Laurent Series and z-Transform
- Geometric Series
Double Pole Examples A

20180308

2 formulas of z

① $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$\Big\Updownarrow\ z^{-1}$$

② $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$\boxed{\frac{-1}{(z-1)(z-2)} \overset{z^{-1}}{\longleftrightarrow} \frac{-0.5z^2}{(z-1)(z-0.5)}}$$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \left(\frac{1}{z^{-1}-1} - \frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{z}{1-z} - \frac{z}{1-2z}\right)$$

$$= \left(\frac{-z}{z-1} + \frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-1}{z-1} + \frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-0.5z}{(z-1)(z-0.5)}\right)$$

$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

Ⓐ $f(z)$ Ⓑ $X(z)$

① $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

①-Ⓐ $|z| < 1$ $$f(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$f(z)$

$|z| > 2$ $$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

①-Ⓑ $|z| < 1$ $$X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$X(z)$

$|z| > 2$ $$X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

②-Ⓐ $|z| < 0.5$ $$f(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$f(z)$

$|z| > 1$ $$f(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

②-Ⓑ $|z| < 0.5$ $$X(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$X(z)$

$|z| > 1$ $$X(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
Ⓐ $f(z)$	$\lvert z\rvert < p$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
	$\lvert z\rvert > q$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$
Ⓑ $X(z)$	$\lvert z\rvert < p$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$
	$\lvert z\rvert > q$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
	$X(z)$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$
	$X(z)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	causal $(n \geq 0)$	causal $(n \geq 1)$
$\lvert z\rvert > q$	$f(z)$	anticausal $(n<0)$	anticausal $(n<1)$
$\lvert z\rvert < p$	$X(z)$	anticausal $(n<1)$	anticausal $(n<0)$
$\lvert z\rvert > q$	$X(z)$	causal $(n \geq 1)$	causal $(n \geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	causal $(n \geq 0)$	causal $(n \geq 1)$
$\lvert z\rvert < p$	$X(z)$	anticausal $(n<1)$	anticausal $(n<0)$
$\lvert z\rvert > q$	$f(z)$	anticausal $(n<0)$	anticausal $(n<1)$
$\lvert z\rvert > q$	$X(z)$	causal $(n \geq 1)$	causal $(n \geq 0)$

I

$$① \quad \frac{-1}{(z-1)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \quad \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2}\right) \qquad \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$\boxed{z} \quad -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{z} \quad +\frac{z}{1-z} - \frac{z}{1-2z}$$

$$\boxed{|z|<1} \quad |z|<1 \quad |0.5z|<1 \qquad \boxed{|z|<0.5} \quad |z|<1 \quad |2z|<1$$

$$\boxed{z^{-1}} \quad \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{z^{-1}} \quad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$\boxed{|z|>2} \quad |z^{-1}|<1 \quad |2z^{-1}|<1 \qquad \boxed{|z|>1} \quad |z^{-1}|<1 \quad |0.5z^{-1}|<1$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad +\frac{z}{1-z} - \frac{z}{1-2z}$$

$$\cdot\frac{1}{z} \downarrow\uparrow \cdot z \qquad \cdot\frac{2}{z} \downarrow\uparrow \cdot\frac{z}{2} \qquad \cdot\frac{1}{z} \downarrow\uparrow \cdot z \qquad \cdot\frac{1}{2z} \downarrow\uparrow \cdot 2z$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

① $\dfrac{-1}{(z-1)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$\left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$\left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$\boxed{z}$ $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$

$\boxed{|z|<1}$ $f(z)$ causal $(n \geq 0)$

$\boxed{z}$ $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$

$X(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

$\boxed{|z|>1}$

$X(z)$ causal $(n \geq 0)$

$$\text{①}\ \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \text{②}\ \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2}\right) \qquad \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$\boxed{z}$ $\quad -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ $\qquad$ $\boxed{z}$ $\quad +\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

$\boxed{|z|<1}$ $\quad f(z)$ causal $(n \geq 0)$ $\qquad$ $\boxed{|z|<0.5}$ $\quad f(z)$ causal $(n \geq 1)$

$X(z)$ anticausal $(n \leq 0)$ $\qquad$ $X(z)$ anticausal $(n \leq -1)$

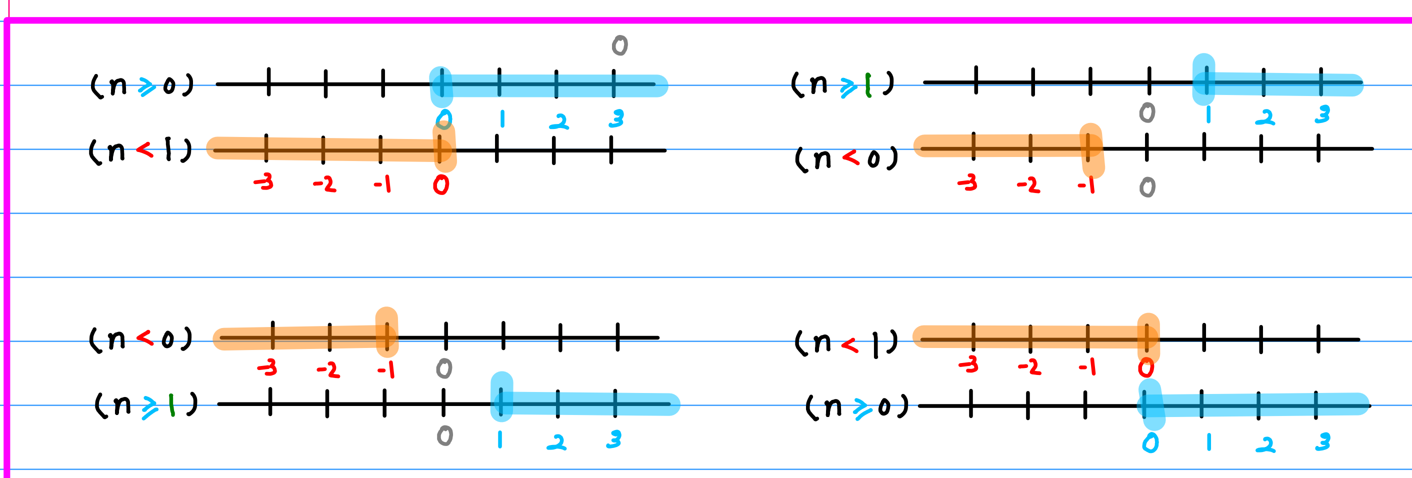
$\boxed{z^{-1}}$ $\quad \dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\qquad$ $\boxed{z^{-1}}$ $\quad -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

$\boxed{|z|>2}$ $\quad f(z)$ anticausal $(n \leq -1)$ $\qquad$ $\boxed{|z|>1}$ $\quad f(z)$ anticausal $(n \leq 0)$

$X(z)$ causal $(n \geq 1)$ $\qquad$ $X(z)$ causal $(n \geq 0)$

$$(n \geq 0) \xrightarrow{-n} (n \leq 0) \equiv (n < 1)$$

$$(n \geq 1) \xrightarrow{-n} (n \leq -1) \equiv (n < 0)$$



① $\dfrac{-1}{(z-1)(z-2)}$ $\xleftrightarrow{\;z^{-1}\;}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$|z| < 0.5$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \quad -2^{n-1}$$

$|z| > 2$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \quad -2^{n-1}$$

$*\; n = \quad 1 \quad 2 \quad 3$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$*\; n = \quad 0 \quad 1 \quad 2$

V

① $\dfrac{-1}{(z-1)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$|z|<1$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] \quad -1^{n-1}$$
$$+\left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right] \quad +2^{n-1}$$

$n=$ 0 -1 -2

$|z|<0.5$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \quad -2^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^{-1} z^1 + \left(\tfrac{1}{1}\right)^{-2} z^2 + \left(\tfrac{1}{1}\right)^{-3} z^3 + \cdots\right] \quad +1^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$n=$ -1 -2 -3

$\boxed{2 = \left(\tfrac{1}{2}\right)^{-1} \quad \left(\tfrac{1}{2}\right) = 2^{-1}}$

$|z|>2$

$$\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^{-1} z^{-2} + \left(\tfrac{1}{1}\right)^{-2} z^{-3} + \cdots\right] \quad +1^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \quad -2^{n-1}$$

$n=$ 1 2 3

$|z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] \quad -1^{n-1}$$
$$+\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] \quad +2^{n-1}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$n=$ 0 1 2

$$① \ \frac{-1}{(z-1)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

$|z| < 0.5$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^{-1} z^1 + \left(\tfrac{1}{1}\right)^{-2} z^2 + \left(\tfrac{1}{1}\right)^{-3} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$|z| > 2$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^{-1} z^{-2} + \left(\tfrac{1}{1}\right)^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$\mathrm{ROC}(z) \qquad f(z) \Longleftrightarrow a_n \qquad \mathrm{RNG}(n)$$

$$|z| < p \qquad\qquad n \geq 0$$

I

$$\boxed{\begin{array}{llll} \mathrm{ROC}(z^{-1}) & f(z) \Longleftrightarrow -a_n & \overline{\mathrm{RNG}}(n) \\ |z| > \frac{1}{p} & & n < 0 \end{array}}$$

$$\mathrm{ROC}(z) \qquad f(z) \Longleftrightarrow a_n \qquad \mathrm{RNG}(n)$$

$$|z| < p \qquad\qquad n \geq 0$$

II

$$\boxed{\begin{array}{llll} \mathrm{ROC}(z^{-1}) & f(z^{-1}) \Longleftrightarrow a_{-n} & \mathrm{RNG}(-n) \\ |z| > \frac{1}{p} & & n < 1 \end{array}}$$

$$\mathrm{ROC}(z) \qquad f(z) \Longleftrightarrow a_n \qquad \mathrm{RNG}(n)$$

$$|z| < p \qquad\qquad n \geq 0$$

III

$$\boxed{\begin{array}{llll} \mathrm{ROC}(z) & f(z^{-1}) \Longleftrightarrow -a_{-n} & \ll \mathrm{RNG}(n) \gg \\ |z| < p & & n \geq 1 \end{array}}$$

I + II

$$\mathrm{ROC}(z) \qquad f(z) \Longleftrightarrow a_n \qquad \mathrm{RNG}(n)$$

$$|z| < p \qquad\qquad n \geq 0$$

IV

$$\boxed{\begin{array}{llll} \mathrm{ROC}(z) & X(z) \Longleftrightarrow a_{-n} & \mathrm{RNG}(-n) \\ |z| < p & & n < 1 \end{array}}$$

$$(\mathrm{III}) = (\mathrm{I}) + (\mathrm{II})$$

(III)

ROC(z)	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	« RNG(n) »	(I) + (II)
$\lvert z \rvert < p$				$n \geq 1$	

ROC(z)	$f(z)$	$\Longleftrightarrow$	a_n	RNG(n)
$\lvert z \rvert < p$				$n \geq 0$

(I)

ROC(z^{-1})	$f(z)$	$\Longleftrightarrow$	$-a_n$	$\overline{\text{RNG}}(n)$
$\lvert z \rvert > \frac{1}{p}$				$n < 0$

$n \leq -1$

(II)

ROC(z)	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{\text{RNG}}(-n)$
$\lvert z \rvert < p$				$n \geq 1$

Compare I with IV

$\mathrm{ROC}(z) \quad f(z) \Longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad\qquad n \geq 0$

I

$\mathrm{ROC}(z^{-1}) \quad f(z) \Longleftrightarrow -a_n \quad \overline{\mathrm{RNG}}(n)$

$|z| > \frac{1}{p} \qquad\qquad n < 0$

$-1 \qquad\qquad$ complement

IV

$\mathrm{ROC}(z) \quad X(z) \Longleftrightarrow a_{-n} \quad \mathrm{RNG}(-n)$

$|z| < p \qquad\qquad n < 1$

$-n \qquad\qquad -n$

symmetrical

Ⓘ ROC(z^{-1}) $\lvert z\rvert > \frac{1}{p}$	$f(z) \Longleftrightarrow -a_n$	$\overline{RNG}(n)$ $n<0$

z^{-1}

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1} + (\frac{1}{2})^{n+1}$ $(n\geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n\geq 1)$
		$\updownarrow$ -1	$\updownarrow$ -1
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1} - (\frac{1}{2})^{n+1}$ $(n<0)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$

z^{-1}

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$			
	$X(z)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$	$+1^{n+1} - (\frac{1}{2})^{n+1}$ $(n<0)$
		$\updownarrow$ -1	$\updownarrow$ -1
$\lvert z\rvert > q$	$X(z)$	$+1^{n-1} - 2^{n-1}$ $(n\geq 1)$	$-1^{n+1} + (\frac{1}{2})^{n+1}$ $(n\geq 0)$

Ⅱ

$ROC(z^{-1})$	$f(z^{-1})$	a_{-n}	$RNG(-n)$
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

z^{-1}

		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
		$-n$	$-n$
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

z^{-1}

z^{-1}

		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$			
	$X(z)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
		$-n$	$-n$
$\lvert z\rvert > q$	$X(z)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

z^{-1}

$$2^{-n+1} = \left(\tfrac{1}{2}\right)^{n} \cdot 2 = \left(\tfrac{1}{2}\right)^{n-1} \qquad \left(\tfrac{1}{2}\right)^{-n-1} = 2^{n} \cdot 2 = 2^{n+1}$$

$$\left(\tfrac{1}{2}\right)^{-n+1} = 2^{n} \cdot \tfrac{1}{2} = 2^{n-1} \qquad 2^{-n-1} = \left(\tfrac{1}{2}\right)^{n} \cdot \tfrac{1}{2} = \left(\tfrac{1}{2}\right)^{n+1}$$

Ⅲ | ROC(z) $|z| < p$ | $f(z^{-1}) \longleftrightarrow -a_{-n}$ | RNG(−n) $n \geq 1$ | Ⅰ+Ⅱ

z^{-1}

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$|z| < p$	$f(z)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1}-2^{n-1}$ $(n \geq 1)$
		$-n,$	-1
$|z| > q$	$f(z)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-1^{n-1}+2^{n-1}$ $(n<1)$
		$-n,$	-1
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

z^{-1}

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$|z| < p$		$-n,$	-1
	$X(z)$	$-1^{n-1}+2^{n-1}$ $(n<1)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
$|z| > q$		$-n,$	-1
	$X(z)$	$+1^{n-1}-2^{n-1}$ $(n \geq 1)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

Ⅳ

ROC(z)	X(z) ⟺ a_{-n}	RNG(-n)
$\lvert z\rvert < p$		$n < 1$

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	f(z)	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-n$
	X(z)	$-1^{n-1} + 2^{n-1}$ $(n < 1)$	
$\lvert z\rvert > q$	f(z)	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$-n$
	X(z)	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	

		$(p,q)=(1,2)$	$(p,q)=(0.5,\ 1)$
		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	f(z)	$-n$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
	X(z)		$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$
$\lvert z\rvert > q$	f(z)	$-n$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$
	X(z)		$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

① − Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-(\underset{n=0}{1^1} + \underset{n=1}{1^2 z} + \underset{n=2}{1^3 z^2} + \cdots) + (\underset{n=0}{(\tfrac{1}{2})} + \underset{n=1}{(\tfrac{1}{2})^2 z} + \underset{n=2}{(\tfrac{1}{2})^3 z^2} + \cdots)$$

$|z| > 2$

$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) - (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$$

$$(\underset{n=-1}{1^0 z^{-1}} + \underset{n=-2}{1^{-1} z^{-2}} + \underset{n=-3}{1^{-2} z^{-3}} + \cdots) - (\underset{n=-1}{(\tfrac{1}{2})^0 z^{-1}} + \underset{n=-2}{(\tfrac{1}{2})^{-1} z^{-2}} + \underset{n=-3}{(\tfrac{1}{2})^{-2} z^{-3}} + \cdots)$$

② − Ⓐ

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$|z| < 0.5$

$$f(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$+(\underset{n=1}{1^0 z^1} + \underset{n=2}{1^1 z^2} + \underset{n=3}{1^2 z^3} + \cdots) - (\underset{n=1}{2^0 z} + \underset{n=2}{2^1 z^2} + \underset{n=3}{2^2 z^3} + \cdots)$$

$|z| > 2$

$$f(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$-(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots) + ((\tfrac{1}{2})^1 z^0 + (\tfrac{1}{2})^2 z^{-1} + (\tfrac{1}{2})^3 z^{-2} + \cdots)$$

$$-(\underset{n=0}{1^{-1} z^0} + \underset{n=-1}{1^{-2} z^{-1}} + \underset{n=-2}{1^{-3} z^{-2}} + \cdots) + (\underset{n=0}{2^{-1} z^0} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal causal

① - Ⓑ $\quad \dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$|z| < 0.5 \quad X(z) = -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z} \quad \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1)$

$-\left(1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$

$-\left(1^{-1} z^0 + 1^{-2} z^1 + 1^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2 \quad X(z) = \dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}} \quad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$+\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

② - Ⓑ $\quad \dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z| < 0.5 \quad X(z) = +\dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$

$+\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right)$

$+\left(1^0 z + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right) - \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 2 \quad X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \quad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$-\left(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

(I) $\boxed{\begin{array}{ll} ROC(z^{-1}) & f(z) \Longleftrightarrow -a_n \quad RNG(n) \\ |z| > \frac{1}{p} & \qquad\qquad\qquad\qquad n<0 \end{array}}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z|<0.5$ $\quad X(z) \qquad a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$

$|z|>2$ $\quad X(z) \qquad b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n\geq 1)$

$\{|z|<0.5\} \cap \{|z|>2\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z|<a \qquad X(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n<0 \quad n<1$

ROC'

$|z|>a^{-1} \qquad X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad a^{n+1} \qquad n<0 \quad n<1 \quad n\geq 0 \quad n\geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} = -\sum_{k=-1}^{-\infty} a^{k+1} z^{k}$$

$$\frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

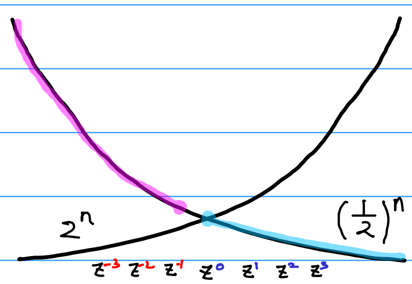
$$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n} = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

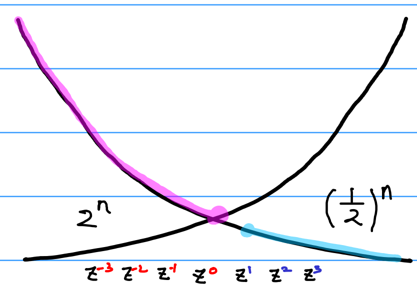
$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots$

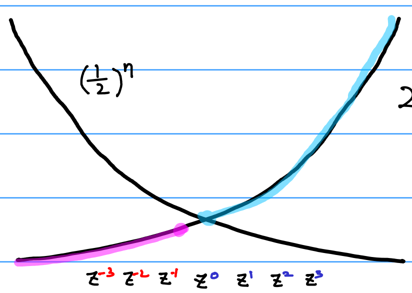
$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots$

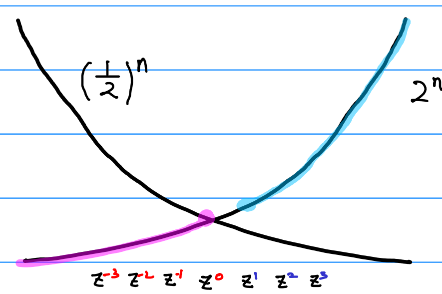
$z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$

$a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$









Ⅳ

ROC(z)	X(z)	⟷	a_{-n}	RNG(-n)
$\lvert z\rvert < p$				$n \le 0$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$

$-n$

ROC $\quad X(z) = \displaystyle\sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\quad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\quad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

Ⅱ

ROC(z^{-1})	f(z^{-1})	a_{-n}	RNG(-n)
$\|z\| > \frac{1}{p}$			$n < 1$

ROC

$|z| < a$ $\quad f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\|$

ROC'

$|z| > a^{-1}$ $\quad f(z^{-1}) = \frac{a^{-1}}{1-a^{-1}z^{-1}} = \sum_{n=0}^{\infty} a^{-n-1} z^{-n}$ $\quad a^{-n-1}$ $\quad n < 1 \quad n < 0 \quad n \geq 0 \quad n \geq 0$

$\left|\frac{a}{z}\right| < 1$ $\quad = \sum_{k=0}^{-\infty} a^{k-1} z^{k}$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$$\text{ROC} \big\updownarrow z^{-1} \qquad f(z) \big\updownarrow z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad a^{n+1} \big\downarrow -n$$

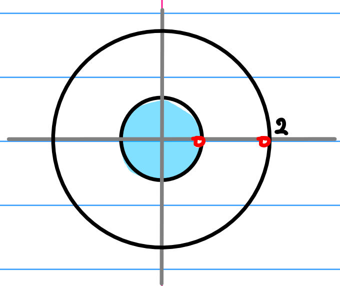
ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1} = a^{n-1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 2$

$$\frac{3}{2}\ \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z|<2 \quad |z|<0.5$



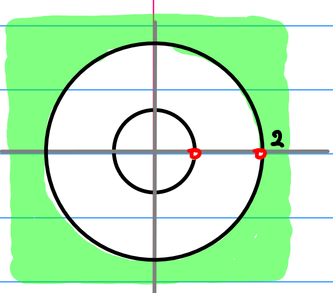
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad (|z|<0.5)$$

$$a_n = (-2)\ 2^n + (0.5)\ \left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\ \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z|>2$$

$|z|>2 \quad |z|>0.5$



$$X(z) = 0.5\ \frac{z}{z-0.5} - 2\ \frac{z}{z-2} \qquad (|z|>2)$$

$$a_n = (0.5)\ \left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

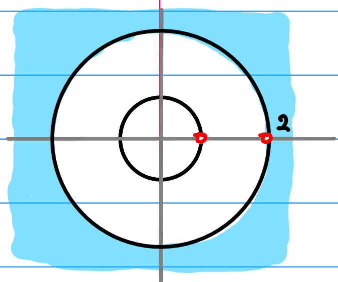
$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

Anti-causal $\qquad f(z) \qquad X(z)$

$\qquad |z|>2 \qquad |z|<0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$\qquad |z|>2 \qquad |z|>0.5$



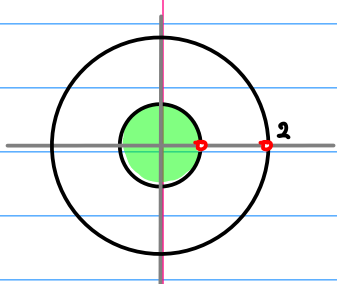
$$f(z) = (-2)\frac{-0.5}{0.5-z} + (0.5)\frac{-2}{2-z} \qquad (|z|>0.5)$$

$$a_n = (+2)\;2^n - (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n<0)$$

$$+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z|<2$$

$\qquad |z|<2 \qquad |z|<0.5$



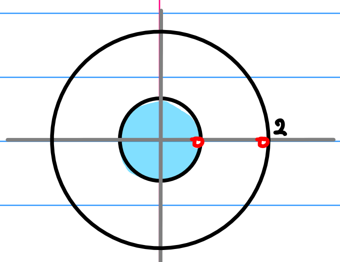
$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z|<2)$$

$$a_n = -(0.5)\left(\tfrac{1}{2}\right)^n + 2\cdot 2^n \qquad (n<0)$$

$$-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$$

① - Ⓐ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

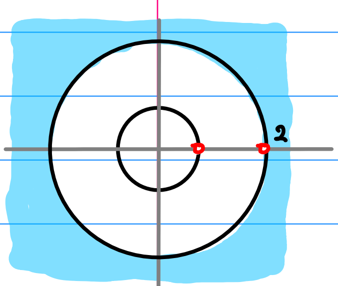


$|z| < 0.5$

$$\begin{aligned} f(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1}(z)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}$$

$(n \geq 0)$ $a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

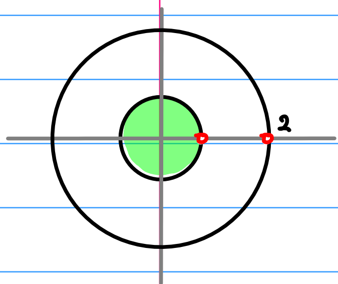
$$\begin{aligned} f(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \qquad \neq \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \\ &= \sum_{n=-1}^{-\infty} (2)^{n+1} z^n - \sum_{n=-1}^{-\infty} \left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}$$

$(n < 0)$ $a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

① - Ⓑ $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$ $\quad |z| < 0.5$ $\quad |z| > 2$

anticausal $\quad$ causal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



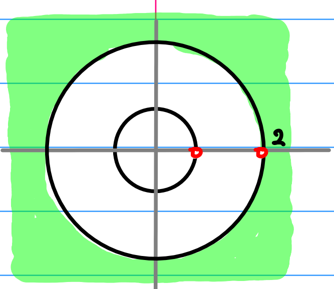
$|z| < 0.5$

$$X(z) = \boxed{\begin{aligned} &\frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} (z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1}\, z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty} (2)^{n-1}\, z^{-n}$$

$(n \leq 0) \quad a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



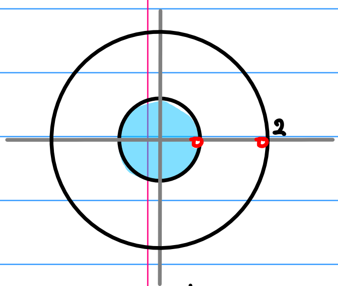
$|z| > 2$

$$X(z) = \boxed{\begin{aligned} &\frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1}\, z^{-n} \end{aligned}} \qquad \neq$$

$(n > 0) \quad a_n = \left(\frac{1}{2}\right)^{n-1} - (2)^{n-1}$

②-Ⓐ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $\qquad |z| < 0.5$ causal $\qquad |z| > 2$ anticausal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



$|z|<0.5$

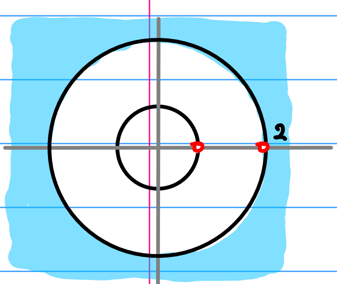
$$f(z) = -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \qquad \neq$$

$$= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n(z)^{n+1}$$

$$= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^n$$

$(n > 0) \qquad a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z|>2$

$$f(z) = \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n$$

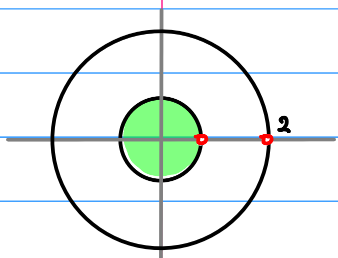
$$= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n}$$

$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0) \qquad a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - Ⓑ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



$|z| < 0.5$

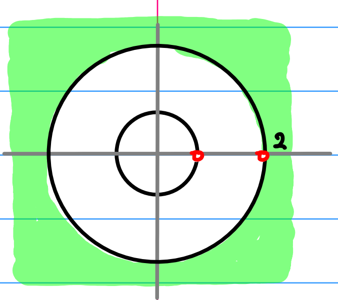
$$\boxed{\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \end{aligned}}$$

$\neq$

$$= -\sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} + \sum_{n=-1}^{-\infty}(2)^{n+1}z^{-n}$$

$(n < 0)$ $a_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$