Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

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Sum of two random variables

Definition

Let W be a random variable equal to
the sum of two independent random variables X and Y

$$W = X + Y$$

the probability distribution function is defined by

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$$

$$x + y \leq wF_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y)dxdy$$

Convoltuion of density functions

Definition

Let $W = X + Y$
the probability distribution function is defined by

$$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y)dxdy$$

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(t-x)dx$$

the density function of the sum of
two statistically independent random variables
is the convolution of their individual density functions

Sum of two random variables (1)

Definition

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$$

$$x + y \leq w$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_{X,Y}(x,y) dy dx$$

$$f_W(w) = \frac{\partial}{\partial w} F_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(t-x) dx$$

Sum of two random variables (2)

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_{X,Y}(x,y) dydx$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_X(x) f_Y(y) dydx$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{-\infty}^{w} f_X(x) f_Y(t-x) dtdx$$

$$F_W(w) = \int_{-\infty}^{w} \int_{-\infty}^{\infty} f_X(x) f_Y(t-x) dxdt$$

$$f_W(w) = \frac{\partial}{\partial w} F_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(t-x) dx$$

https://math.stackexchange.com/questions/1919545/sum-of-random-variables-and-convolution

Sum of several random variables (1)

Definition

the sum Y of 3 independent random variables X_1, X_2, X_3

Let $Y_1 = X_1 + X_2$, then

$$f_{Y_1}(y_1) = f_{X_2}(x_2) * f_{X_1}(x_1)$$

Let $Y_2 = X_3 + Y_1$, then

$$f_{Y_2}(y_2) = f_{X_3}(x_3) * f_{Y_1}(y_1) = f_{X_3}(x_3) * f_{X_2}(x_2) * f_{X_1}(x_1)$$

5

Sum of several random variables (2)

Definition

the sum Y of N independent random variables $X_1, X_2, ..., X_N$
Let $Y_1 = X_1 + X_2 + \cdots + X_N$, then

$$f_Y(y) = f_{X_N}(x_N) * f_{X_{N-1}}(x_{N-1})) * \cdots * f_{X_1}(x_1)$$