Functor (1A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Typeclasses and Instances

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

such *behavior* is defined by

- **function definition**
- function **type declaration** only

a function definition

```
(==) :: a -> a -> Bool
x == y = not (x /= y)
```

- **a type declaration**

a function type

```
(==) :: a -> a -> Bool
```

- **a type declaration**

A function definition can be **overloaded**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Typeclasses and Type

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

a **type** is an **instance** of a **typeclass** implies

the function types declared by the **typeclass**
are defined (implemented) in the **instance**

so that we can use the functions
that the **typeclass** defines with that **type**

No relation with classes in Java or C++

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

A Concrete Type and a Type Constructor

a : a concrete type

Maybe : not a concrete type
: a **type constructor** that takes one parameter
in order to produces a concrete type.

Maybe a : a concrete type

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe

Maybe : Algebraic Data Type (ADT)

Widely used because it effectively extends a type Integer
into a new **context** in which it has an extra value (**Nothing**)
that represents *a lack of value*

can check for that extra value *before accessing* the possible Integer

good for debugging

Many other languages have this sort of "***no-value***" value via NULL references.

The Haskel **Maybe** type handle this ***no-value*** more effectively.

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

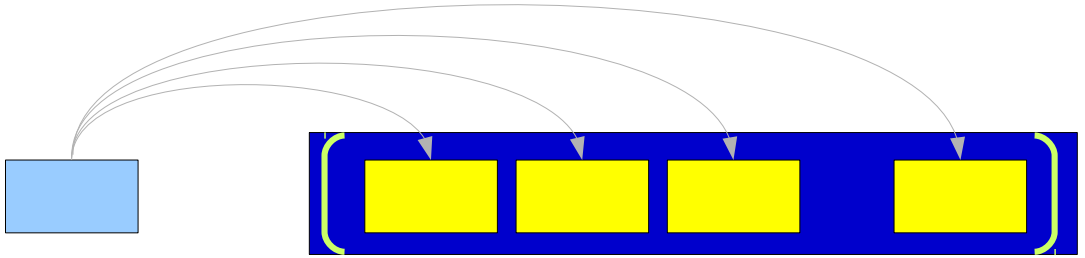
Functor typeclass – “mapped over”

the **Functor typeclass** is basically

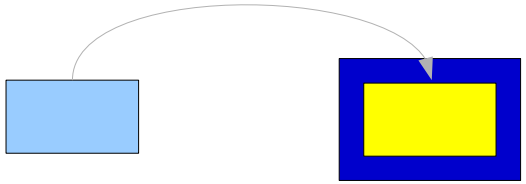
for things that can be *mapped over*

ex) mapping over **list**s

the **list** type is a Functor typeclass



List Functor



Maybe Functor

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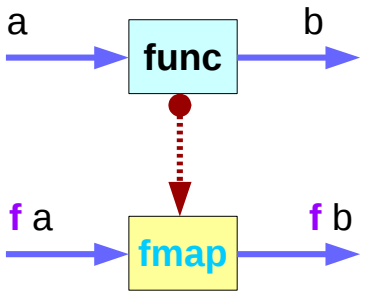
Functor typeclass – instances

```
class Functor f

instance Functor Maybe
instance Functor [ ]
```




```
function fmap
function func
type constructor  f
```

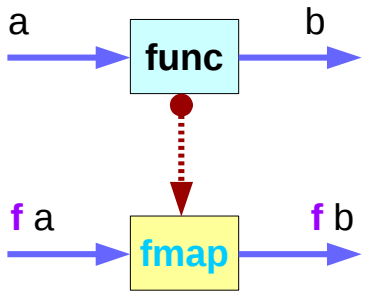
f is a **type constructor** taking <u>one</u> type parameter

Maybe instance of the **Functor typeclass**
[ ] instance of the **Functor typeclass**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Functor typeclass – fmap defined

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```



The **Functor typeclass**

defines the function **fmap**

without a default implementation

function fmap
function func
type constructor f

the **type variable f**

f is not a concrete type (**f** alone cannot hold a **value**)

f is a **type constructor** taking one type parameter

Maybe Int : a concrete type (a concrete type can hold a **value**)

Maybe : a **type constructor** that takes one type as the parameter

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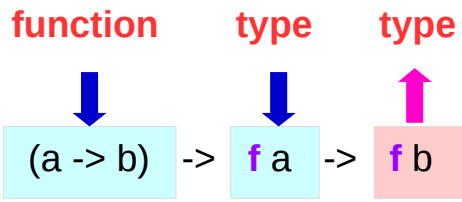
Function map & fmap

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```



func

fmap takes

- a **function** from **one type** to **another** (a -> b)
- a **Functor f** applied with **one type** (**f** a)

fmap returns

- a **Functor f** applied with **another type** (**f** b)

```
map :: (a -> b) -> [a] -> [b]
```

map takes

- a function from one type to another (* 2)
- take a list of one type [1, 2, 3]
- returns a list of another type [2, 4, 6]

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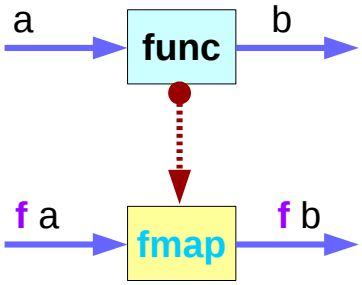
List : an instance of the Functor typeclass

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
    map :: (a -> b) -> [a] -> [b]
```

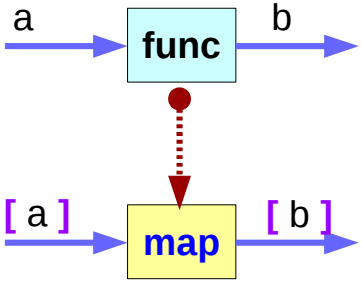


function fmap
function func
type constructor f

map is just a **fmap** that works only on **list**s

a **list** is an **instance** of the **Functor typeclass**.

```
instance Functor [ ] where
    fmap = map
```



f : a type constructor that takes one type
[ ] : a type constructor that takes one type
[a] : a concrete type ([Int], [String] or [[String]])

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List Examples

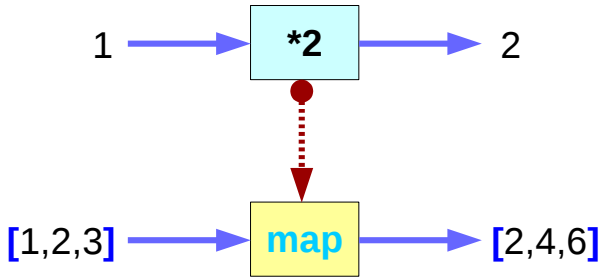
```
class Functor f where
   fmap :: (a -> b) -> f a -> f b
```

```
   map :: (a -> b) -> [a] -> [b]
```

```
instance Functor [ ] where
   fmap = map
```

```
map :: (a -> b) -> [a] -> [b]
ghci> fmap (*2) [1..3]
[2,4,6]
ghci> map (*2) [1..3]
[2,4,6]
```

function fmap map
function func (*2)
type constructor f [ ]



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Maybe : an instance of the Functor typeclass

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```

f	⟺	Maybe
f a	⟺	Maybe a
f b	⟺	Maybe b

(a -> b)	⟺	func

instance : implementing fmap

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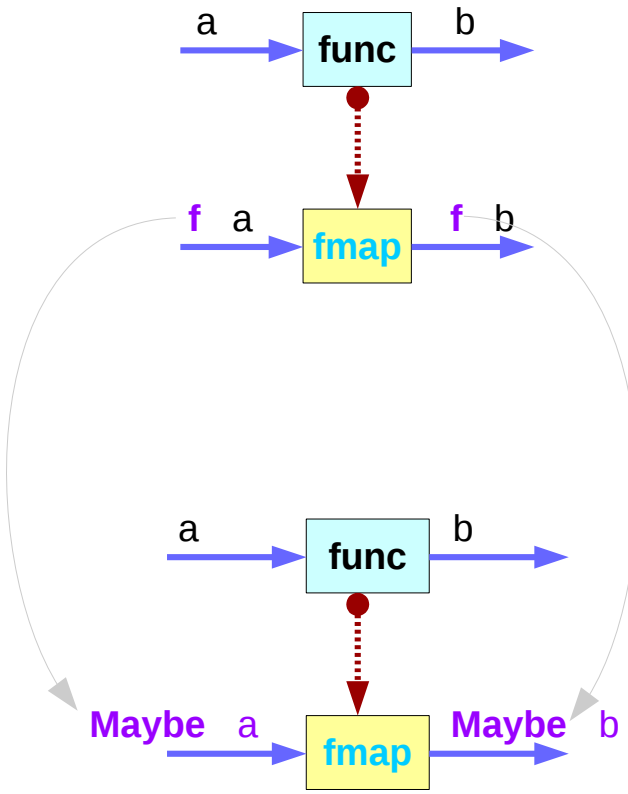
f : a type variable

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

f : a type variable

f ⟺ **Maybe**

```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```



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f : a type constructor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

f : a **type constructor** taking one type parameter

type **f a** ⟺ **Maybe a** type

type **f b** ⟺ **Maybe b** type

f a **Maybe** a

```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

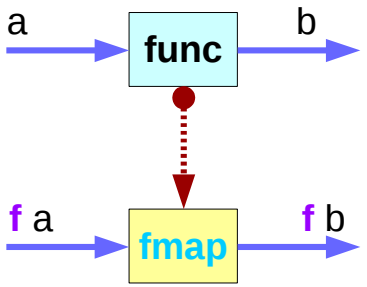
f ~~type~~ **Maybe** ~~type~~

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

f and Maybe

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

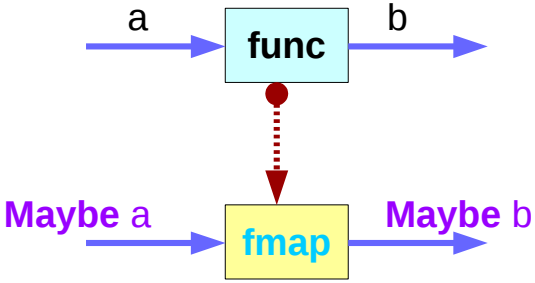
```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```



f : a **type variable**

f : a **type constructor** taking one type parameter

Maybe : an **instance** of **Functor** typeclass



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Maybe : an argument to fmap, together with a

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

```
fmap :: (a -> b) -> f a -> f b
```

```
fmap :: (a -> b) -> Maybe a -> Maybe b
fmap func (Just x) = Just (func  x)
fmap func Nothing = Nothing
```

```
fmap :: (a -> b) -> Maybe a -> Maybe b
fmap f (Just x) = Just (f  x)
fmap f Nothing = Nothing
```

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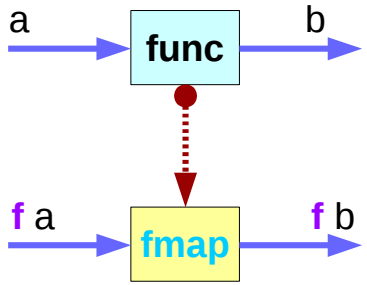
Maybe : an argument to fmap, together with a

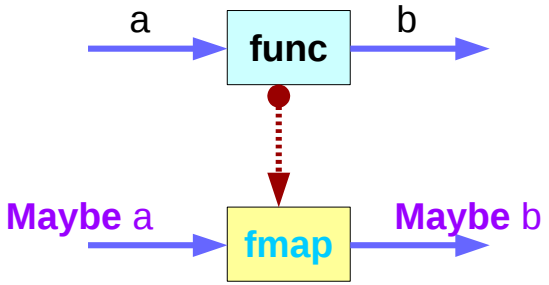
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```

```
fmap :: (a -> b) -> f a -> f b
```




```
fmap :: (a -> b) -> Maybe a -> Maybe b
```



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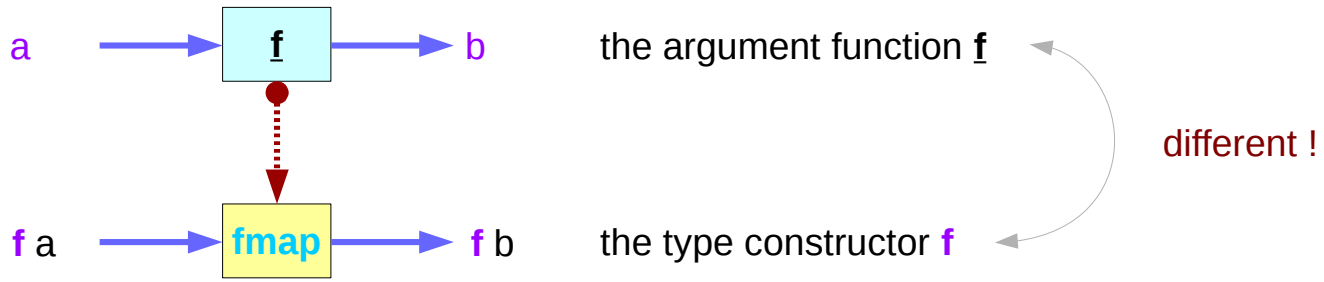
The distinct two f's

```
class Functor  f  where
   fmap :: (a -> b) ->  f  a ->  f  b
```

Functor f

```
instance Functor Maybe where
   fmap  f  (Just x) = Just ( f  x)
   fmap  f  Nothing = Nothing
```

f :: (a->b)



An argument f to fmap vs. Functor f

```
class Functor  f  where
   fmap :: (a -> b) ->  f  a ->  f  b
```

```
instance Functor Maybe where
   fmap func (Just x) = Just (func  x)
   fmap func Nothing = Nothing
```

```
instance Functor Maybe where
   fmap  f  (Just x) = Just ( f  x)
   fmap  f  Nothing = Nothing
```

f

func

f

f : a **type variable**

f : a **type constructor** taking one type parameter

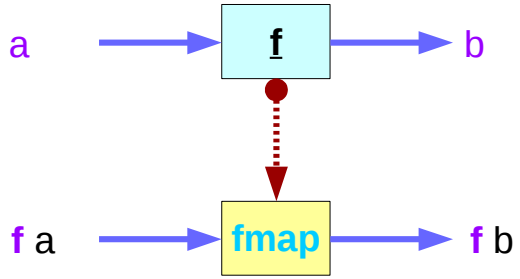
f an argument function to fmap

f is different from the type constructor f

f : a -> b ⟺ func : a -> b

Maybe Functor

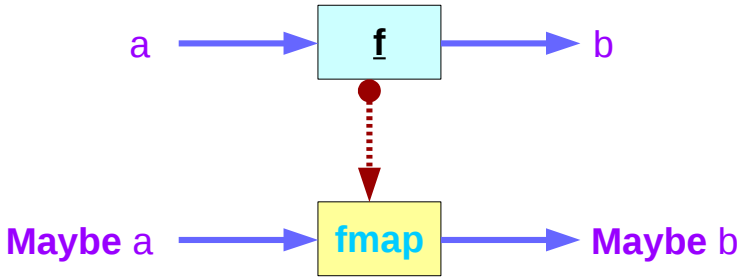
Type Class

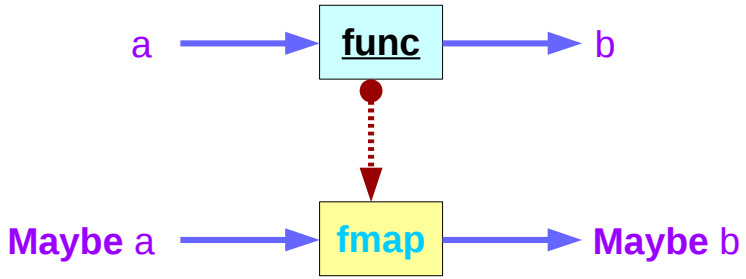



```
class Functor f

instance Functor Maybe
instance Functor [ ]
```

Instance





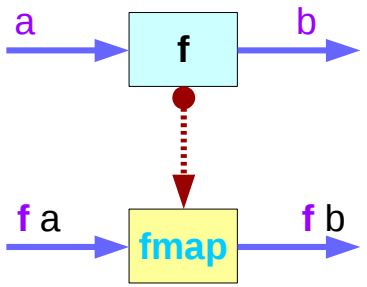
Maybe Functor Examples (1)

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

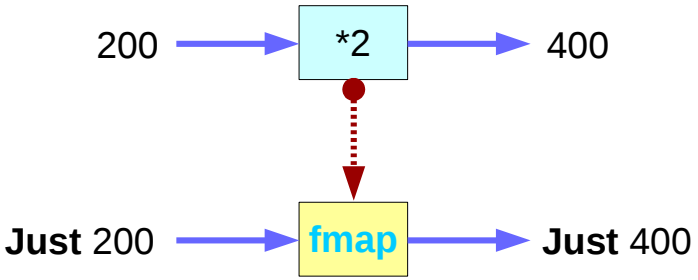
f

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

f




```
ghci> fmap (*2) (Just 200)
Just 400
ghci> fmap (*2) Nothing
Nothing
```



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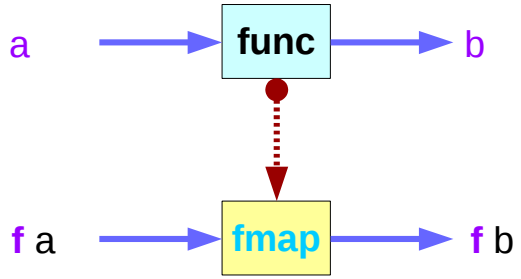
Maybe Functor Examples (2)

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

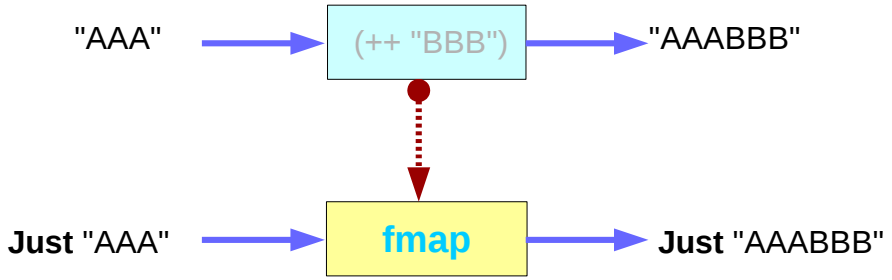
f

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

f




```
ghci> fmap (++ "BBB") (Just "AAA")
Just "AAABBB"
ghci> fmap (++ "BBB") Nothing
Nothing
```



http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe as a functor

Functor typeclass:

- transforming one type to another
- transforming operations of one type to those of another

Maybe a is an instance of a **functor** type class

Functor provides **fmap** method
maps functions of the base type (such as Integer)
to *functions* of the lifted type (such as **Maybe** Integer).

m_father :: Person -> **Maybe** Person
m_mother :: Person -> **Maybe** Person

Fmap

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

A *function* **f** transformed with **fmap**
can work on a **Maybe** value

```
case maybeVal of
  Nothing   -> Nothing          -- there is nothing, so just return Nothing
  Just val  -> Just (f val)     -- there is a value, so apply the function to it
```

```
class Functor f where
   fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
   fmap f (Just x) = Just (f x)
   fmap f Nothing = Nothing
```

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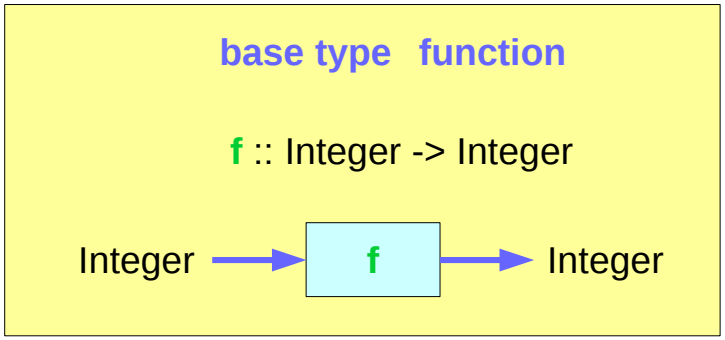
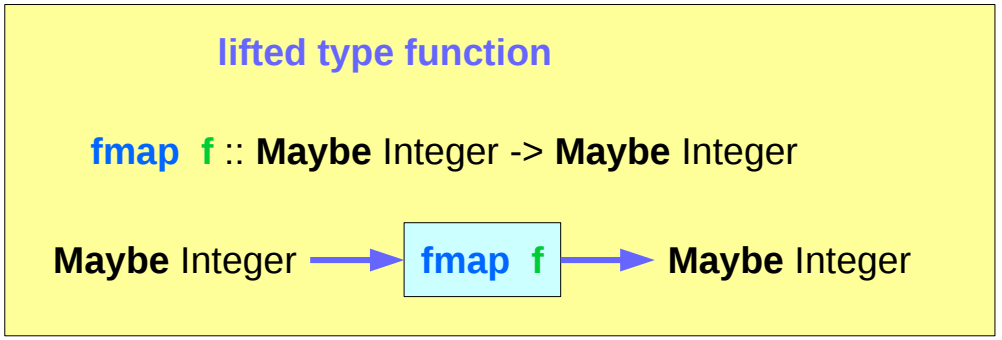




https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

A *function* **f** transformed with **fmap**
to work on a **Maybe** value

```
f      ::       Int    ->        Int
fmap f :: Maybe Integer -> Maybe Integer
```

m_x : a **Maybe** **Integer** value

```
fmap f  m_x
```

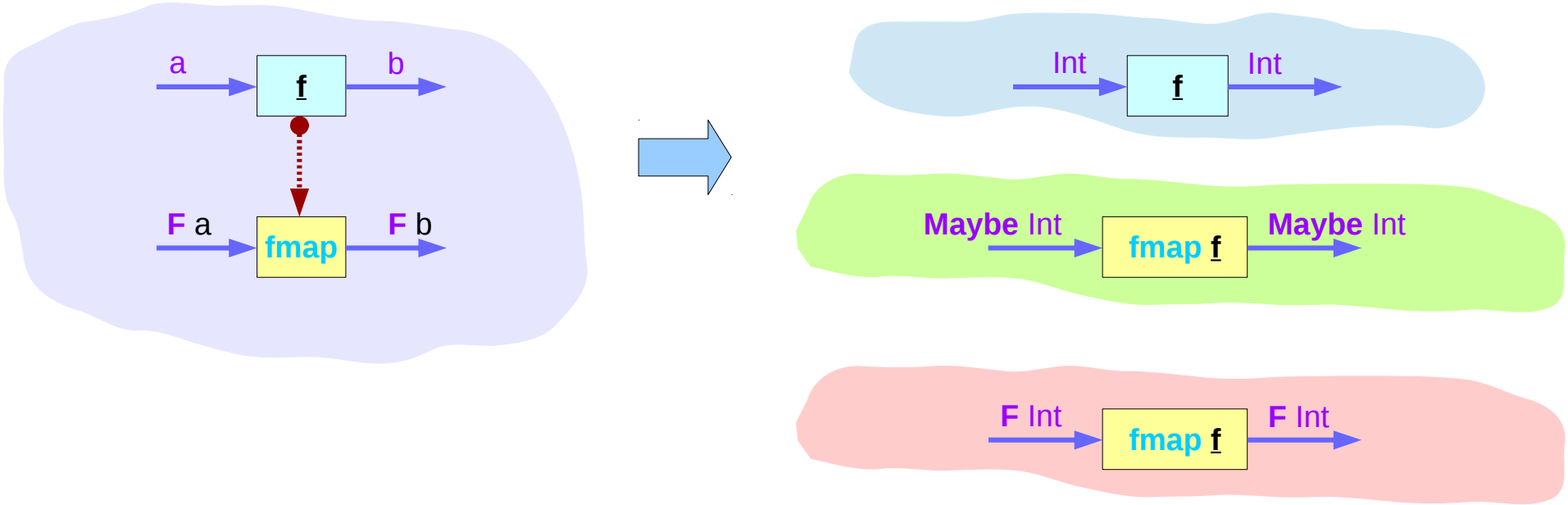
https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Transforming operations

Functor provides **fmap** method

maps *functions* of the base type (such as Integer)

to *functions* of the lifted type (such as **Maybe** Integer).



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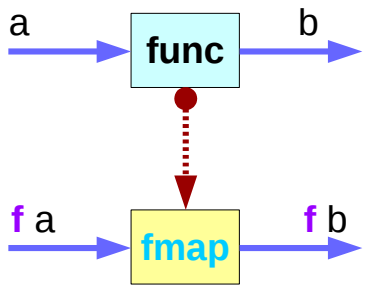
fmap func

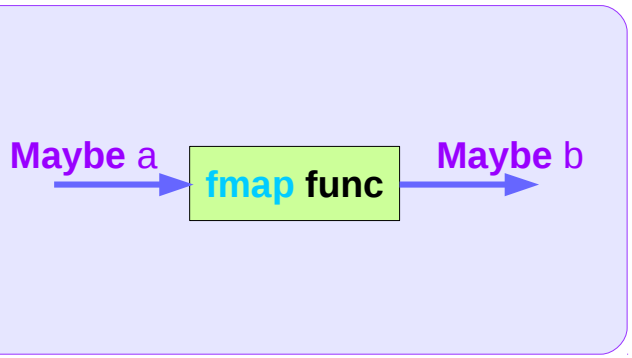
```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

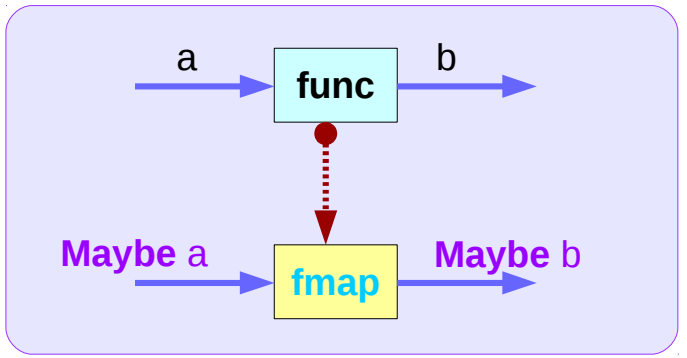
```
instance Functor Maybe where
  fmap func (Just x)  = Just (func  x)
  fmap func Nothing = Nothing
```



fmap func Just x

fmap func Just x





http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Apply a function to lifted type values

m_x :: Maybe Integer (Just 101, Nothing, …)

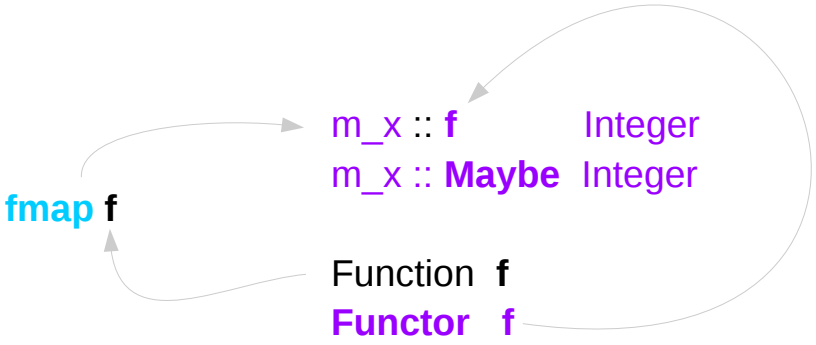
f :: Int -> Int

fmap f m_x

to apply the function **f** directly to the Maybe Integer
without concerning whether it is Nothing or not

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```



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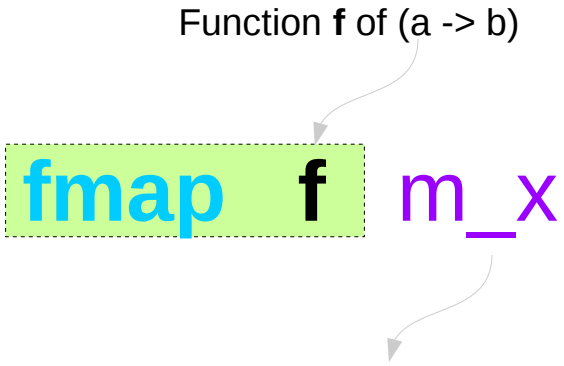
Maybe as a functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

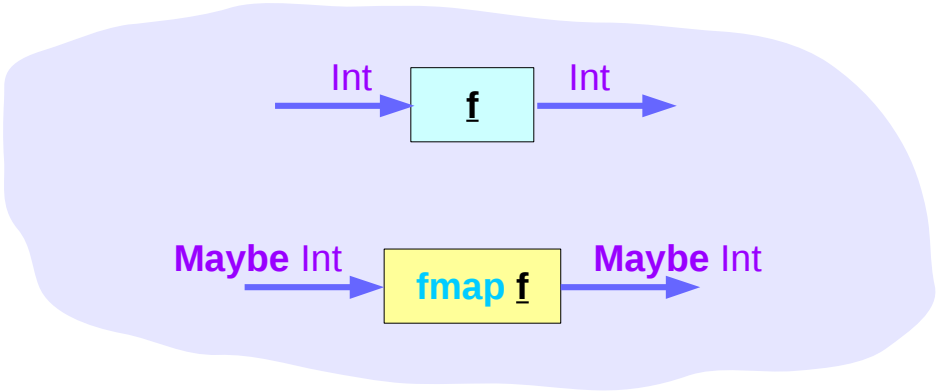


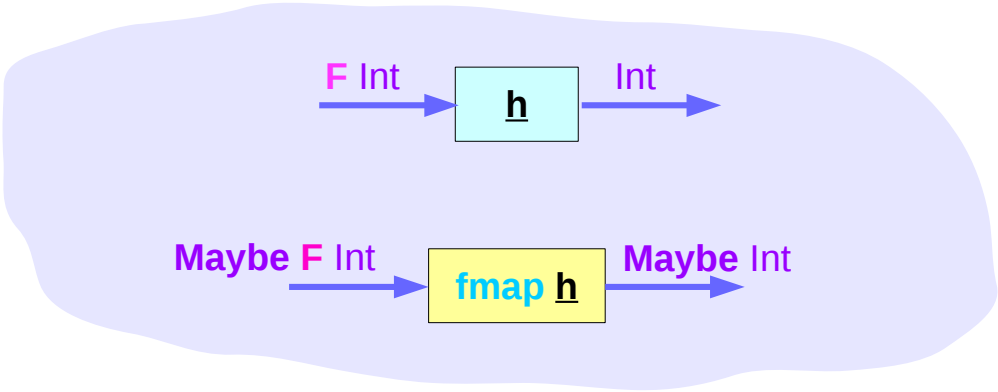

```
m_x :: f       Integer
m_x :: Maybe   Integer
```

Function **f**
Functor f

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor





Can apply a whole chain of
lifted Integer -> Integer functions
to Maybe Integer values
and only have to worry about
explicitly checking for **Nothing**
once when you're finished.

```
class Functor f where
  fmap :: (F Int -> Int) -> f Int -> f Int
```

```
instance Functor Maybe where
  fmap :: (F Int -> Int) -> Maybe F Int -> Maybe Int

  fmap h (Just f_x) = Just (h f_x)
  fmap h Nothing  = Nothing
```

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

So if you have a Maybe Integer value m_x and an Int -> Int function f,
you can do **fmap f** m_x to apply the function **f** directly to the Maybe Integer
without worrying if it's actually got a value or not.

In fact, you could apply a whole chain of lifted Integer -> Integer functions to Maybe Integer
values and only have to worry about explicitly checking for Nothing once when you're finished.

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe instances

Maybe is

- an instance of **Eq** and **Ord** (as a base type)
- an instance of **Functor**
- an instance of **Monad**

https://wiki.haskell.org/Maybe

Maybe class

The Maybe type definition

```
data Maybe a = Just a | Nothing
   deriving (Eq, Ord)
```

Maybe is
an instance of **Eq** and **Ord** (as a base type)

https://wiki.haskell.org/Maybe

Maybe Functor

For **Functor**, the **fmap f**
moves inside the **Just** constructor
is identity on the **Nothing** constructor.

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

fmap f (Just x) = Just (f x)
fmap f Nothing = Nothing

https://wiki.haskell.org/Maybe

maybe library function

maybe :: b -> (a->b) -> Maybe a -> b

The maybe function takes
    a default value (b),
    a function (a->b), and
    a Maybe value (Maybe a).

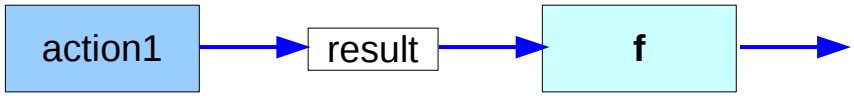
If the Maybe value is **Nothing**,
    the function returns the default value.
Otherwise, it applies the function
    to the value inside the **Just** and returns the result.

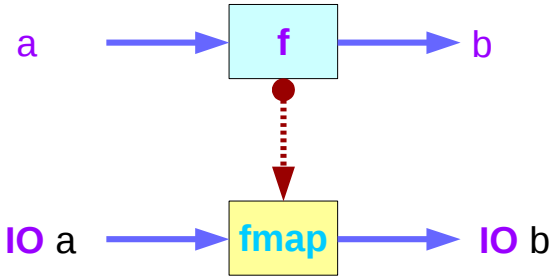
```
>>> maybe False odd (Just 3)
True

>>> maybe False odd Nothing
False
```

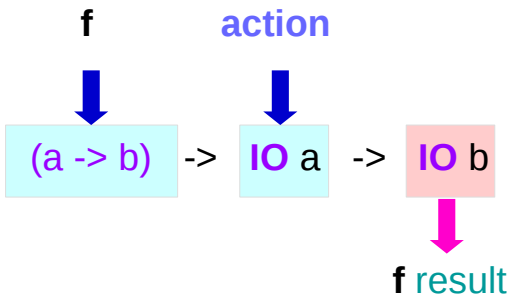
https://hackage.haskell.org/package/base-4.10.0.0/docs/Data-Maybe.html

IO Functor

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```








```
instance Functor Maybe where
  fmap func (Just x) = Just (func  x)
  fmap func Nothing = Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

IO Functor Example

```
main = do line <- getLine
          let line' = reverse line
          putStrLn $ "You said " ++ line' ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line' ++ " backwards!"

main = do line <- fmap reverse getLine
          putStrLn $ "You said " ++ line ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line ++ " backwards!"
```

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```

```
fmap reverse getLine = do
  result <- getLine
  return (reverse result)
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Typeclass Examples (3)

Functor (->) r

The function type r -> a 2 + 3

can be rewritten as (->) r a (+) 2 3

When we look at it as (->) r a, we can see (->) in a slighty different light, because we see that it's just a type constructor that takes two type parameters, just like Either. B

A function takes any thing and returns any thing

```
g :: a -> b
g :: r -> a
```

```
fmap :: (a -> b) -> f a -> f b
fmap :: (a -> b) -> ((->) r a) -> ((->) r b)
fmap :: (a -> b) -> (r -> a) -> (r -> b)
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

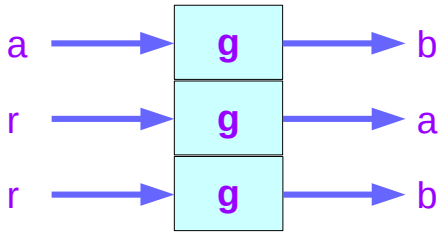
Functor Typeclass Examples (3)

```
instance Functor ((->) r) where
  fmap f g =  (\x -> f (g x))
```

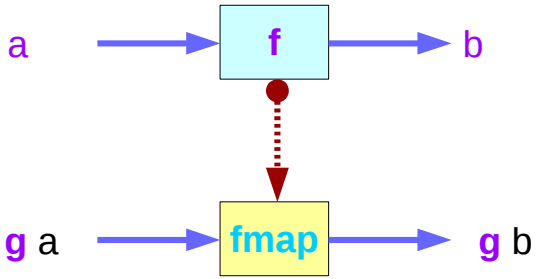
```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

A function takes any thing and returns any thing

```
g :: a -> b
g :: r -> a
```




```
fmap :: (a -> b) -> f a -> f b
fmap :: (a -> b) -> ((->) r a) -> ((->) r b)
fmap :: (a -> b) -> (r -> a) -> (r -> b)
```



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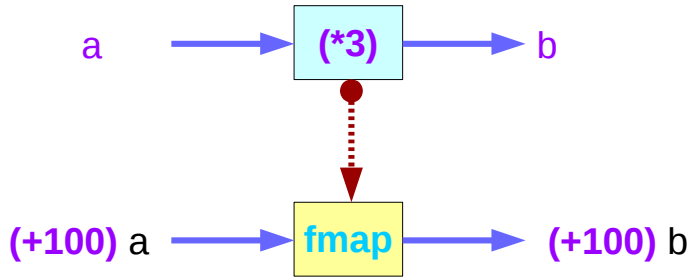
Functor Typeclass Examples (4)

```
instance Functor ((->) r) where
  fmap f g =  (\x -> f (g x))

instance Functor ((->) r) where
  fmap = ( . )
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

```
ghci> :t fmap (*3) (+100)
fmap (*3) (+100) :: (Num a) => a -> a
ghci> fmap (*3) (+100) 1
303
ghci> (*3) `fmap` (+100) $ 1
303
ghci> (*3) . (+100) $ 1
303
ghci> fmap (show . (*3)) (*100) 1
"300"
```



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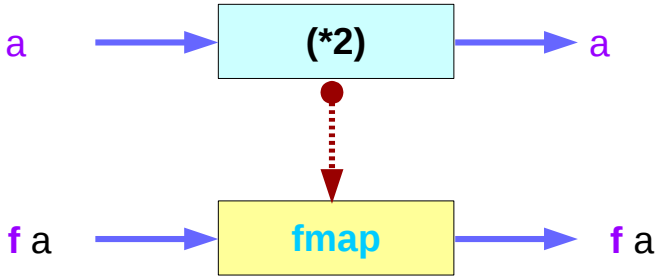
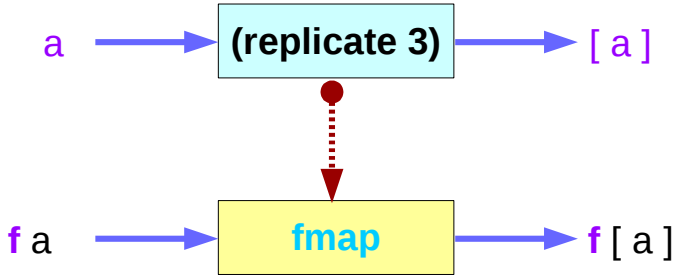
Functor Typeclass Examples (5)

```
ghci> :t fmap (*2)
fmap (*2) :: (Num a, Functor f) => f a -> f a
```




```
ghci> :t fmap (replicate 3)
fmap (replicate 3) :: (Functor f) => f a -> f [a]
```



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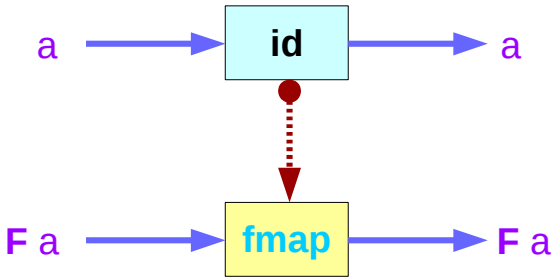
Functor Typeclass Examples (6)

```
ghci> fmap (replicate 3) [1,2,3,4]
[[1,1,1],[2,2,2],[3,3,3],[4,4,4]]

ghci> fmap (replicate 3) (Just 4)
Just [4,4,4]

ghci> fmap (replicate 3) (Right "blah")
Right ["blah","blah","blah"]

ghci> fmap (replicate 3) Nothing
Nothing

ghci> fmap (replicate 3) (Left "foo")
Left "foo"
```

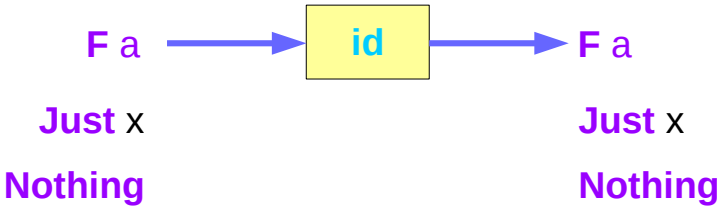
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Laws

```
fmap id = id

id :: a -> a
id    x = x
```




```
instance Functor Maybe where
   fmap func (Just x) = Just (func  x)
   fmap func Nothing = Nothing

instance Functor Maybe where
   fmap f (Just x) = Just (f x)
   fmap f Nothing = Nothing

instance Functor Maybe where
   fmap id (Just x) = Just (id x)
   fmap id Nothing = Nothing
```



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Functor Typeclass

```
ghci> fmap id (Just 3)
Just 3
ghci> id (Just 3)
Just 3
ghci> fmap id [1..5]
[1,2,3,4,5]
ghci> id [1..5]
[1,2,3,4,5]
ghci> fmap id []
[]
ghci> fmap id Nothing
Nothing
```

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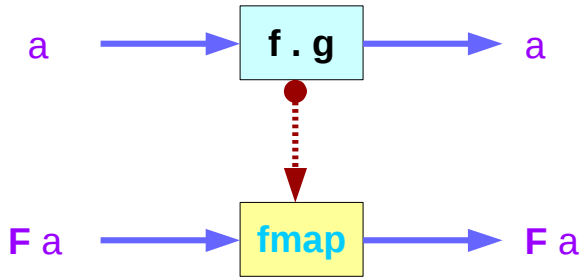
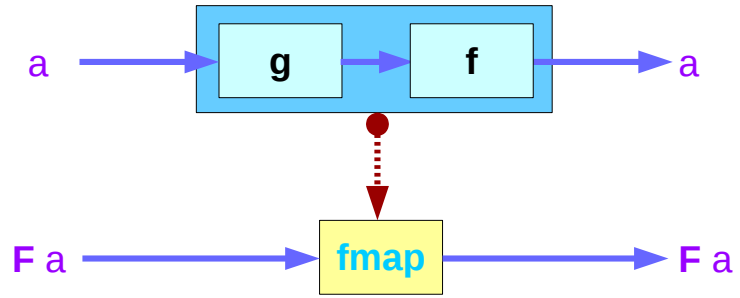
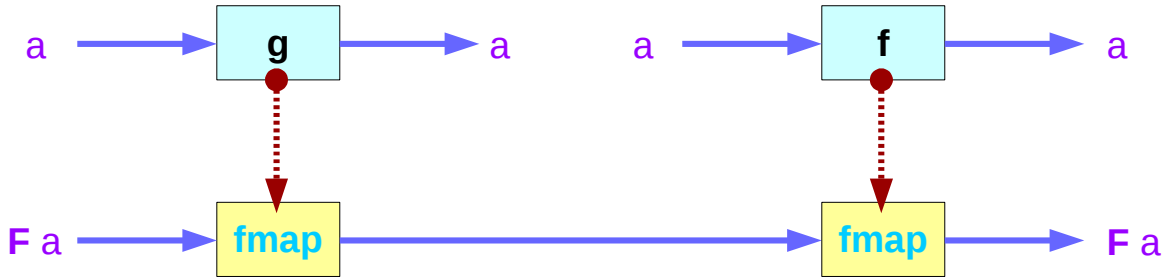
Functor Laws

fmap (f . g) = fmap f . fmap g

fmap (f . g) F = fmap f (fmap g F)







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Functor Laws

```
fmap (f . g) = fmap f . fmap g
fmap (f . g) F = fmap f (fmap g F)

  instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

```
fmap (f . g) Nothing = Nothing
fmap f (fmap g Nothing) = Nothing

fmap (f . g) (Just x) = Just ((f . g) x) = Just (f (g x))
fmap f (fmap g (Just x)) = fmap f (Just (g x)) = Just (f (g x))
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf