

Digital Signal Octave Codes (0B)

- Aliasing and Folding Frequencies

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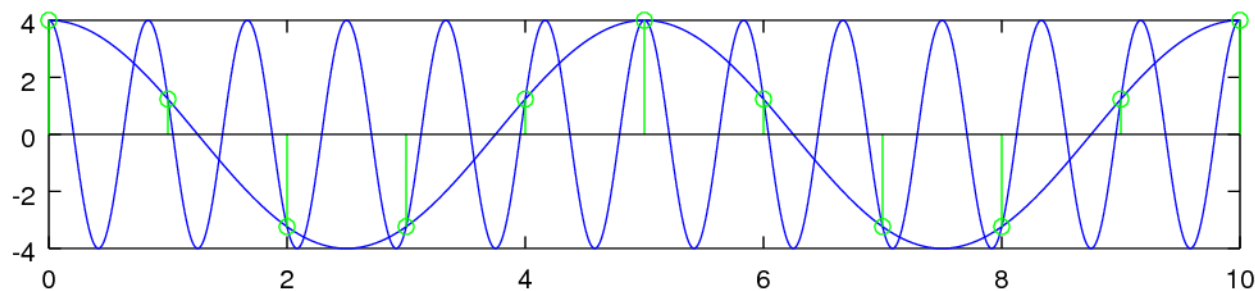
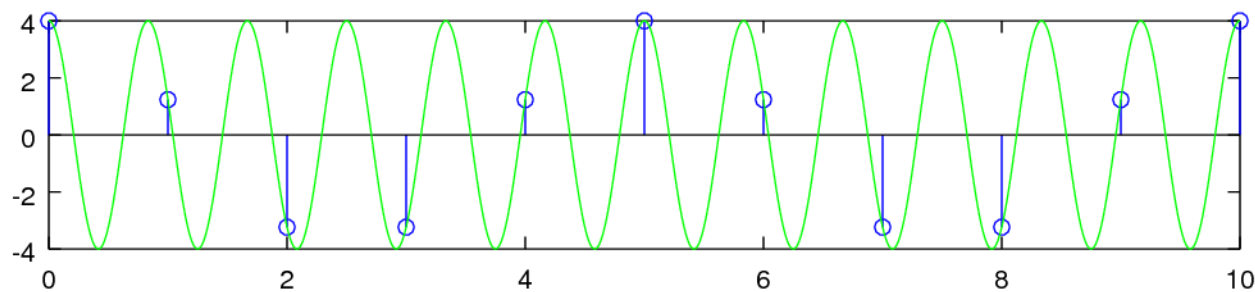
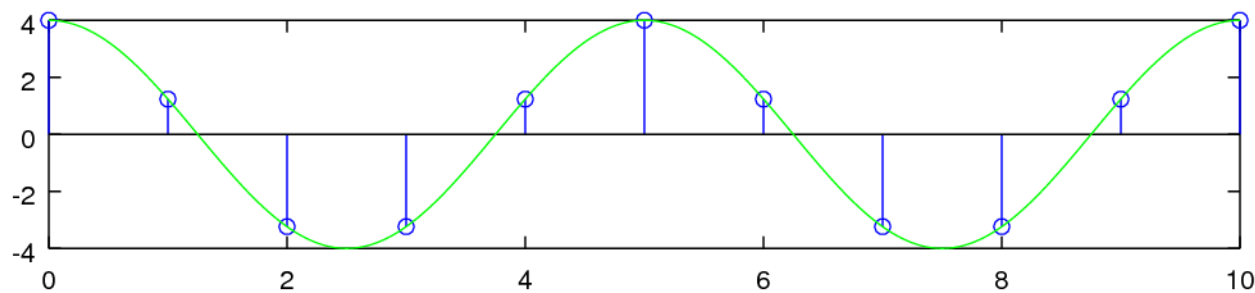
Based on

M.J. Roberts, Fundamentals of Signals and Systems

S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed

S.D. Stearns, Digital Signal Processing with Examples in MATLAB

Aliasing Condition Examples

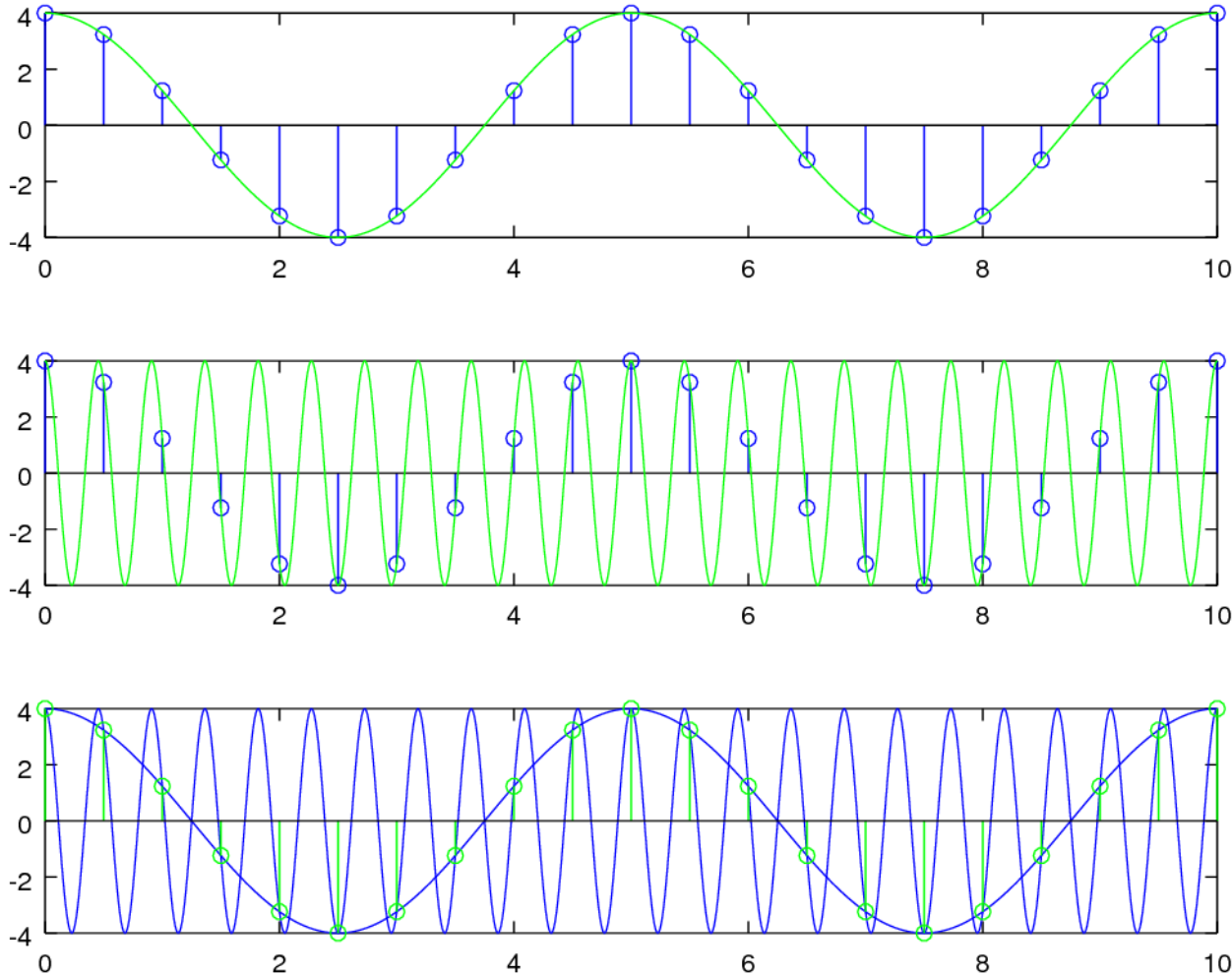


```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
```

```
subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples

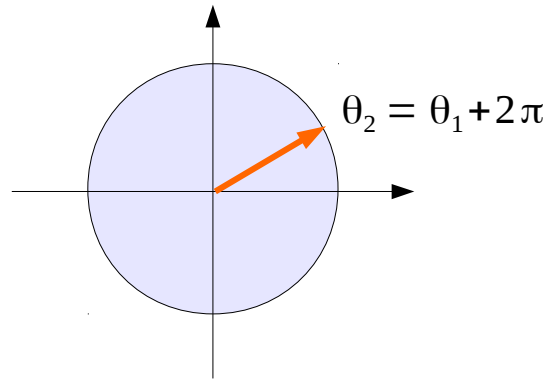
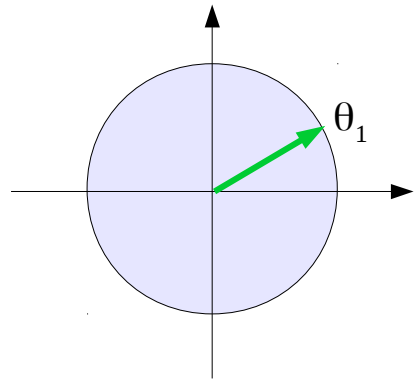


```
clf  
n = [0:0.5:10];  
t = [0:1000]/100;  
y1 = 4*cos(2*pi*(1/5)*n);  
y2 = 4*cos(2*pi*(11/5)*n);  
yt1 = 4*cos(2*pi*(1/5)*t);  
yt2 = 4*cos(2*pi*(11/5)*t);
```

```
subplot(3,1,1);  
stem(n, y1); hold on;  
plot(t, yt1, 'g');  
subplot(3,1,2);  
stem(n, y2); hold on;  
plot(t, yt2, 'g');  
subplot(3,1,3);  
plot(t, yt1); hold on;  
plot(t, yt2);  
stem(n, y1, 'g');
```

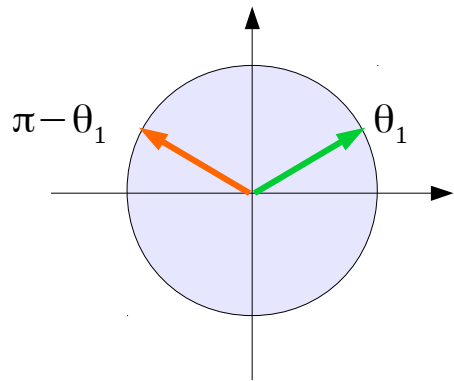
M.J. Roberts, Fundamentals of Signals and Systems

Identical Sine values and Cosine Values



$$\omega_1 t - \omega_2 t = 2\pi$$

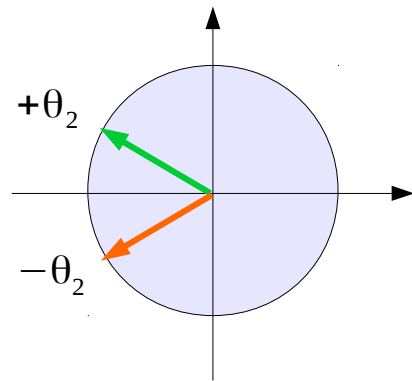
periodic condition



$$\omega_1 t + \omega_2 t = +\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

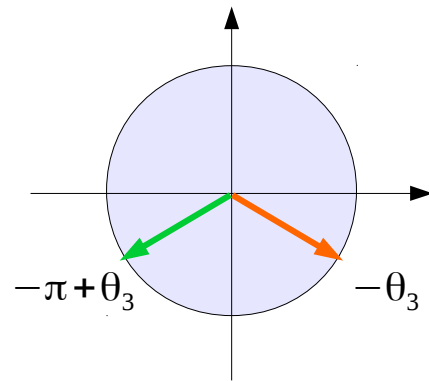
$$-\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = 0$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

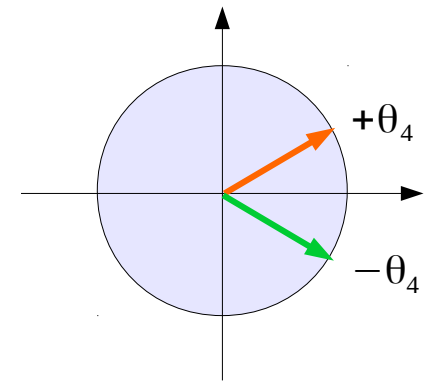
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = -\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$-\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = 0$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

Identical Sine values and Cosine Values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

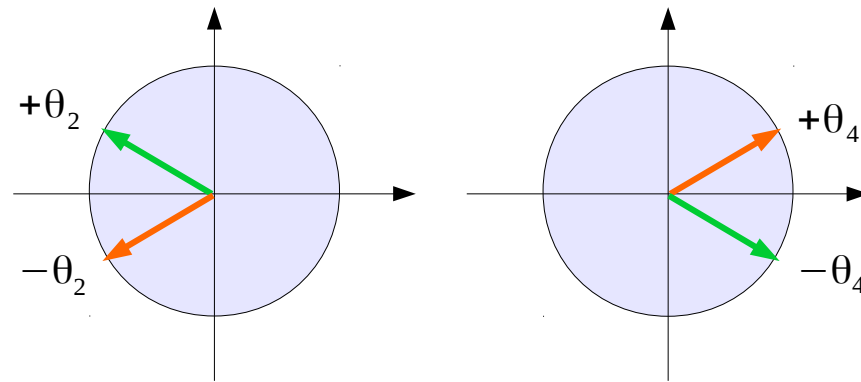
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



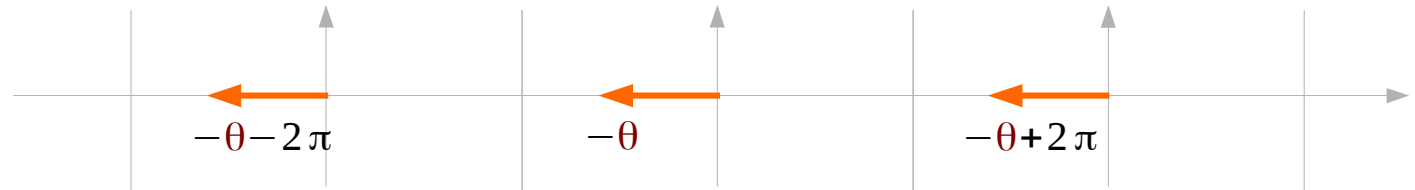
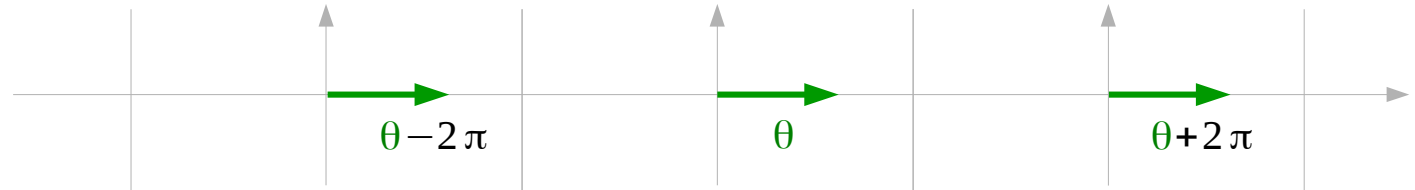
Angles of identical trigonometric values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

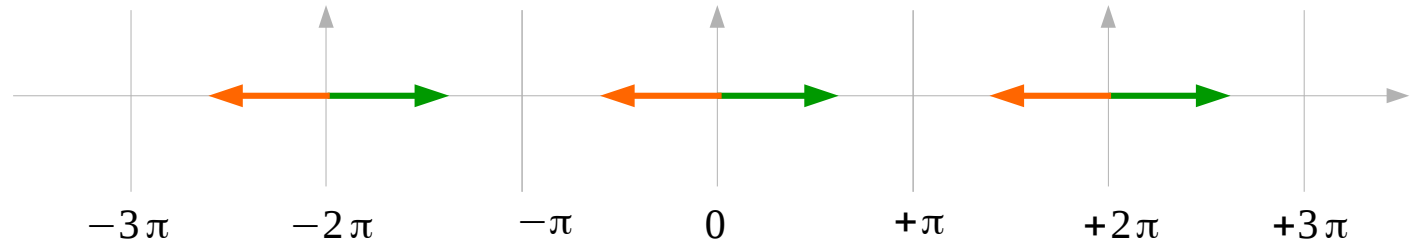
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



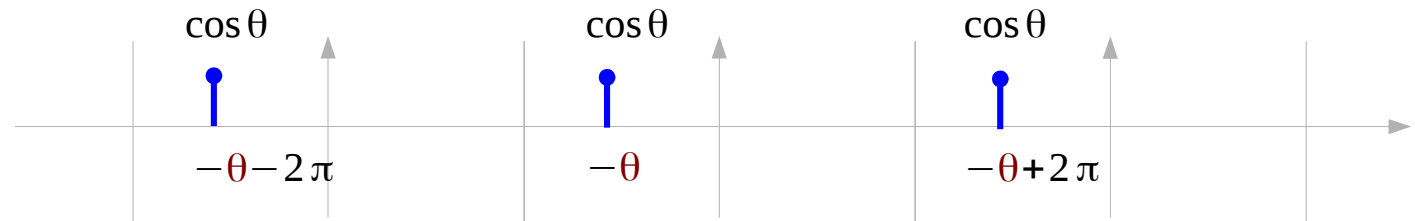
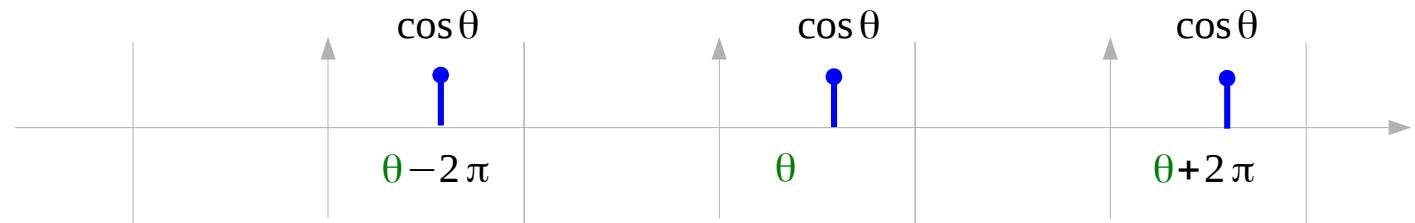
Identical Cosine Values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

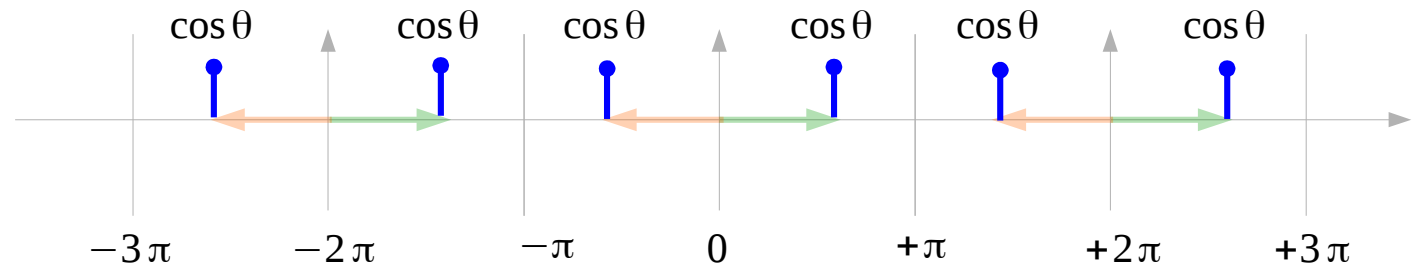
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



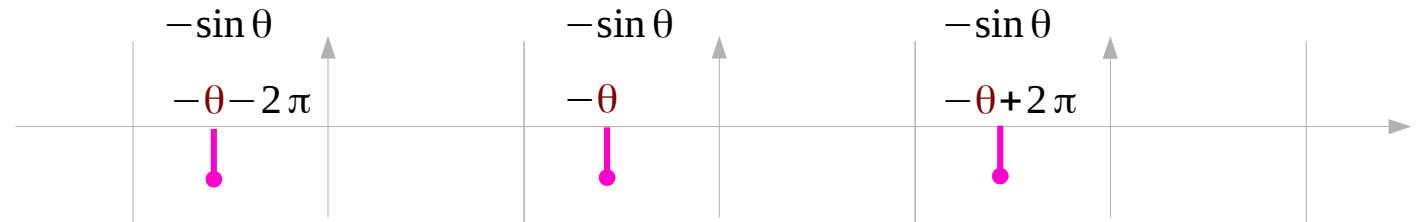
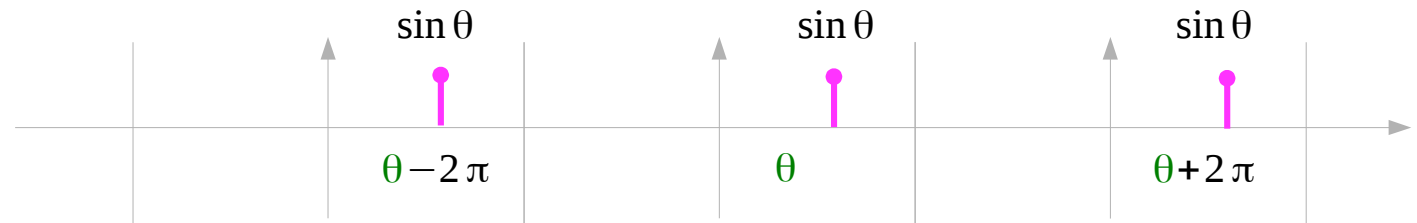
Identical Absolute Sine values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

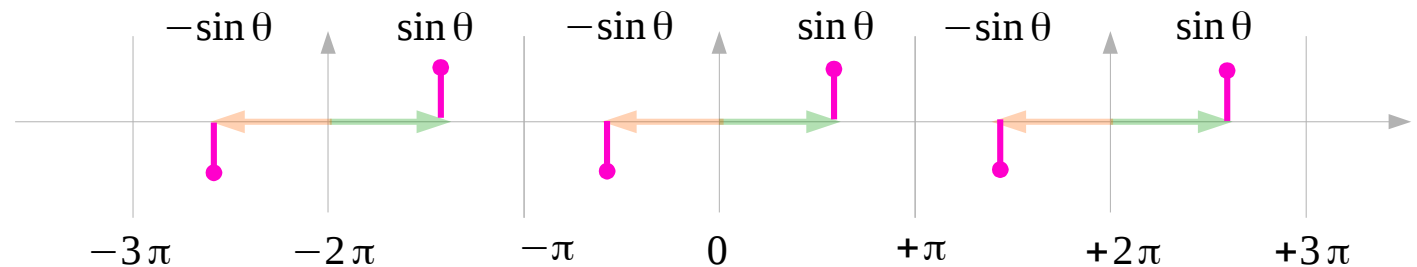
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Spectrum Representation

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

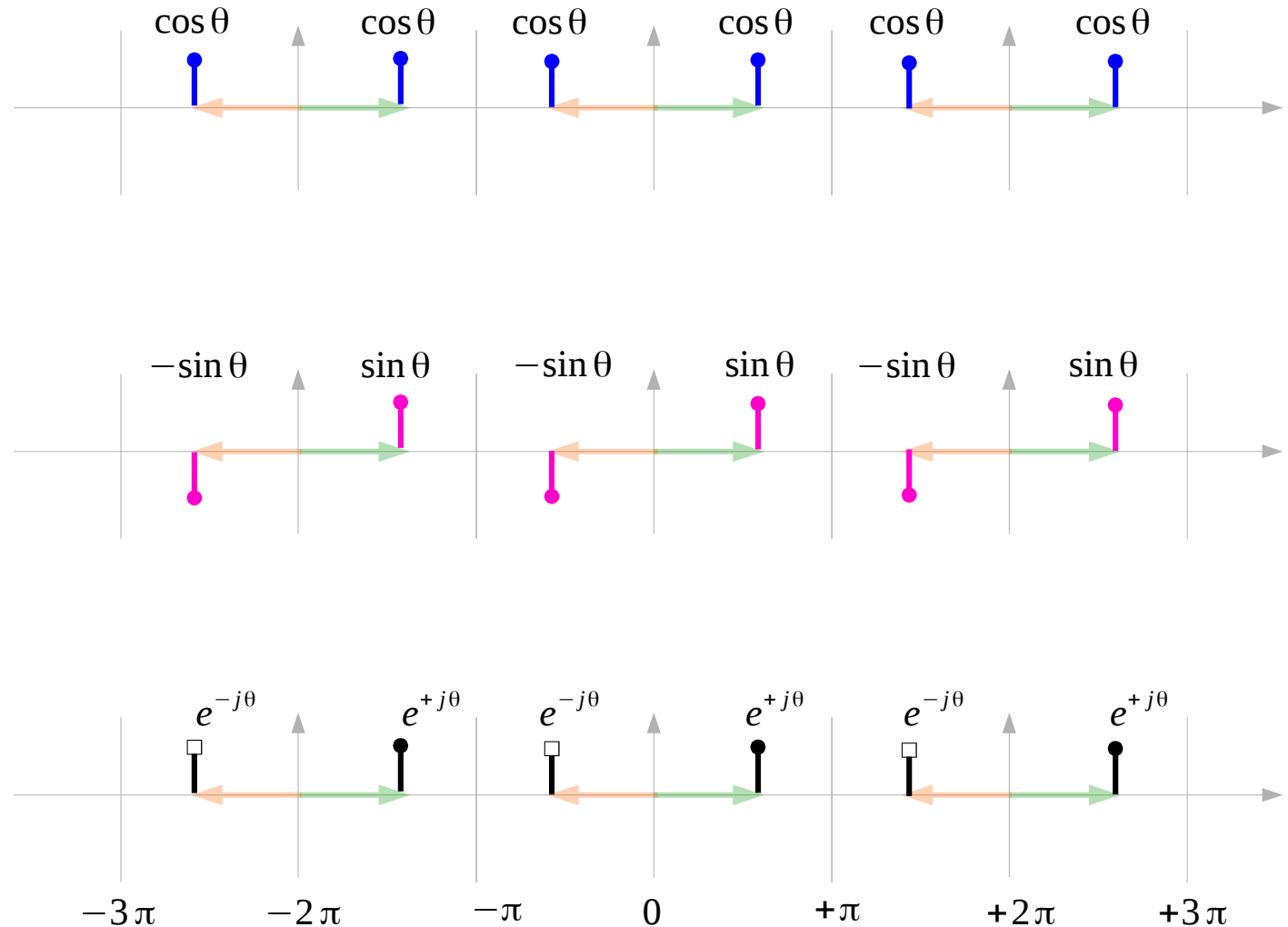
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



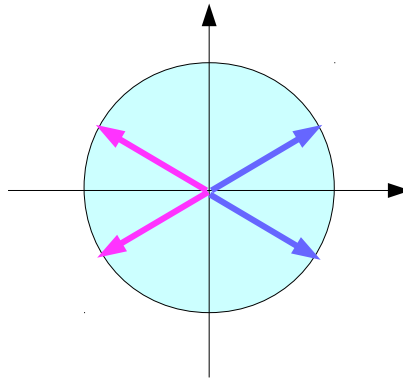
((()))

$$\cos(2\pi f_1 t) = \cos(2\pi f_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

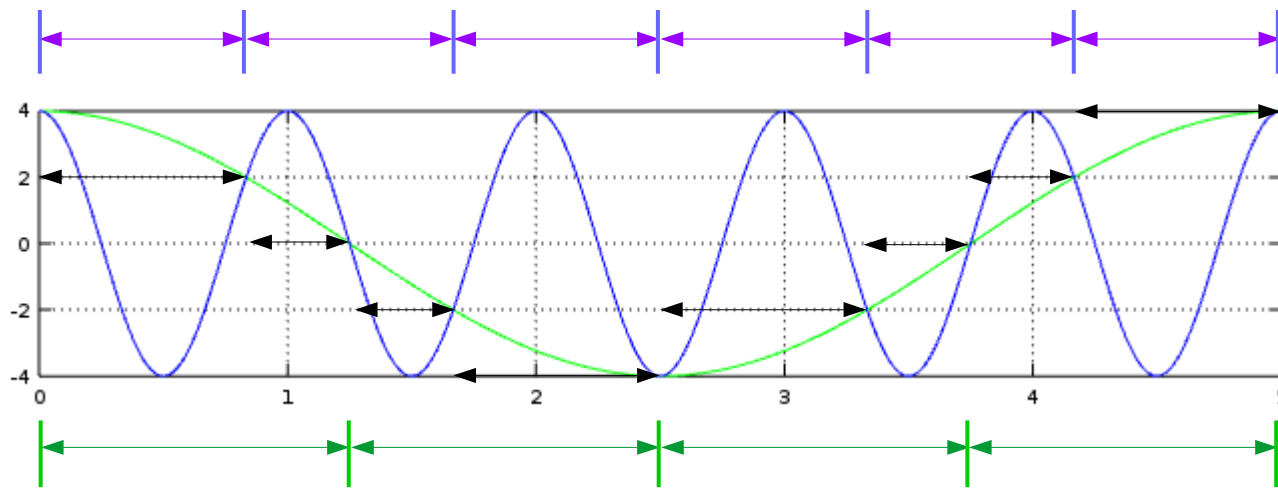
$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases}$$

$$\begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = n \end{cases}$$










$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \dots \quad \omega_1 t + \omega_2 t = 2n\pi$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \dots \quad \omega_1 t - \omega_2 t = 2n\pi$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

Example Frequency Pairs

$f_1 = \frac{2}{5}$	2 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{3}{5}$	3 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{4}{5}$	4 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{5}{5}$	5 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{6}{5}$	6 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{7}{5}$	7 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec
$f_1 = \frac{8}{5}$	8 cycles/5 sec		$f_2 = \frac{1}{5}$	1 cycles/5 sec

Identical Cosine Value Conditions

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$\omega_1 t - \omega_2 t = 2n\pi$$

Plotting the same valued cosine samples

```
clf
t = [0:500]/100;
```

```
n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on
```

```
subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');
```

```
subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

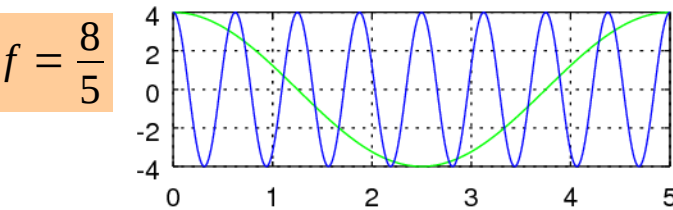
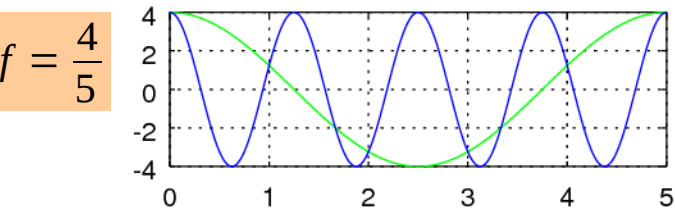
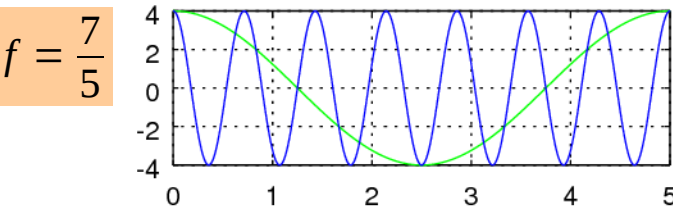
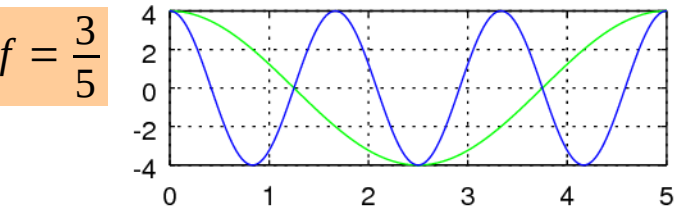
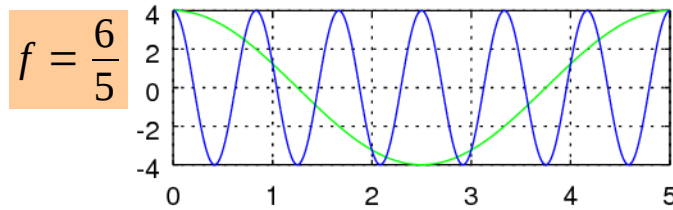
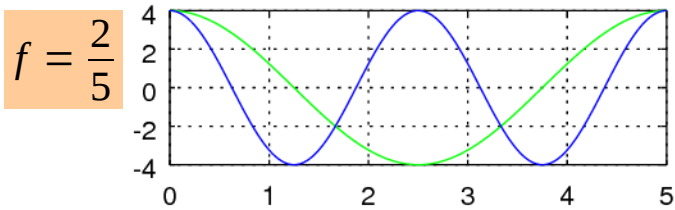
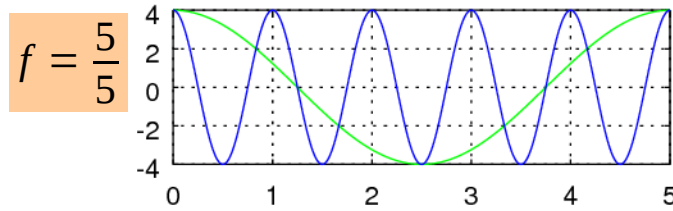
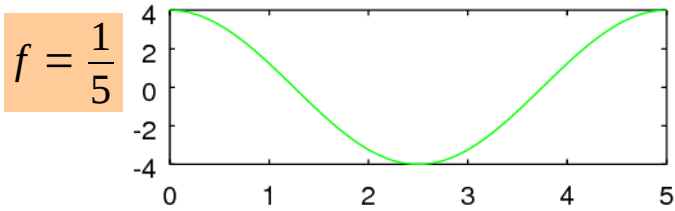
```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');
```

```
subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');
```

```
subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');
```

```
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

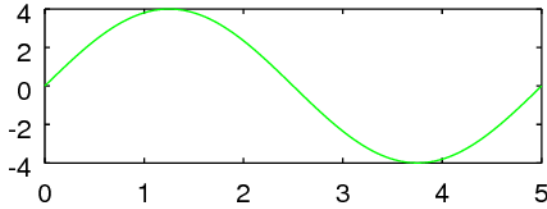

Graphs of $\cos(2\pi(n/5)t)$ & $\cos(2\pi(1/5)t)$



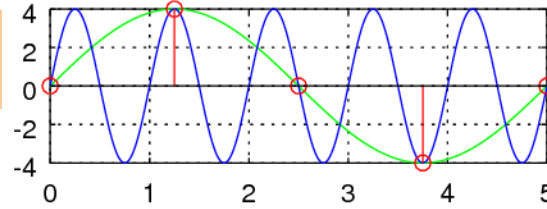
```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

Cosine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$

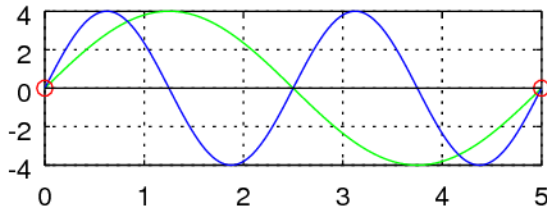
$$f = \frac{1}{5}$$



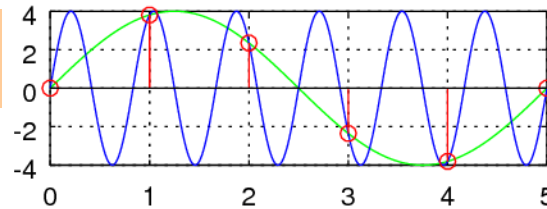
$$f = \frac{5}{5}$$



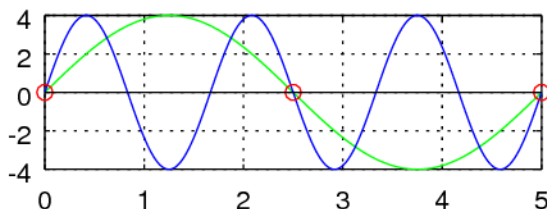
$$f = \frac{2}{5}$$



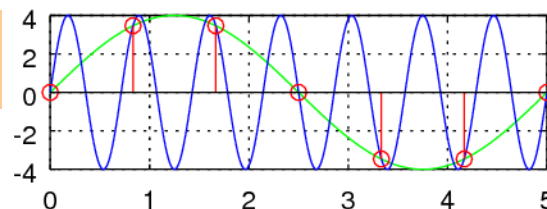
$$f = \frac{6}{5}$$



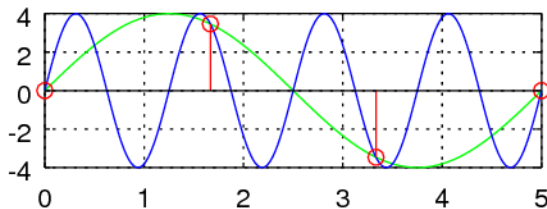
$$f = \frac{3}{5}$$



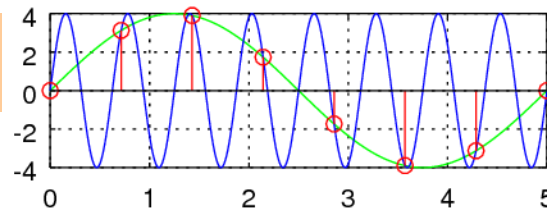
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



$$f = \frac{8}{5}$$



$$\omega_1 t + \omega_2 t = 2n\pi$$

- n2 = 0: 5/3 : 5;
- n3 = 0: 5/4 : 5;
- n4 = 0: 5/5 : 5;
- n5 = 0: 5/6 : 5;
- n6 = 0: 5/7 : 5;
- n7 = 0: 5/8 : 5;
- n8 = 0: 5/9 : 5;

$$y2 = 4 \cdot \cos(2 \cdot \pi \cdot (2/5) \cdot n2);$$

$$y3 = 4 \cdot \cos(2 \cdot \pi \cdot (3/5) \cdot n3);$$

$$y4 = 4 \cdot \cos(2 \cdot \pi \cdot (4/5) \cdot n4);$$

$$y5 = 4 \cdot \cos(2 \cdot \pi \cdot (5/5) \cdot n5);$$

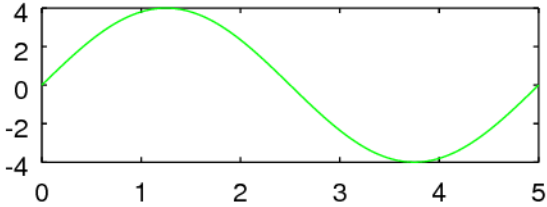
$$y6 = 4 \cdot \cos(2 \cdot \pi \cdot (6/5) \cdot n6);$$

$$y7 = 4 \cdot \cos(2 \cdot \pi \cdot (7/5) \cdot n7);$$

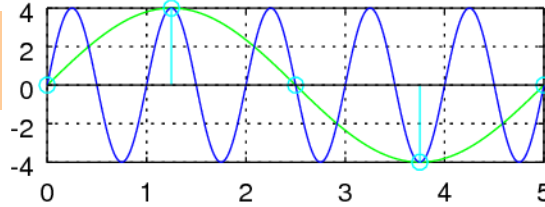
$$y8 = 4 \cdot \cos(2 \cdot \pi \cdot (8/5) \cdot n8);$$

Cosine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

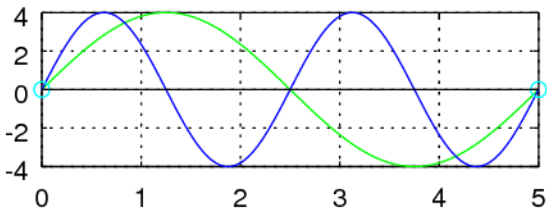
$$f = \frac{1}{5}$$



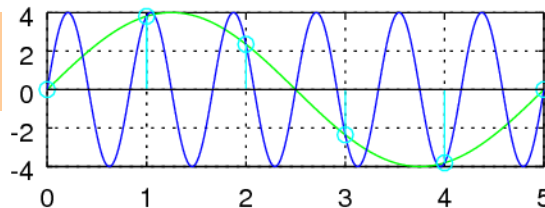
$$f = \frac{5}{5}$$



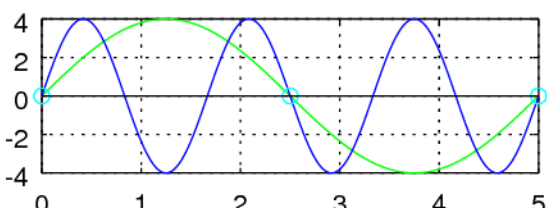
$$f = \frac{2}{5}$$



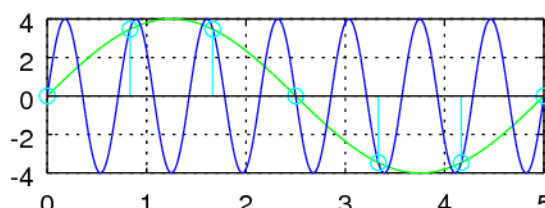
$$f = \frac{6}{5}$$



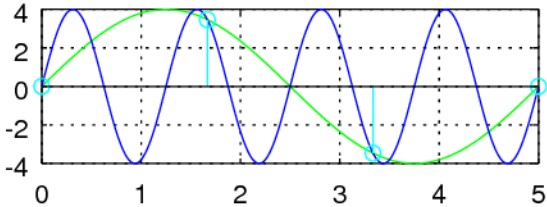
$$f = \frac{3}{5}$$



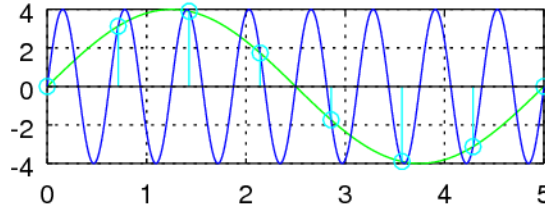
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



$$f = \frac{8}{5}$$



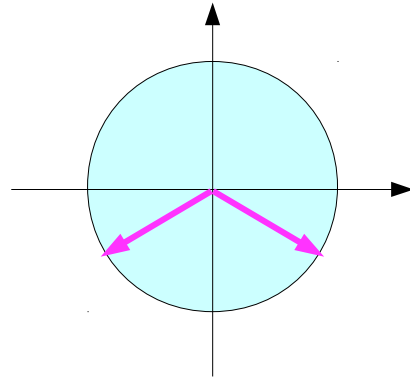
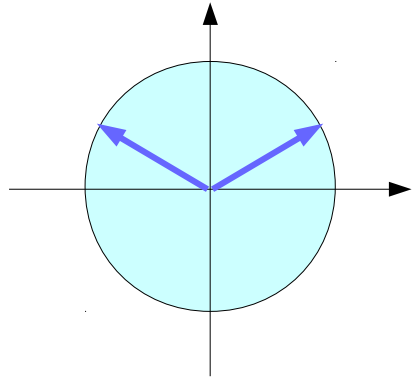
$$\omega_1 t - \omega_2 t = 2n\pi$$

- n2 = 0: 5/1 : 5;
- n3 = 0: 5/2 : 5;
- n4 = 0: 5/3 : 5;
- n5 = 0: 5/4 : 5;
- n6 = 0: 5/5 : 5;
- n7 = 0: 5/6 : 5;
- n8 = 0: 5/7 : 5;

- y2 = 4*cos(2*pi*(2/5)*n2);
- y3 = 4*cos(2*pi*(3/5)*n3);
- y4 = 4*cos(2*pi*(4/5)*n4);
- y5 = 4*cos(2*pi*(5/5)*n5);
- y6 = 4*cos(2*pi*(6/5)*n6);
- y7 = 4*cos(2*pi*(7/5)*n7);
- y8 = 4*cos(2*pi*(8/5)*n8);

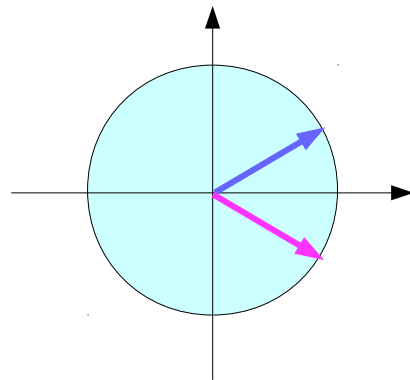
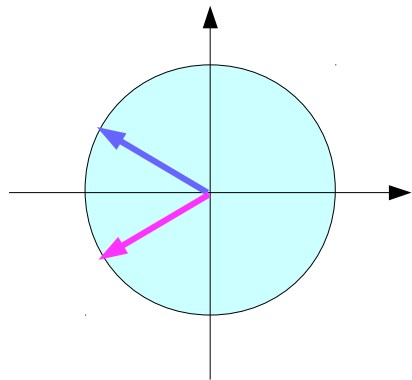
((()))

$2\pi f_1 t + 2\pi f_2 t = 2n\pi, (2n+1)\pi$ Conditions



$$\omega_1 t + \omega_2 t = (2n+1)\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$



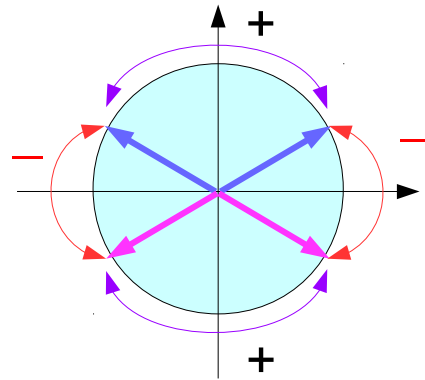
$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\sin(2\pi f_1 t) = \pm \sin(2\pi f_2 t)$$

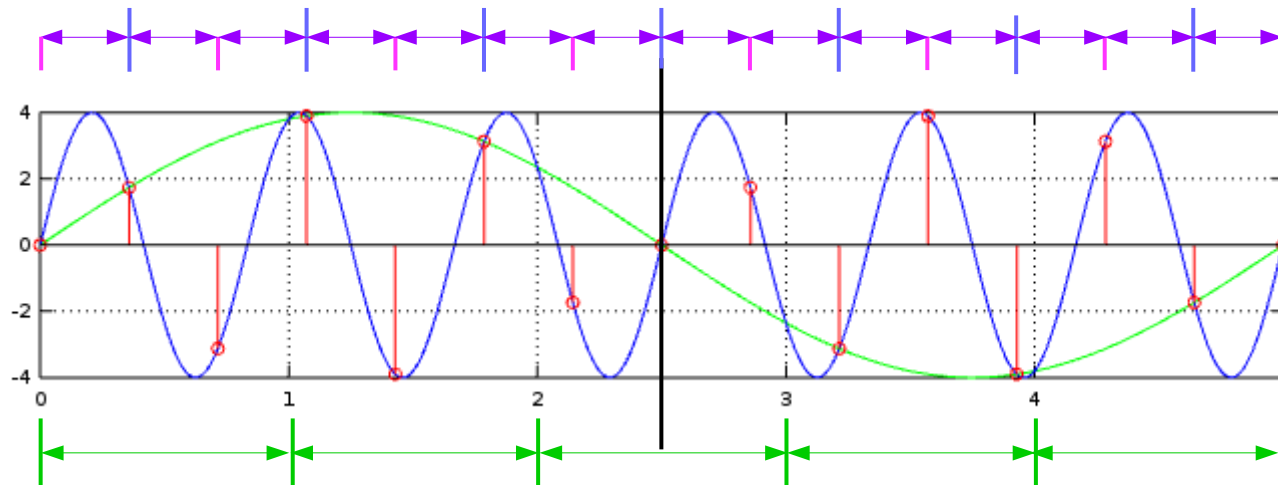
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\begin{aligned} \omega_1 t - \omega_2 t &= 2n\pi \\ \omega_1 t + \omega_2 t &= n\pi \end{aligned}$$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = \frac{n}{2} \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases} \quad \begin{cases} \frac{14}{5}t = n \\ \frac{5}{5}t = n \end{cases}$$

$$\pm \sin(\omega_1 t) = \sin(\omega_2 t)$$



$$\frac{5}{14}, \frac{10}{14}, \frac{15}{14}, \dots \quad \omega_1 t + \omega_2 t = n\pi$$

$$\pm \sin(\omega_1 t) = \sin(\omega_2 t)$$

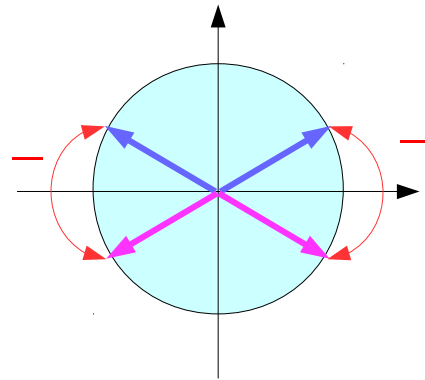
$$1, 2, 3, \dots \quad \omega_1 t - \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\sin(2\pi f_1 t) = -\sin(2\pi f_2 t)$$

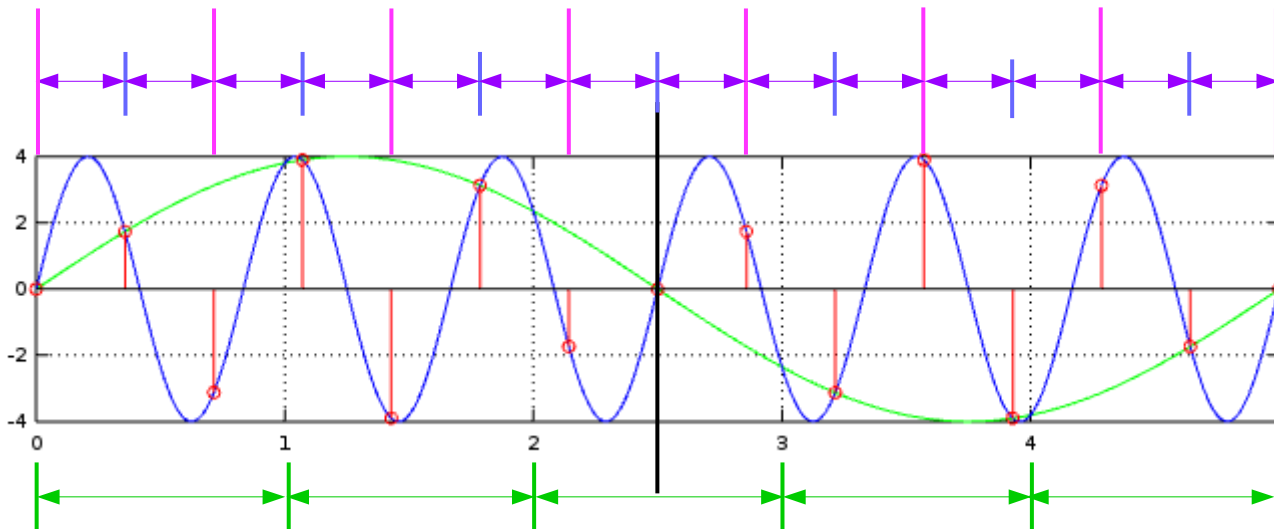
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$\omega_1 t - \omega_2 t = 2n\pi$
$\omega_1 t + \omega_2 t = 2n\pi$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = n \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases} \quad \begin{cases} \frac{7}{5}t = n \\ \frac{5}{5}t = n \end{cases}$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$



$$\frac{5}{7}, \frac{10}{7}, \frac{15}{7}, \dots \quad \omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$1, 2, 3, \dots \quad \omega_1 t - \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

Plotting the same valued sine samples

```
clf  
t = [0:500]/100;
```

```
n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;  
n3 = 0: 5/4 : 5;  
n4 = 0: 5/5 : 5;  
n5 = 0: 5/6 : 5;  
n6 = 0: 5/7 : 5;  
n7 = 0: 5/8 : 5;  
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;  
n3 = 0: 5/2 : 5;  
n4 = 0: 5/3 : 5;  
n5 = 0: 5/4 : 5;  
n6 = 0: 5/5 : 5;  
n7 = 0: 5/6 : 5;  
n8 = 0: 5/7 : 5;
```

```
yt1 = -4*sin(2*pi*(1/5)*t);  
yt2 = 4*sin(2*pi*(2/5)*t);  
yt3 = 4*sin(2*pi*(3/5)*t);  
yt4 = 4*sin(2*pi*(4/5)*t);  
yt5 = 4*sin(2*pi*(5/5)*t);  
yt6 = 4*sin(2*pi*(6/5)*t);  
yt7 = 4*sin(2*pi*(7/5)*t);  
yt8 = 4*sin(2*pi*(8/5)*t);
```

```
y2 = 4*sin(2*pi*(2/5)*n2);  
y3 = 4*sin(2*pi*(3/5)*n3);  
y4 = 4*sin(2*pi*(4/5)*n4);  
y5 = 4*sin(2*pi*(5/5)*n5);  
y6 = 4*sin(2*pi*(6/5)*n6);  
y7 = 4*sin(2*pi*(7/5)*n7);  
y8 = 4*sin(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);  
plot(t, yt1, 'g'); hold on
```

```
subplot(4,2,3);  
plot(t, yt1, 'g'); hold on  
plot(t, yt2, 'b'); grid on  
stem(n2, y2, 'r');
```

```
subplot(4,2,5);  
plot(t, yt1, 'g'); hold on  
plot(t, yt3, 'b'); grid on  
stem(n3, y3, 'r');
```

```
subplot(4,2,7);  
plot(t, yt1, 'g'); hold on  
plot(t, yt4, 'b'); grid on  
stem(n4, y4, 'r');
```

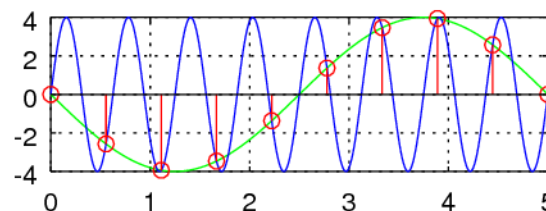
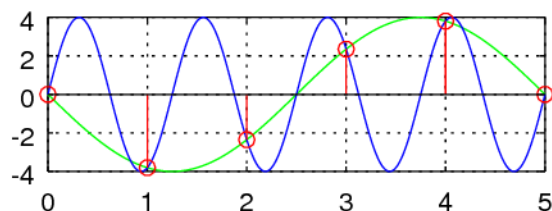
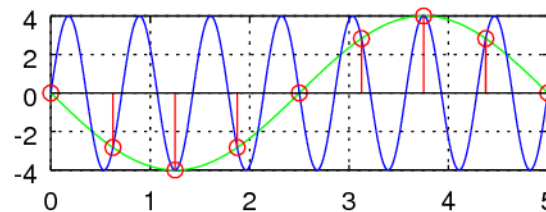
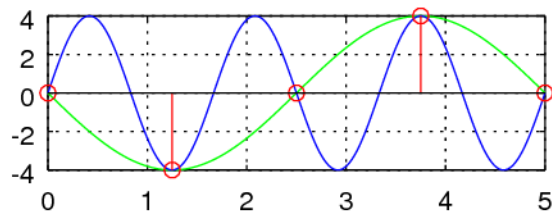
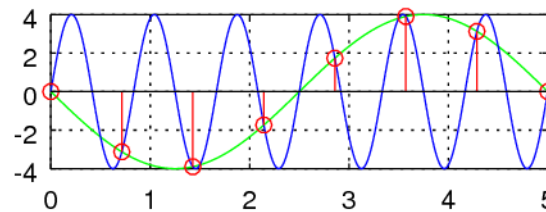
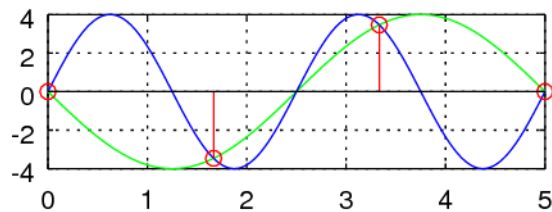
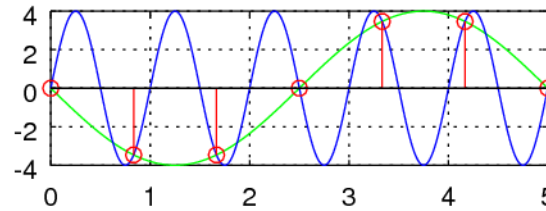
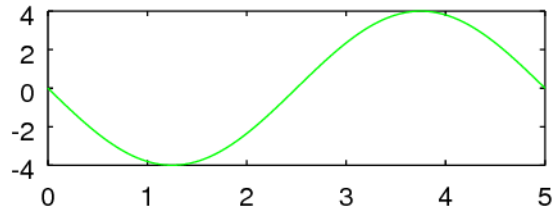
```
subplot(4,2,2);  
plot(t, yt1, 'g'); hold on  
plot(t, yt5, 'b'); grid on  
stem(n5, y5, 'r');
```

```
subplot(4,2,4);  
plot(t, yt1, 'g'); hold on  
plot(t, yt6, 'b'); grid on  
stem(n6, y6, 'r');
```

```
subplot(4,2,6);  
plot(t, yt1, 'g'); hold on  
plot(t, yt7, 'b'); grid on  
stem(n7, y7, 'r');
```

```
subplot(4,2,8);  
plot(t, yt1, 'g'); hold on  
plot(t, yt8, 'b'); grid on  
stem(n8, y8, 'r');
```

Sine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$



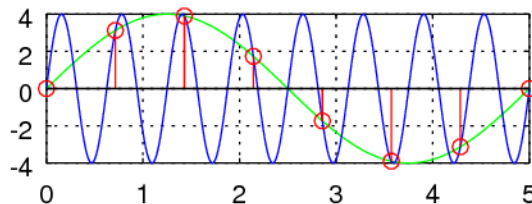
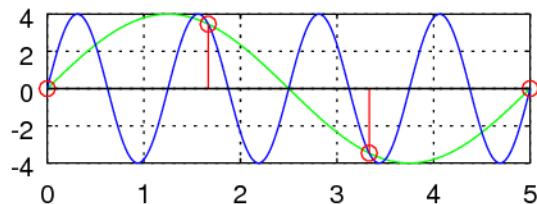
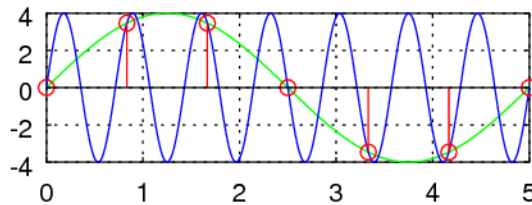
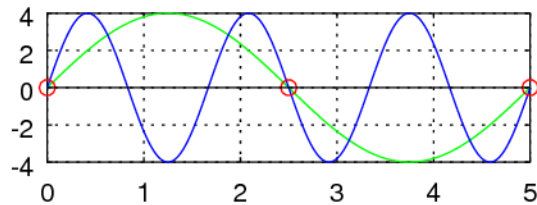
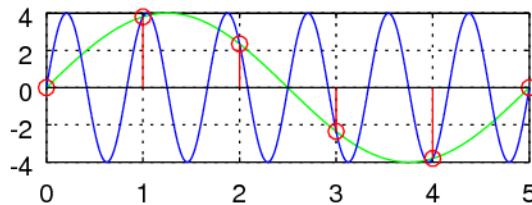
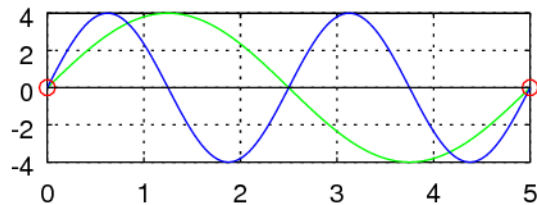
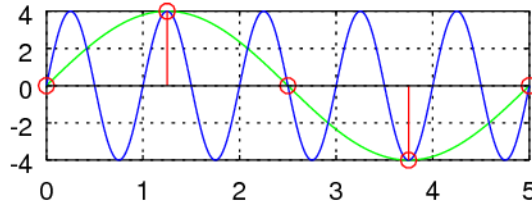
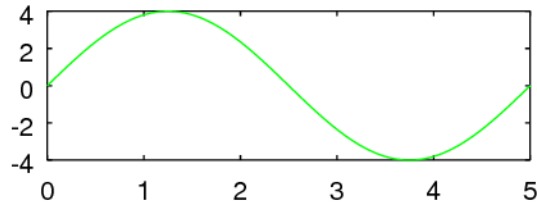
$$\omega_1 t + \omega_2 t = 2n\pi$$

- $n2 = 0: 5/3 : 5;$
- $n3 = 0: 5/4 : 5;$
- $n4 = 0: 5/5 : 5;$
- $n5 = 0: 5/6 : 5;$
- $n6 = 0: 5/7 : 5;$
- $n7 = 0: 5/8 : 5;$
- $n8 = 0: 5/9 : 5;$

- $y2 = 4*\sin(2*\pi*(2/5)*n2);$
- $y3 = 4*\sin(2*\pi*(3/5)*n3);$
- $y4 = 4*\sin(2*\pi*(4/5)*n4);$
- $y5 = 4*\sin(2*\pi*(5/5)*n5);$
- $y6 = 4*\sin(2*\pi*(6/5)*n6);$
- $y7 = 4*\sin(2*\pi*(7/5)*n7);$
- $y8 = 4*\sin(2*\pi*(8/5)*n8);$

$$yt1 = -4*\sin(2*\pi*(1/5)*t);$$

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$



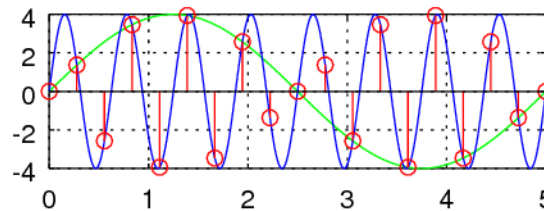
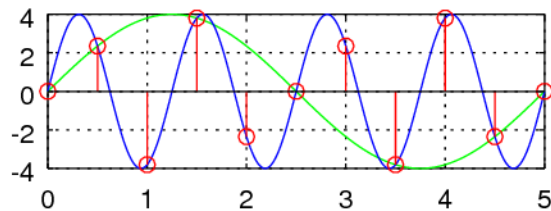
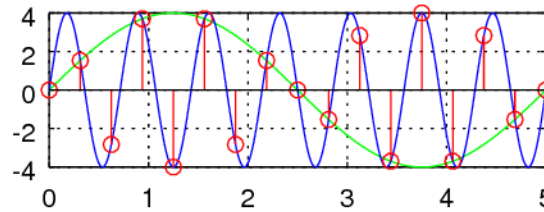
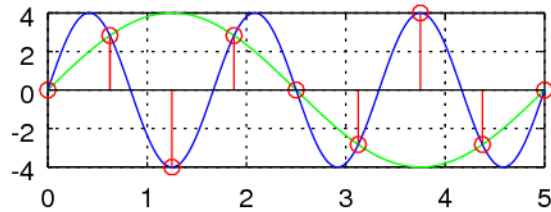
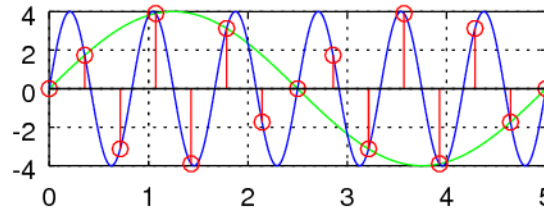
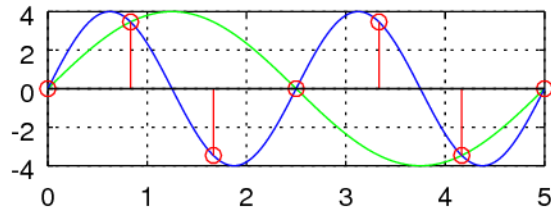
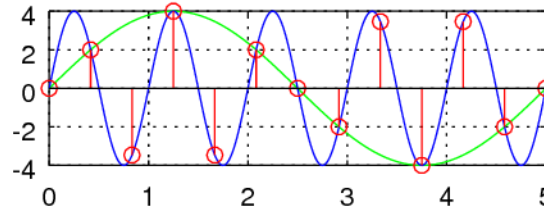
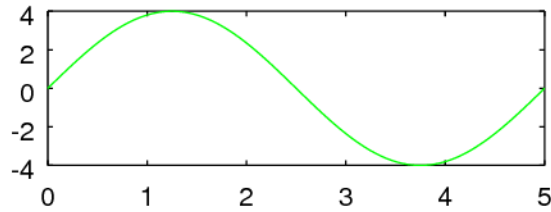
$$\omega_1 t - \omega_2 t = 2n\pi$$

- n2 = 0: 5/1 : 5;
- n3 = 0: 5/2 : 5;
- n4 = 0: 5/3 : 5;
- n5 = 0: 5/4 : 5;
- n6 = 0: 5/5 : 5;
- n7 = 0: 5/6 : 5;
- n8 = 0: 5/7 : 5;

- y2 = 4*sin(2*pi*(2/5)*n2);
- y3 = 4*sin(2*pi*(3/5)*n3);
- y4 = 4*sin(2*pi*(4/5)*n4);
- y5 = 4*sin(2*pi*(5/5)*n5);
- y6 = 4*sin(2*pi*(6/5)*n6);
- y7 = 4*sin(2*pi*(7/5)*n7);
- y8 = 4*sin(2*pi*(8/5)*n8);

$$yt1 = +4*\sin(2*\pi*(1/5)*t);$$

Sine values at $2\pi f_1 t + 2\pi f_2 t = n\pi$



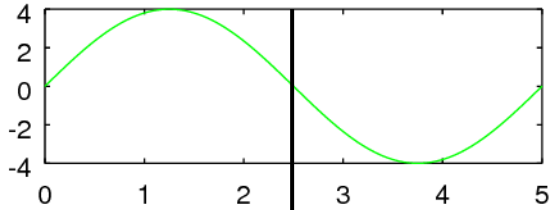
$$\omega_1 t + \omega_2 t = 2n\pi$$

- $n2 = 0: (1/2)5/3 : 5;$
- $n3 = 0: (1/2)5/4 : 5;$
- $n4 = 0: (1/2)5/5 : 5;$
- $n5 = 0: (1/2)5/6 : 5;$
- $n6 = 0: (1/2)5/7 : 5;$
- $n7 = 0: (1/2)5/8 : 5;$
- $n8 = 0: (1/2)5/9 : 5;$

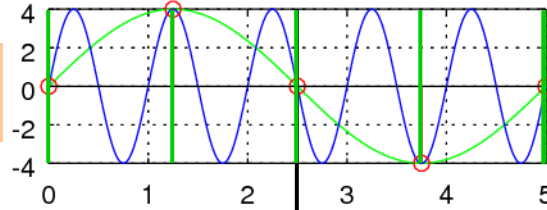
- $y2 = 4*\sin(2*\pi*(2/5)*n2);$
- $y3 = 4*\sin(2*\pi*(3/5)*n3);$
- $y4 = 4*\sin(2*\pi*(4/5)*n4);$
- $y5 = 4*\sin(2*\pi*(5/5)*n5);$
- $y6 = 4*\sin(2*\pi*(6/5)*n6);$
- $y7 = 4*\sin(2*\pi*(7/5)*n7);$
- $y8 = 4*\sin(2*\pi*(8/5)*n8);$

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

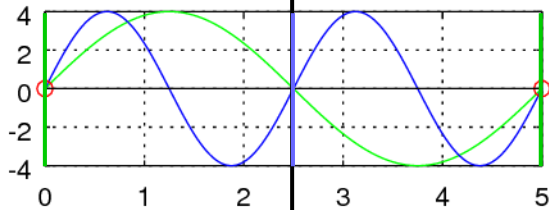
$$f = \frac{1}{5}$$



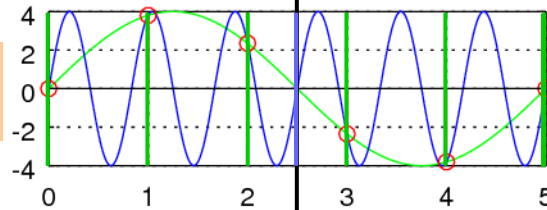
$$f = \frac{5}{5}$$



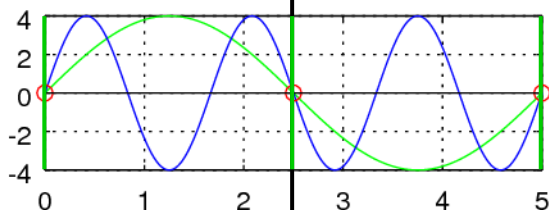
$$f = \frac{2}{5}$$



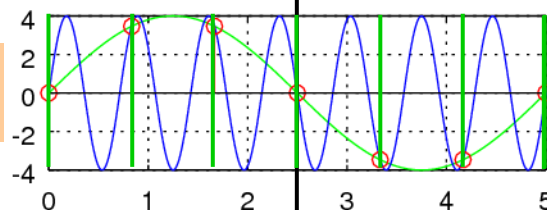
$$f = \frac{6}{5}$$



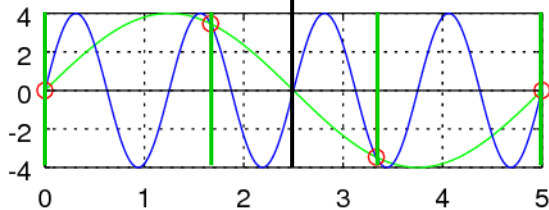
$$f = \frac{3}{5}$$



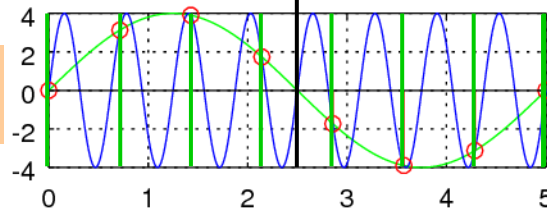
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



$$f = \frac{8}{5}$$



$$\omega_1 t - \omega_2 t = 2n\pi$$

- n2 = 0: 5/1 : 5;
- n3 = 0: 5/2 : 5;
- n4 = 0: 5/3 : 5;
- n5 = 0: 5/4 : 5;
- n6 = 0: 5/5 : 5;
- n7 = 0: 5/6 : 5;
- n8 = 0: 5/7 : 5;

$$y2 = 4 * \sin(2 * \pi * (2/5) * n2);$$

$$y3 = 4 * \sin(2 * \pi * (3/5) * n3);$$

$$y4 = 4 * \sin(2 * \pi * (4/5) * n4);$$

$$y5 = 4 * \sin(2 * \pi * (5/5) * n5);$$

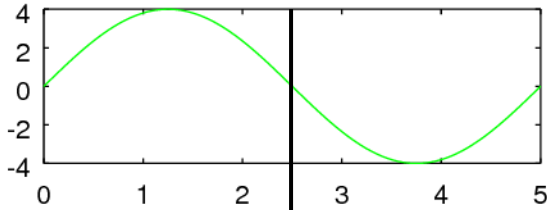
$$y6 = 4 * \sin(2 * \pi * (6/5) * n6);$$

$$y7 = 4 * \sin(2 * \pi * (7/5) * n7);$$

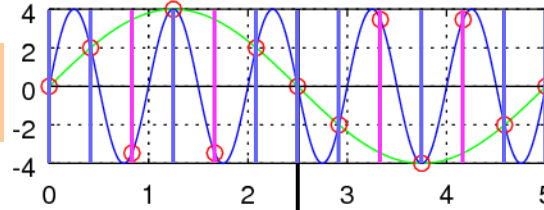
$$y8 = 4 * \sin(2 * \pi * (8/5) * n8);$$

Aliasing Condition Examples (1)

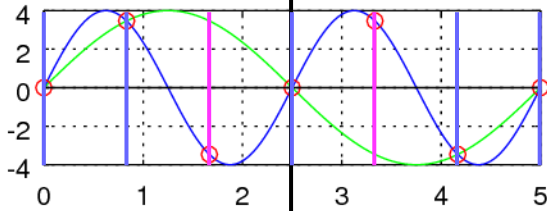
$$f = \frac{1}{5}$$



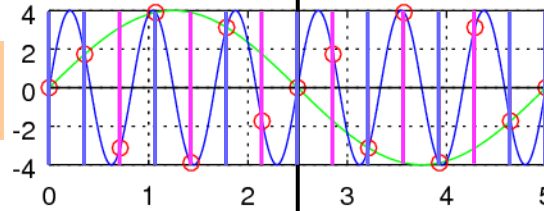
$$f = \frac{5}{5}$$



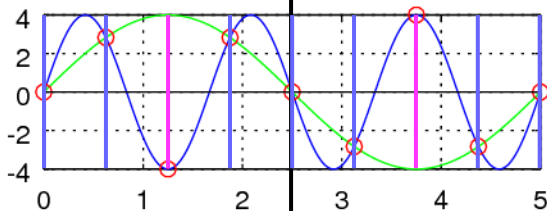
$$f = \frac{2}{5}$$



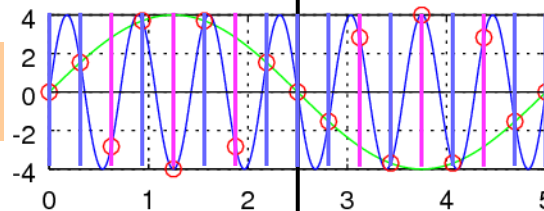
$$f = \frac{6}{5}$$



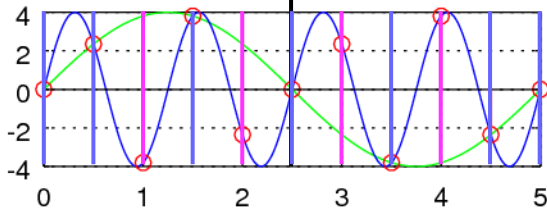
$$f = \frac{3}{5}$$



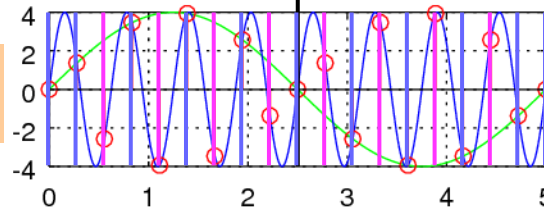
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



$$f = \frac{8}{5}$$

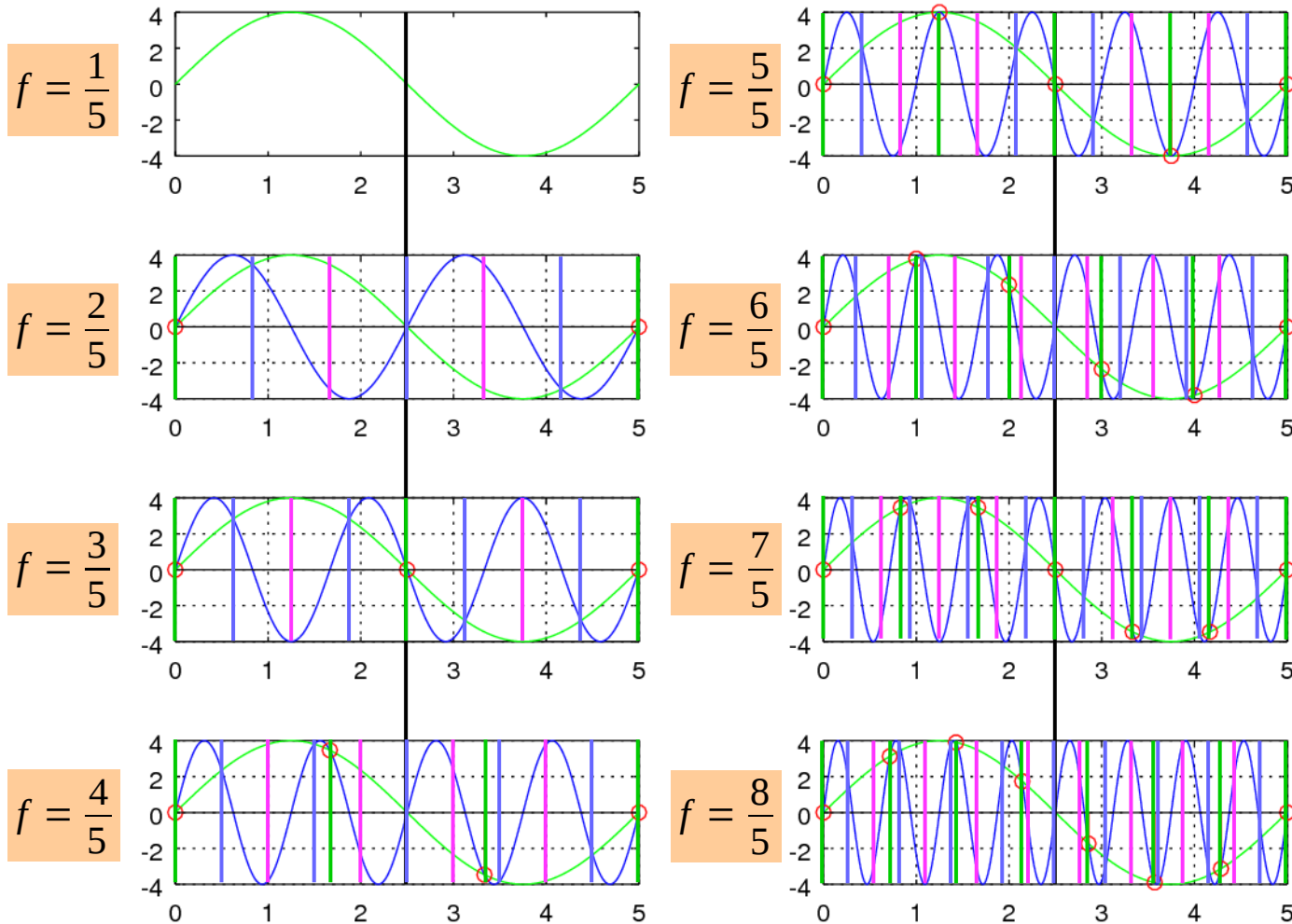


$$\omega_1 t + \omega_2 t = 2n\pi$$

$$\begin{aligned} n2 = 0: & (1/2)5/3 : 5; \\ n3 = 0: & (1/2)5/4 : 5; \\ n4 = 0: & (1/2)5/5 : 5; \\ n5 = 0: & (1/2)5/6 : 5; \\ n6 = 0: & (1/2)5/7 : 5; \\ n7 = 0: & (1/2)5/8 : 5; \\ n8 = 0: & (1/2)5/9 : 5; \end{aligned}$$

$$\begin{aligned} y2 &= 4 * \sin(2 * \pi * (2/5) * n2); \\ y3 &= 4 * \sin(2 * \pi * (3/5) * n3); \\ y4 &= 4 * \sin(2 * \pi * (4/5) * n4); \\ y5 &= 4 * \sin(2 * \pi * (5/5) * n5); \\ y6 &= 4 * \sin(2 * \pi * (6/5) * n6); \\ y7 &= 4 * \sin(2 * \pi * (7/5) * n7); \\ y8 &= 4 * \sin(2 * \pi * (8/5) * n8); \end{aligned}$$

Aliasing Condition Examples (2)



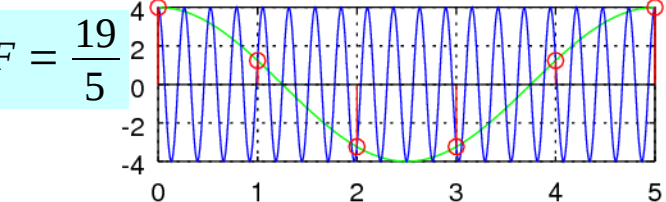
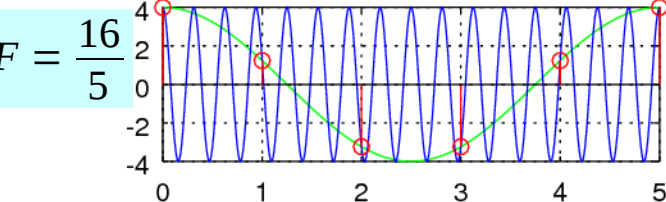
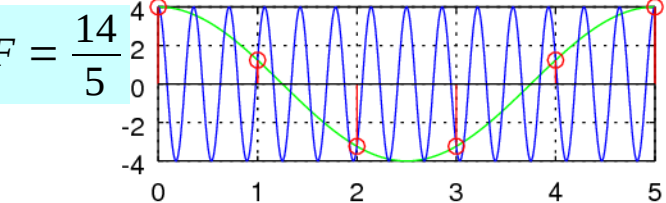
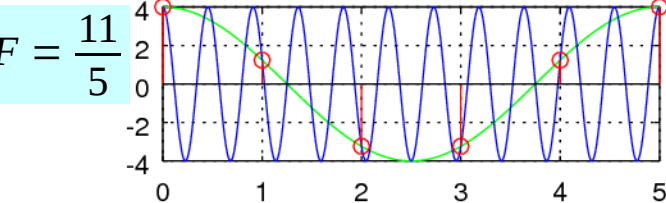
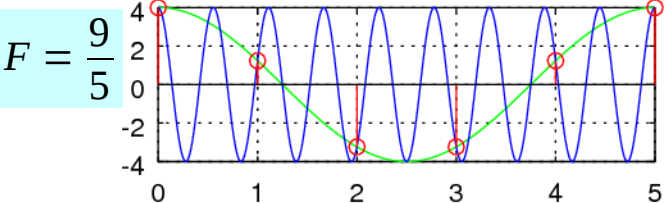
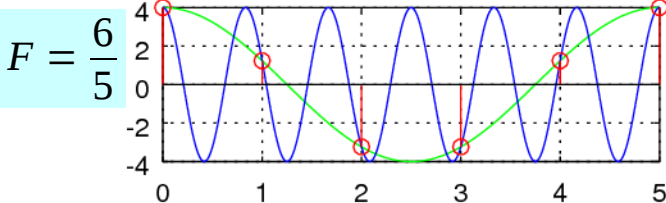
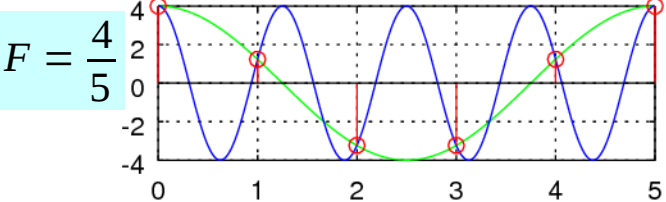
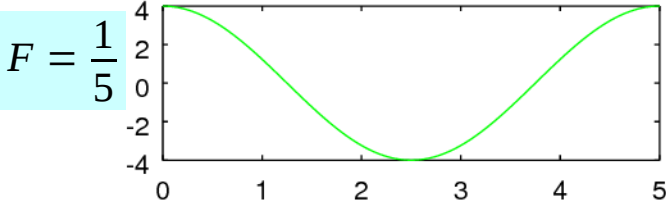
$$\omega_1 t - \omega_2 t = 2n\pi$$

- $n_2 = 0: 5/1 : 5;$
- $n_3 = 0: 5/2 : 5;$
- $n_4 = 0: 5/3 : 5;$
- $n_5 = 0: 5/4 : 5;$
- $n_6 = 0: 5/5 : 5;$
- $n_7 = 0: 5/6 : 5;$
- $n_8 = 0: 5/7 : 5;$

- $y_2 = 4 * \sin(2 * \pi * (2/5) * n_2);$
- $y_3 = 4 * \sin(2 * \pi * (3/5) * n_3);$
- $y_4 = 4 * \sin(2 * \pi * (4/5) * n_4);$
- $y_5 = 4 * \sin(2 * \pi * (5/5) * n_5);$
- $y_6 = 4 * \sin(2 * \pi * (6/5) * n_6);$
- $y_7 = 4 * \sin(2 * \pi * (7/5) * n_7);$
- $y_8 = 4 * \sin(2 * \pi * (8/5) * n_8);$

((()))

Aliasing and Folding



Plotting of Aliasing & Folding Frequencies

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;

y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);

subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

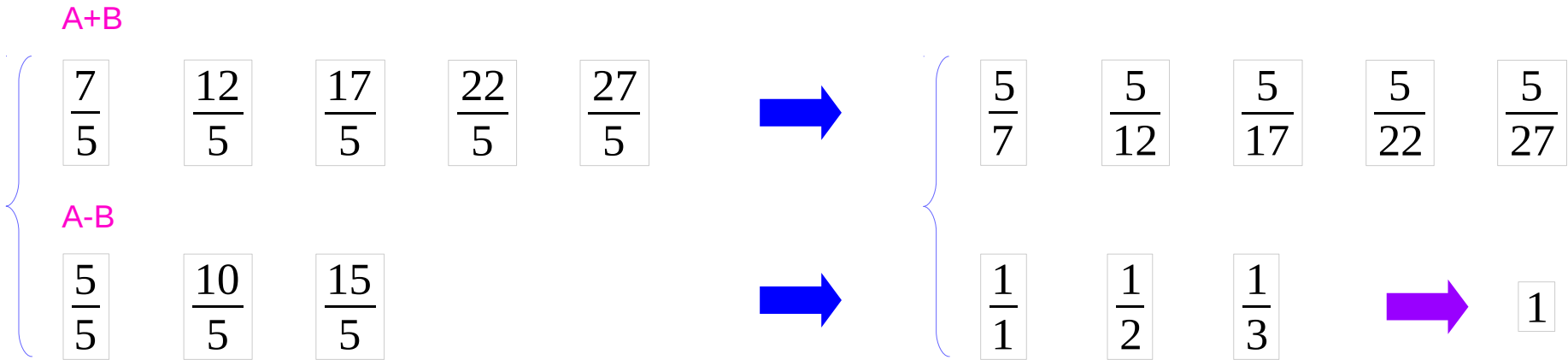
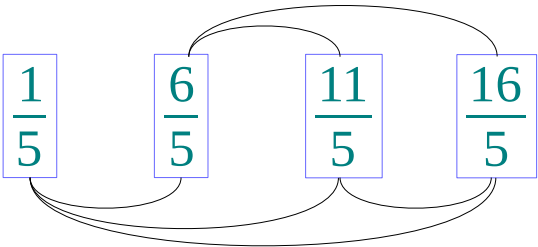
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

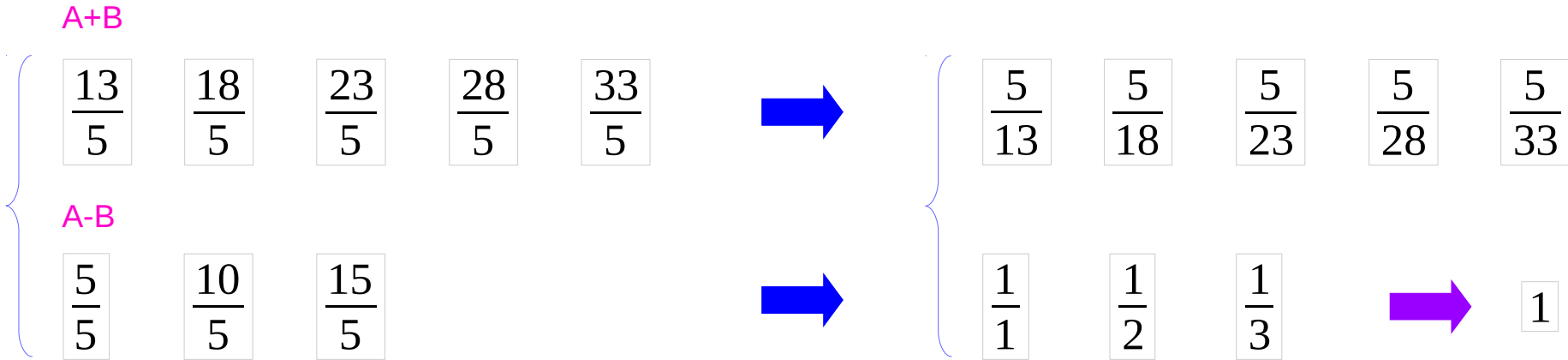
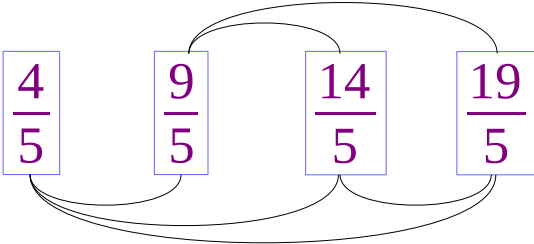
subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

Aliasing Frequencies (1)

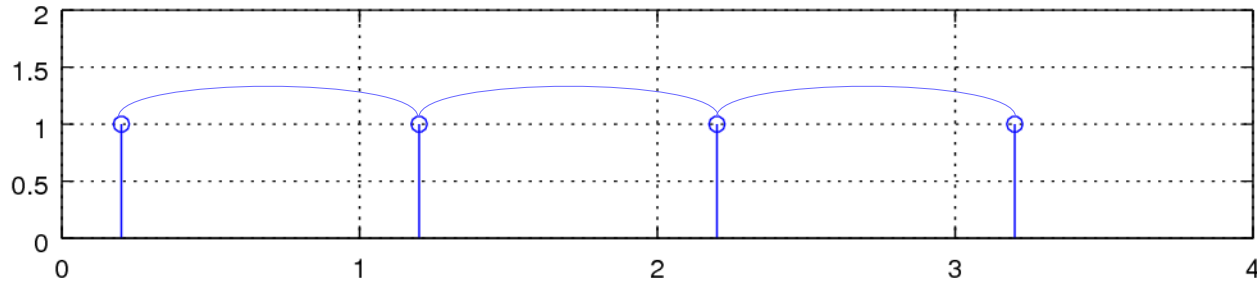


Aliasing Frequencies (2)



Aliasing and Folding Frequencies

Aliasing frequencies

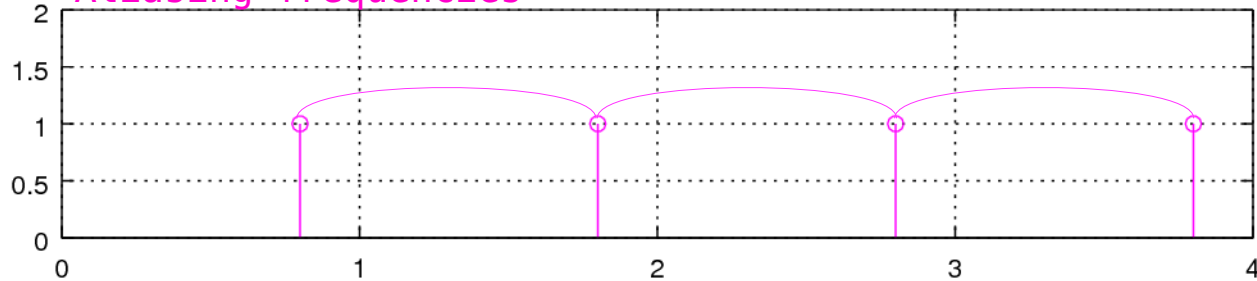


```
n1 = [1/5, 6/5, 11/5, 16/5];  
n2 = [4/5, 9/5, 14/5, 19/5];
```

```
y1 = [1, 1, 1, 1];  
y2 = [1, 1, 1, 1];
```

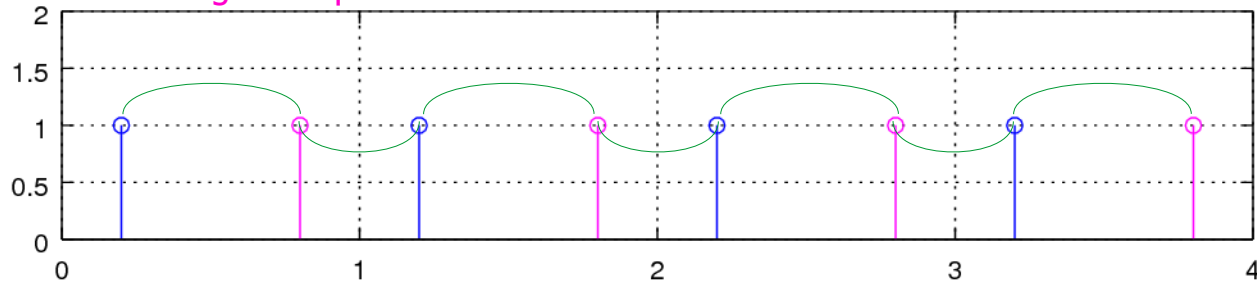
```
subplot(3, 1, 1)  
stem(n1, y1, 'b'); grid on;  
axis([0, 4, 0, 2]);
```

Aliasing frequencies



```
subplot(3, 1, 2)  
stem(n2, y2, 'm'); grid on;  
axis([0, 4, 0, 2]);
```

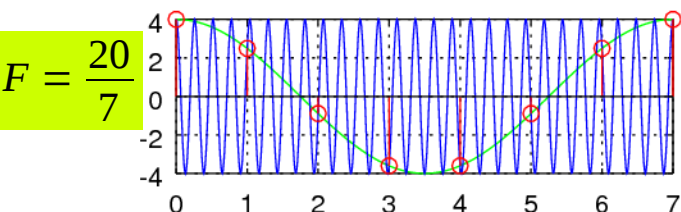
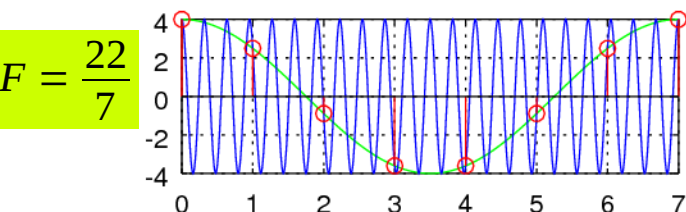
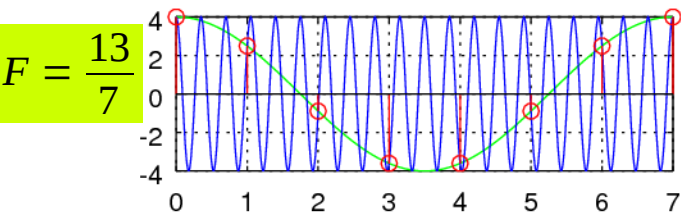
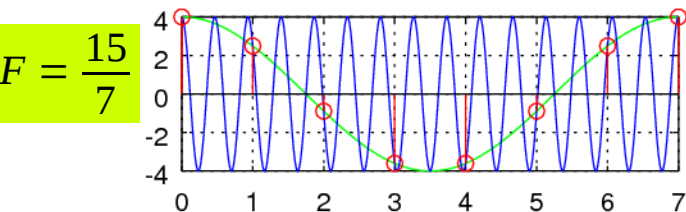
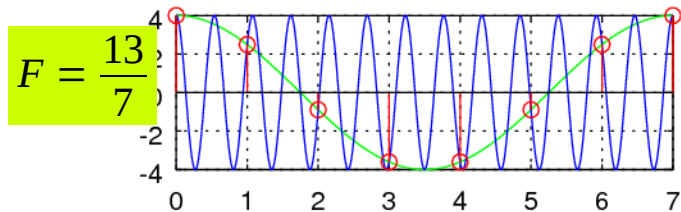
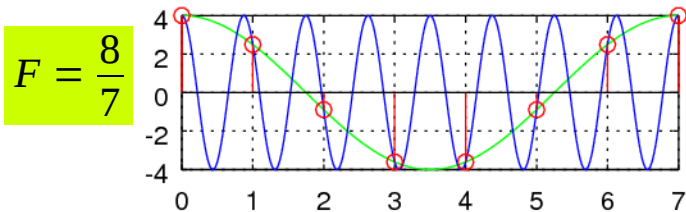
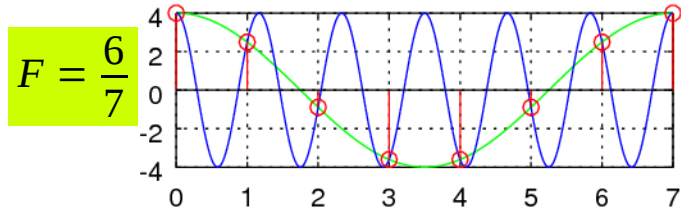
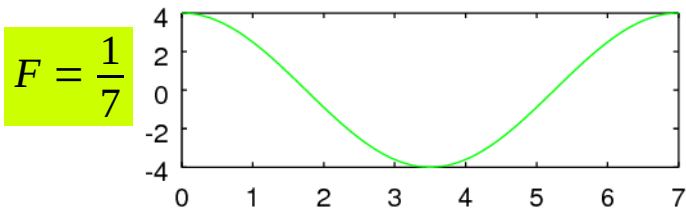
Folding frequencies



```
subplot(3, 1, 3)  
stem(n1, y1, 'b'); hold on;  
stem(n2, y2, 'm'); grid on;  
axis([0, 4, 0, 2]);
```

J.H. McClellan, et al., Signal Processing First

Graphs of $\cos(2\pi(n/7)t)$ & $\cos(2\pi(1/7)t)$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann