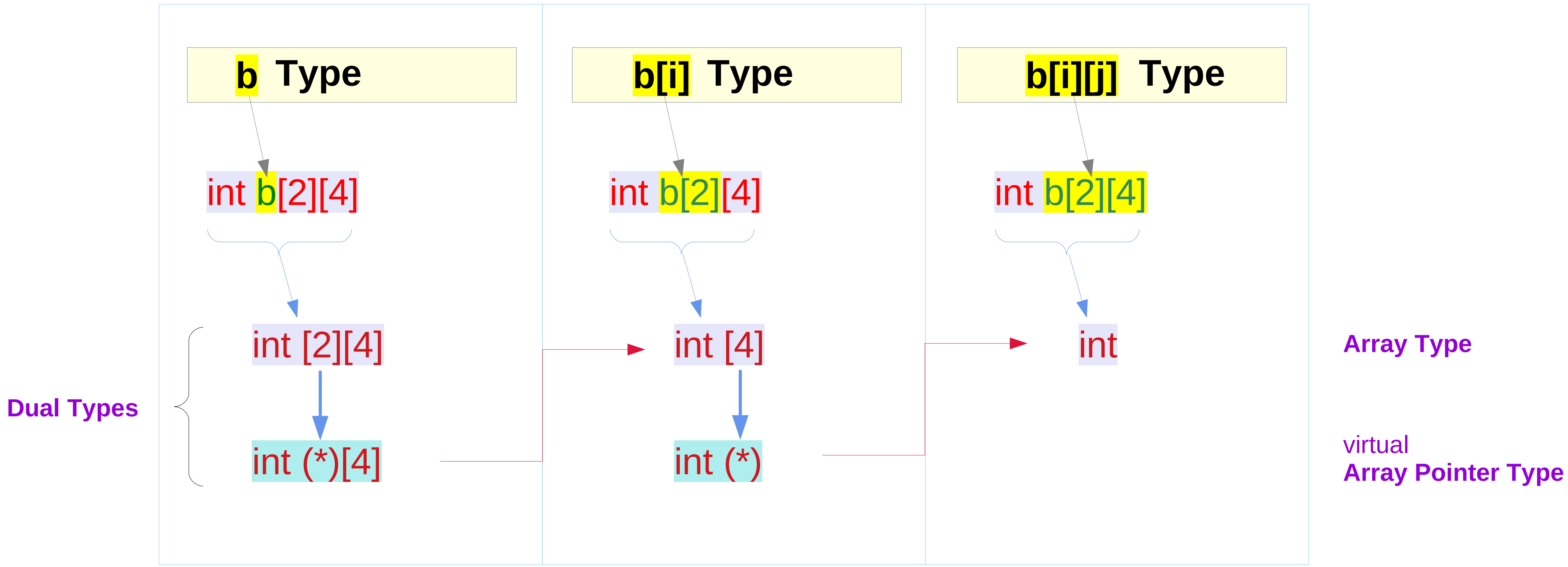
int [4] — **1-d** array
more constrained type

relaxing the 1st dimension
generalization
↓ ↑
a specific instance

int (*) — **0-d** array pointer
more general type

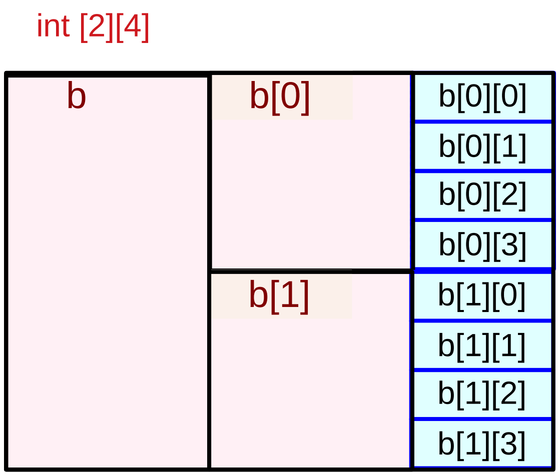
# Subarray types in a **2-d** array

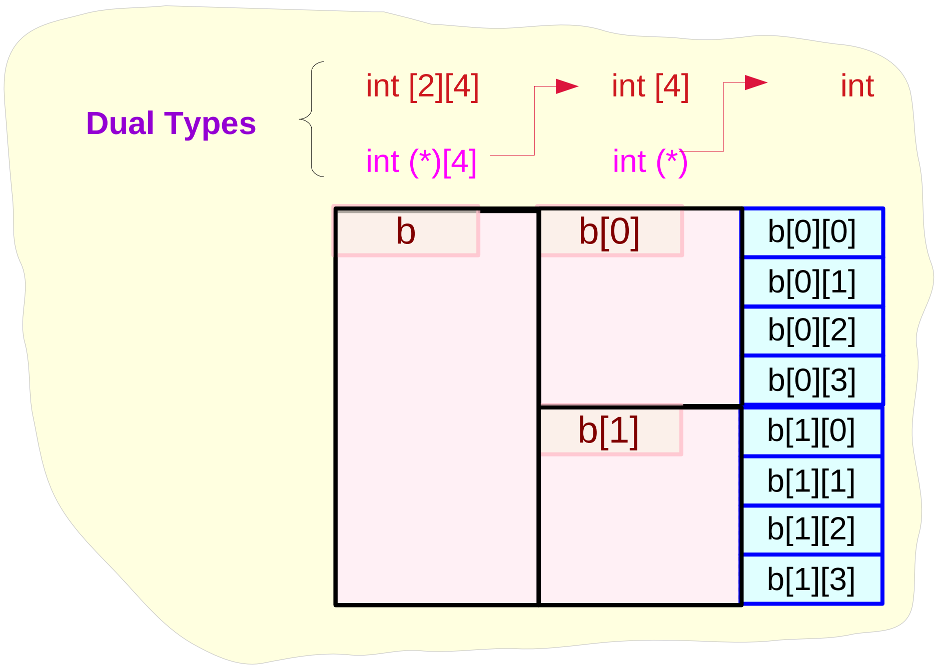
int b[2][4]; **2-d** array b

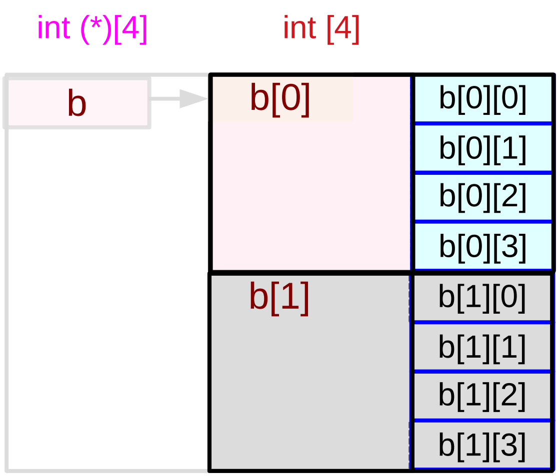


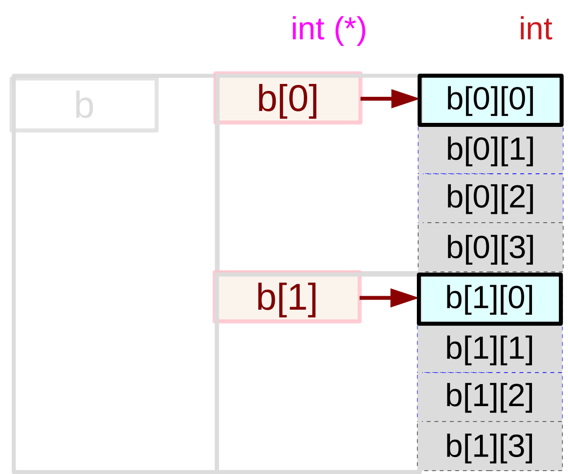
# Dual types in a **2-d** array

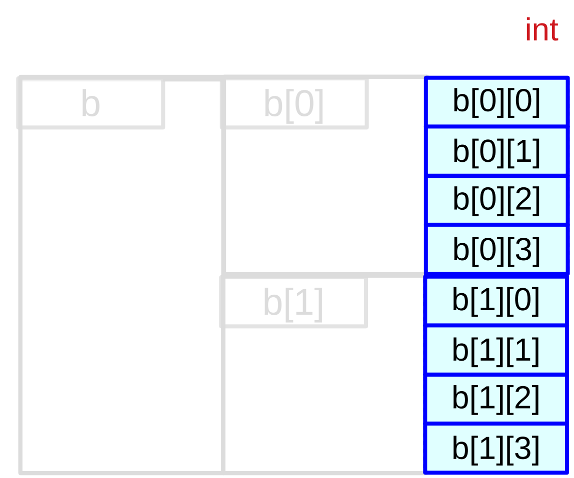
int b[2][4]; **2-d** array b



# Subarray type examples

int a[4];

| | | | relaxed type | virtual |
|---|---|---|---|---|
| a | int [4] | **1-d** array type | int (*) | **0-d** array pointer type |
| a[i] | int | **0-d** array type | | |

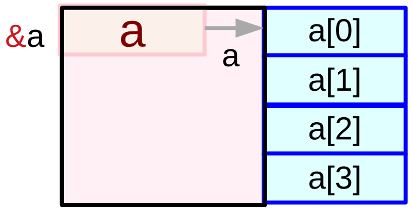
int b[2][4];

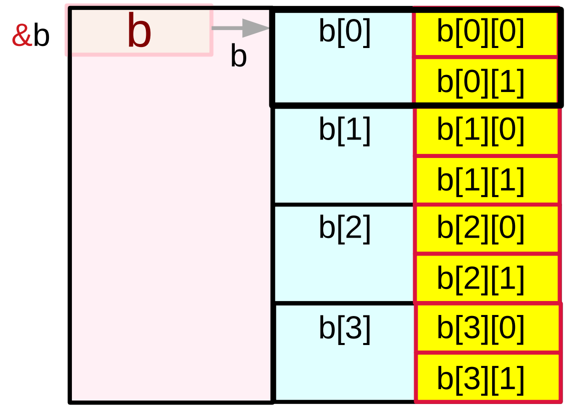
| | | | relaxed type | virtual |
|---|---|---|---|---|
| b | int [2][4] | **2-d** array type | int (*)[4] | **1-d** array pointer type |
| b[i] | int [4] | **1-d** array type | int (*) | **0-d** array pointer type |
| b[i][j] | int | **0-d** array type | | |

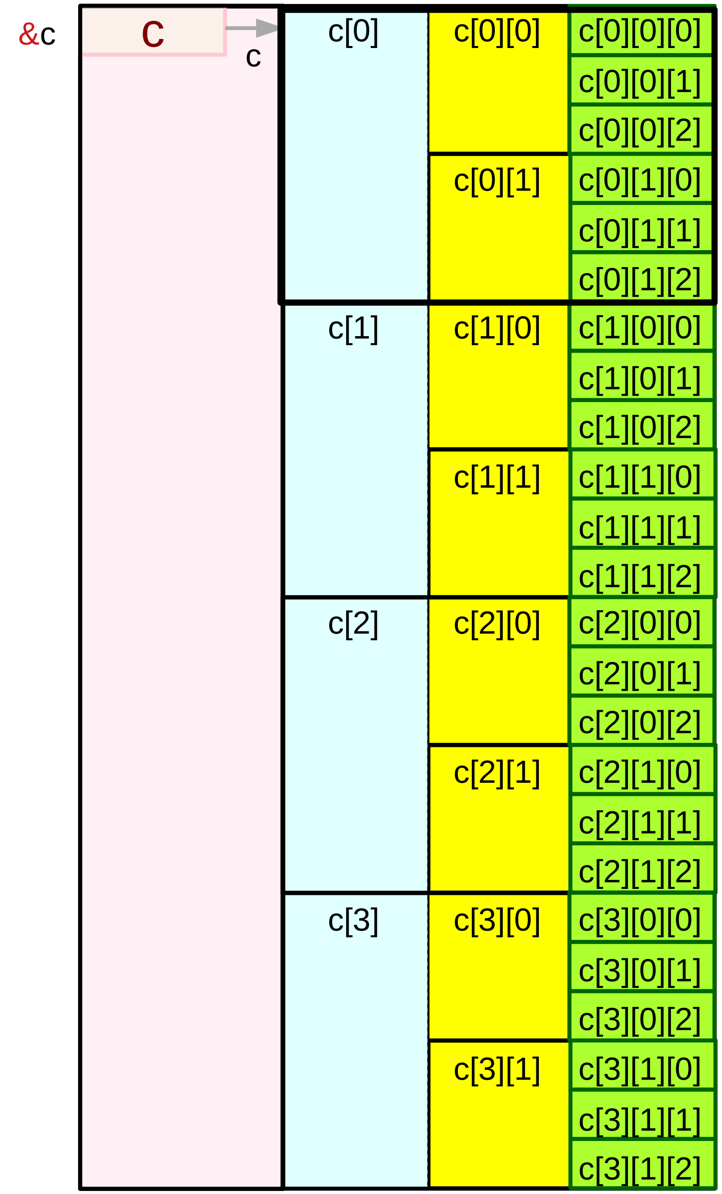
int c[4][2][3];

| | | | relaxed type | virtual |
|---|---|---|---|---|
| c | int [4][2][3] | **3-d** array type | int (*)[2][3] | **2-d** array pointer type |
| c[i] | int [4][2] | **2-d** array type | int (*)[2] | **1-d** array pointer type |
| c[i][j] | int [4] | **1-d** array type | int (*) | **0-d** array pointer type |
| c[i][j][k] | int | **0-d** array type | | |

# Types of **a**, **b**, **c** arrays



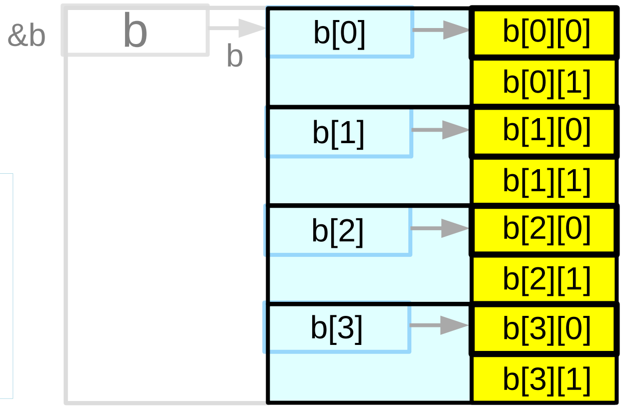

```
int a[4];
int b[2][4];
int c[4][2][3];
```

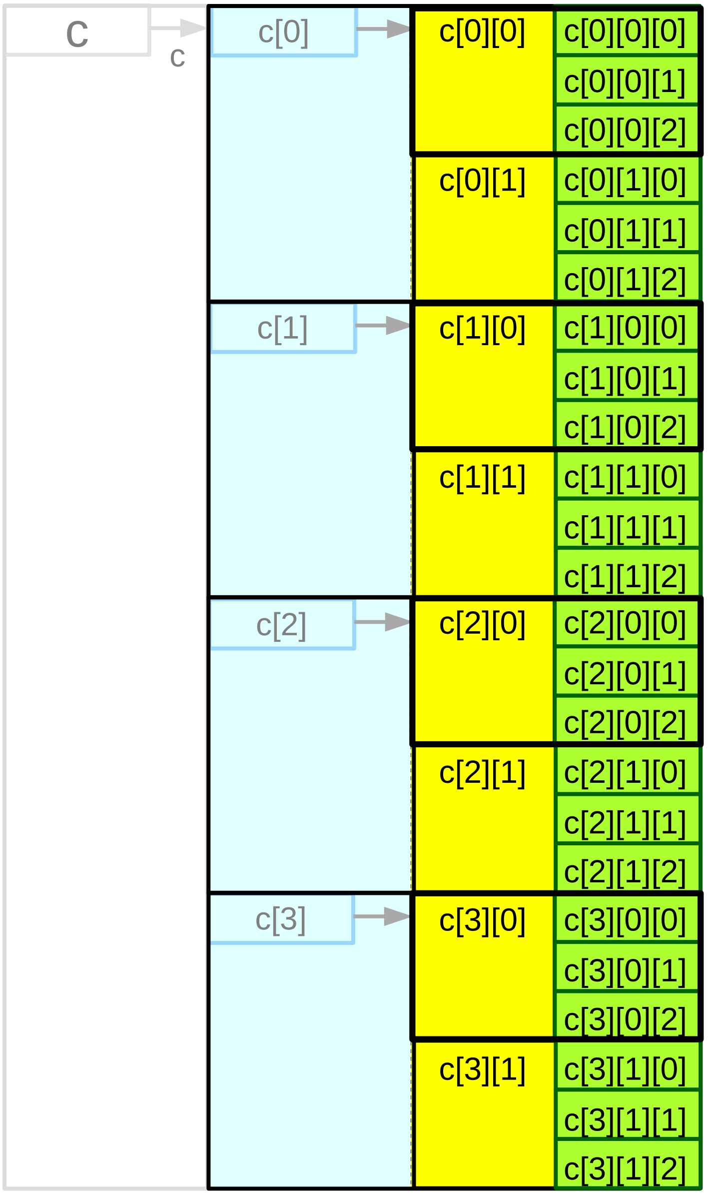
**dual types**

| | | |
|---|---|---|
| int [4]<br>int (*) | **1-d** array **a**<br>**0-d** array pointer **a** (virtual) | **a[i]**<br>***(a+i)** |
| int [4][2];<br>int (*)[2]; | **2-d** array **b**<br>**1-d** array pointer **b** (virtual) | **b[i]**<br>***(b+i)** |
| int [4][2][3];<br>int (*)[2][3]; | **3-d** array **c**<br>**2-d** array pointer **c** (virtual) | **c[i]**<br>***(c+i)** |

# Types of **b[i]**, **c[i]** subarrays

```
int a[4];
int b[2][4];
int c[4][2][3];
```



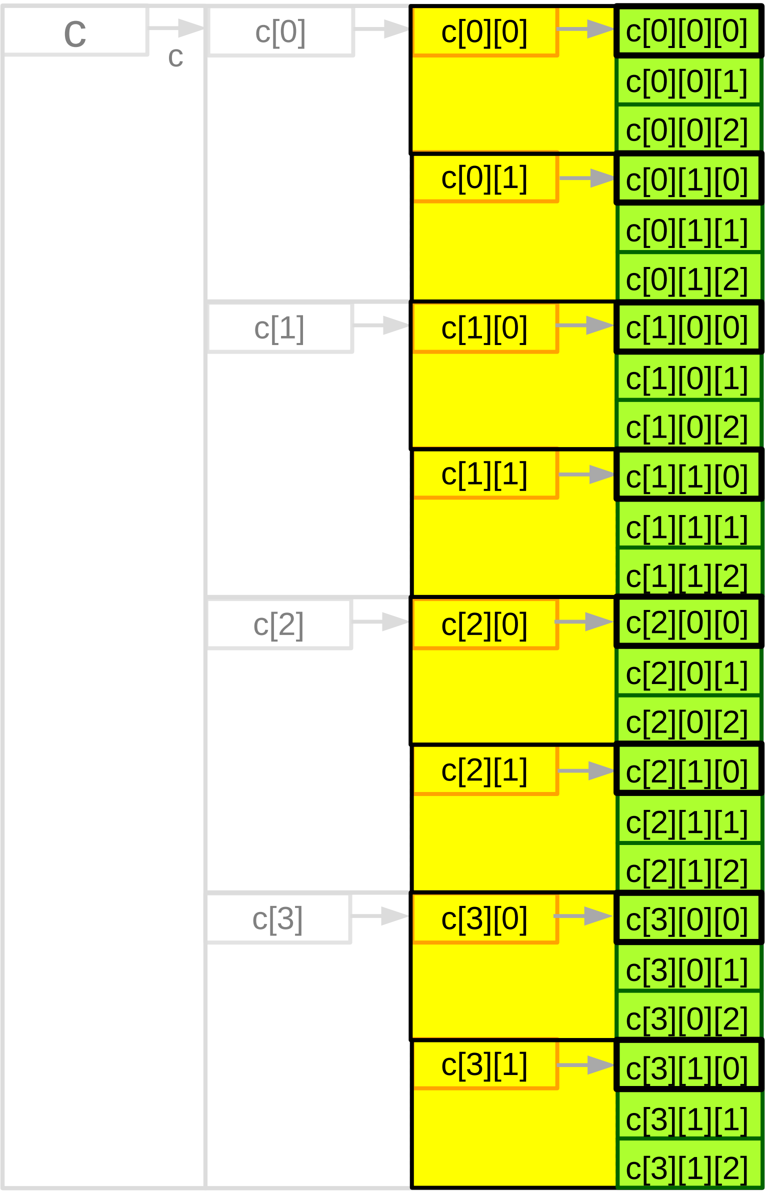




## dual types

| | | |
|---|---|---|
| int [2] | **1-d** array **b[i]** | **b[i][j]** |
| int (*) | **0-d** array pointer **b[i]** (virtual) | ***(b[i]+j)** |
| int [2][3]; | **2-d** array **c[i]** | **c[i][j]** |
| int (*)[3]; | **1-d** array pointer **c[i]** (virtual) | ***(c[i]+j)** |

# Types of **c[i][j]** subarrays

int a[4];
int b[2][4];
int c[4][2][3];




**dual types**

| | | |
|---|---|---|
| int [3] | **1-d** array **c[i][j]** | **c[i][j][k]** |
| int (*) | **0-d** array pointer **c[i][j]** (virtual) | ***(c[i][j]+k)** |

# Types of a **4-d** array and its subarrays

## int d[4][2][3][4];

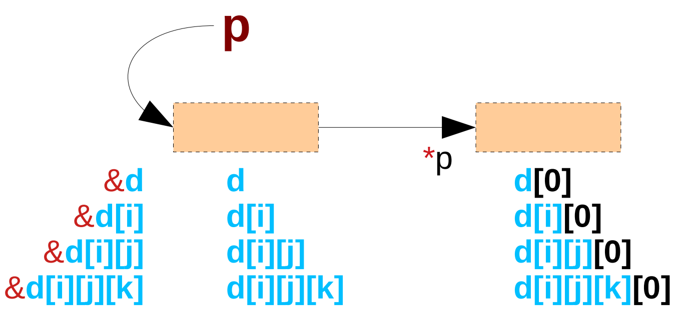
types

| | | | | |
|---|---|---|---|---|
| **d** | consider d[4][2][3][4] | int [4][2][3][4] | **4-d** array | |
| | relax the 1st dimension | int (*)[2][3][4] | **3-d** array pointer (virtual) | |
| **d[i]** | consider d[i][2][3][4] | int [2][3][4] | **3-d** array | |
| | relax the 1st dimension | int (*)[3][4] | **2-d** array pointer (virtual) | |
| **d[i][j]** | consider d[i][j][3][4] | int [3][4] | **2-d** array | |
| | relax the 1st dimension | int (*)[4] | **1-d** array pointer (virtual) | |
| **d[i][j][k]** | consider d[i][j][k][4] | int [4] | **1-d** array | |
| | relax the 1st dimension | int (*) | **0-d** array pointer (virtual) | |

i,j,k are specific index values i=[0..3], j=[0..1], k=[0..2]

# Initializing ***n*-d** array pointers with ***n*-d** subarrays

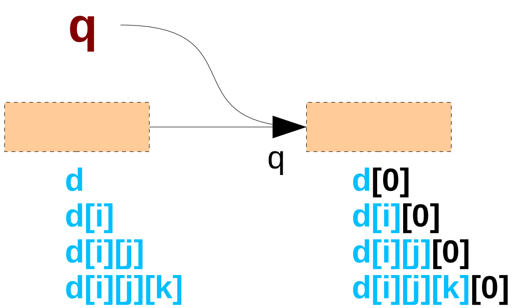
int d[4][2][3][4];




| | | | | |
|---|---|---|---|---|
| d | **4-d** array | d[4][2][3][4] | p = &d | abstract data |
| p | **4-d** array pointer | (*p)[4][2][3][4] | int (*p)[4][2][3][4] = &d; | |
| | | | (*p)[i][j][k][l] ≡ d[i][j][k][l] | |
| d[i] | **3-d** array | d[i][2][3][4] | p = &d[i] | abstract data |
| p | **3-d** array pointer | (*p)[2][3][4] | int (*p)[3][4] = &d[i]; | |
| | | | (*p)[j][k][l] ≡ d[i][j][k][l] given i | |
| d[i][j] | **2-d** array | d[i][j][3][4] | p = &d[i][j] | abstract data |
| p | **2-d** array pointer | (*p)[3][4] | int (*p)[4] = &d[i][j]; | |
| | | | (*p)[k][l] ≡ d[i][j][k][l] given i, j | |
| d[i][j][k] | **1-d** array | d[i][j][k][4] | p = &d[i][j][k] | abstract data |
| p | **1-d** array pointer | (*p)[4] | int (*p) = &d[i][j][k]; | |
| | | | (*p)[l] ≡ d[i][j][k][l] given i, j, k | |

# Initializing **(*n*-1)-d** array pointers with ***n*-d** subarrays

int d[4][2][3][4];




| | | | | |
|---|---|---|---|---|
| **d** | **4-d** array | d[4][2][3][4] | **q = d** | virtual pointer |
| **q** | **3-d** array pointer | (***q**)[2][3][4] | int (***q**)[2][3][4] = **d**; | |
| | | | **q**[i][j][k][l] ≡ **d**[i][j][k][l] | |
| **d[i]** | **3-d** array | d[i][2][3][4] | **q = d[i]** | virtual pointer |
| **q** | **2-d** array pointer | (***q**)[3][4] | int (***q**)[3][4] = **d[i]**; | |
| | | | **q**[j][k][l] ≡ **d[i]**[j][k][l] | given i |
| **d[i][j]** | **2-d** array | d[i][j][3][4] | **q = d[i][j]** | virtual pointer |
| **q** | **1-d** array pointer | (***q**)[4] | int (***q**)[4] = **d[i][j]**; | |
| | | | **q**[k][l] ≡ **d[i][j]**[k][l] | given i, j |
| **d[i][j][k]** | **1-d** array | d[i][j][k][4] | **q = d[i][j][k]** | virtual pointer |
| **q** | **0-d** array pointer | (***q**) | int (***q**) = **d[i][j][k]**; | |
| | | | **q**[l] ≡ **d[i][j][k]**[l] | given i, j, k |

# Aggregate Data Types
# Abstract Data Types
# Virtual Array Pointers

# Aggregate data type

**an aggregate type**

consists of **N** elements

each element
- starting address
- size

int [4][4]

dual type

**a virtual pointer**

the address and value of
a virtual pointer
are the *same*

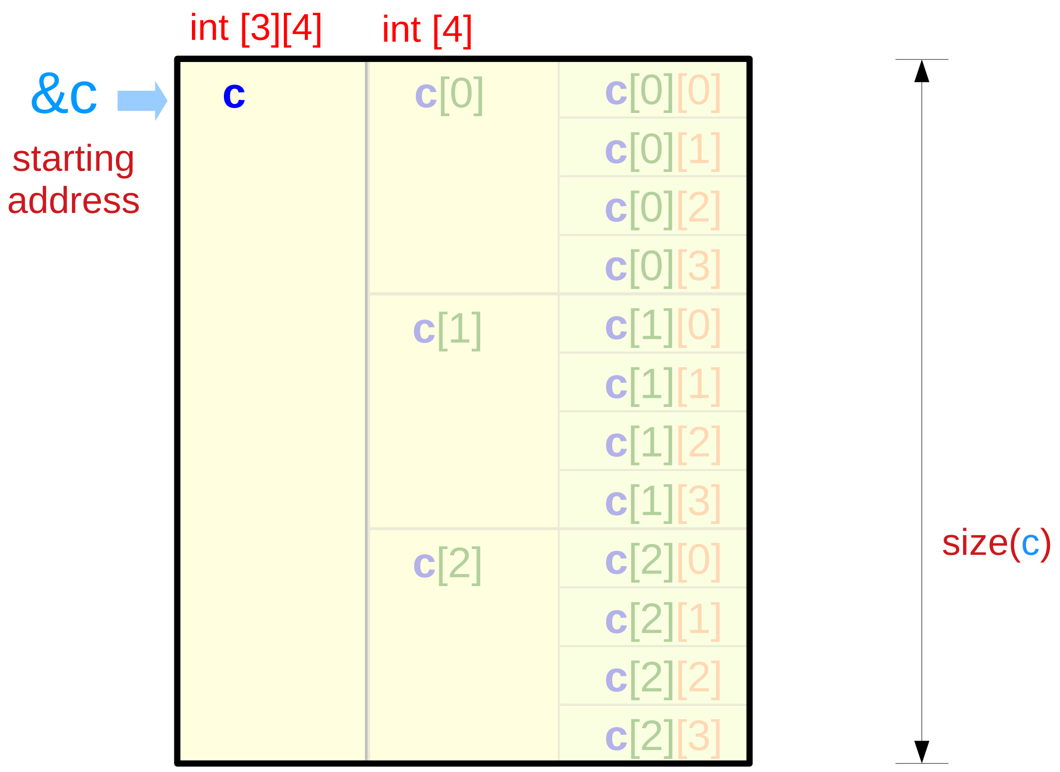
int (*)[4]

**an abstract type**

is considered as one unit

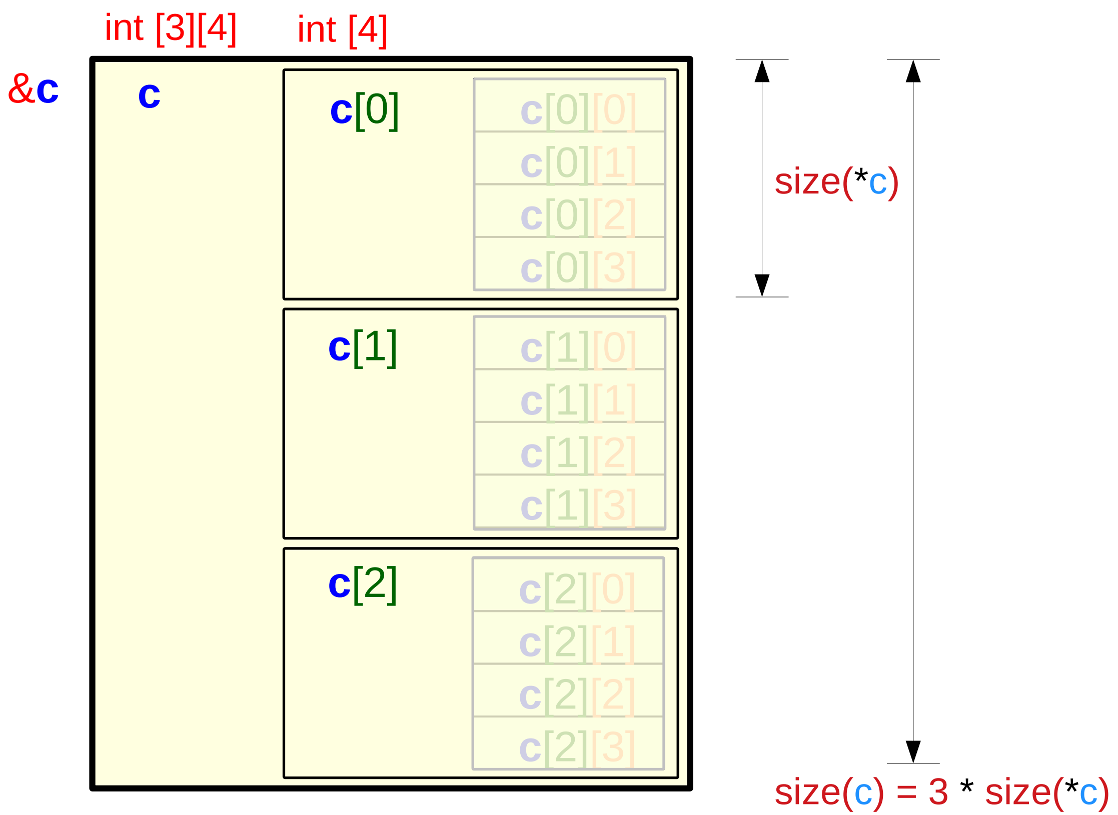
- starting address
- size

# Abstract data **c**




| | |
|---|---|
| an abstract data | c |
| - start address | &c |
| - size | sizeof(c) |

# Aggregate data **c**

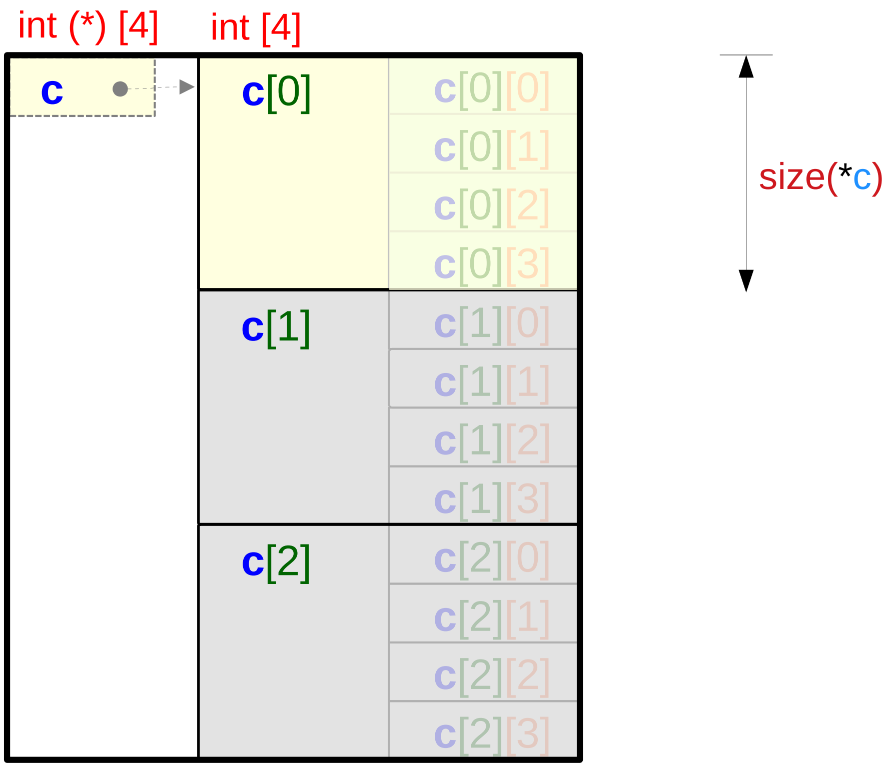



an aggregate type data **c**

- 1st element **c**[0]
- 2nd element **c**[1]
- 3rd element **c**[2]

# Virtual pointer **c**

&c = c = &c[0]




a virtual pointer **c**
- pointer address **&c**
- pointer value **c** = &**c**[0]

with the constraint
**c** = &**c**

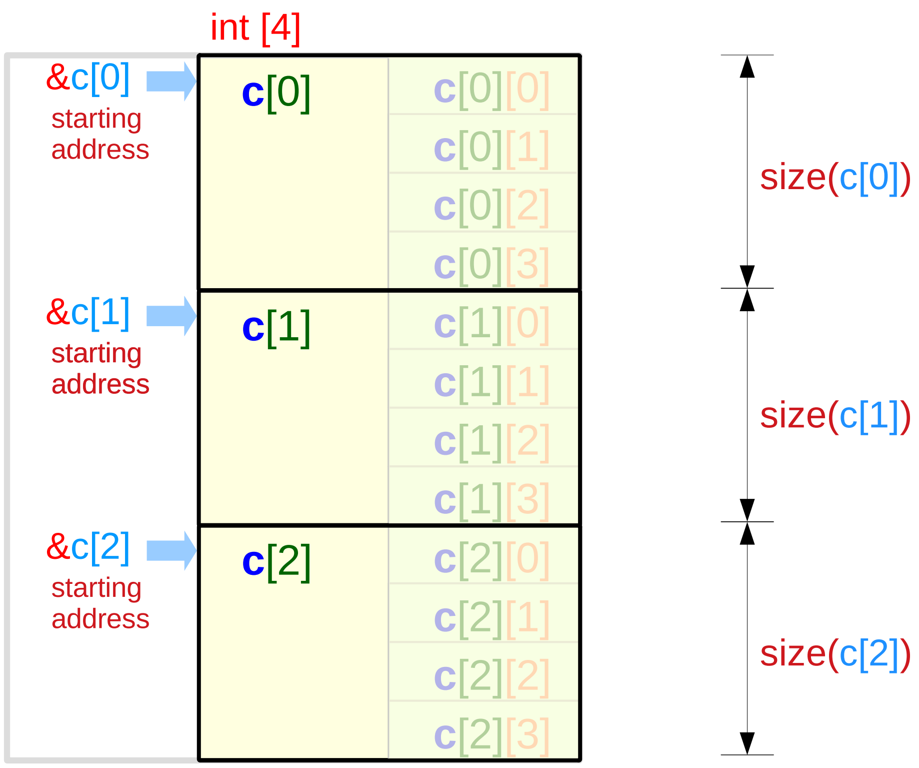
an abstract data **c**[0] = ***c**
- start address &**c**[0] = **c**
- size sizeof(**c**[0])

virtual pointer **c** points
to abstract data **c**[0]

virtual pointers

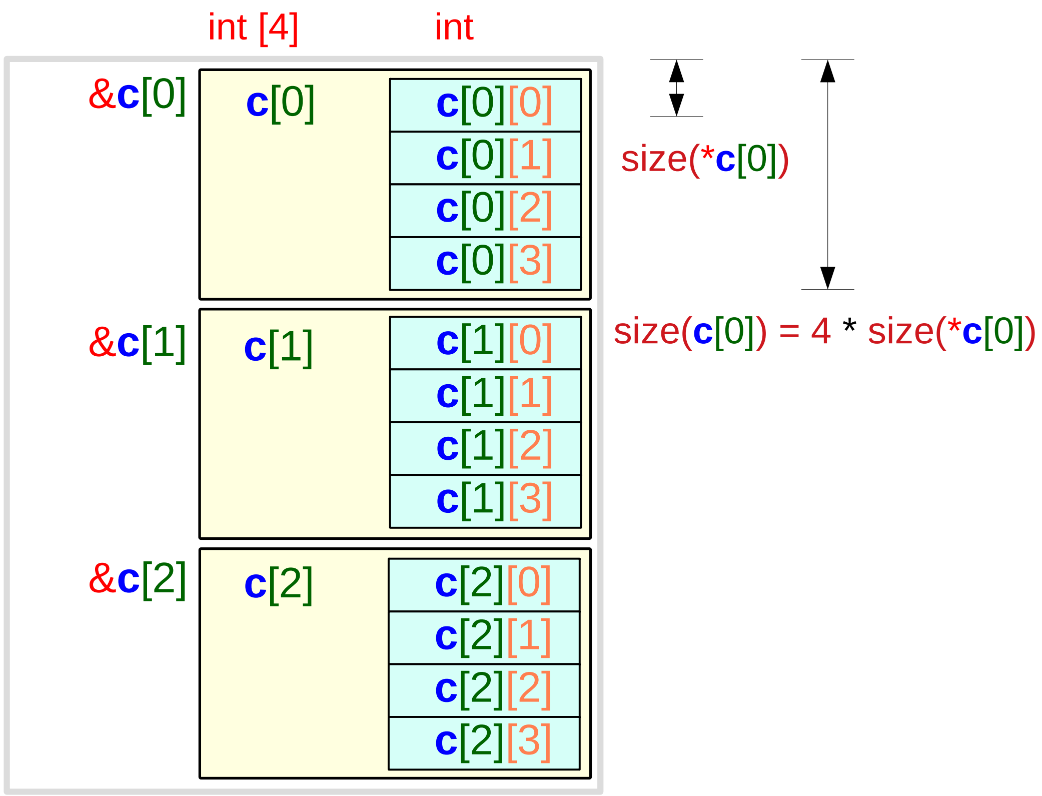
- no physical memory locations are allocated
- address and data have the same value

# Abstract data **c[i]**




- an abstract data c[0]
  - start address &c[0]
  - size sizeof(c[0])
- an abstract data c[1]
  - start address &c[1]
  - size sizeof(c[1])
- an abstract data c[2]
  - start address &c[2]
  - size sizeof(c[2])

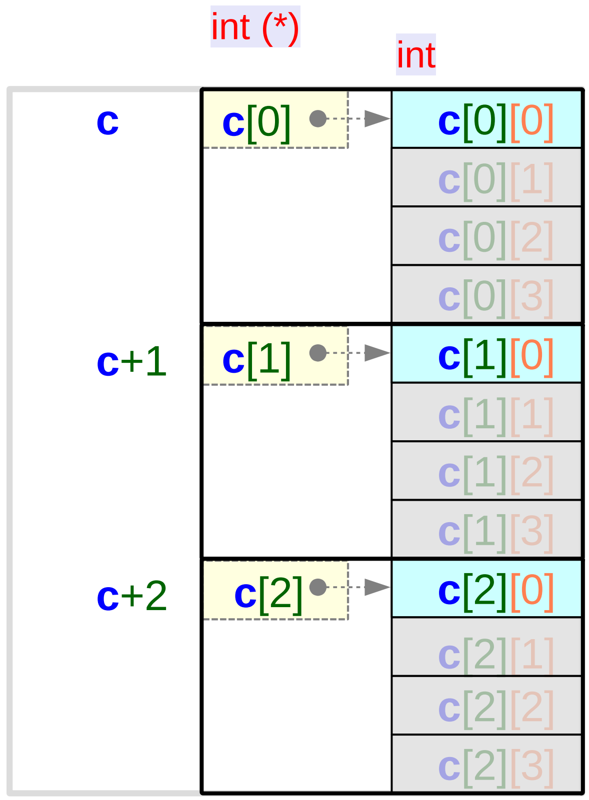
# Aggregate data **c[i]**




an aggregate type data c[i]

- 1st element c[i][0]
- 2nd element c[i][1]
- 3rd element c[i][2]
- 4th element c[i][3]

# Virtual pointer **c[i]**




a virtual pointer c[i]

- pointer address &c[i]
- pointer value c+i = &c[i]

with the constraint

c[i] = &c[i]

an primitive data c[i][0] = *c[i]

- start address &c[i][0] = c[i]
- size sizeof(c[i][0])

virtual pointer c[i]

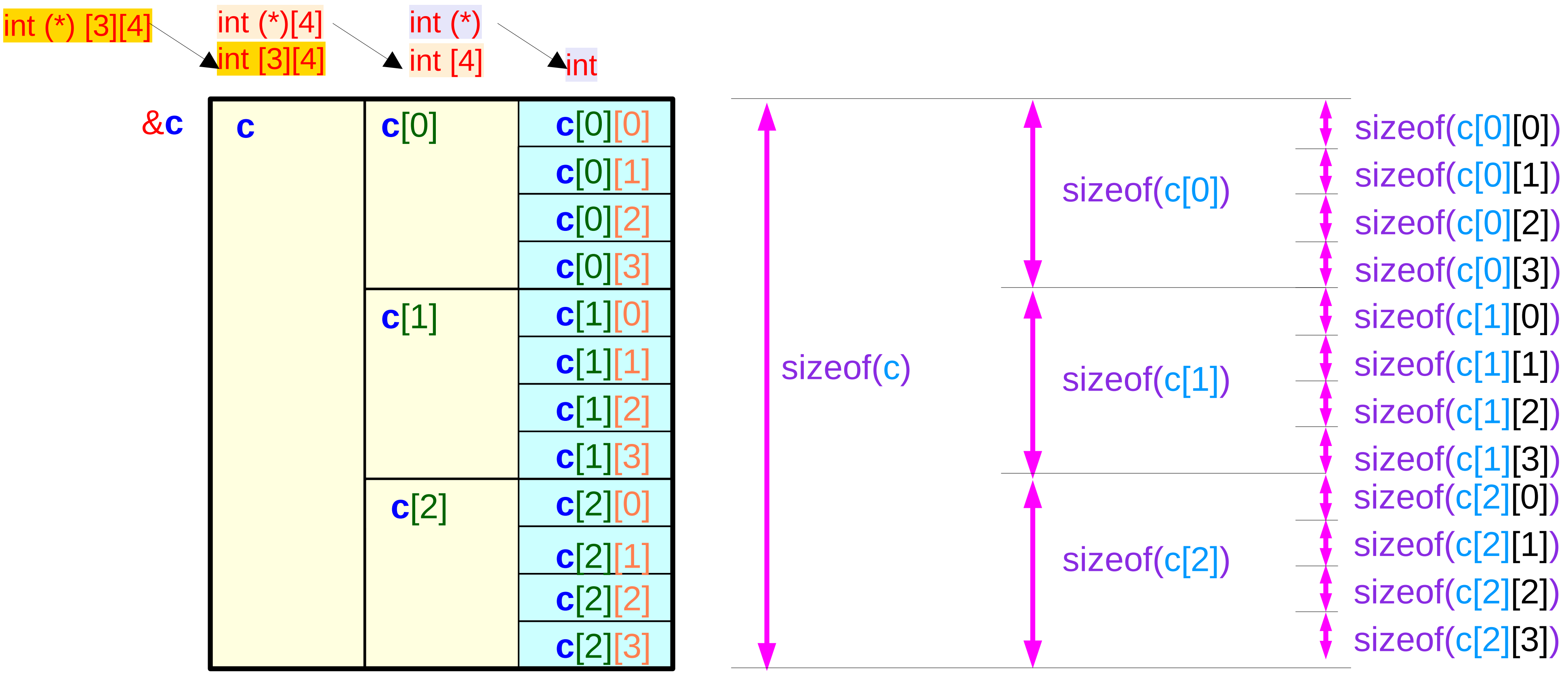
points to primitive data c[i][0]

virtual pointers

- no physical memory locations are allocated
- address and data have the same value

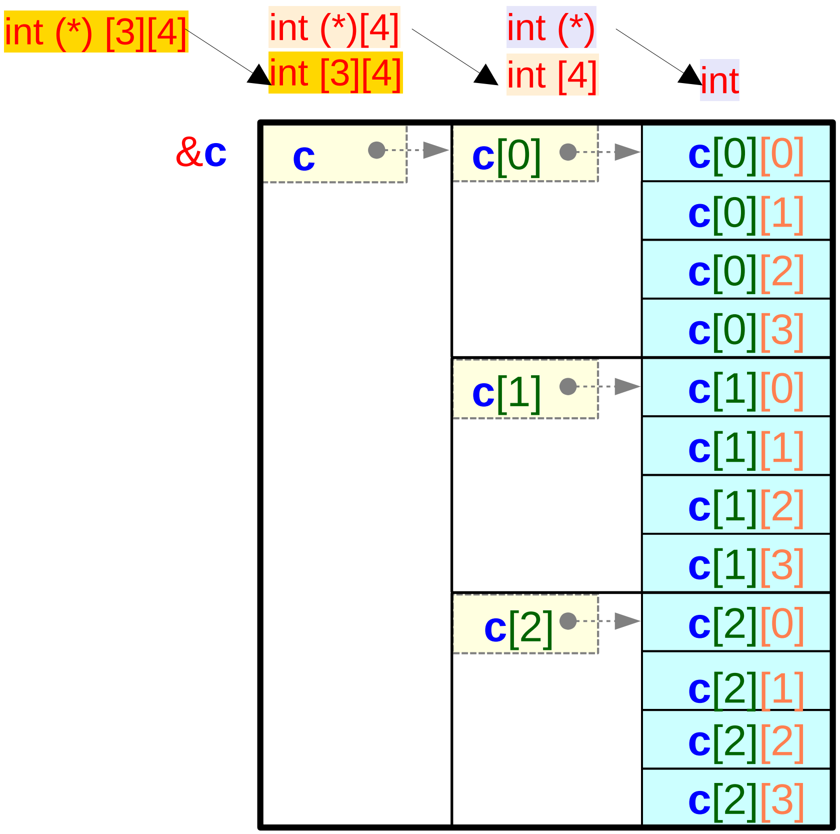
# A **2-d** array and its **1-d** sub-arrays – a size view

int c[3][4];



# A **2-d** array and its **1-d** sub-arrays – a virtual pointer view

int c[3][4];




value( c) = value( c[0]) = value(&c[0][0])
value(&c) = value(&c[0]) = value(&c[0][0])

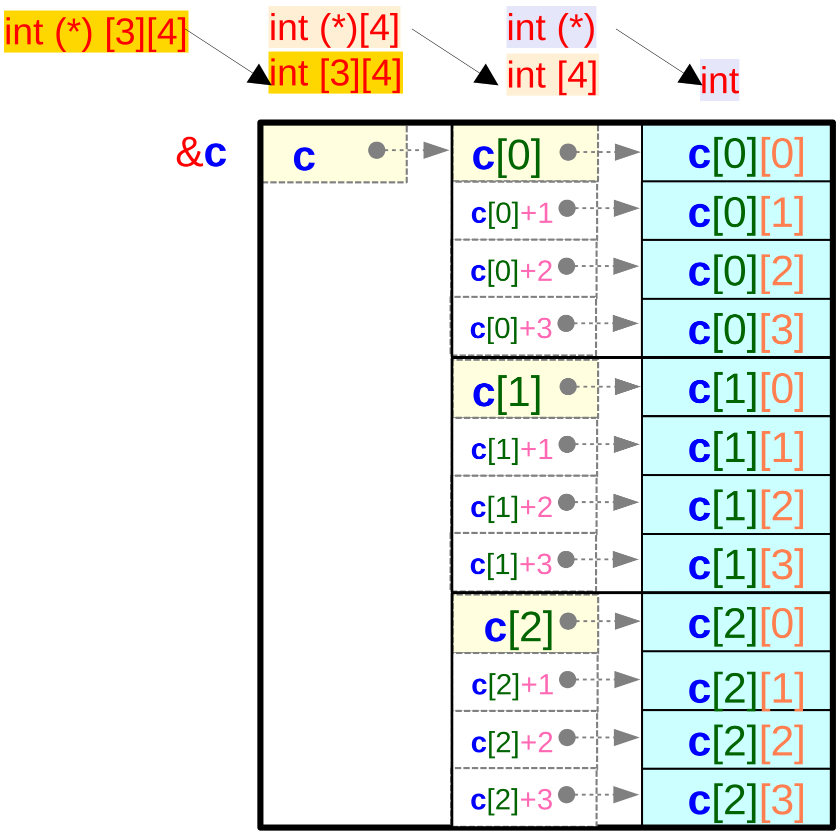
value( c[1]) = value(&c[1][0])
value(&c[1]) = value(&c[1][0])

value( c[2]) = value(&c[2][0])
value(&c[2]) = value(&c[2][0])

address(c) = address(c[0]) = address(c[0][0])
address(c[1]) = address(c[1][0])
address(c[2]) = address(c[2][0])

# A **2-d** array and its **1-d** sub-arrays – size relation

int c[3][4];




sizeof(c) = sizeof(c[0]) * 3 … *leading element*
sizeof(c+1) = pointer size (4/8 bytes)
sizeof(c+2) = pointer size (4/8 bytes)

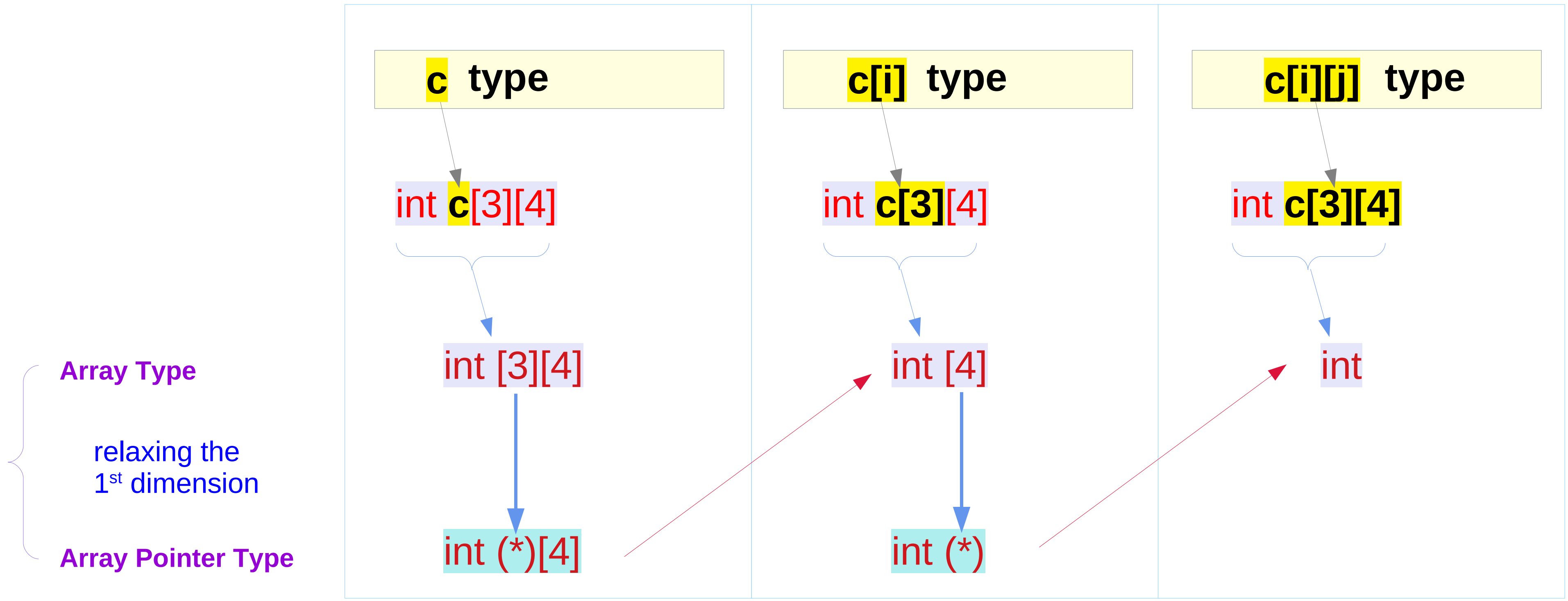
sizeof(c[0]) = sizeof(c[0][0]) * 4 … *leading element*
sizeof(c[0]+1) = pointer size (4/8 bytes)
sizeof(c[0]+2) = pointer size (4/8 bytes)
sizeof(c[0]+3) = pointer size (4/8 bytes)

sizeof(c[1]) = sizeof(c[1][0]) * 4 … *leading element*
sizeof(c[1]+1) = pointer size (4/8 bytes)
sizeof(c[1]+2) = pointer size (4/8 bytes)
sizeof(c[1]+3) = pointer size (4/8 bytes)

sizeof(c[2]) = sizeof(c[2][0]) * 4 … *leading element*
sizeof(c[2]+1) = pointer size (4/8 bytes)
sizeof(c[2]+2) = pointer size (4/8 bytes)
sizeof(c[2]+3) = pointer size (4/8 bytes)

# Sub-array types in a **2-d** array

int c[3][4];  **2-d** array c




**Dual Types**

- **Identifying nested arrays**

  **in a 2-d array declaration**

# Nested arrays in a 2-d array declaration

int c[3] [4] ;

int **c[3]** [4] ;

**c** : a 3 element array
**c[i]** : each element

**int** c[3] **[4] ;**

**c[i]**'s type 1 : int [4]
an array of 4 integers

relaxed dimension

**int** c[3] [4] ;

**c[i]**'s type 2: int (*)
a pointer to an integer

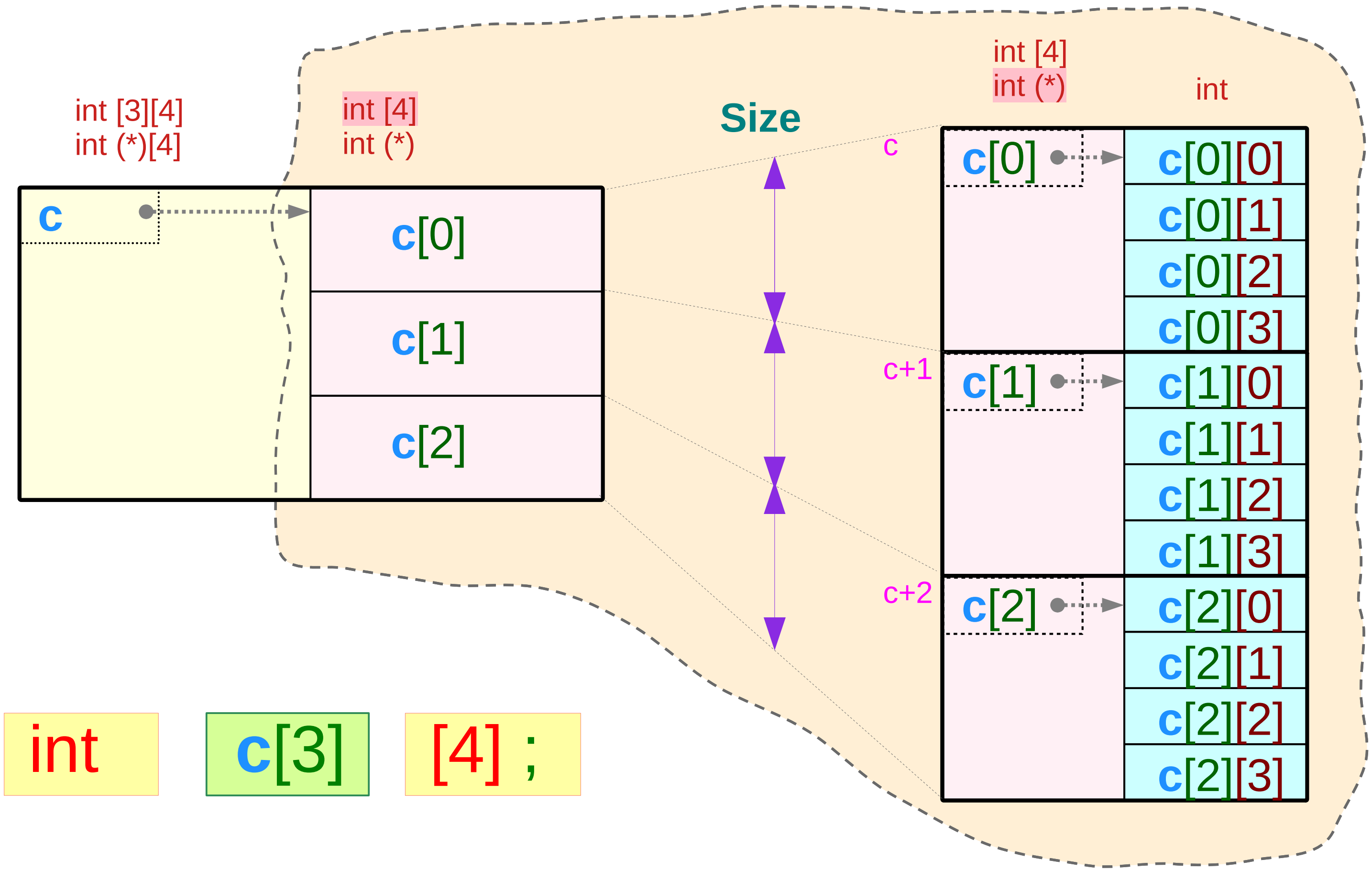
# Nested arrays

c[3]

**c** : a 3 element array
**c[i]** : each element

int [4] ;

**c[i]**'s type 1 : int [4]
**c[i]**'s type 2 : int (*)




**Address**

&c[0][0] ➡ c[0] ➡ c

&c[1][0] ➡ c[1]

&c[2][0] ➡ c[2]

int c[3] [4] ;

# c : 3-element array

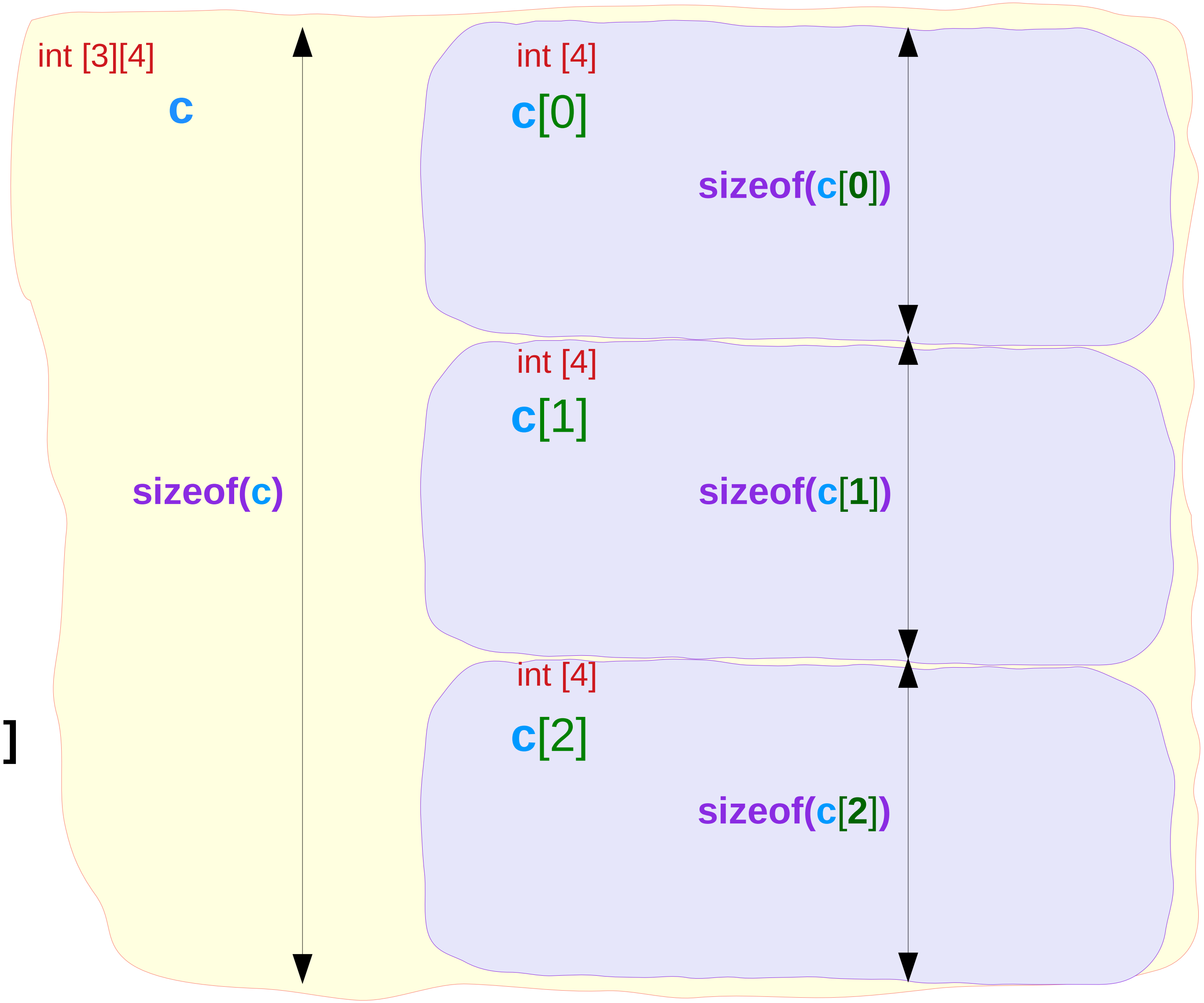
| | | |
|---|---|---|
| c | **2-d** array | int [3][4] |
| c[i] | **1-d** array | int [4] |

int c [3] [4] ;

# 3-element array c

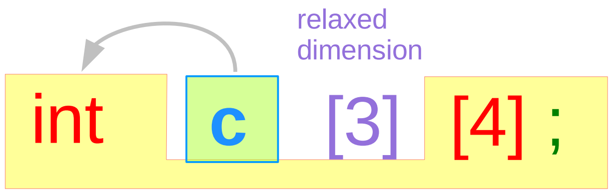
abstract data element **c[i]**

each element **c[i]** has the
1-d array type int [4]



# c : pointer to a 4-element array

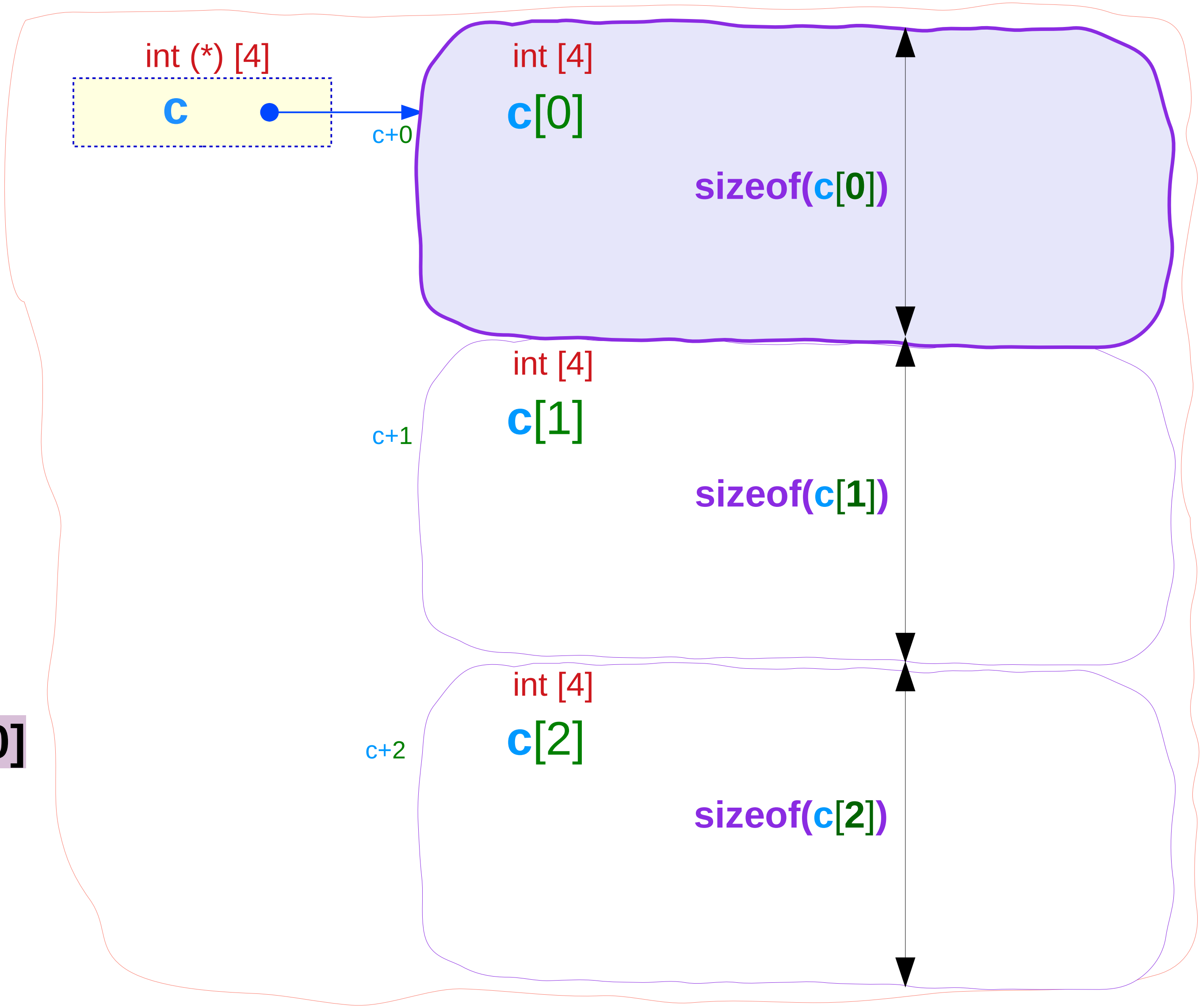
| | | |
|---|---|---|
| c | **1-d** array pointer | int (*)[4] |
| c[i] | **1-d** array | int [4] |




# pointer c

abstract data element **c[0]**

each element **c[i]** has the
1-d array type int [4]



# c[i] : 4-element array

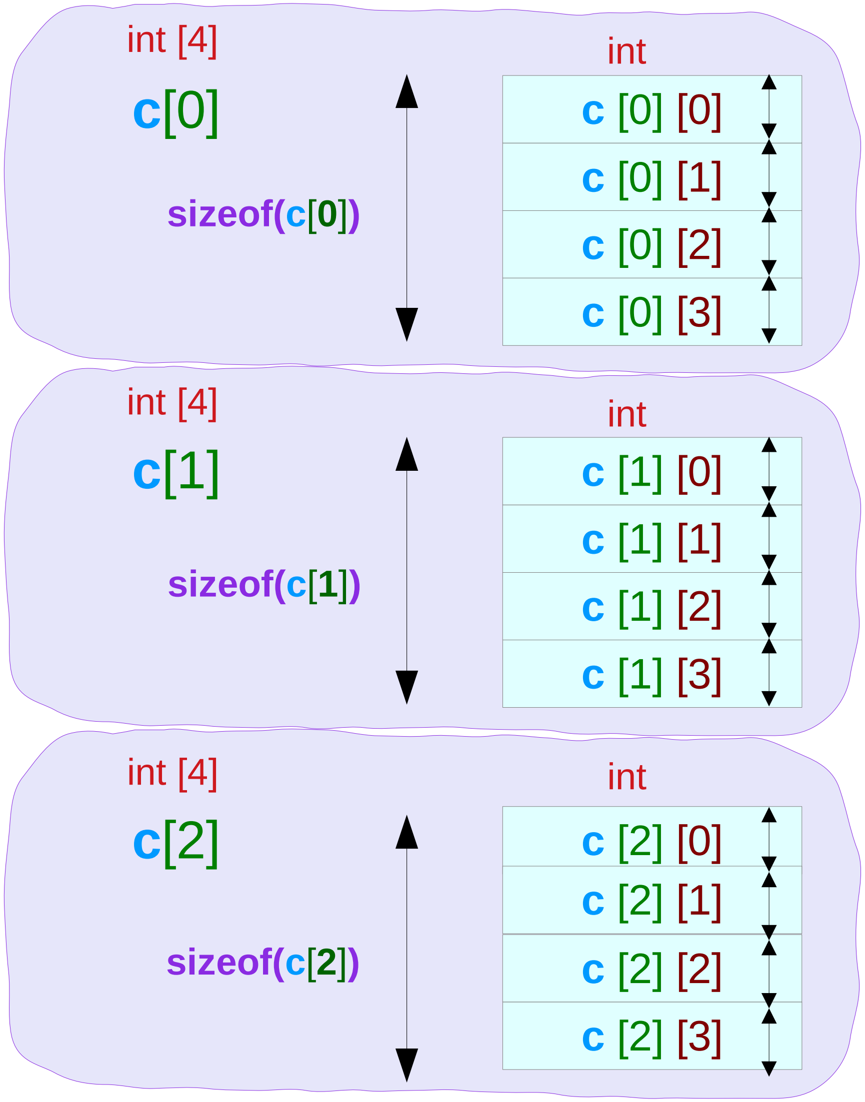
| | | |
|---|---|---|
| c[i] | **1-d** array | int [4] |
| c[i][j] | **0-d** array | int |

int c [3] [4] ;

## 4-element array c[i]

primitive data element **c[i][j]**

each element **c[i][j]** has
the primitive type int



# c[i] : pointer to a primitive data

| | | |
|---|---|---|
| c[i] | **0-d** array pointer | int (*) |
| c[i][j] | **0-d** array | int |

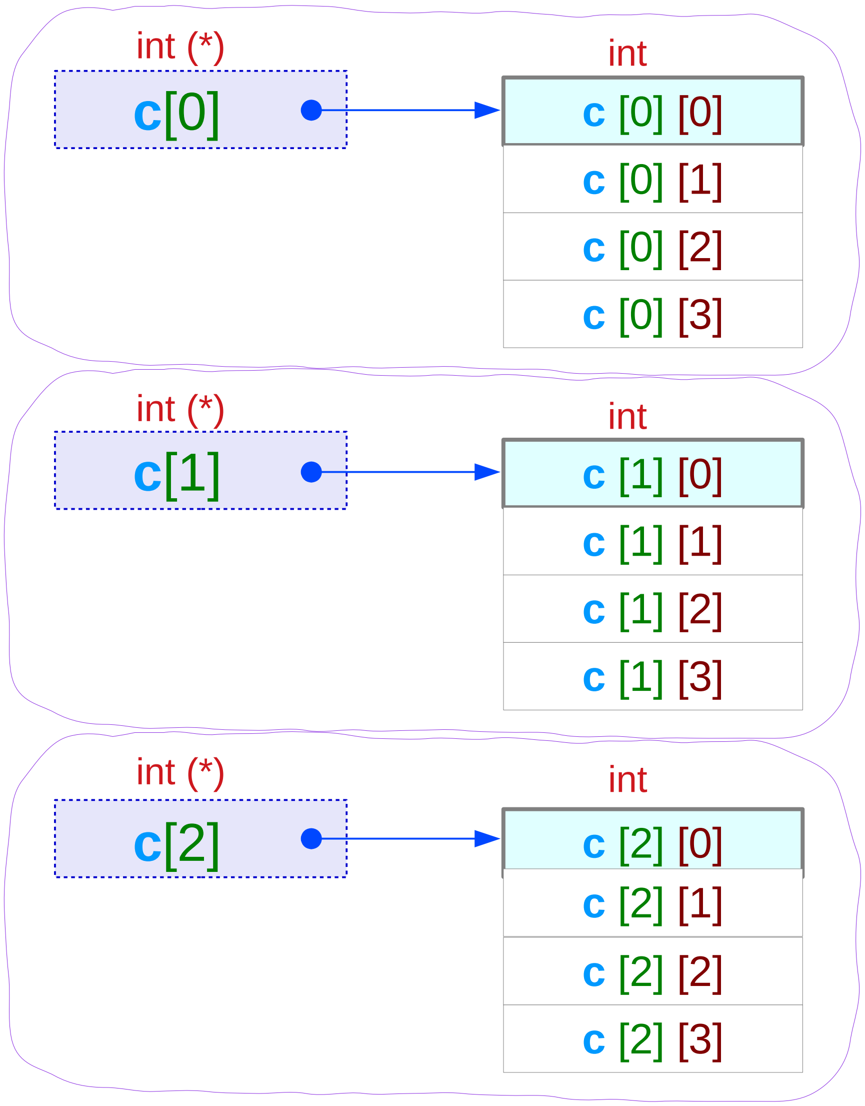
relaxed dimension

int c [3] [4] ;

# pointer c[i]

primitive data element **c[i][0]**

each element **c[i][j]** has
the primitive type int



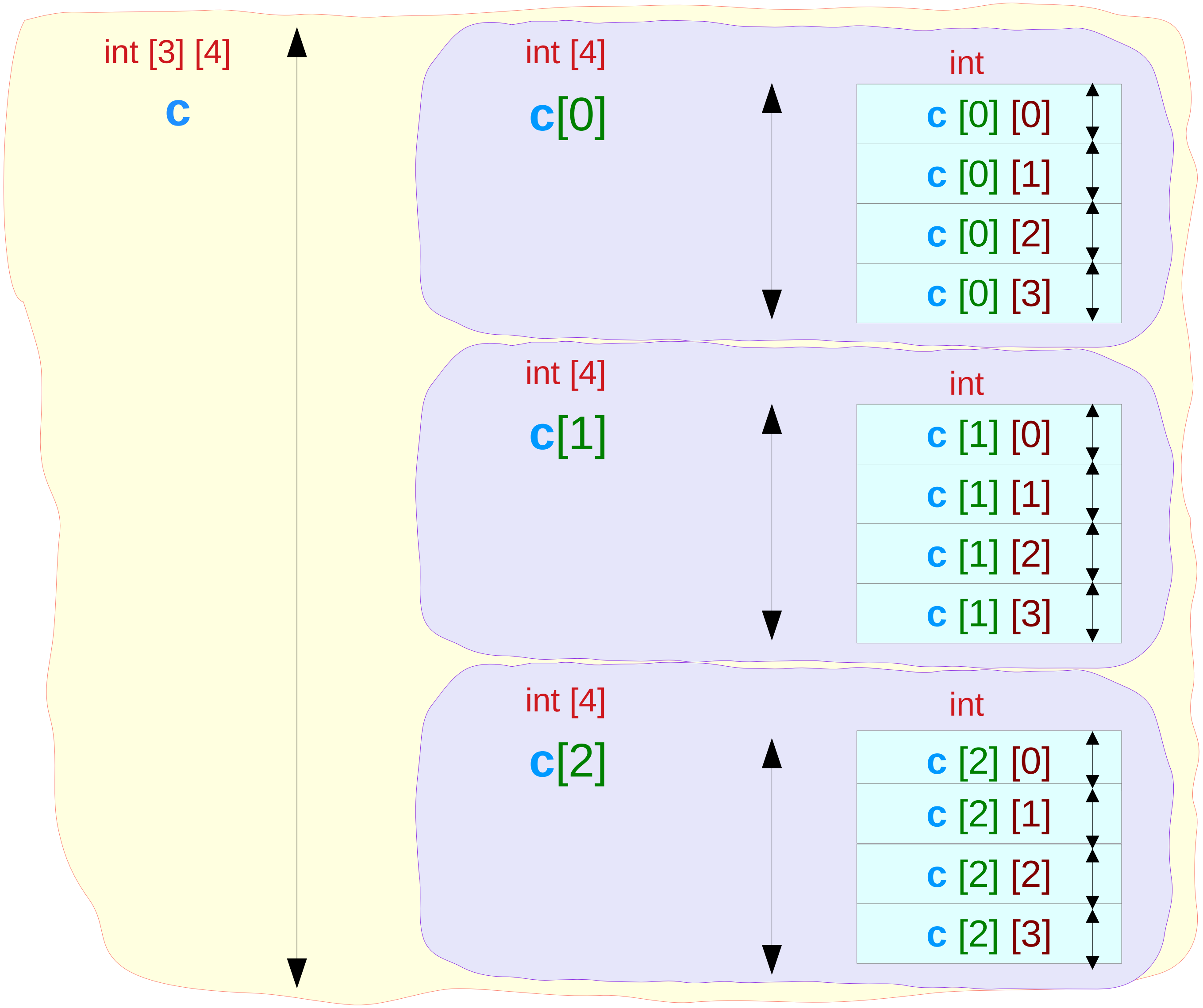
# Recursive data view

| | | |
|---|---|---|
| c | **2-d** array | int [3][4] |
| c | **1-d** array pointer | int (*)[4] |
| c[i] | **1-d** array | int [4] |
| c[i] | **0-d** array pointer | int (*) |
| c[i][j] | **0-d** array | int |

int　c[3]　[4] ;

**3-element array c**

**4-element array c[i]**



# Pointer view

| | | |
|---|---|---|
| **c** | **2-d** array | int [3][4] |
| **c** | **1-d** array pointer | int (*)[4] |
| **c[i]** | **1-d** array | int [4] |
| **c[i]** | **0-d** array pointer | int (*) |
| **c[i][j]** | **0-d** array | int |

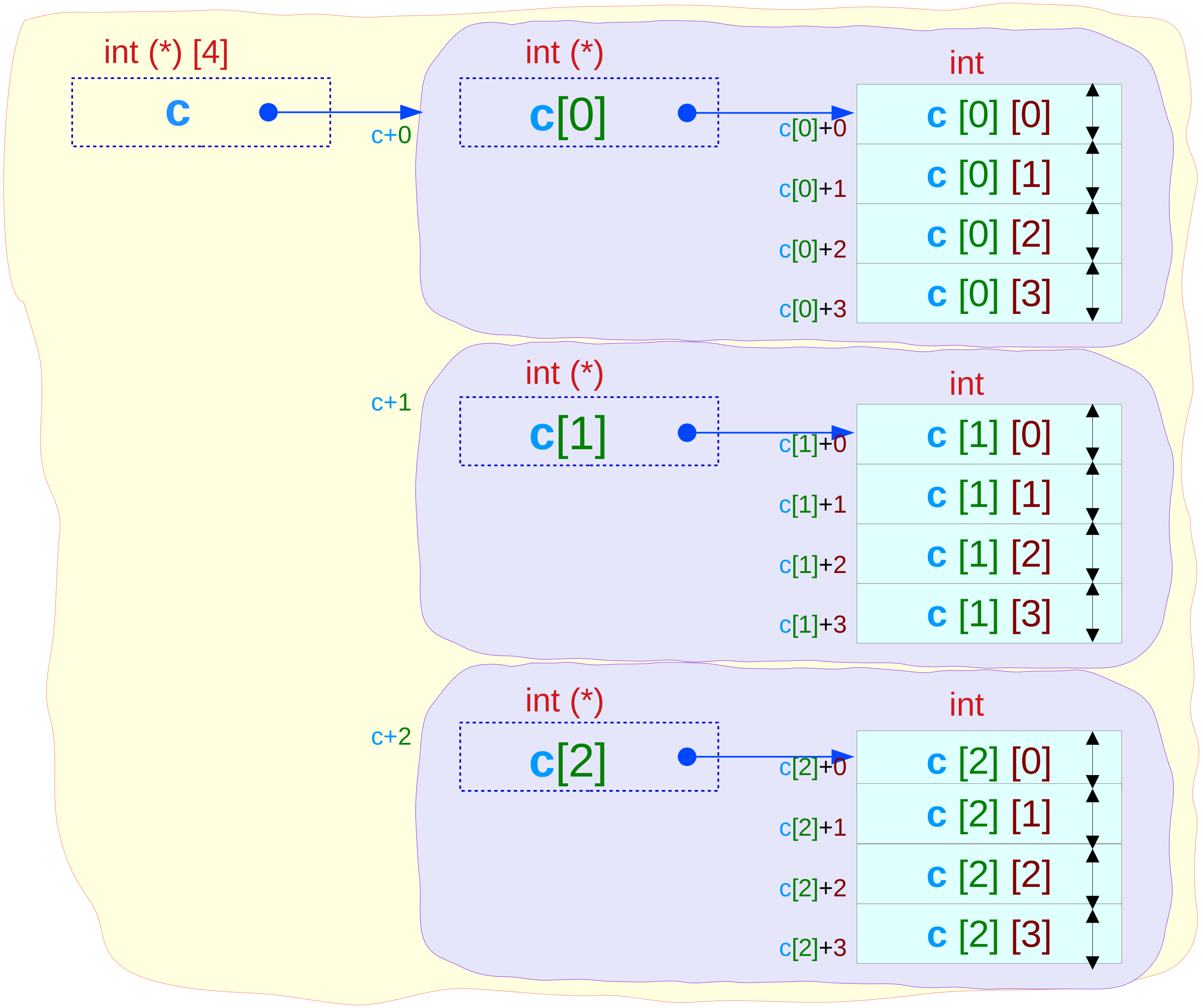
int c[3] [4] ;

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

v(c[2]) = v(&c[2][0])

v ≡ value



# 1-d array pointer

int (*p) [4] ;

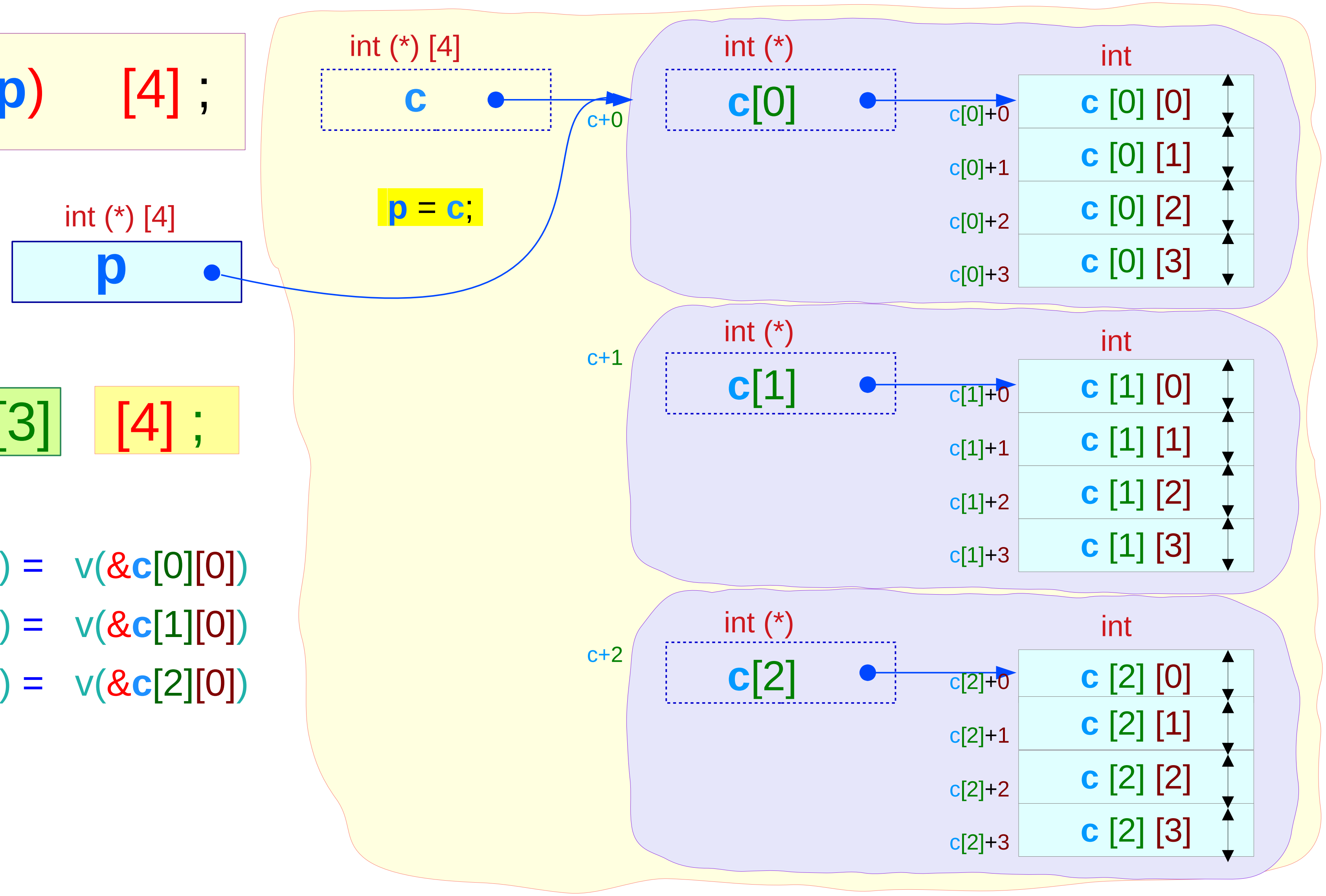
int c[3] [4] ;

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

v(c[2]) = v(&c[2][0])

v ≡ value

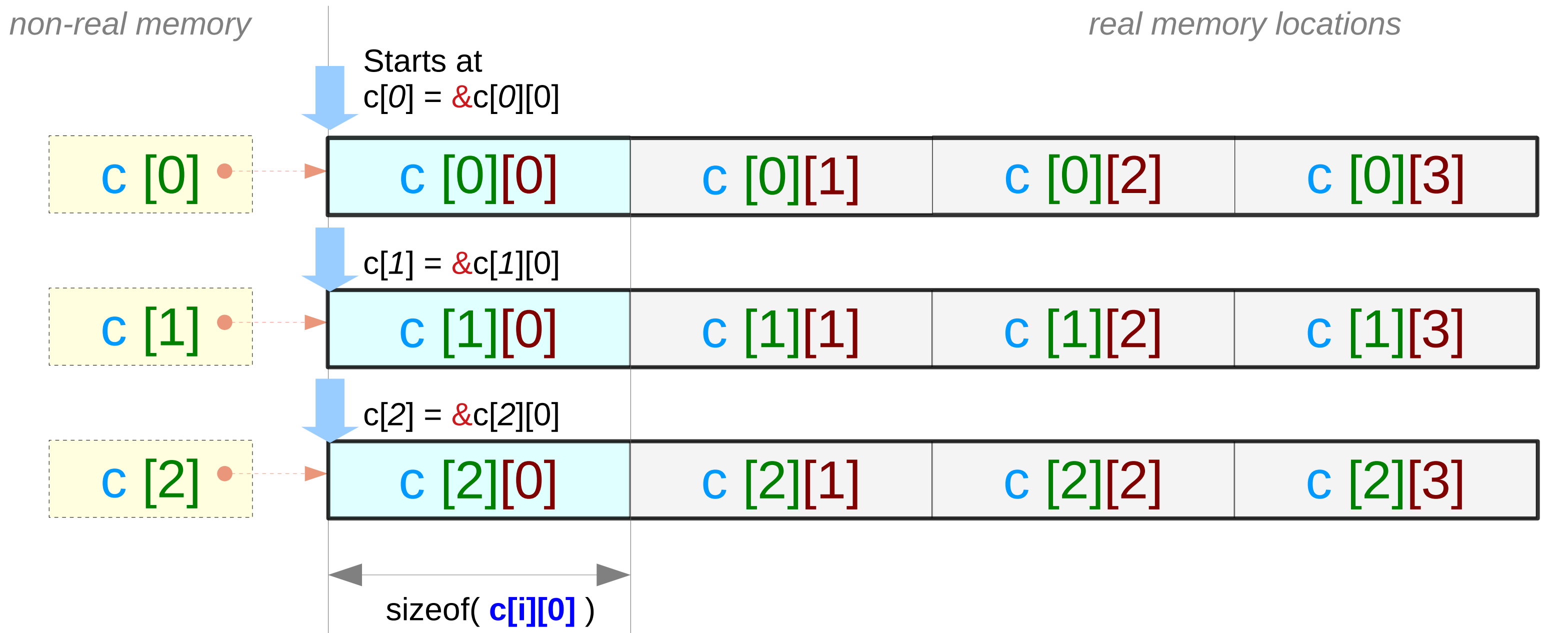


# Pointer **c[i]** and integer **c[i][0]**

int **c[3]** [4];

non-real pointer **c[i]** : value(**c[i]**)= &**c[i][0]**

**0-d** array pointer




**virtual pointer c[i]**
**0-d array pointer : int (*)**

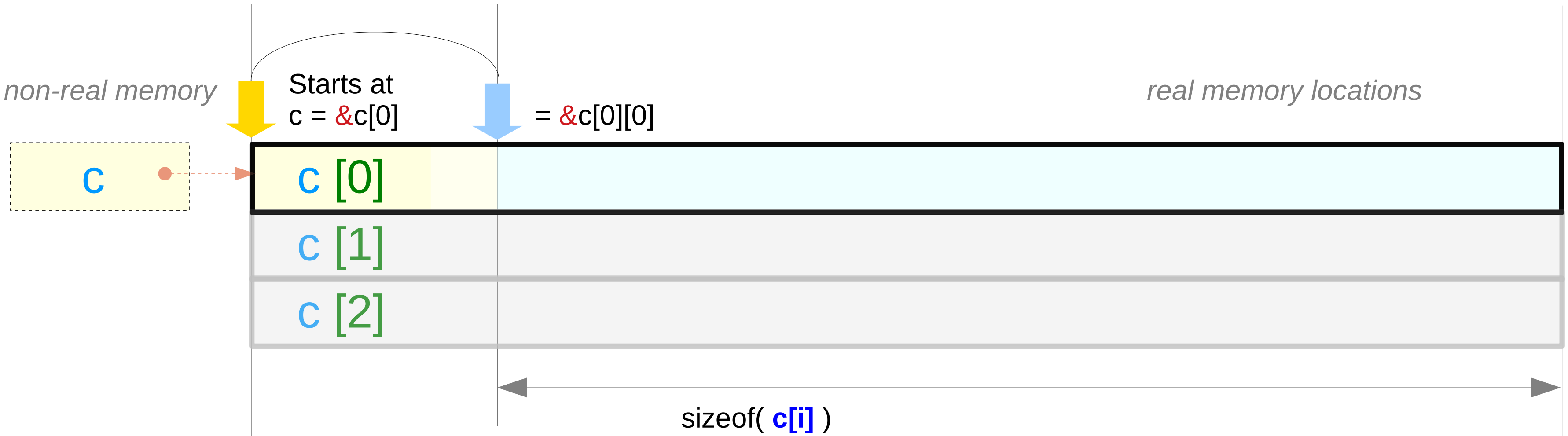
**integer c[i][0]**
**0-d array : int**

**c[i]** points to the 1st element **c[i][0]**

# Pointer **c** and abstract data **c[i]**

```
int c [3] [4];
```

non-real pointer **c** : value(**c**) = &**c[0]** = &**c[0][0]** — **1-d** array pointer
abstract data **c[i]** : sizeof(**c[i]**) = 4 * sizeof(**int**) — **1-d** array




**virtual pointer c**
**1-d array pointer : int (*)[4]**

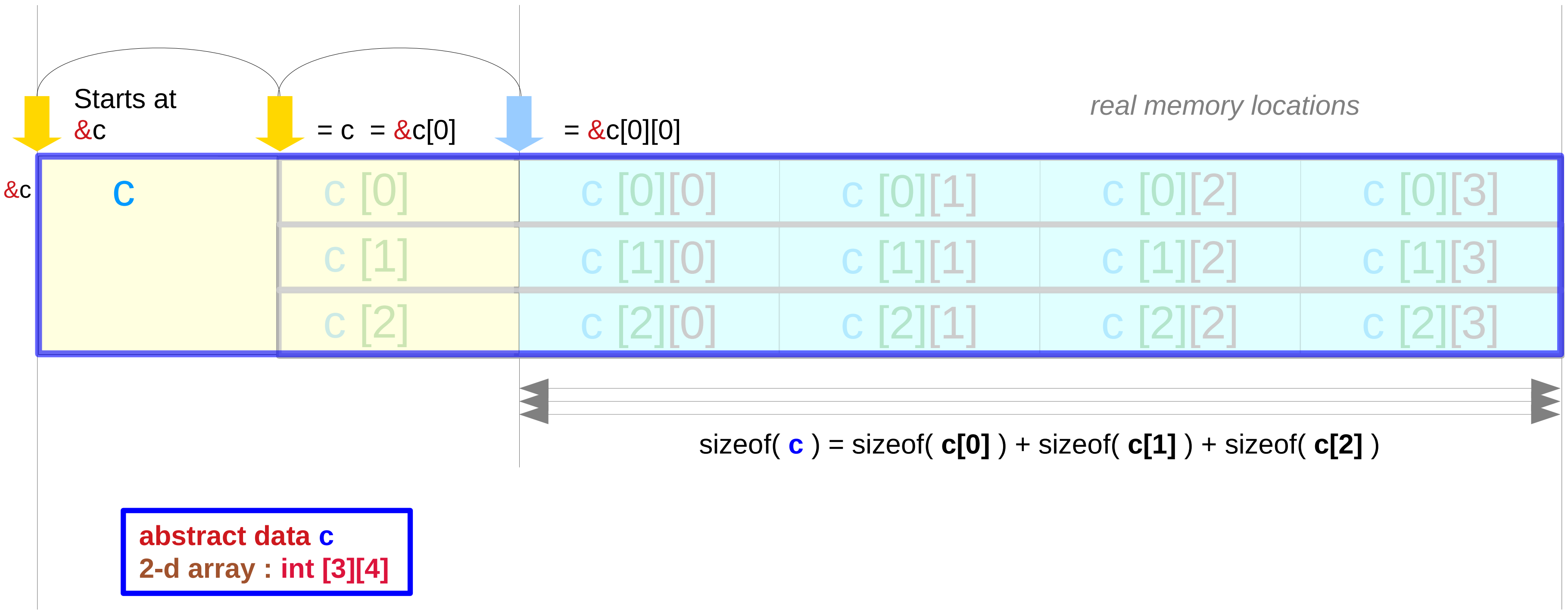
**abstract data c[i]**
**1-d array : int [4]**

**c** points to the 1st element **c[0]**

# Abstract data **c**

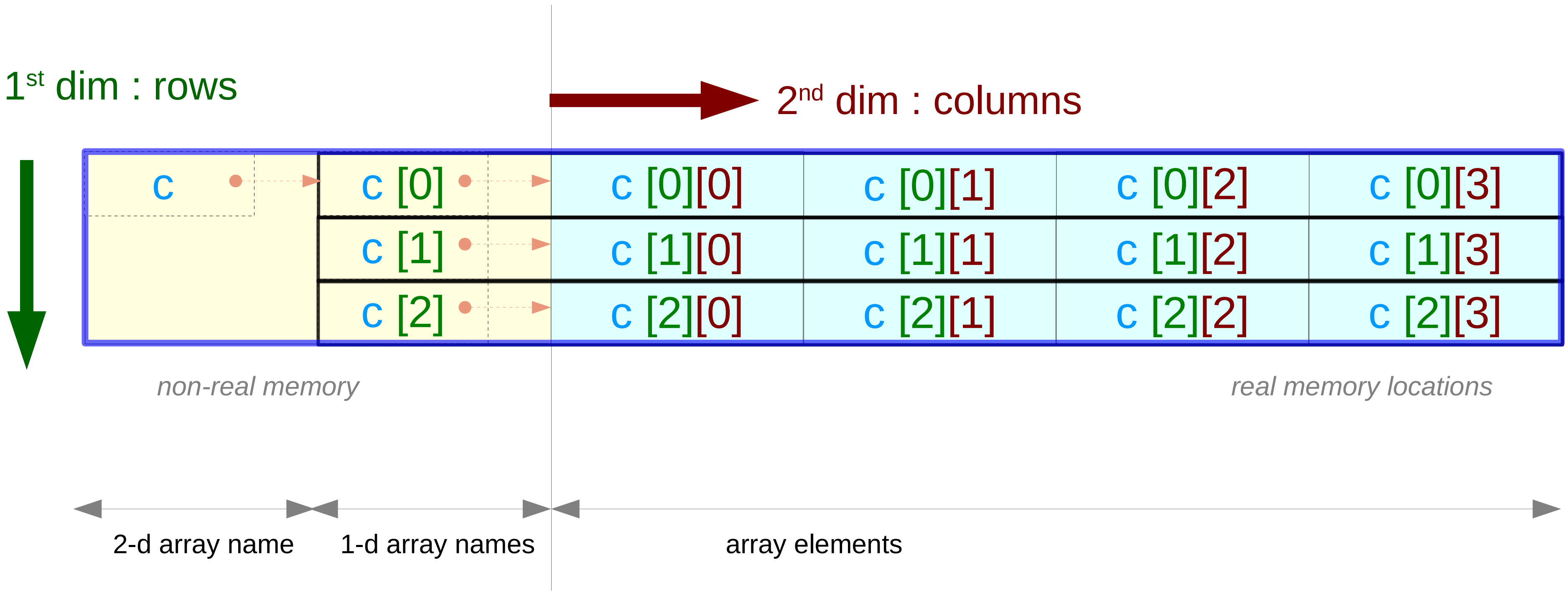
```
int c [3] [4];
```

abstract data **c:** sizeof(**c**) = 3 * sizeof(**c[i]**) **2-d** array



# Rows and columns of a **2-d** array **c**

```
int c[3] [4];
```



# The name of a 2-d array

```
int         a [4];

int      c [4] [4];
```

**1. the name of the nested array (recursive definition)**

**2. a double pointer**

**3. a pointer to an array**

# **2-d** array **c** and **1-d** array **q**

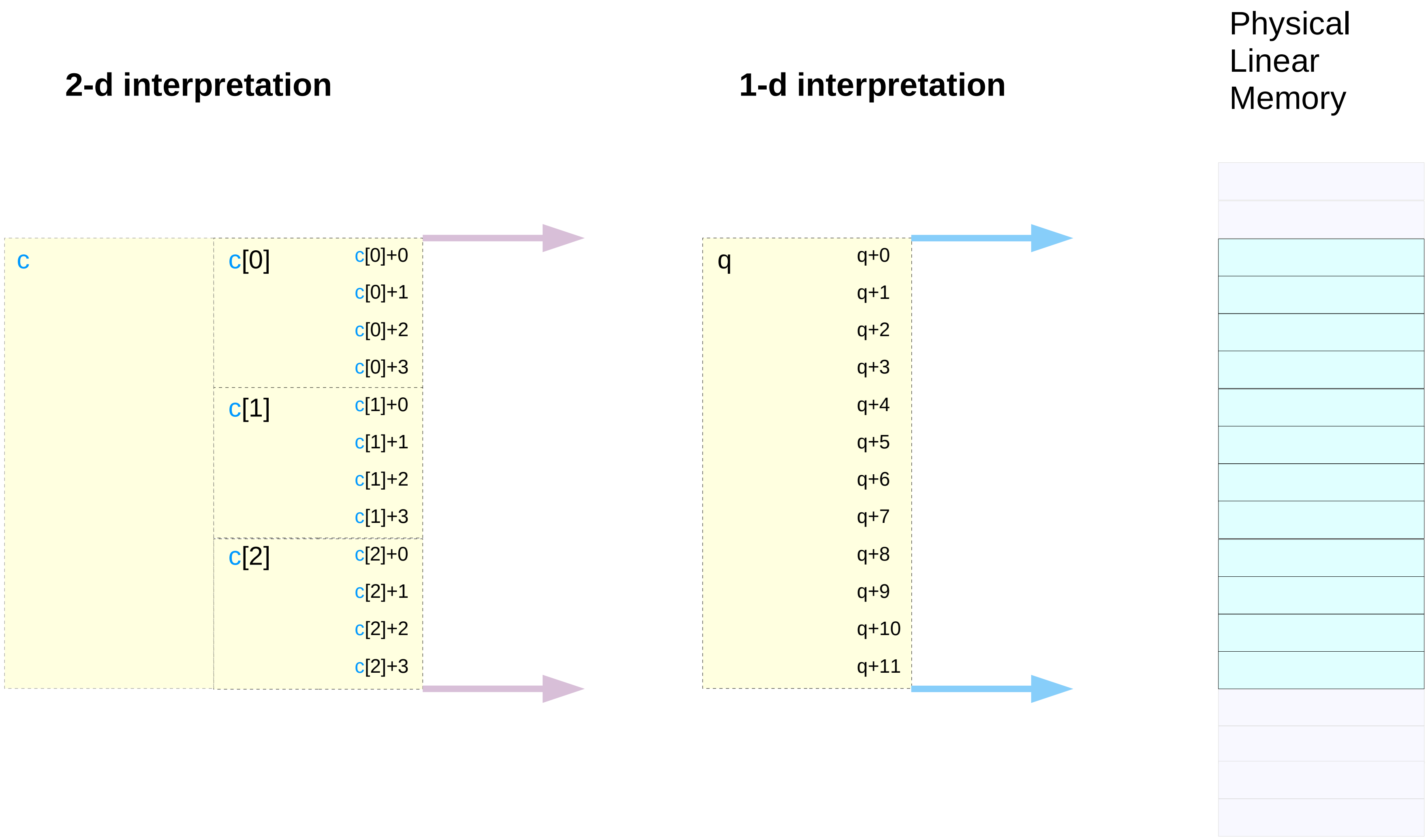
```
int c [3] [4];
```

```
int q [3*4];
```

| | | | |
|---|---|---|---|
| c | c[0] | c[0]+0 | c[0][0] |
| | | c[0]+1 | c[0][1] |
| | | c[0]+2 | c[0][2] |
| | | c[0]+3 | c[0][3] |
| | c[1] | c[1]+0 | c[1][0] |
| | | c[1]+1 | c[1][1] |
| | | c[1]+2 | c[1][2] |
| | | c[1]+3 | c[1][3] |
| | c[2] | c[2]+0 | c[2][0] |
| | | c[2]+1 | c[2][1] |
| | | c[2]+2 | c[2][2] |
| | | c[2]+3 | c[2][3] |

| | | |
|---|---|---|
| q | q+0 | q[0*4+0] |
| | q+1 | q[0*4+1] |
| | q+2 | q[0*4+2] |
| | q+3 | q[0*4+3] |
| | q+4 | q[1*4+0] |
| | q+5 | q[1*4+1] |
| | q+6 | q[1*4+2] |
| | q+7 | q[1*4+3] |
| | q+8 | q[2*4+0] |
| | q+9 | q[2*4+1] |
| | q+10 | q[2*4+2] |
| | q+11 | q[2*4+3] |

# 2-d and 1-d interpretations of linear memories



# A **2-d** array stored as a **1-d** array (row major order)

int c [3] [4];

c[i][j]

**[i*4+j]**

**[k]**

| | | | |
|---|---|---|---|
| c | c[0] | c[0]+0 | c[0][0] |
| | | c[0]+1 | c[0][1] |
| | | c[0]+2 | c[0][2] |
| | | c[0]+3 | c[0][3] |
| | c[1] | c[1]+0 | c[1][0] |
| | | c[1]+1 | c[1][1] |
| | | c[1]+2 | c[1][2] |
| | | c[1]+3 | c[1][3] |
| | c[2] | c[2]+0 | c[2][0] |
| | | c[2]+1 | c[2][1] |
| | | c[2]+2 | c[2][2] |
| | | c[2]+3 | c[2][3] |

index values

| | |
|---|---|
| 0 | =[0*4+0] |
| 1 | =[0*4+1] |
| 2 | =[0*4+2] |
| 3 | =[0*4+3] |
| 4 | =[1*4+0] |
| 5 | =[1*4+1] |
| 6 | =[1*4+2] |
| 7 | =[1*4+3] |
| 8 | =[2*4+0] |
| 9 | =[2*4+1] |
| 10 | =[2*4+2] |
| 11 | =[2*4+3] |

| | | |
|---|---|---|
| q | q+0 | q[0] |
| | q+1 | q[1] |
| | q+2 | q[2] |
| | q+3 | q[3] |
| | q+4 | q[4] |
| | q+5 | q[5] |
| | q+6 | q[6] |
| | q+7 | q[7] |
| | q+8 | q[8] |
| | q+9 | q[9] |
| | q+10 | q[10] |
| | q+11 | q[11] |

# 2-d array access via a single pointer

```
int    *p = c[0] ;        ⇐    int c [3][4];

p[ i*4 + j ]              ⇐    c[ i ][ j ]

*(p+ i*4 + j)             ⇐    *(*(c+i)+ j)
```

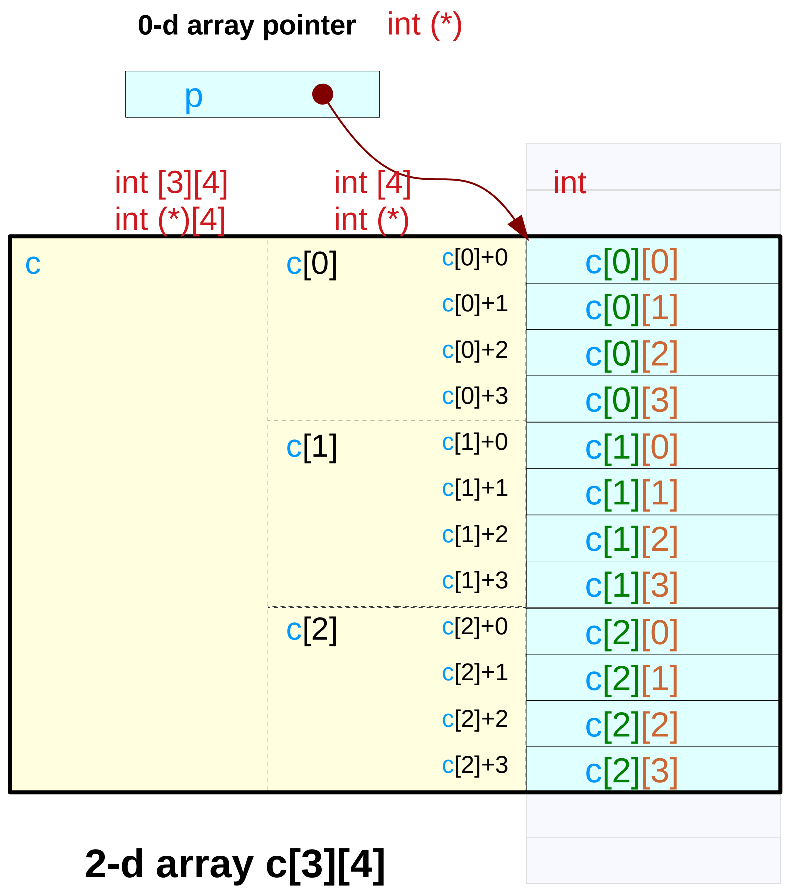
```
*(p +k)        i = k / 4;
               j = k % 4;
```

# View a **2-d** array as a **1-d** array

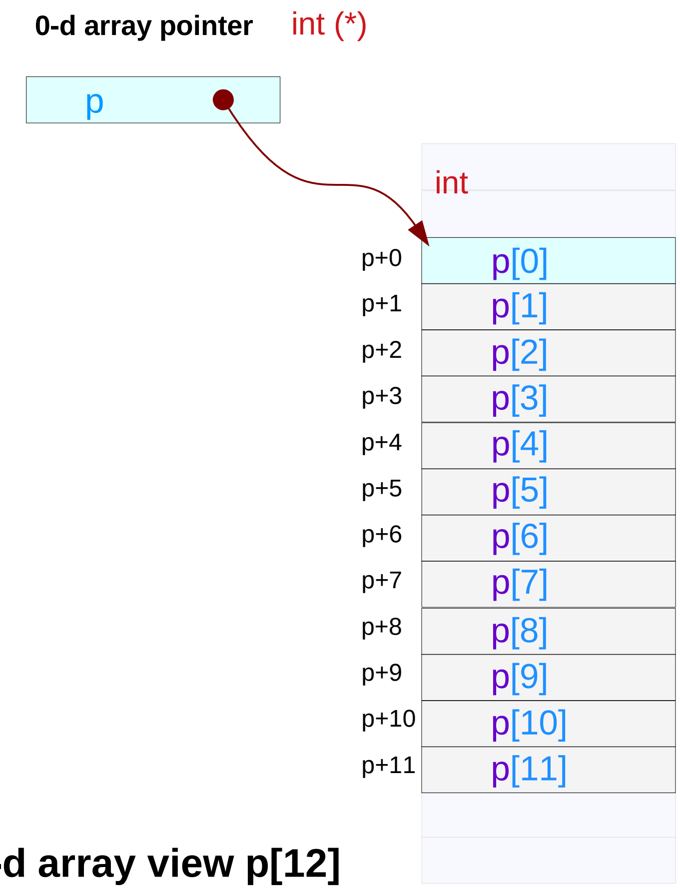
```
int c [3][4];
```

```
int *p = c[0];
```

**c, c[0], &c[0][0]**




**2-d array c[3][4]**



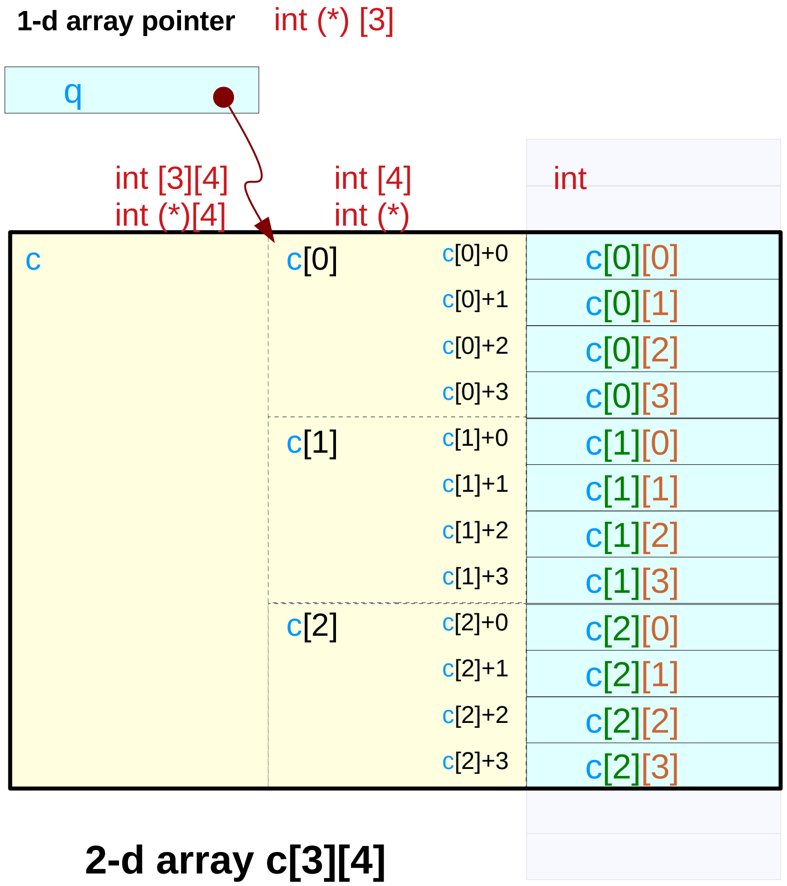

**1-d array view p[12]**

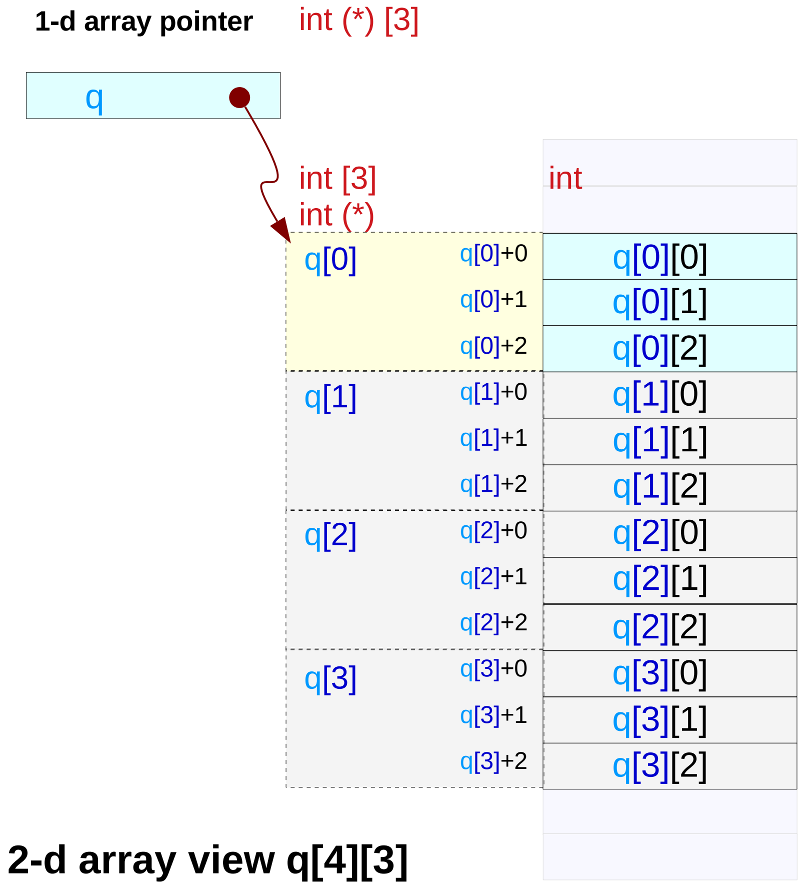
# View a **2-d** array as another **2-d** array

int c [3][4];

int (*q) [3] = (int (*) [3]) c;

**c**, **c[0]**, **&c[0][0]**



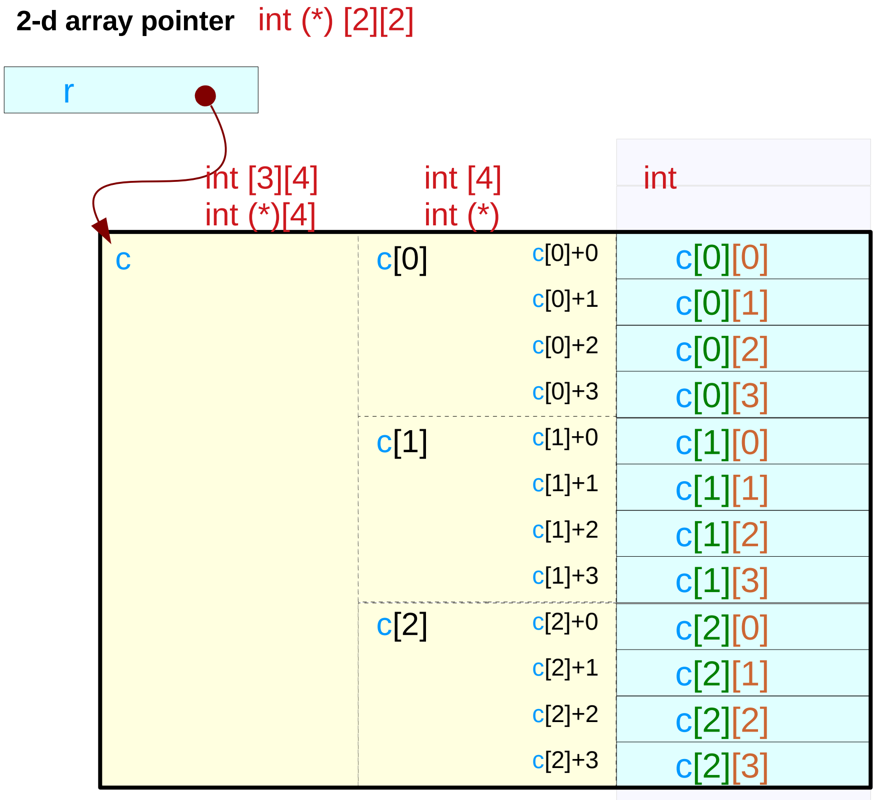



# A **2-d** array stored as a **1-d** array (row major order)

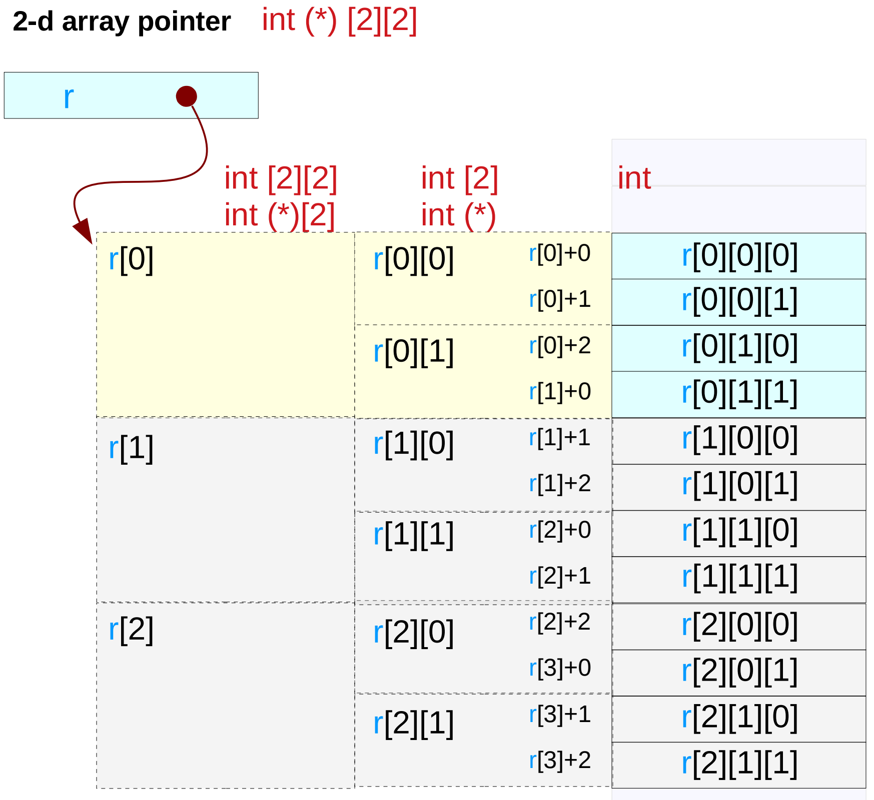
```
int c [3] [4];
```

```
int (*r) [2][2] = (int (*) [2][2]) c;
```

c, c[0], &c[0][0]




**2-d array c[3][4]**




**3-d array view r[3][2][2]**

# **2-d** array access via pointers

```
int c [3][4];
```

```
int     *p = c[0] ;
```

**1. recursive pointers**

```
c [ i ][ j ]
```

```
(*(c+i))[ j ]   ⇨   int (*p)[4];
```

```
*(c[ i ]+ j)
```

```
*(*(c+i)+ j)    ⇨   int **q;
```

**2. linear array pointers**

```
p[ i*4 + j ]
```

```
*(p+ i*4 + j )
```

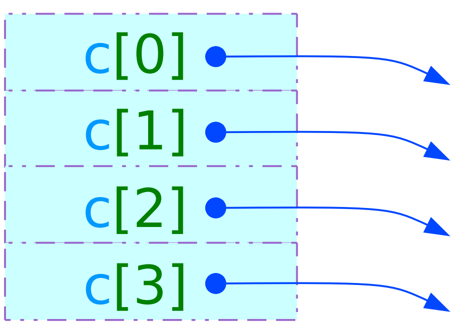
# Static Allocation of a 2-d Array

int A [3][4];

A in %eax,
i  in %edx,
j  in %ecx

```
sall    $2, %ecx                   ;; j * 4
leal    (%edx, %edx, 2), %edx      ;; i * 3
leal    (%ecx, %edx, 4), %edx      ;; j * 4 + i * 12
movl    (%eax, %edx), %eax         ;; read M[ XA+4(3i +j) ]
```

| | |
|---|---|
| c[0]+0 | *(c [0]+0) |
| c[0]+1 | *(c [0]+1) |
| c[0]+2 | *(c [0]+2) |
| c[0]+3 | *(c [0]+3) |
| c[1]+0 | *(c [1]+0) |
| c[1]+1 | *(c [1]+1) |
| c[1]+2 | *(c [1]+2) |
| c[1]+3 | *(c [1]+3) |
| c[2]+0 | *(c [2]+0) |
| c[2]+1 | *(c [2]+1) |
| c[2]+2 | *(c [2]+2) |
| c[2]+3 | *(c [2]+3) |




The pointer array :
not allocated
in the memory

# Pointers, arrays, and operator precedence

# Address-of & and dereference * operators

## Address-of operation

| &X | = | value(&X) | rvalue |
|---|---|---|---|
| C Expressions | | Mixed Expressions | |

rvalue: &X — lvalue: X

&X evaluates the address value of a variable X

& is a mathematical operator (the inverse operator of *)

value(&X) = value(value(&X)) = value(&X) = &X

## Dereference operation

| *X | = | *value(X) | lvalue |
|---|---|---|---|
| C Expressions | | Mixed Expressions | |

rvalue: X — lvalue: *X

lvalue must be evaluated to rvalue

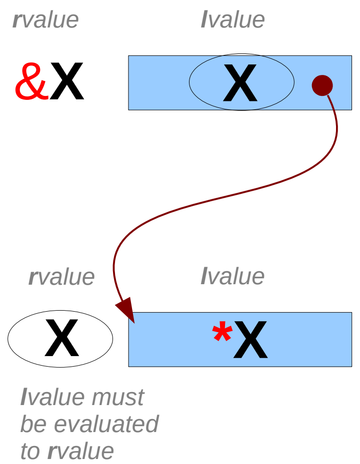
X must be evaluated to an address before de-referencing

# Equivalences in address replications

## Eqivalences in Address Replications

*a pointer variable* ***X***

value(&X) ≡ value(X)




*&X and X have different types*
*but have the same value*

*at the pointed address* ***X***

value(X) ≡ *value(X)

*X and *X have different types*
*but have the same value*

value(&X) ≡ value(X) ≡ *value(X)

# Equivalences in array notations

X[n] → *(X+n)

&X[n] → value (X+n)

C operator &
≠ inverse of *

math operator &
= inverse of *

value(&X[n])
= value(&*(X+n))
= value(X+n)

X[0] → *(X+0)

&X[0] → value (X+0)

C operator &
≠ inverse of *

math operator &
= inverse of *

value(&X[0])
= value(&*(X+0))
= value(X+0)

# Pointer Arithmetic

- increment / decrement **++X, ––X, X++, X––**
- addition of an integer **X + i**
- subtraction of an integer **X – i**
- subtracting two pointers of the same type **X – Y**
- comparison of pointers **==, !=, >, >=, <, <=**

- adding two pointers are not allowed ~~**X + Y**~~

pointer variables: **X**, **Y**

integer variables : **i**

(**int**, **short**, **char**, ...)

# Pointer Addition / Subtraction

pointer variables: **X**, **Y**

primitive variables : **A, B**

| | |
|---|---|
| **X + A** | the variable **A** must have integer compatible types, otherwise error |
| **X – A** | |
| **X + Y** | error! |
| **X – Y** | o.k. |

value(**X** + **A**) = value(**X**) $+_a$ **A** * sizeof(***X**)

value(**X – A**) = value(**X**) $-_a$ **A** * sizeof(***X**)

value(**X + Y**) error!

value(**X – Y**) = value(**X**) $-_a$ value(**Y**)

value(**X**) is used to avoid confusion between pointer additions and arithmetic additions

| | |
|---|---|
| value(**A**) = **A** | primitive variable **A** |
| value(**X**) ≠ **X** | pointer variable **X** |

**X** : may involved in pointer additions + or pointer subtractions **–**

value(**X**) : may involved in arithmetic additions $+_a$ arithmetic subtractions $-_a$

# Pointer Addition / Subtraction

pointer variables: **X**, **Y**

primitive variables : **A, B**

value(**A**) = **A** primitive variable **A**

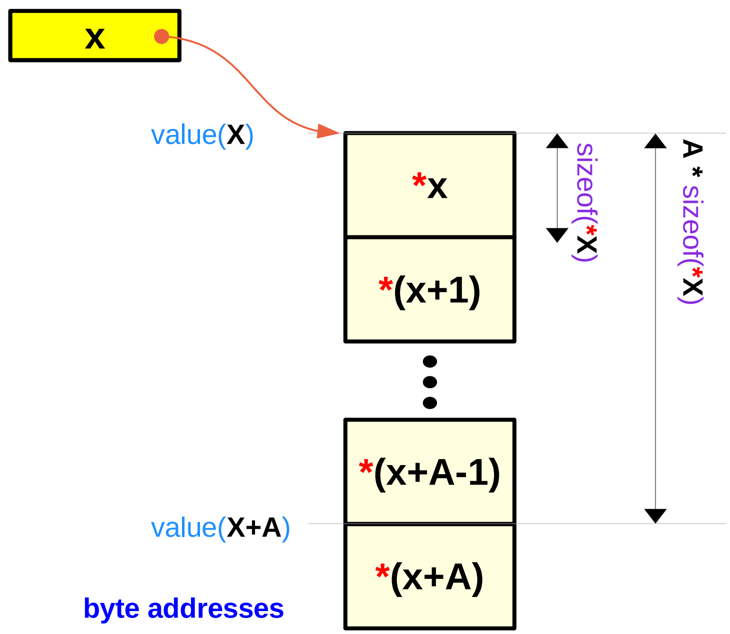
value(**X**) ≠ **X** pointer variable **X**

| | | |
|---|---|---|
| **X** | C expression | + ➡ pointer additions<br>– ➡ pointer subtractions |
| value(**X**) | Math expression | + ➡ arithmetic additions $+_a$<br>– ➡ arithmetic subtractions $-_a$ |
| **A** | C expression | + ➡ arithmetic additions $+_a$<br>– ➡ arithmetic subtractions $-_a$ |
| value(**A**) | Math expression | + ➡ arithmetic additions $+_a$<br>– ➡ arithmetic subtractions $-_a$ |

# Pointer Addition / Subtraction

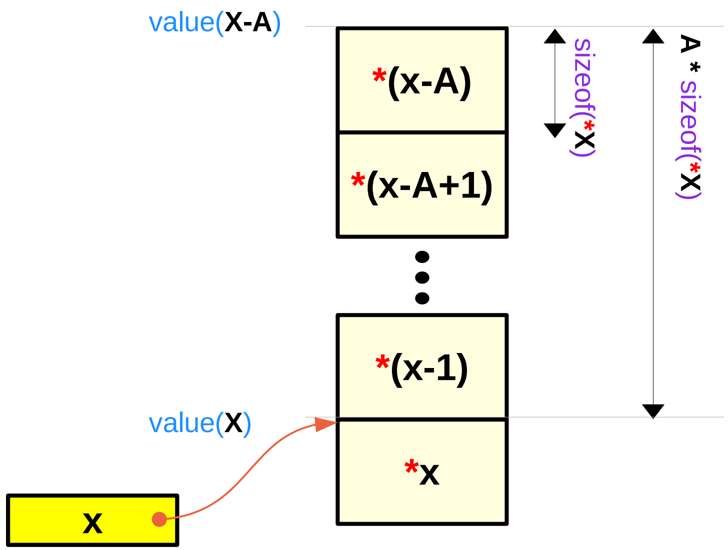
**X + A : pointer addition**

value(**X** + **A**) = value(**X**) + **A** * sizeof(*__X__)




**X – A : pointer subtraction**

value(**X – A**) = value(**X**) – **A** * sizeof(*__X__)



# Subscript [ ] and dereference * notations (1a)

| | | |
|---|---|---|
| value(**A** + **B**) = value(**A**) $+_a$ value(**B**)<br>value(**X** + **B**) ≠ value(**X**) $+_a$ value(**B**) | ⇒ | value(**A** + **B**) = **A** $+_a$ **B**<br>value(**X** + **B**) = value(**X**) $+_a$ **B** * sizeof(***X**) |
| value(**A** $+_a$ **B**) = value(**A**) $+_a$ value(**B**)<br>value(**X** $+_a$ **B**) = value(**X**) $+_a$ value(**B**) | ⇒ | value(**A** $+_a$ **B**) = **A** $+_a$ **B**<br>value(**X** $+_a$ **A**) = value(**X**) $+_a$ **B** |

value(**A**) = **A** primitive variable

value(**X**) ≠ **X** pointer variable

# Subscript [ ] and dereference * notations (1a)

value(value(**A**)) = value(**A**) = **A**

value(value(**X**)) = value(**X**) ≠ **X**

value(value(**X** + **i**))

= value(value(**X**) $+_a$ **i** * sizeof(*__X__))

= value(value(**X**)) $+_a$ value(**i** * sizeof(*__X__))

= value(**X**) $+_a$ **i** * sizeof(*__X__)

= value(**X**) + **i** * sizeof(*__X__)  in math expression

# Operator Precedence of * and [ ]

| | | |
|---|---|---|
| *x[m] | ≡ | *(x[m]) |
| x[m][n] | ≡ | (x[m])[n] |
| **x | ≡ | *(*x) |

[ ] has a higher priority than *

[ ] has left-to-right associativity

* has right-to-left associativity

(*x)[m][n] ⟺ ((*x)[m])[n]

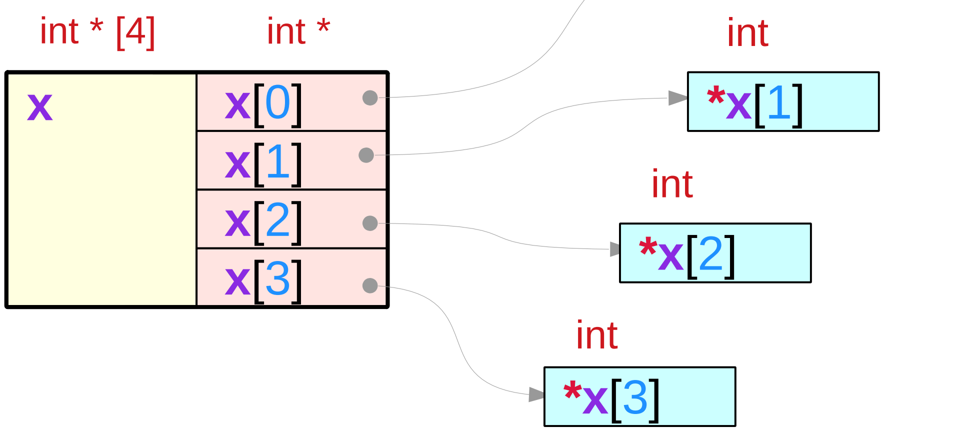
(*x[m])[n] ⟺ (*(x[m]))[n]

red parentheses ( ) must not be removed

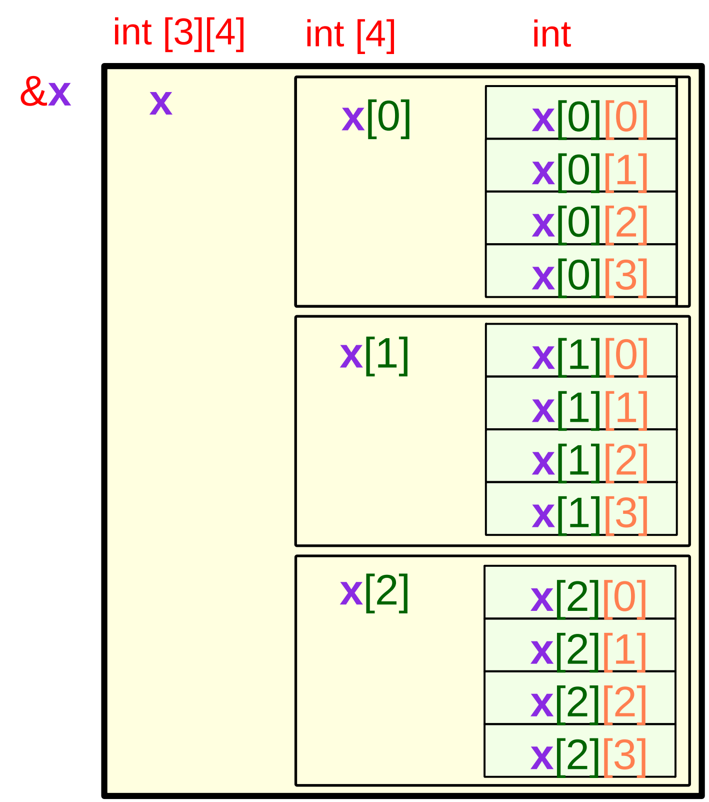
gray parentheses ( ) can be removed

# Operator Precedence of * and [ ]

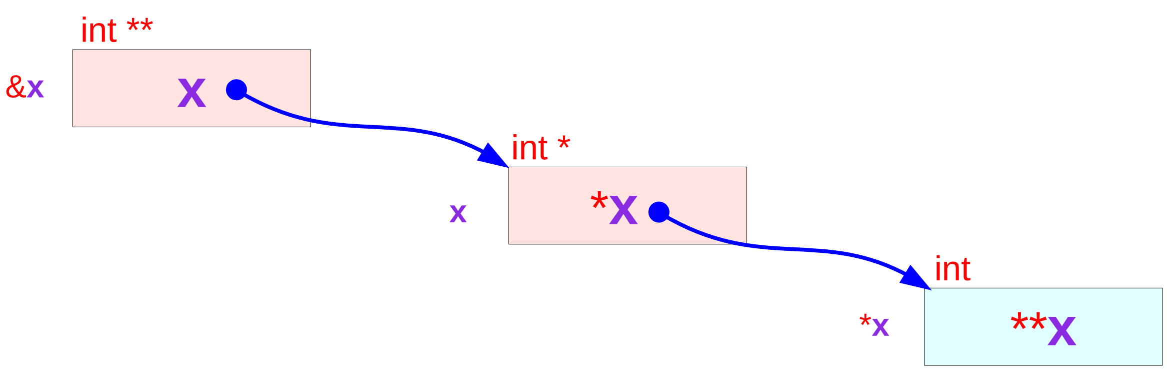
*x[m] ≡ *(x[m])



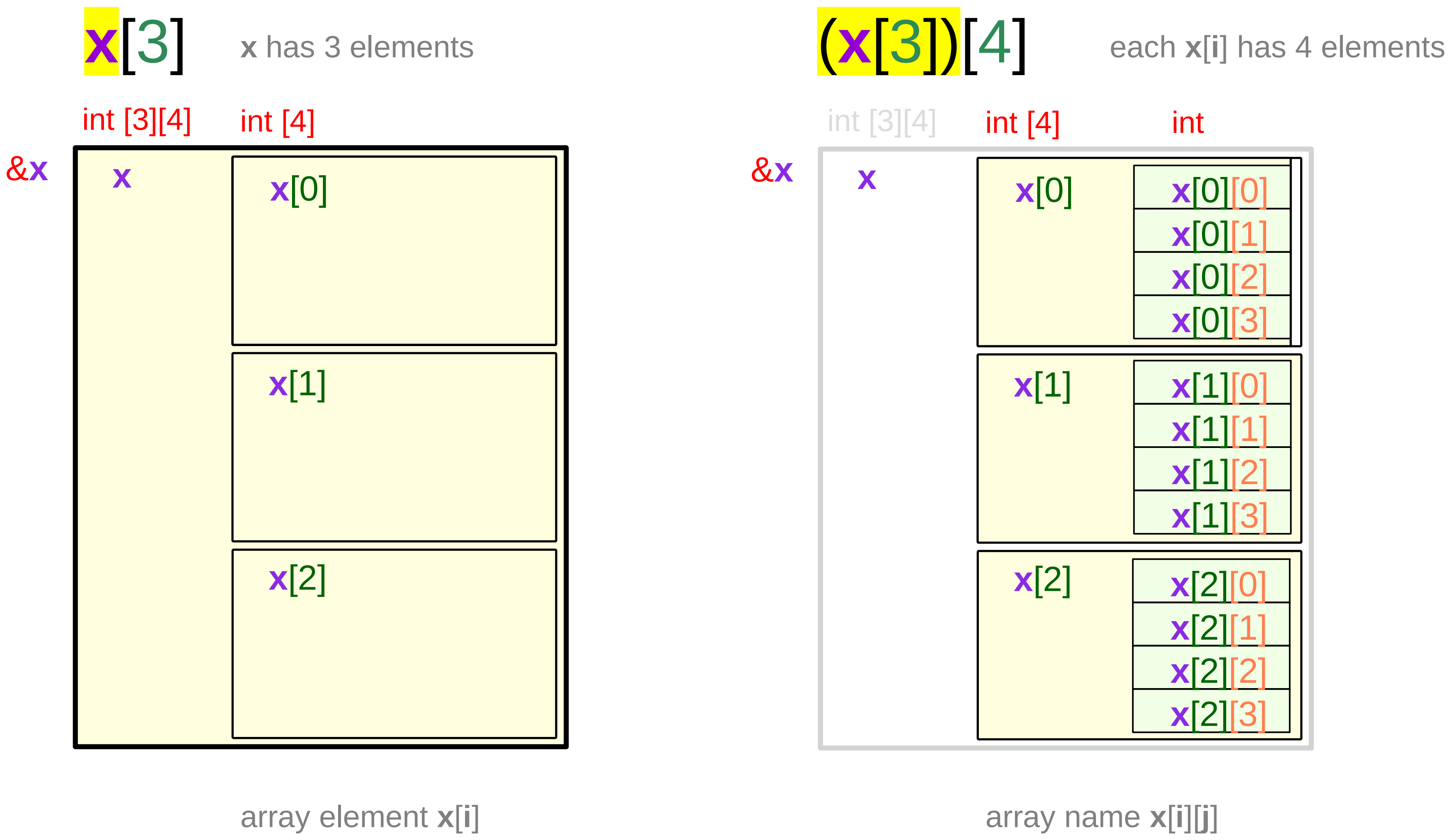

x[m][n] ≡ (x[m])[n]



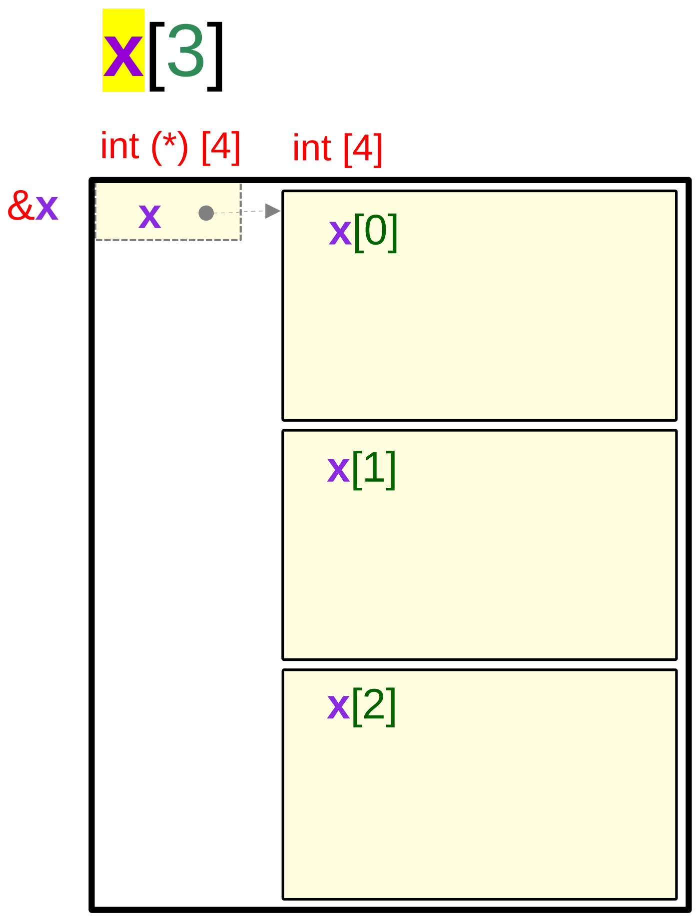

**x ≡ *(*x)



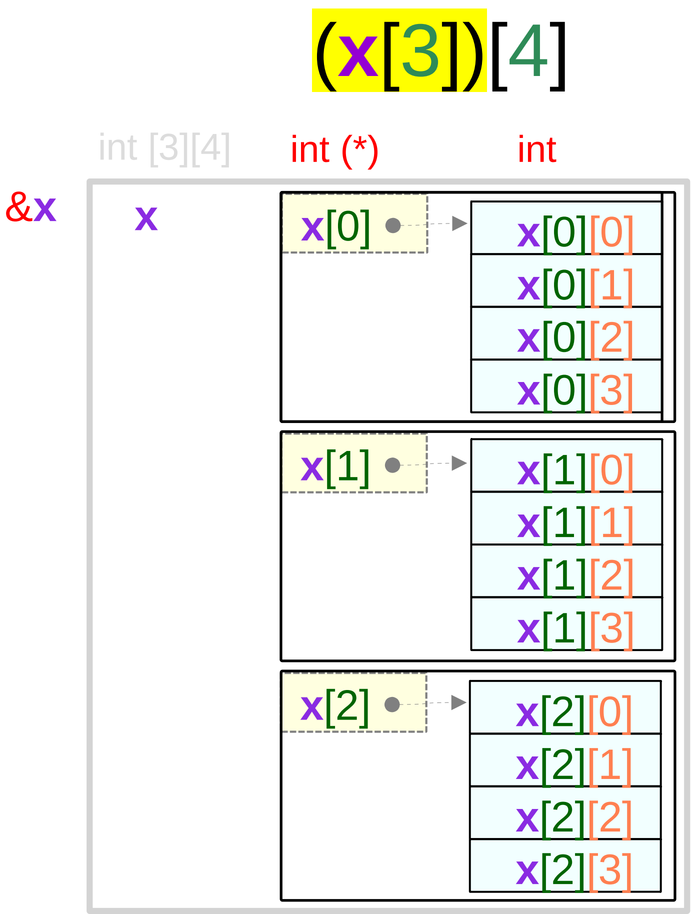
# Abstract Data **x** and **x**[**i**]



# Virtual Pointers x and x[i]



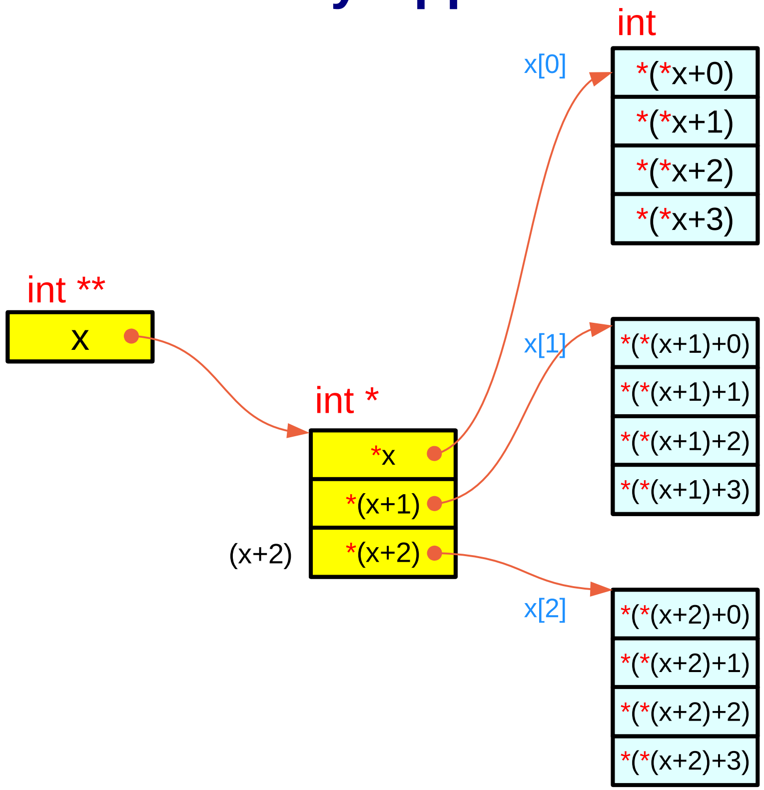

array name **x** ………...virtual pointer
array element **x**[**i**] …... abstract data




array name **x**[**i**] ……… virtual pointer
array element **x**[**i**][**j**] …. primitive data

# Virtual Pointers **x** and **x[i]**

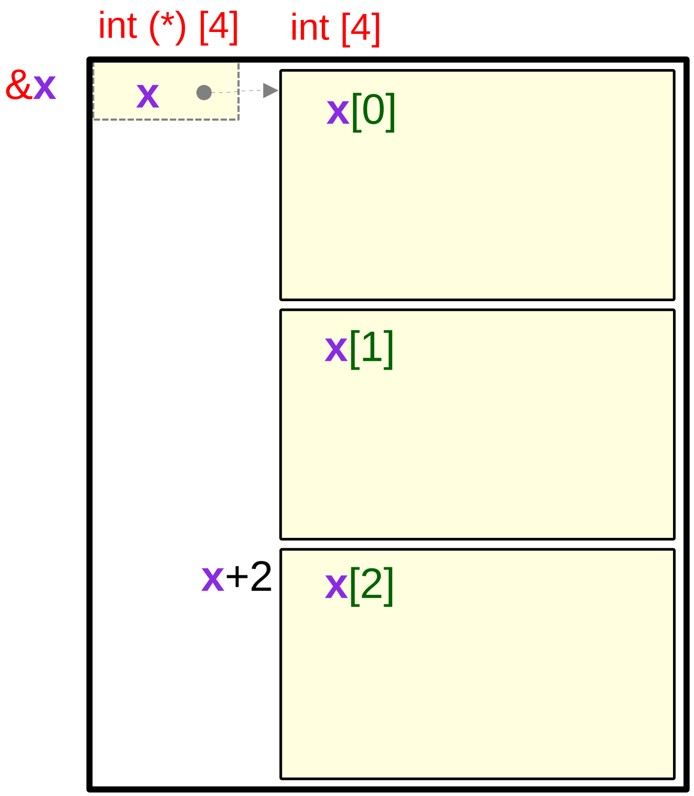
## Pointer Array Approach




value(**X** + **i**)

= value(**X**) $+_a$ **i** * sizeof(***X**)

## Array Pointer Approach

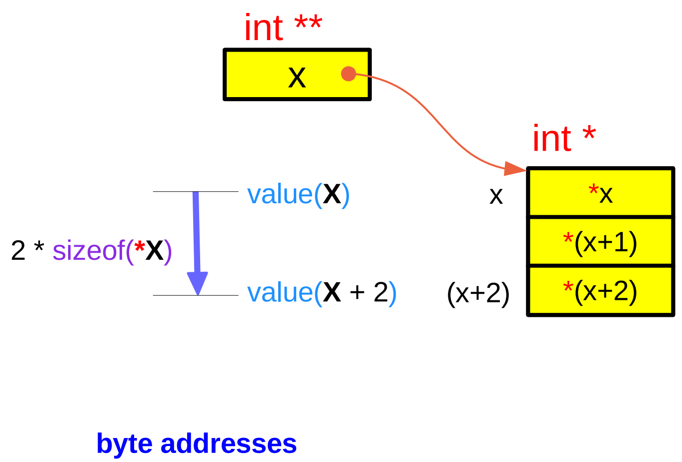


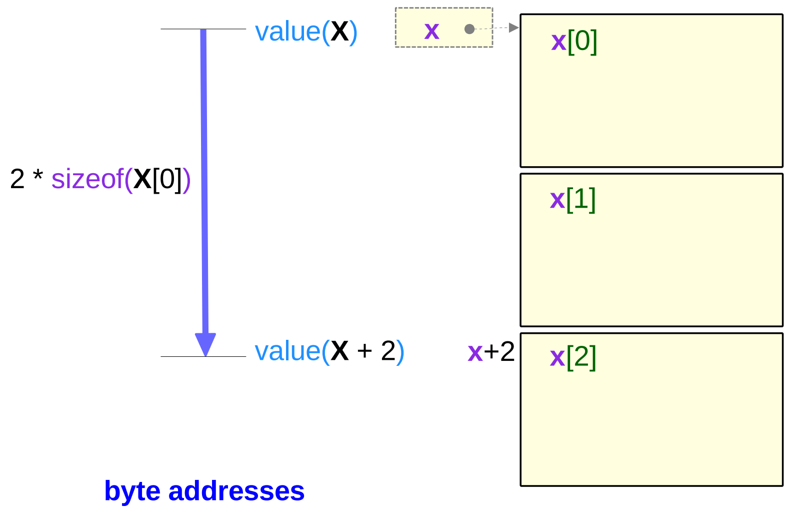

value(**X** + **i**)

= value(**X**) $+_a$ **i** * sizeof(**X**[0])

# Base and offset in byte addresses

| pointer variable **X** | array variable **X** |
| --- | --- |
| value(**X** + **i**) — pointer addition + | value(**X** + **i**) — pointer addition + |
| = value(**X**) $+_a$ **i** * sizeof(*__X__) — arithmetic addition $+_a$ | = value(**X**) $+_a$ **i** * sizeof(**X**[0]) — arithmetic addition $+_a$ |



# Base and offset in byte addresses

| pointer variable **X** | |
|---|---|
| value(**X** + **i**) | pointer addition + |
| = value(**X**) $+_a$ **i** * sizeof(*__X__) | arithmetic addition $+_a$ |
| = value(**X**) + **i** * sizeof(*__X__) | in math expression |

| array variable **X** | |
|---|---|
| value(**X** + **i**) | pointer addition + |
| = value(**X**) $+_a$ **i** * sizeof(**X**[0]) | arithmetic addition $+_a$ |
| = value(**X**) + **i** * sizeof(**X**[0]) | in math expression |

arithmetic addition $+_a$ notation
can be replaced with the ordinary + notation
when there is no possible misunderstanding
e.g. in math expressions

# Subscript [ ] and dereference * notations (1a)

**pointer variable X**

value(*(**X** + **i**) + **j**)

= *value(**X** + **i**) $+_a$ **j** * sizeof(****X**)

= *value(**X**) $+_a$ **i** * sizeof(***X**) $+_a$ **j** * sizeof(****X**)

= *value(**X**) + **i** * sizeof(***X**) + **j** * sizeof(****X**)

in math expressions

**array variable X**

value(**X**[**i**] + **j**)

= value(**X**[**i**]) $+_a$ **j** *sizeof(**X**[**i**][**j**])

= *value(**X**) $+_a$ **i** *sizeof(**X**[0]) $+_a$ **j***sizeof(**X**[0][0])

= value(**X**) $+_a$ **i** *sizeof(**X**[0]) $+_a$ **j** *sizeof(**X**[0][0])

= value(**X**) + **i** * sizeof(**X**[0]) + **j** * sizeof(**X**[0][0])

in math expressions

# Subscript [ ] and dereference * notations (1a)

$value(X + i) \neq value(X) + i$

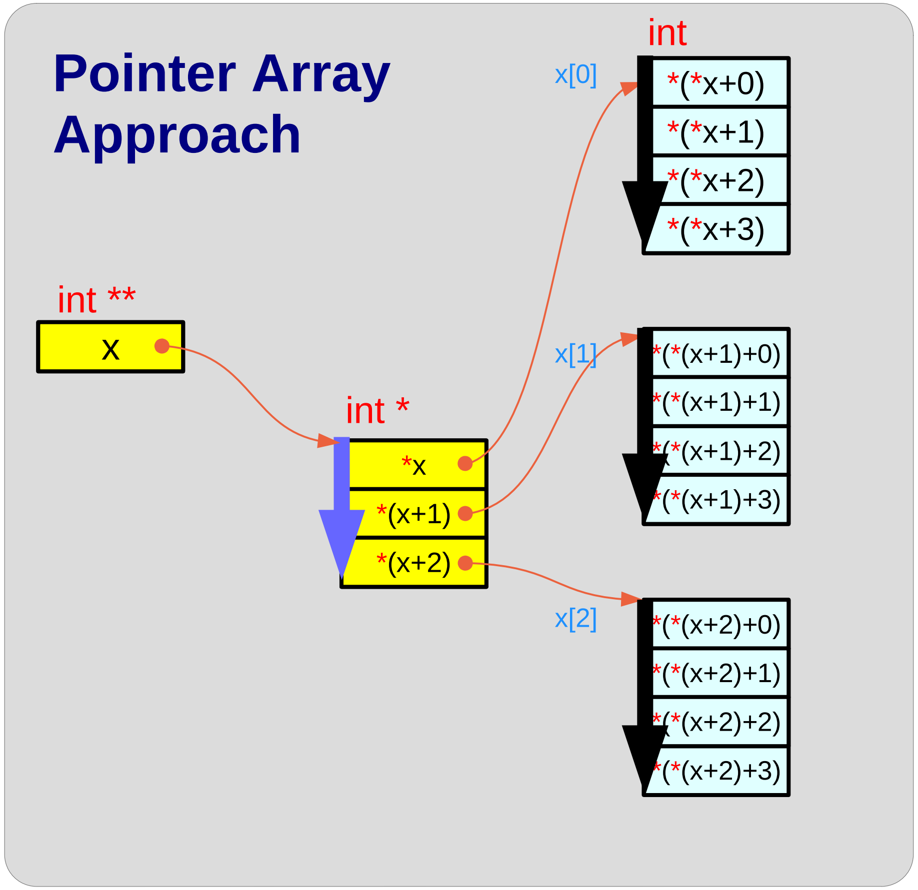
$value((X + i) + j) \neq value(X) + i + j$

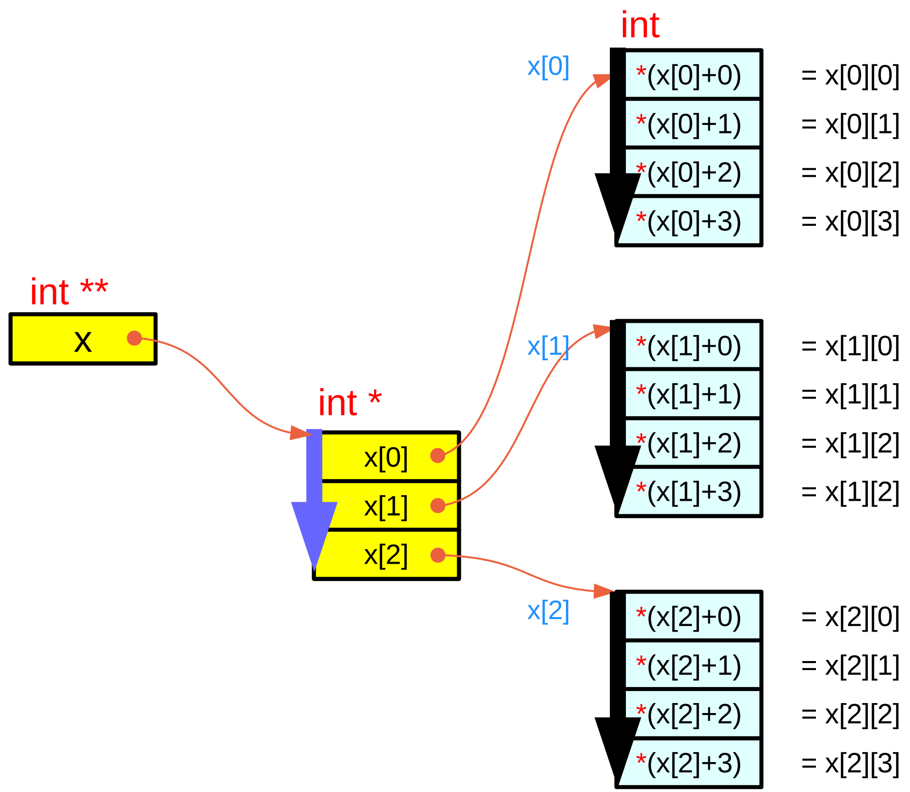
$value(value(X + i)) = value(X) + i * sizeof(*X)$

$value(X + i) = value(X) + i * sizeof(*X)$

$value((X + i) + j) = value(X + i) + j * sizeof(**X)$
$= value(X) + i * sizeof(*X) + j * sizeof(**X)$

# * into [ ] notations – Pointer Array Approach

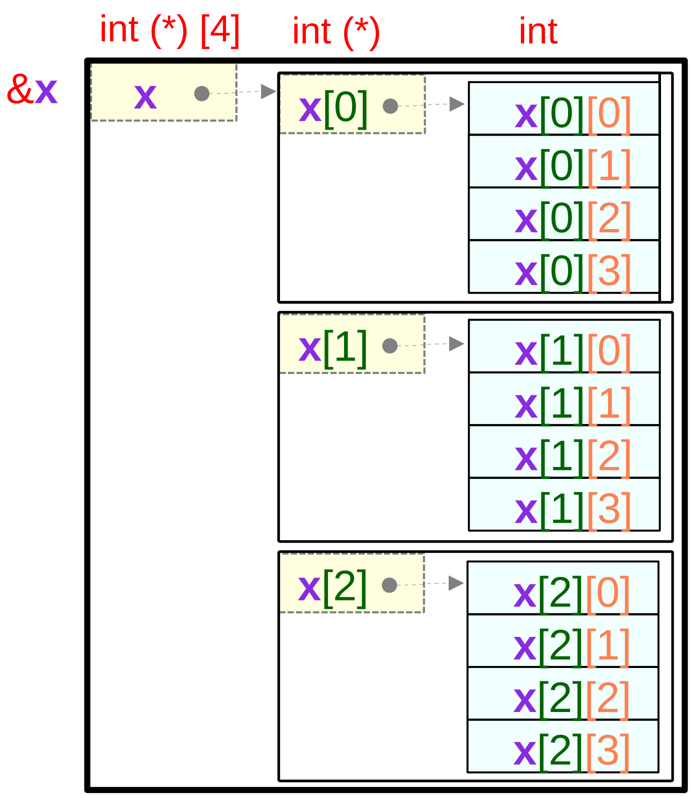


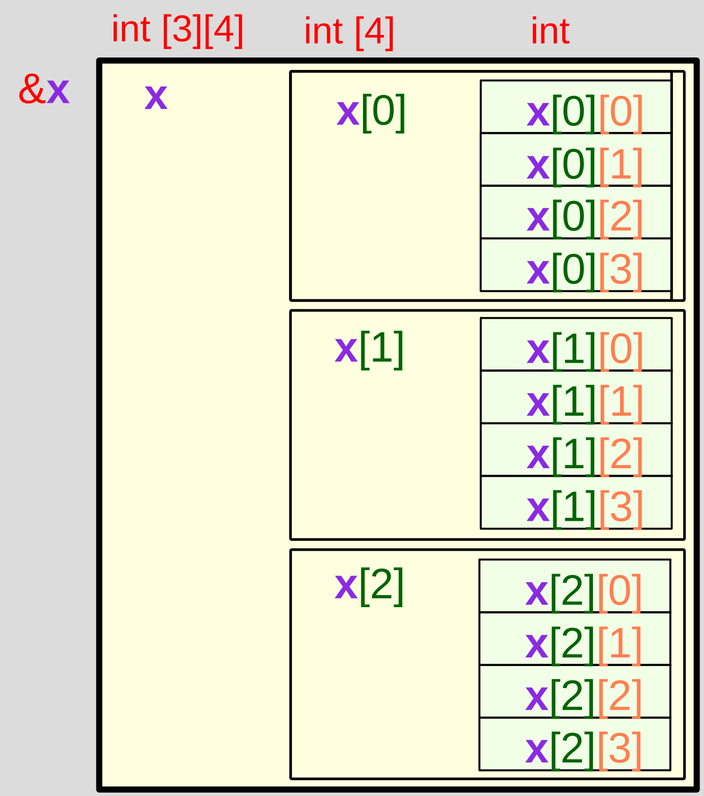




C expression $*(*(x+i)+j) \rightarrow x[i][j]$

Math expression $*(*(x+i)_{1\cdot 4}+j)_{1\cdot 4}$

# * and [ ] notations – Array Pointer Approach



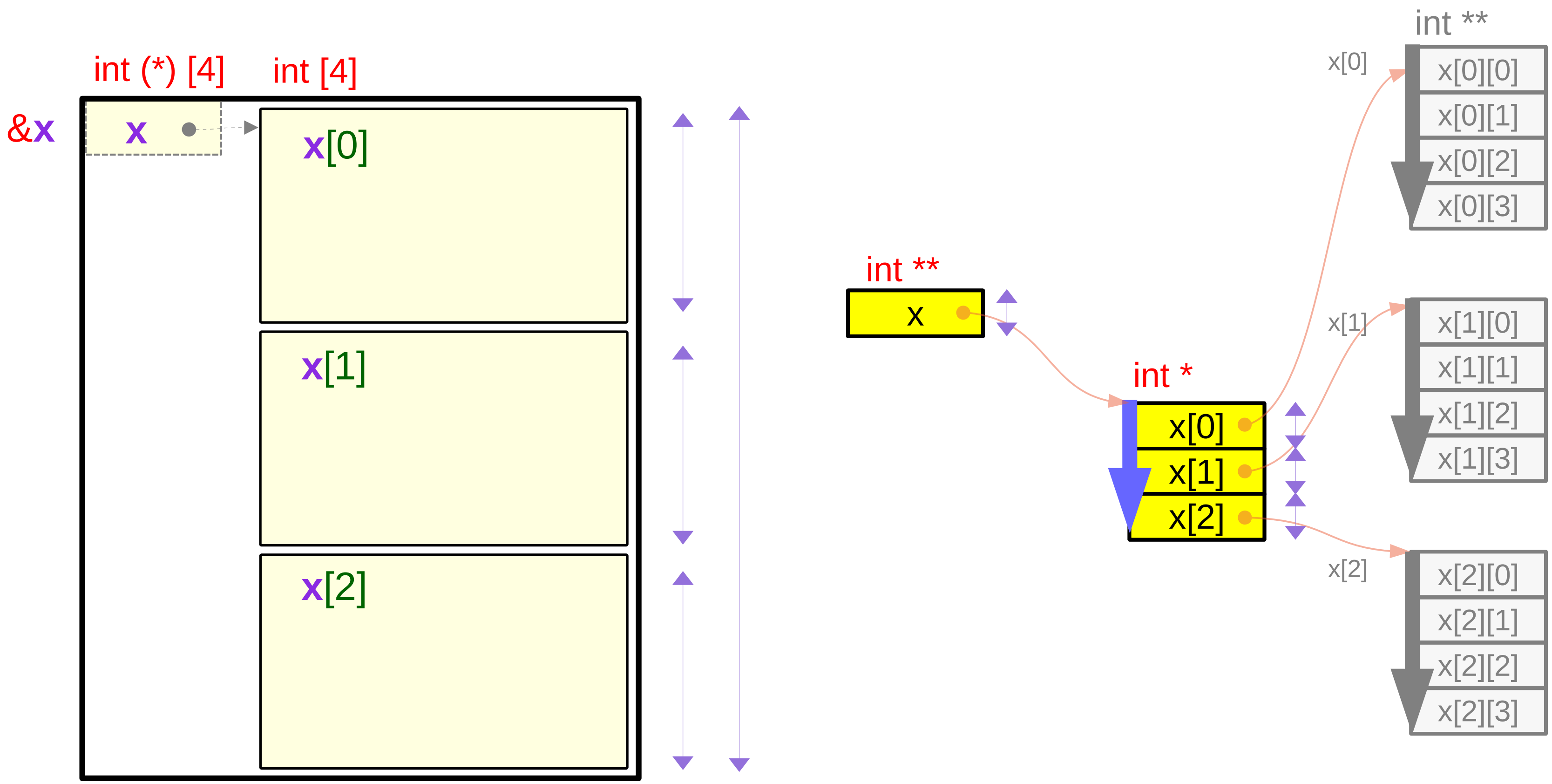




**Array Pointer Approach**

C expression: `*(*(x+i)+j)` ← `x[i][j]`

Math expression: $*(*(x+i)_{4\cdot 4}+j)_{1\cdot 4}$

# Virtual pointers vs. real pointers (1)




value(&x) = value(x)

sizeof(x) = 3 * sizeof(*x)

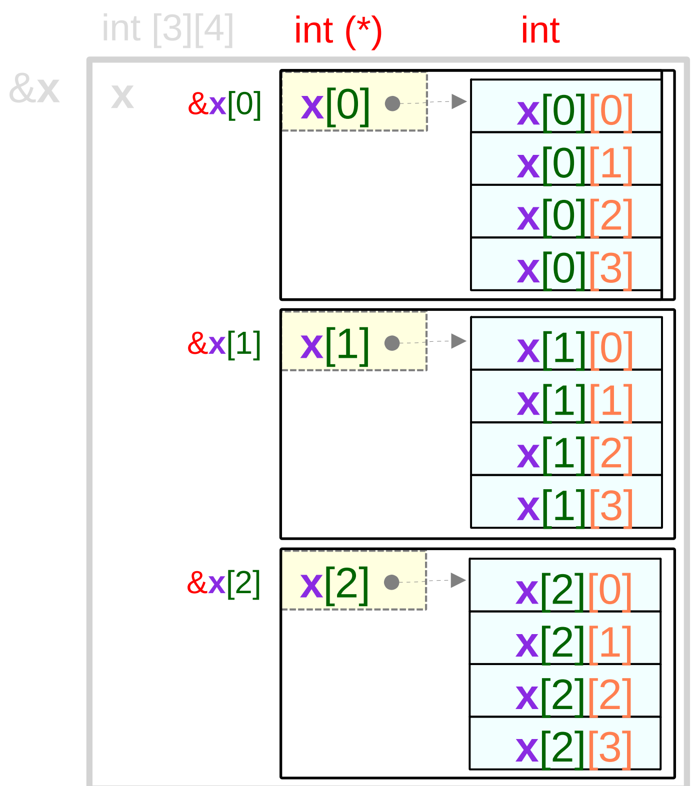
value(x+i) = value(x) + i * sizeof(*x)
= value(x) + i * 4 *4

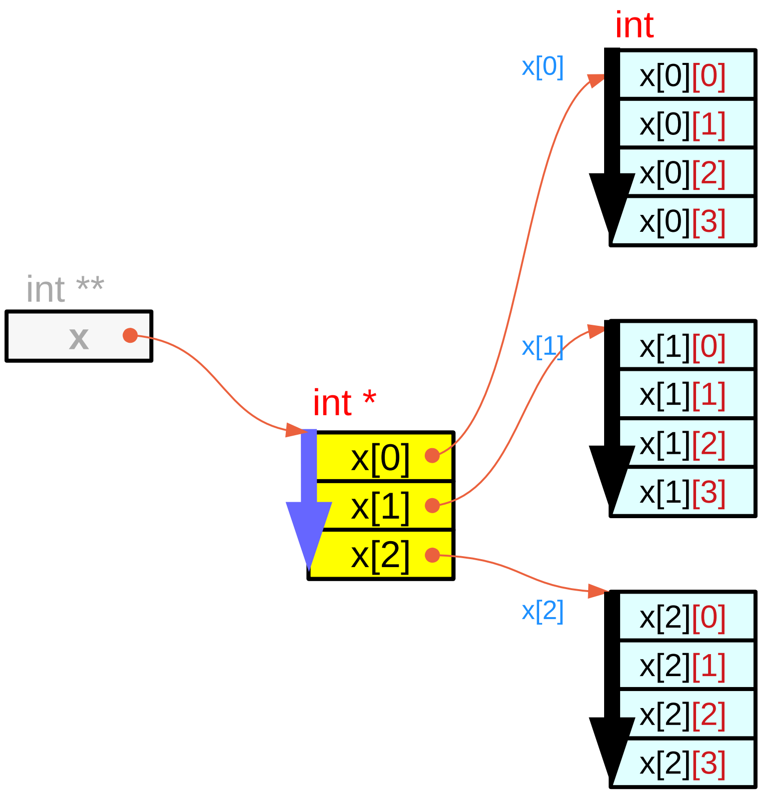
value(&x) ≠ value(x)

sizeof(x) = sizeof(*x) = 4

value(x+i) = value(x) + i * sizeof(*x)
= value(x) + i * 4

# Virtual pointers vs. real pointers (2)




value(&**x**[**i**]) = value(**x**[**i**])

sizeof(**x**[**i**]) = 4 * sizeof(***x**[**i**])

value(**x**[**i**]+**j**) = value(**x**[**i**]) + **j** * sizeof(***x**[**i**])
= value(**x**[**i**]) + **j** * 4

value(&**x**[**i**]) ≠ value(**x**[**i**])

sizeof(**x**[**i**]) = sizeof(***x**[**i**]) = 4

value(**x**[**i**]+**j**) = value(**x**[**i**]) + **j** * sizeof(***x**[**i**])
= value(**x**[**i**]) + **j** * 4

# Left-to-right and right-to-left associative operators

| | | | | |
|---|---|---|---|---|
| p[i] | ≡ | p[i] | ⇒ | *(p+i) |
| p[i][j] | ≡ | (p[i])[j] | ⇒ | *(*(p+i)+j) |
| p[i][j][k] | ≡ | ((p[i])[j])[k] | ⇒ | *(*(*(p+i)+j)+k) |

| | | | | |
|---|---|---|---|---|
| *p | ≡ | *(p) | ⇒ | p[0] |
| **p | ≡ | *(*(p)) | ⇒ | (p[0])[0] |
| ***p | ≡ | *(*(*(p))) | ⇒ | ((p[0])[0])[0] |

# Relaxing the outermost dimension

| | | |
|---|---|---|
| p[i] | ≡ | *(p+i) |
| p[i][j] | ≡ | *(p[i]+j) |
| p[i][j][k] | ≡ | *(p[i][j]+k) |

| | | |
|---|---|---|
| &p[i] | ≡ | value(p+i) |
| &p[i][j] | ≡ | value(p[i]+j) |
| &p[i][j][k] | ≡ | value(p[i][j]+k) |

| | | |
|---|---|---|
| p[0] | ≡ | *p |
| p[i][0] | ≡ | *p[i] |
| p[i][j][0] | ≡ | *p[i][j] |

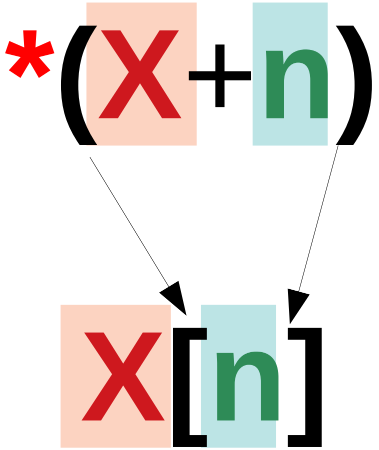
| | | |
|---|---|---|
| &p[0] | ≡ | value(p) |
| &p[i][0] | ≡ | value(p[i]) |
| &p[i][j][0] | ≡ | value(p[i][j]) |

valid for proper **i**, **j**, **k** values

# Relaxing all the dimensions

| | | | | |
|---|---|---|---|---|
| p[i] | ≡ | *(p+i) | ➡ | *(p+i) |
| p[i][j] | ≡ | *(p[i]+j) | ➡ | *(*(p+i)+j) |
| p[i][j][k] | ≡ | *(p[i][j]+k) | ➡ | *(*(*(p+i)+j)+k) |

*(X+n) ≡ X[n]




valid for proper **i**, **j**, **k** values

# Equivalences on relaxing all the dimensions

| | | |
|---|---|---|
| p[i] | ≡ | *(p+i) |
| p[i][j] | ≡ | *(*(p+i)+j) |
| p[i][j][k] | ≡ | *(*(*(p+i)+j)+k) |

| | | |
|---|---|---|
| &p[i] | ≡ | value(p+i) |
| &p[i][j] | ≡ | value(*(p+i)+j) |
| &p[i][j][k] | ≡ | value(*(*(p+i)+j)+k) |

*value(X) = *X

| | | |
|---|---|---|
| p[0] | ≡ | *p |
| p[0][0] | ≡ | **p |
| p[0][0][0] | ≡ | ***p |

| | | |
|---|---|---|
| &p[0] | ≡ | value(p) |
| &p[0][0] | ≡ | value(*p) |
| &p[0][0][0] | ≡ | value(**p) |

valid for proper **i**, **j**, **k** values

# Address Calculation (1) Array Pointer Approach

int c [2][3][4] ;

c[i] ≡ *(c + i)
c[i][j] ≡ *(c[i] + j)
c[i][j][k] ≡ *(c[i][j] + k)

&c[i] ≡ value(c + i)
&c[i][j] ≡ value(c[i] + j)
&c[i][j][k] ≡ value(c[i][j] + k)

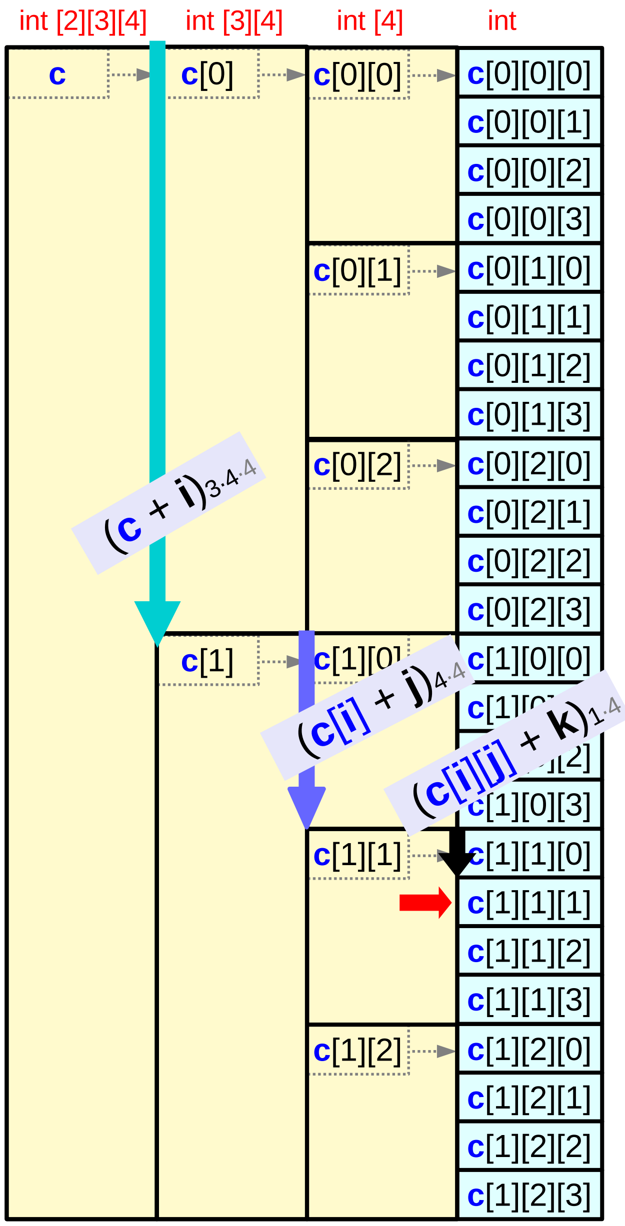
**address replication**

value(c[i][j][k]) ≠ value(&c[i][j][k]) ← primitive data & address

value(c[i][j]) = value(&c[i][j]) = value(&c[i][j][0])
value(c[i]) = value(&c[i]) = value(&c[i][0][0])
value(c) = value(&c) = value(&c[0][0][0])

skip **i** elements of **c**
skip **j** elements of **c[i]**
skip **k** elements of **c[i][j]**

skip **i*3*4** primitive elements of **c**
skip **j*4** primitive elements of **c**
skip **k** primitive elements of **c**



# Address Calculation (2) Pointer Array Approach

```
int ** c [2] ;
int *  b [2*3] ;
int    a [2*3*4] ;
```

c[i] = &b[3*i]  (= b + 3*i)
b[j] = &a[4*j]  (= a + 4*j)

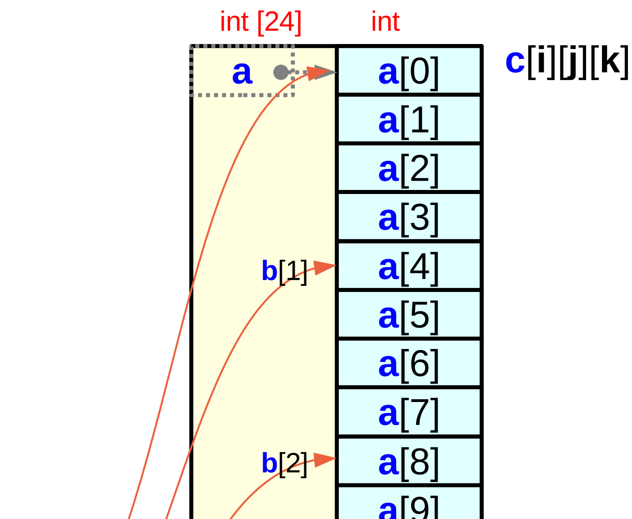
b[j]  ≡ (a+j*4)
*(b[j]+k)  = *(a+j*4+k);
b[j][k]  ≡ a[j*4+k]

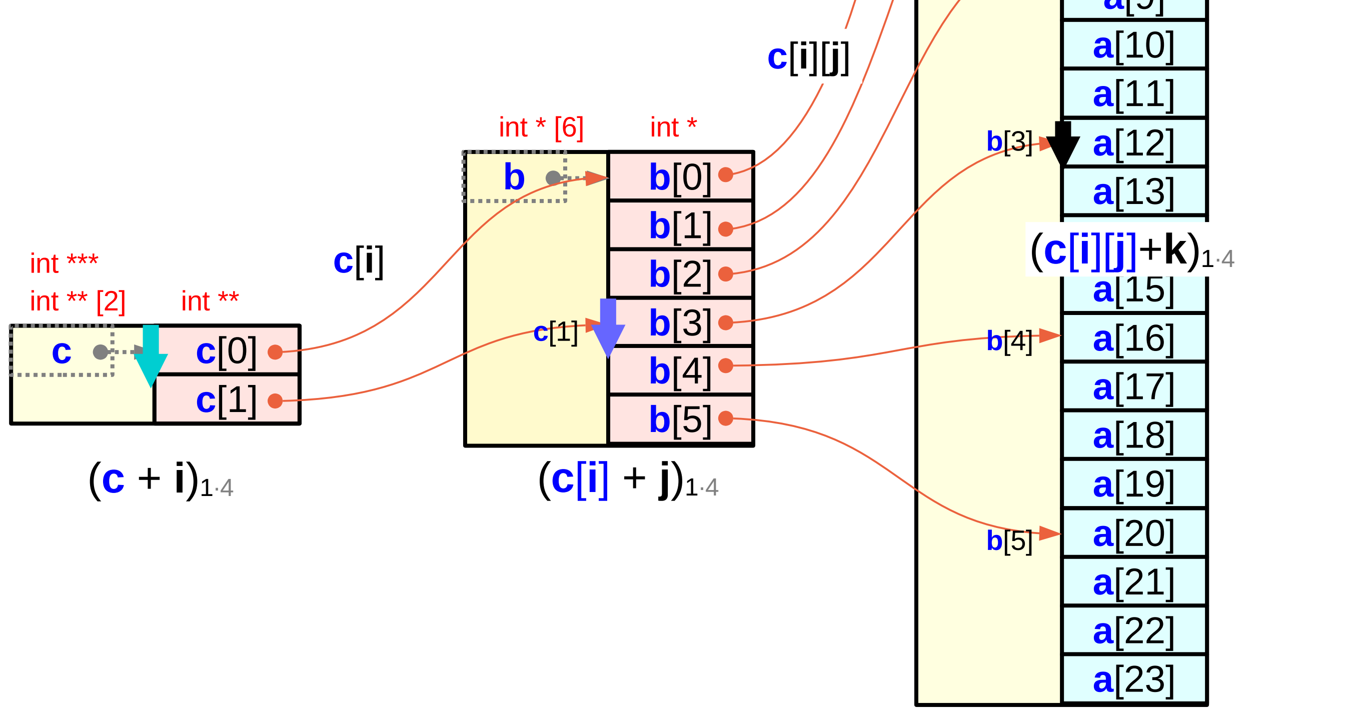
c[i]  ≡ (b+i*3)
*(c[i]+j)  = *(b+i*3+j);
c[i][j]  ≡ b[i*3+j]

c[i][j]  ≡ (a+(i*3+j)*4)
*(c[i][j]+k)  = *(a+(i*3+j)*4+k);
c[i][j][j]  ≡ a[(i*3+j)*4+k]

skip **i** elements of **c**
skip **j** elements of **b**
skip **k** elements of **a**

skip **i*3*4** elements of **a**
skip **j*4** elements of **a**
skip **k** elements of **a**



# Address Calculation (3)

value(c + i) = value(c) + i * 3 * 4 * 4
value(c[i] + j) = value(c[i]) + j * 4 * 4
value(c[i][j] + k) = value(c[i][j]) + k * 4

value(c + i) = value(c) + i * sizeof(*c)
value(c[i] + j) = value(c[i]) + j * sizeof(*c[i])
value(c[i][j] + k) = value(c[i][j]) + k * sizeof(*c[i][j])

c[i] ≡ *(c + i)
c[i][j] ≡ *(c[i] + j)
c[i][j][k] ≡ *(c[i][j] + k)

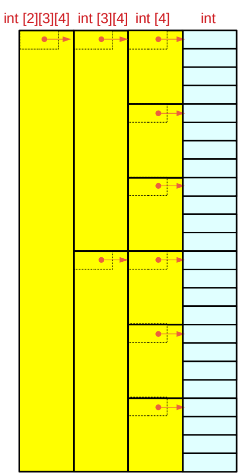
&c[i] ≡ value(c + i)
&c[i][j] ≡ value(c[i] + j)
&c[i][j][k] ≡ value(c[i][j] + k)

**Array Pointer Approach**

$(c + i)_{3 \cdot 4 \cdot 4}$

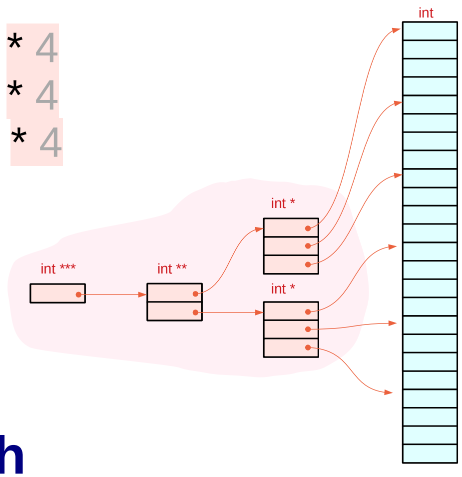
$(c[i] + j)_{4 \cdot 4}$

$(c[i][j] + k)_{1 \cdot 4}$




value(c + i) = value(c) + i * 4
value(c[i] + j) = value(c[i]) + j * 4
value(c[i][j] + k) = value(c[i][j]) + k * 4

$(c + i)_{1 \cdot 4}$ $(c[i] + j)_{1 \cdot 4}$ $(c[i][j] + k)_{1 \cdot 4}$




**Pointer Array Approach**

# Subscript [ ] and dereference * notations (1a)

p[i] ≡ *(p+i)

from p, skip i·M·N integers

&p[i] = value((p + i)$_{M \cdot N \cdot 4}$)
= value(p + i * M·N·4)

p[i][j] ≡ *(*(p+i)+j)

from p[i], skip j·N integers

&p[i][j] = value((p[i] + j)$_{N \cdot 4}$)
= value(p[i] + j * N·4)

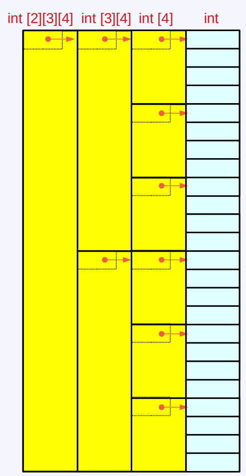
p[i][j][k] ≡ *(*(*(p+i)+j)+k)

from p[i][j], skip k integers

&p[i][j][k] = value((p[i][j]+k)$_{1 \cdot 4}$)
= value(p[i][j] + k * 1·4)

int p[L][M][N]

**Array Pointer Approach**




***address replications***

value(p[i]) = &p[i] = value(p + i)

value(p[i][j]) = &p[i][j] = value(p[i] + j)

value(p[i][j][k]) ≠ &p[i][j][k] = value(p[i][j] + k)

&p[i][j][k] = value(p + i *M·N·4 + j *N·4 + k * 4)

p[i][j][k] = *value(p + i *M·N·4 + j *N·4 + k * 4)

# Subscript [ ] and dereference * notations (1b)

| | | |
|---|---|---|
| p[i] | ≡ | *(p+i) |
| p[i][j] | ≡ | *(*(p+i)+j) |
| p[i][j][k] | ≡ | *(*(*(p+i)+j)+k) |

skip **i** pointers from **p**

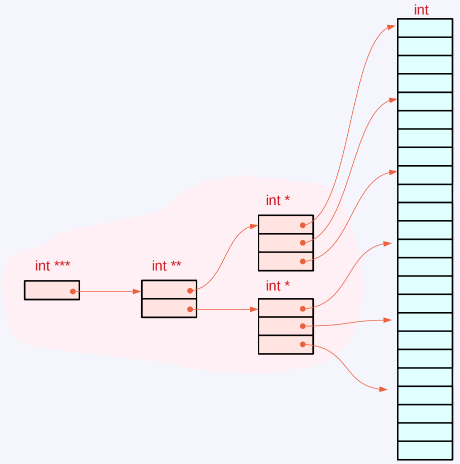
&**p**[**i**] = value(**p** + **i**)$_{1\cdot 4}$
= value(**p** + **i** *4)

skip **j** pointers from **p[i]**

&**p**[**i**][**j**] = value(**p**[**i**] + **j**)$_{1\cdot 4}$
= value(**p**[**i**] + **j** *4)

skip **k** integers from **p[i][j]**

&**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**] + **k**)$_{1\cdot 4}$
= value(**p**[**i**][**j**] + **k** *4)




int ** p [L] ;
int * q [L·M] ;
int r [L·M·N] ;

**Pointer Array Approach**

***address dereferences***

value(**p**[**i**]) = *(&**p**[**i**]) = *value(**p** + **i**)

value(**p**[**i**][**j**]) = *(&**p**[**i**][**j**]) = *value(**p**[**i**] + **j**)

value(**p**[**i**][**j**][**k**]) = *(&**p**[**i**][**j**][**k**]) = *value(**p**[**i**][**j**] + **k**)

&**p**[**i**][**j**][**k**] = value(*value(*value(**p**+**i***4)+**j***4)+**k***4)

# Subscript [ ] and dereference * notations (1a)

$p[i][j][k] = *value(*value(*value(p+i)+j)+k)$ → *address replications*

$= value(value(value(p + i)_{3\cdot4\cdot4}) + j)_{4\cdot4} + k)_4$

$= value(value(value(p) + i *3\cdot4\cdot4) + j *4\cdot4) + k *4)$

$value(X + i) = value(X) + i * sizeof(*X)$

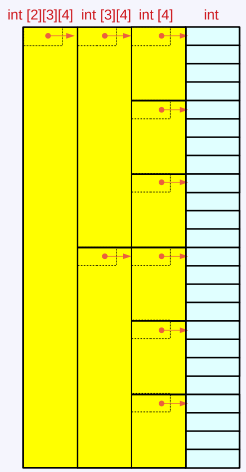
$value((X + i) + j) = value(X + i) + j * sizeof(**X)$

$= value(X) + i * sizeof(*X) + j * sizeof(**X)$

# Subscript [ ] and dereference * notations (1a)

int p[L][M][N]

**Array Pointer Approach**




***address replications***

value(p[i]) = &p[i] = value(p + i)

value(p[i][j]) = &p[i][j] = value(p[i] + j)

value(p[i][j][k]) ≠ &p[i][j][k] = value(p[i][j] + k)

&p[i][j][k] = value(p + i *M·N·4 + j *N·4 + k * 4)

p[i][j][k] = *value(p + i *M·N·4 + j *N·4 + k * 4)

| | | | | | | |
|---|---|---|---|---|---|---|
| abstract data | int [3][4] | value(p[i]) | = &p[i] | = value(p + i) | int (*)[3][4] | virtual pointer |
| abstract data | int [4] | value(p[i][j]) | = &p[i][j] | = value(p[i] + j) | int (*)[4] | virtual pointer |
| primitive data | int | value(p[i][j][k]) | ≠ &p[i][j][k] | = value(p[i][j] + k) | int (*) | virtual pointer |

p[i][j][k] = *value(*value(*value(p+i)+j)+k) → *address replications*

= value(value(value(p + i)$_{3·4·4}$) + j)$_{4·4}$ + k)$_{4}$

= value(p + i *3·4·4 + j *4·4 + k *4)

# Subscript [ ] and dereference * notations (2)

| | | |
|---|---|---|
| p[i] | ≡ | *(p+i) |
| p[i][j] | ≡ | *(*(p+i)+j) |
| p[i][j][k] | ≡ | *(*(*(p+i)+j)+k) |

C Expressions

| | | |
|---|---|---|
| &p[i] | ≡ | value(p+i) |
| &p[i][j] | ≡ | value(*(p+i)+j) |
| &p[i][j][k] | ≡ | value(*(*(p+i)+j)+k) |

C Expressions

int p [L][M][N] ;

value(&X) = value(X) (address replication)

| | | |
|---|---|---|
| p[i] | → | $*(p+i)_{M\cdot N\cdot 4}$ |
| p[i][j] | → | $*(*(p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}$ |
| p[i][j][k] | → | $*(*(*(p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}+k)_{1\cdot 4}$ |

Math Expressions

| | | |
|---|---|---|
| &p[i] | → | $\text{value}(p+i)_{M\cdot N\cdot 4}$ |
| &p[i][j] | → | $\text{value}((p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}$ |
| &p[i][j][k] | → | $\text{value}(((p+i)_{M\cdot N\cdot 4}+j)_{N\cdot 4}+k)_{1\cdot 4}$ |

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

| | | |
|---|---|---|
| p[i] | → | $*(p+i)_{1\cdot 4}$ |
| p[i][j] | → | $*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}$ |
| p[i][j][k] | → | $*(*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}+k)_{1\cdot 4}$ |

Math Expressions

| | | |
|---|---|---|
| &p[i] | → | $\text{value}(p+i)_{1\cdot 4}$ |
| &p[i][j] | → | $\text{value}(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}$ |
| &p[i][j][k] | → | $\text{value}(*(*(p+i)_{1\cdot 4}+j)_{1\cdot 4}+k)_{1\cdot 4}$ |

Math Expressions

# Subscript [ ] and dereference * notations (3)

int p [L][M][N] ;

value(&X) = value(X) (address replication)

&p[i] = value((p + i)$_{M \cdot N \cdot 4}$) = value(p + i * M·N·4)

&p[i][j] = value((p[i] + j)$_{N \cdot 4}$) = value(p[i] + j * N·4)

&p[i][j][k] = value((p[i][j]+k)$_{1 \cdot 4}$) = value(p[i][j] + k * 1·4)

= value(p + i * M·N·4 + j * N·4 + k * 4)

&p[i] → value(p+i)$_{M \cdot N \cdot 4}$

&p[i][j] → value((p+i)$_{M \cdot N \cdot 4}$+j)$_{N \cdot 4}$

&p[i][j][k] → value(((p+i)$_{M \cdot N \cdot 4}$+j)$_{N \cdot 4}$+k)$_{1 \cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

&p[i] = value(p + I)$_{1 \cdot 4}$ = value(p + i * 1·4)

*

&p[i][j] = value(p[i] + j)$_{1 \cdot 4}$ = value(p[i] + j * 1·4)

*

&p[i][j][k] = value(p[i][j] + k)$_{1 \cdot 4}$ = value(p[i][j] + k * 1·4)

= value(*value(*value(p +i*4) +j*4) +k*4)

&p[i] → value(p+i)$_{1 \cdot 4}$

&p[i][j] → value(*(p+i)$_{1 \cdot 4}$+j)$_{1 \cdot 4}$

&p[i][j][k] → value(*(*(p+i)$_{1 \cdot 4}$+j)$_{1 \cdot 4}$+k)$_{1 \cdot 4}$

Math Expressions

# Operator Precedence

| Precedence | Operator | Description | Associativity |
|---|---|---|---|
| 1 | ++ -- | Suffix/postfix increment and decrement | Left-to-right |
| | () | Function call | |
| | [] | Array subscripting | (((x[m])[n])[p]) |
| | . | Structure and union member access | → |
| | -> | member access through pointer | |
| | (type){list} | Compound literal(C99) | |
| 2 | ++ -- | Prefix increment and decrement | Right-to-left |
| | + - | Unary plus and minus | |
| | ! ~ | Logical NOT and bitwise NOT | |
| | (type) | Type cast | |
| | * | Indirection (dereference) | *(*(*(*x))) |
| | & | Address-of | ← |
| | sizeof | Size-of | |
| | _Alignof | Alignment requirement(C11) | |
| | | | |

https://en.cppreference.com/w/c/language/operator_precedence

# Limitations

No index Range Checking

Array Size must be a constant expression

- Variable Array Size

Arrays cannot be Copied or Compared

Aggregate Initialization and Global Arrays

Precedence Rule

Index Type Must be Integral

# References

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[5] https://pdos.csail.mit.edu/6.828/2008/readings/pointers.pdf