

Logical Implication

$P \rightarrow F$

P implies F

If F also takes the value of 1 whenever P equals 1

P is an implicant of F

P is a product term

$$F(x,y,z,w) = xy + yz + w$$

$xy \rightarrow F$

Then $xyz \rightarrow F$, $xyzw \rightarrow F$

In this case, xy is a prime implicant,

And xyz , $xyzw$ is not a prime implicant

Essential prime implicants are

prime implicants that cover an output of the function
that no combination of other prime implicants is able to cover.

A prime implicant that includes one or more **distinguished one cells (★)**.

Essential prime implicants are important

because a minimal sum contains all essential prime implicants.

<http://web.cecs.pdx.edu/~mcnames/ECE171/Lectures/Lecture10.html>

$$F(A,B,C,D) = \text{Sum}(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$

f		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	m0	m1	m3	m2
AB=01	01	m4	m5	m7	m6
AB=11	11	m12	m13	m15	m14
AB=10	10	m8	m9	m11	m10

f		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

BD		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1

AB=01	01		★1	1		essential prime implicant
AB=11	11		1	1		
AB=10	10	1	1	1	1	

		CD=00	CD=01	CD=11	CD=10	
		00	01	11	10	
AB=00	00	1		1	1	
AB=01	01		1	1		
AB=11	11		1	1		
AB=10	10	1	1	1	1	

A **prime implicant** is a product term (eg. B'C, AB' ...)
 Obtained by combining the maximum possible number of adjacent squares
 in the map.
 If a minterm in a square is covered by only one prime implicant,
 That prime implicant is said to be essential. (**essential** prime implicant)

		CD=00	CD=01	CD=11	CD=10	
		00	01	11	10	
AB=00	00	1		1	1	
AB=01	01		1	1		
AB=11	11		1	1		
AB=10	10	1	1	1	1	

		CD=00	CD=01	CD=11	CD=10	
		00	01	11	10	
AB=00	00	★1		1	1	essential prime implicant
AB=01	01		1	1		
AB=11	11		1	1		
AB=10	10	1	1	1	1	

		CD=00	CD=01	CD=11	CD=10	
		00	01	11	10	
AB=00	00	1		1	1	
AB=01	01		1	1		
AB=11	11		1	1		
AB=10	10	1	1	1	1	

f

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

BD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

B'D'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

f

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

CD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

B'C

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	<u>1</u>	<u>1</u>	1

AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	<u>1</u>	<u>1</u>	<u>1</u>	1

CD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	<u>1</u>	
AB=11	11		1	<u>1</u>	
AB=10	10	1	<u>1</u>	<u>1</u>	1

AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		<u>1</u>	<u>1</u>	
AB=10	10	1	<u>1</u>	<u>1</u>	1

CD+AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	<u>1</u>	
AB=11	11		<u>1</u>	<u>1</u>	
AB=10	10	1	<u>1</u>	<u>1</u>	1

CD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	<u>1</u>	<u>1</u>	1

AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	<u>1</u>	<u>1</u>	<u>1</u>	1

CD + AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	<u>1</u>	<u>1</u>	<u>1</u>	1

B'C

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	<u>1</u>	<u>1</u>	1

AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		<u>1</u>	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	<u>1</u>	<u>1</u>	1

B'C+AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

B'C

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

B'C+AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

A **prime implicant** is a product term (eg. B'C, AB' ...)
 Obtained by combining the maximum possible number of adjacent squares in the map.
 If a minterm in a square is covered by only one prime implicant,
 That prime implicant is said to be essential. (**essential** prime implicant)

$$f = w'x' + yz$$

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

$$w'x'$$

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

$$yz$$

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

$$f = w'z + yz$$

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

$$w'z$$

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

yz

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	x	1	1	x
AB=01	01		x	1	
AB=11	11			1	
AB=10	10			1	

BD+B'D'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

BD+B'D' CD+AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

(1) $F = BD + B'D + CD + AB'$

BD+B'D' CD+AD

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1		1	1
AB=01	01		1	1	
AB=11	11		1	1	
AB=10	10	1	1	1	1

(2) $F = BD + B'D + CD + AD$

BD+B'D' B'C+AB'

		CD=00	CD=01	CD=11	CD=10
		00	01	11	10
AB=00	00	1			
AB=01	01		1	1	1
AB=11	11		1	1	
AB=10	10	1	1	1	1

(3) $F = BD + B'D + B'C + AB'$