DT Impulse Function (4B)

- Continuous Time Impulse Function

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This document was produced by using OpenOffice and Octave.

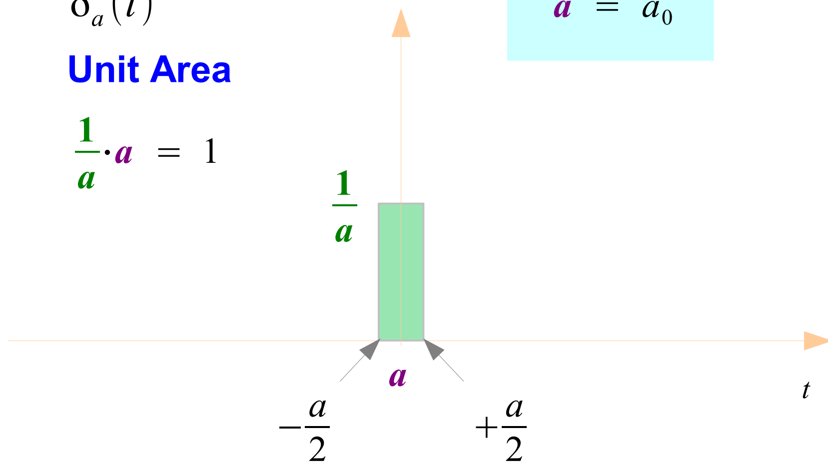
The Delta Function

$\delta_a(t)$

Unit Area

$\frac{1}{a} \cdot a = 1$

$a = a_0$



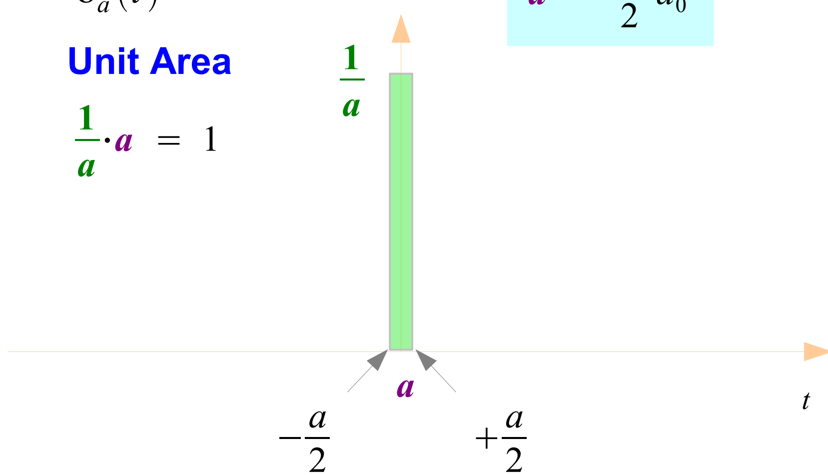
Height	Width	Area
$\frac{1}{a} = \frac{1}{a_0}$	$a = a_0$	$\frac{1}{a} \cdot a = 1$
$\frac{1}{a} = 2 \cdot \frac{1}{a_0}$	$a = \frac{1}{2} \cdot a_0$	$\frac{1}{a} \cdot a = 1$
$\frac{1}{a} \to \infty$	$a \to 0$	$\frac{1}{a} \cdot a = 1$

$\delta_a(t)$

Unit Area

$\frac{1}{a} \cdot a = 1$

$a = \frac{1}{2} \cdot a_0$



$\delta(t)$ **Dirac Delta**

Unit impulse function

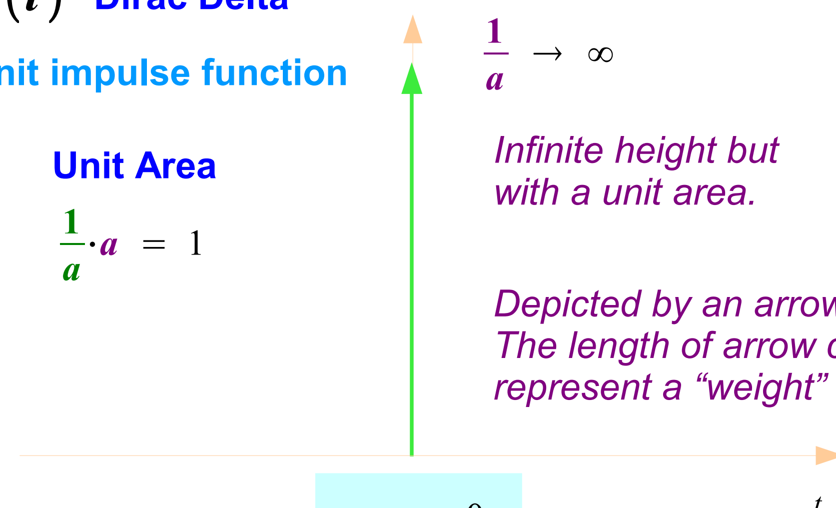
Unit Area

$\frac{1}{a} \cdot a = 1$

$\frac{1}{a} \to \infty$

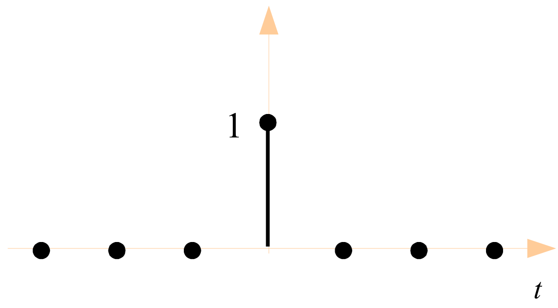
Infinite height but with a unit area.

Depicted by an arrow
The length of arrow can represent a "weight"



$a \to 0$

The Unit Impulse

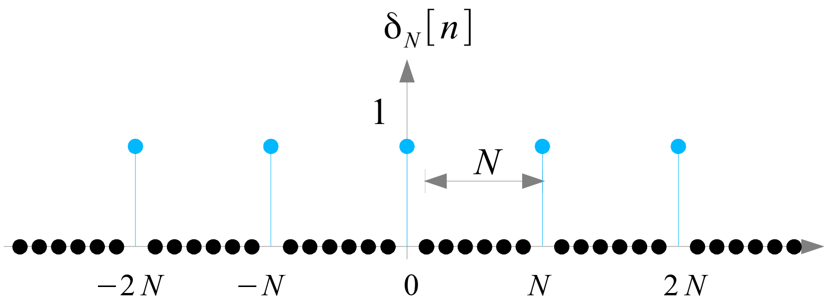


Kronecker Delta

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Impulse Train

$$\delta_N[n] = \sum_{m=-\infty}^{+\infty} \delta[n - mN]$$

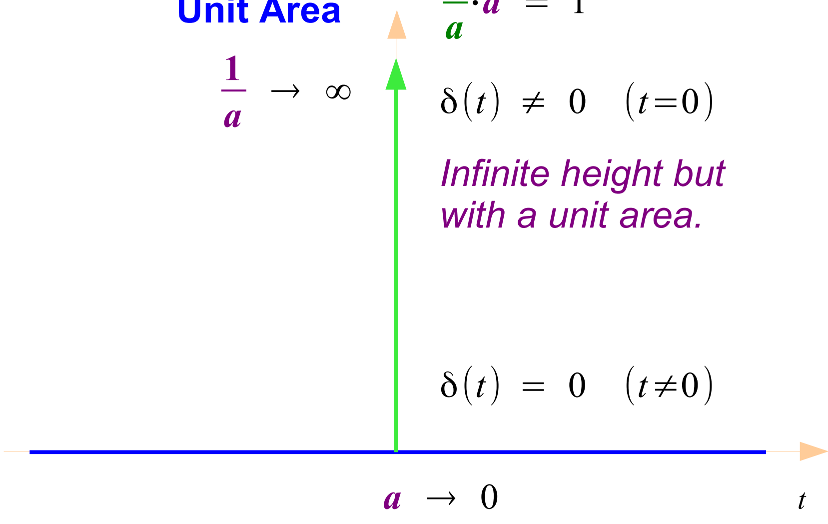


The Properties of the Delta Function

$\delta(t)$

Unit Area

$\frac{1}{a} \cdot a = 1$



The Equivalence Property

$$g(t)\,\delta(t) = g(0)\,\delta(t)$$

$$g(t)\,\delta(t-t_0) = g(t_0)\,\delta(t-t_0)$$

The Sampling Property

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t)\,dt = g(0)$$

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t-t_0)\,dt = g(t_0)$$

An Even Function

$$\delta(-t) = \delta(t)$$

The Replication Property

$$\int_{-\infty}^{+\infty} g(\tau)\,\delta(t-\tau)\,d\tau = g(t)$$

The Scaling Property

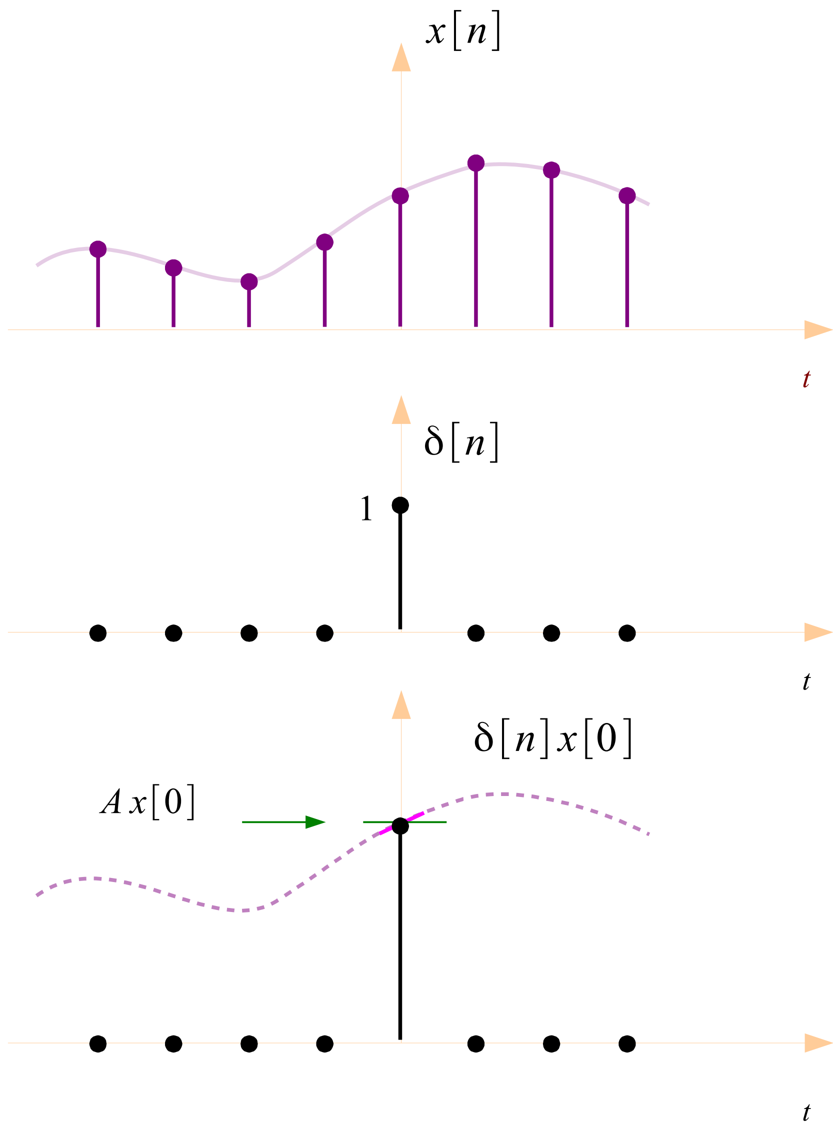
$$\delta(a(t-t_0)) = \frac{1}{|a|}\delta(t-t_0)$$

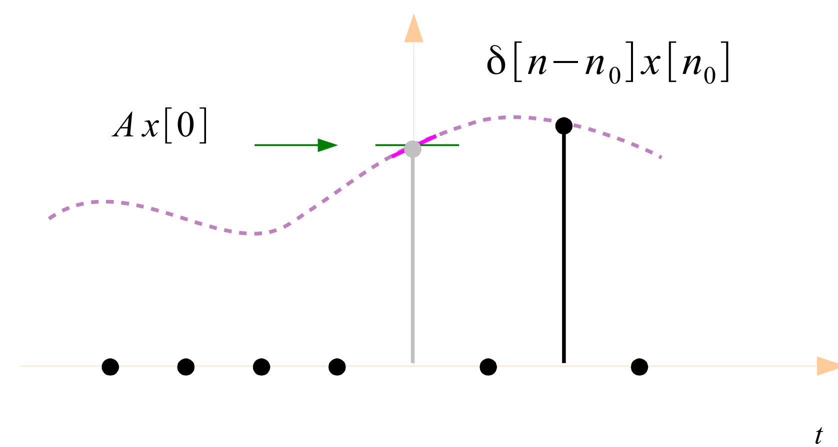
The Fourier Transform Property

$$\int_{-\infty}^{+\infty} \delta(t)\,e^{-j2\pi f t}\,dt = 1$$

The Equivalence Property

$$\sum_{n=-\infty}^{\infty} A\,\delta[n-n_0]\,x[n] \;=\; A\,x[n_0]$$





Octave Impulse Functions

```
function y = Ddelta(n)
    y = double(n == 0);
    nn = find(round(n) ~= n);
    y(nn) = NaN;



function y = DdeltaTrain(N, n)
    if N == round(N),
        y = double(n/N == round(n/N)) ;
        nn = find(round(n) ~= n);
        y(nn) = NaN;
    else
        disp(“N is not an integer’);
    end
```

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M. J. Roberts, Fundamentals of Signals and Systems
[4] S. Haykin, An Introduction to Analog & Digital Communications