

DT Impulse Function (4B)

- Continuous Time Impulse Function

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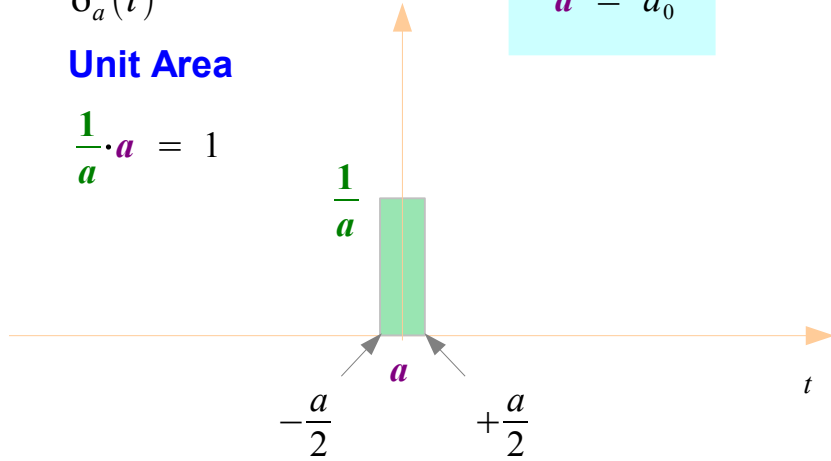
The Delta Function

$$\delta_a(t)$$

Unit Area

$$\frac{1}{a} \cdot a = 1$$

$$a = a_0$$



Height

Width

Area

$\frac{1}{a} = \frac{1}{a_0}$	$a = a_0$
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$\frac{1}{a} \cdot a = 1$

$\frac{1}{a} = 2 \cdot \frac{1}{a_0}$	$a = \frac{1}{2} \cdot a_0$
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$\frac{1}{a} \cdot a = 1$

$\frac{1}{a} \rightarrow \infty$	$a \rightarrow 0$
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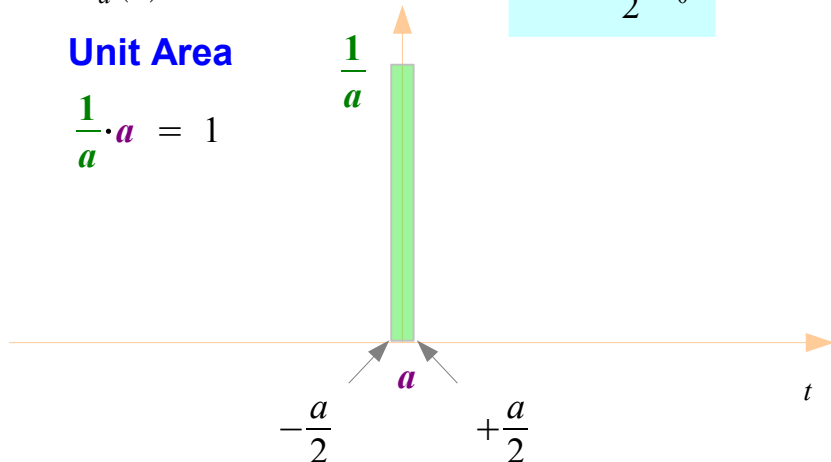
$\frac{1}{a} \cdot a = 1$

$$\delta_a(t)$$

Unit Area

$$\frac{1}{a} \cdot a = 1$$

$$a = \frac{1}{2} \cdot a_0$$



$\delta(t)$ Dirac Delta

Unit impulse function

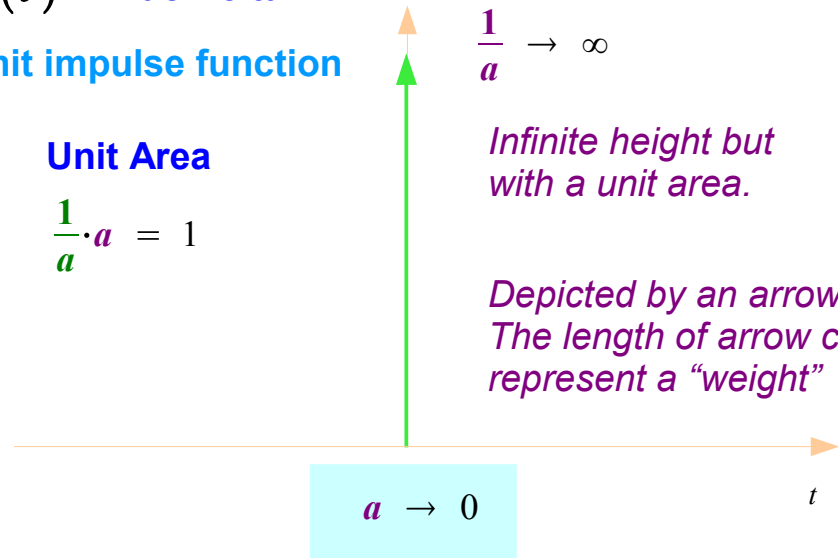
Unit Area

$$\frac{1}{a} \cdot a = 1$$

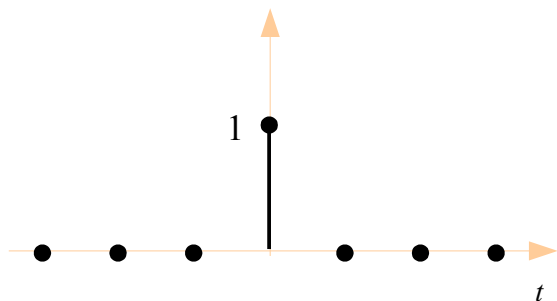
$$\frac{1}{a} \rightarrow \infty$$

Infinite height but with a unit area.

Depicted by an arrow
The length of arrow can represent a "weight"



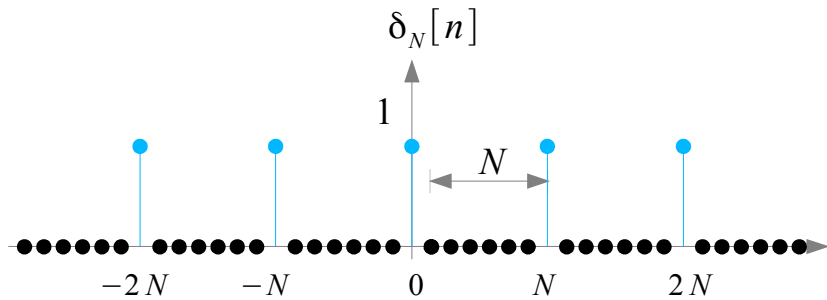
The Unit Impulse



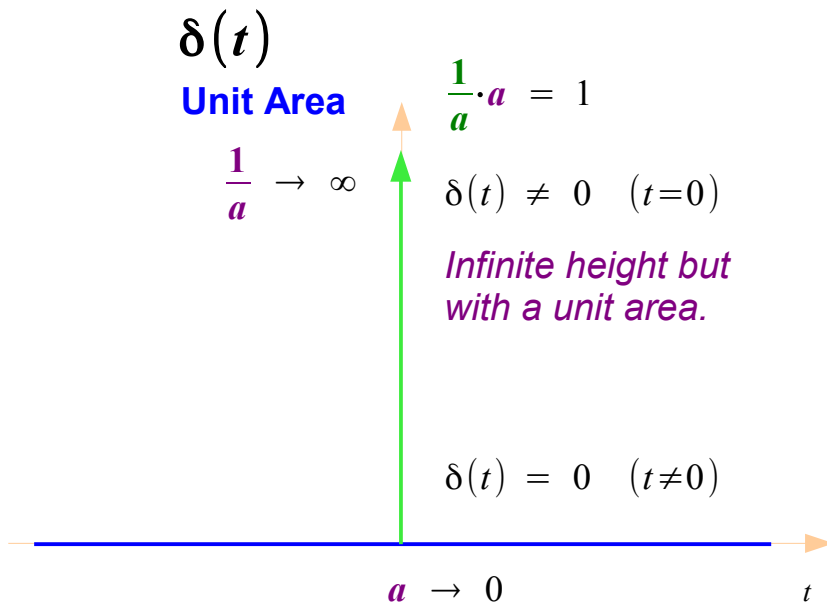
Kronecker Delta $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

Impulse Train

$$\delta_N[n] = \sum_{m=-\infty}^{+\infty} \delta[n - mN]$$



The Properties of the Delta Function



The Equivalence Property

$$g(t) \delta(t) = g(0) \delta(t)$$

$$g(t) \delta(t-t_0) = g(t_0) \delta(t-t_0)$$

The Sampling Property

$$\int_{-\infty}^{+\infty} g(t) \delta(t) dt = g(0)$$

$$\int_{-\infty}^{+\infty} g(t) \delta(t-t_0) dt = g(t_0)$$

An Even Function

$$\delta(-t) = \delta(t)$$

The Replication Property

$$\int_{-\infty}^{+\infty} g(\tau) \delta(t-\tau) d\tau = g(t)$$

The Scaling Property

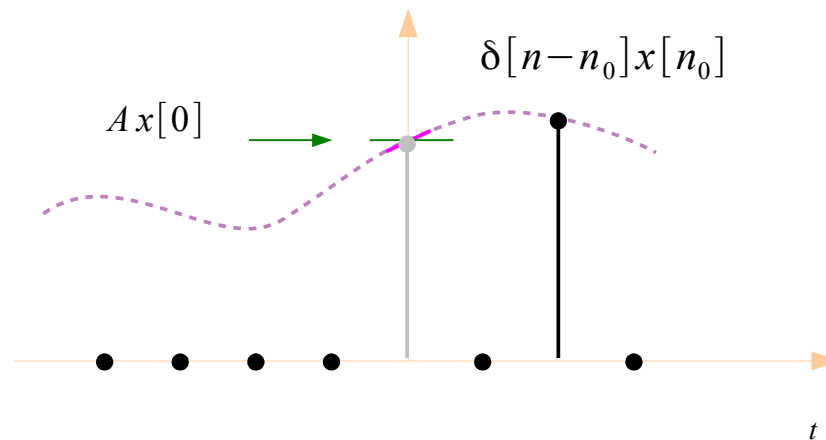
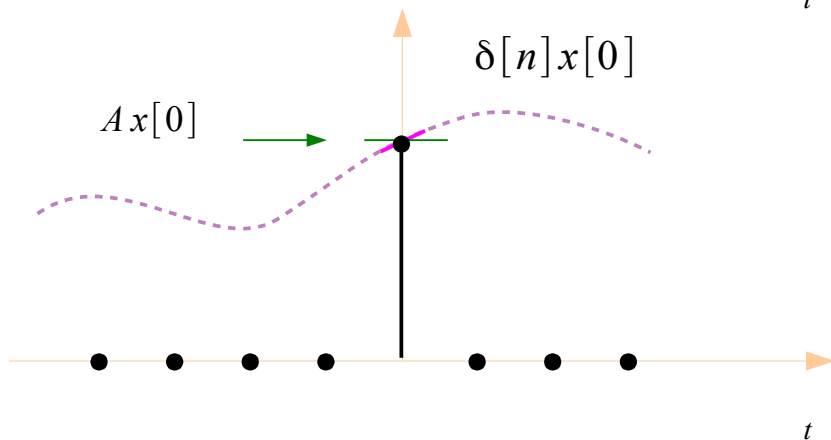
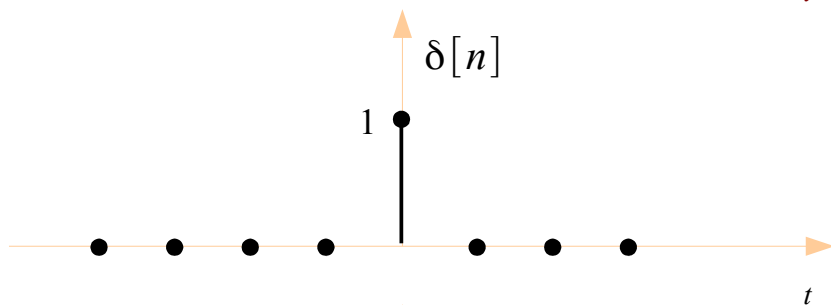
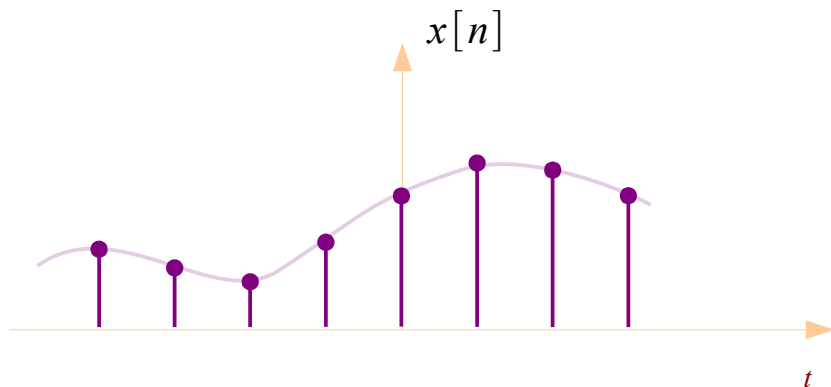
$$\delta(a(t-t_0)) = \frac{1}{|a|} \delta(t-t_0)$$

The Fourier Transform Property

$$\int_{-\infty}^{+\infty} \delta(t) e^{-j2\pi f t} dt = 1$$

The Equivalence Property

$$\sum_{n=-\infty}^{\infty} A\delta[n - n_0]x[n] = Ax[n_0]$$



Octave Impulse Functions

```
function y = Ddelta(n)
    y = double(n == 0);
    nn = find(round(n) ~= n);
    y(nn) = NaN;
```

```
function y = DdeltaTrain(N, n)
    if N == round(N),
        y = double(n/N == round(n/N)) ;
        nn = find(round(n) ~= n);
        y(nn) = NaN;
    else
        disp("N is not an integer");
    end
```


References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M. J. Roberts, Fundamentals of Signals and Systems
- [4] S. Haykin, An Introduction to Analog & Digital Communications