Capacitors in an AC circuit

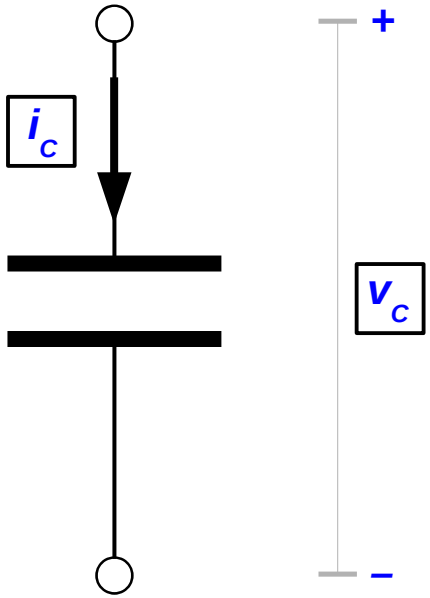
Young Won Lim
10/02/2017

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Young Won Lim
10/02/2017

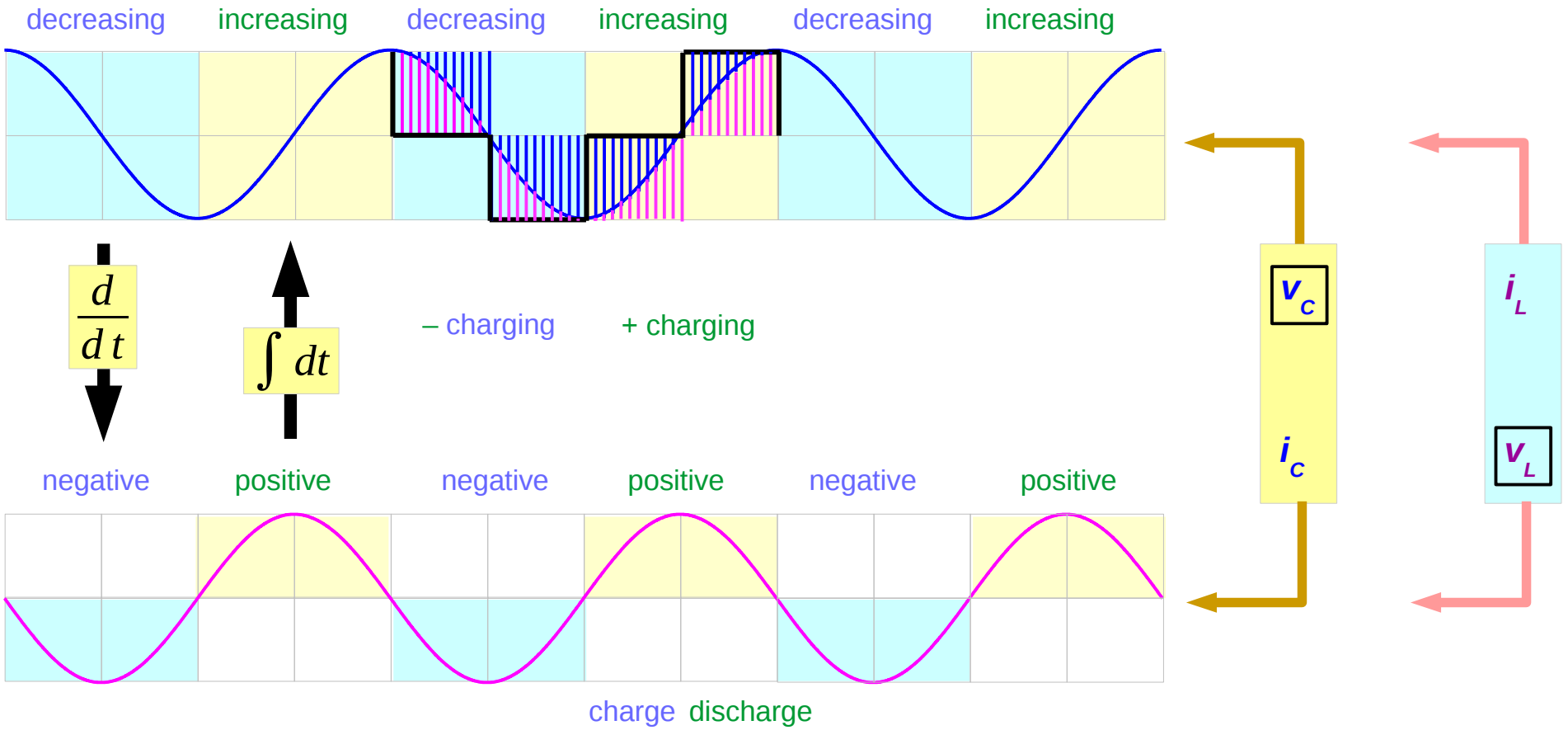
Invertible Functions



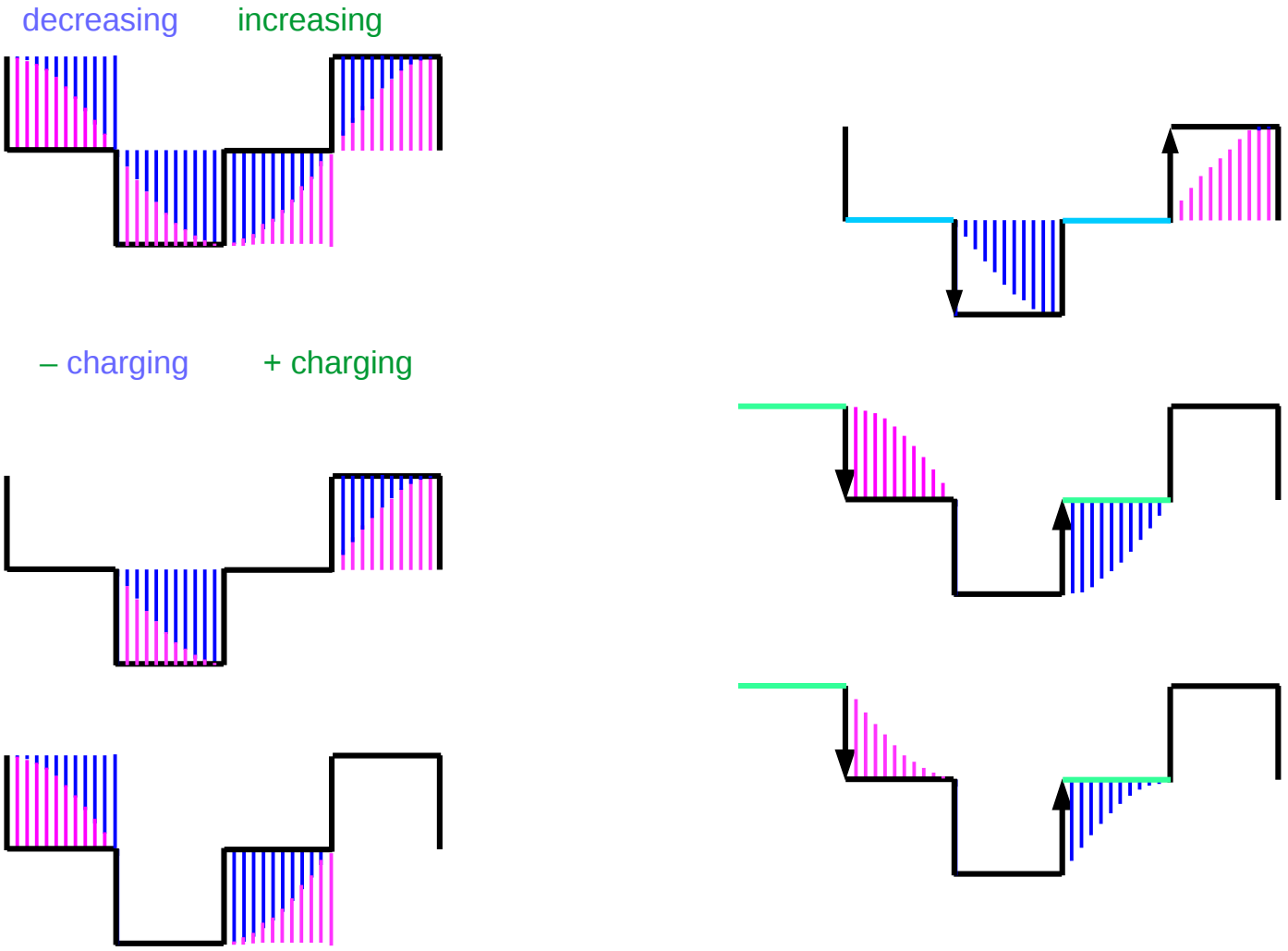
$$v_c(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow i_c(t)$$

$$v_c(t) \leftarrow \boxed{\int \cdot\, dt} \leftarrow i_c(t),\ v_c(0)$$

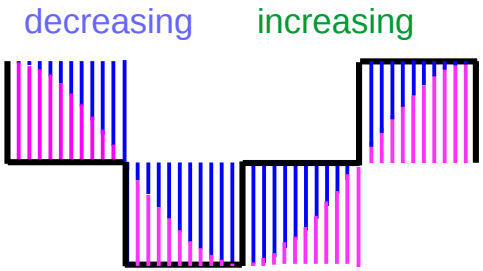
Ever-charging signal pairs

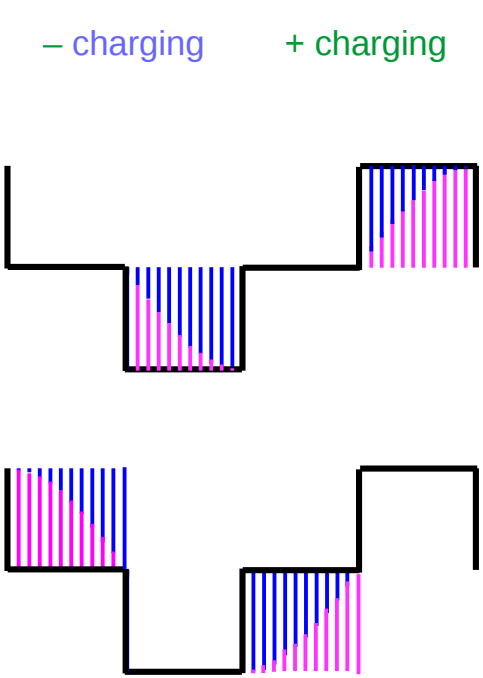


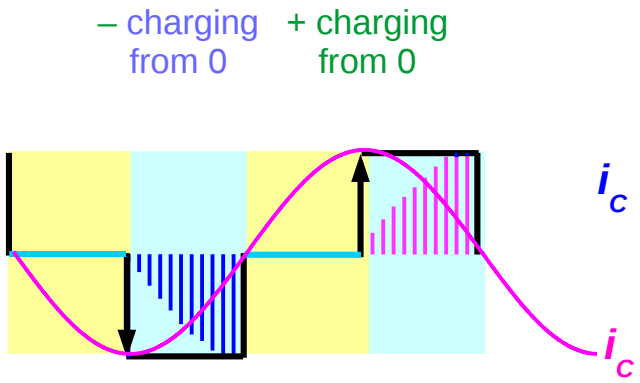
Ever-charging signal pairs

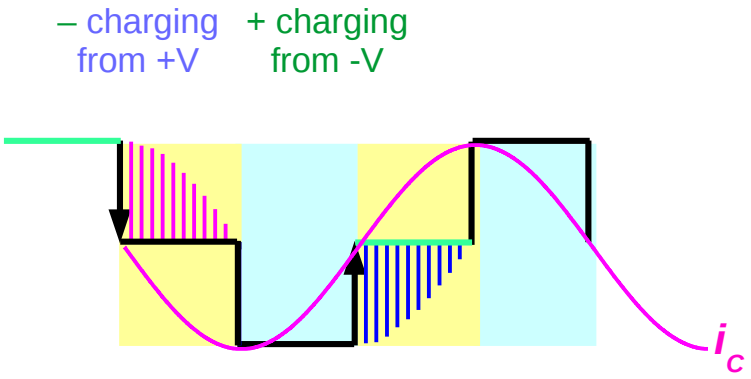


Ever-charging signal pairs

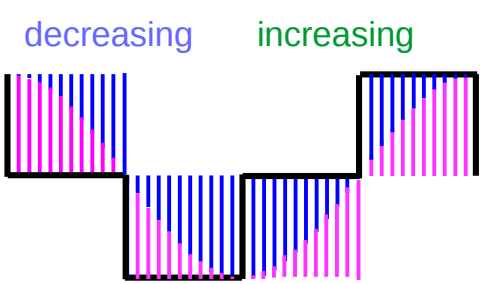


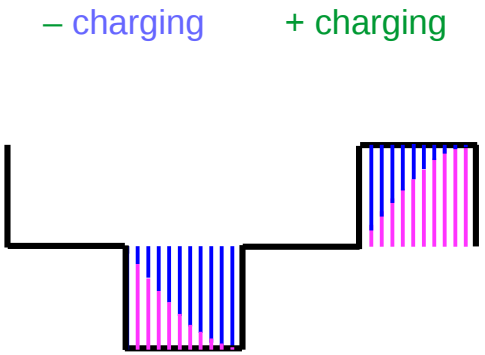


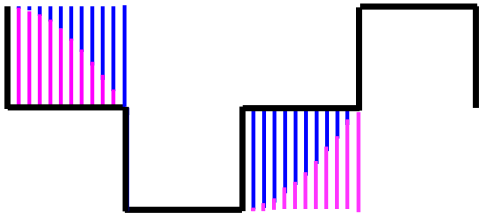


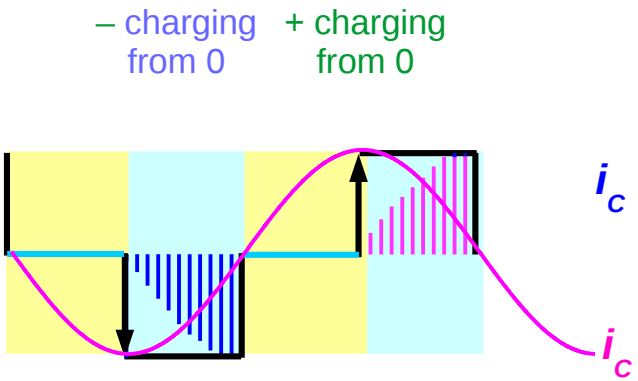


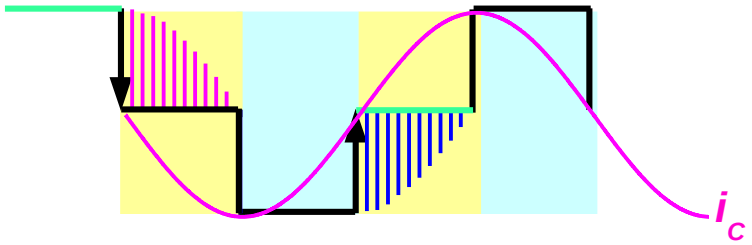
Ever-charging signal pairs

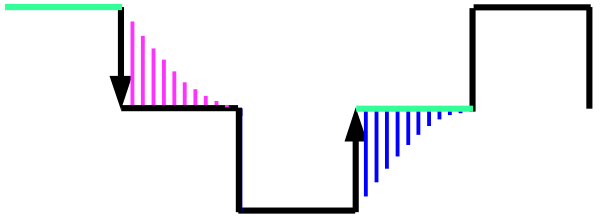




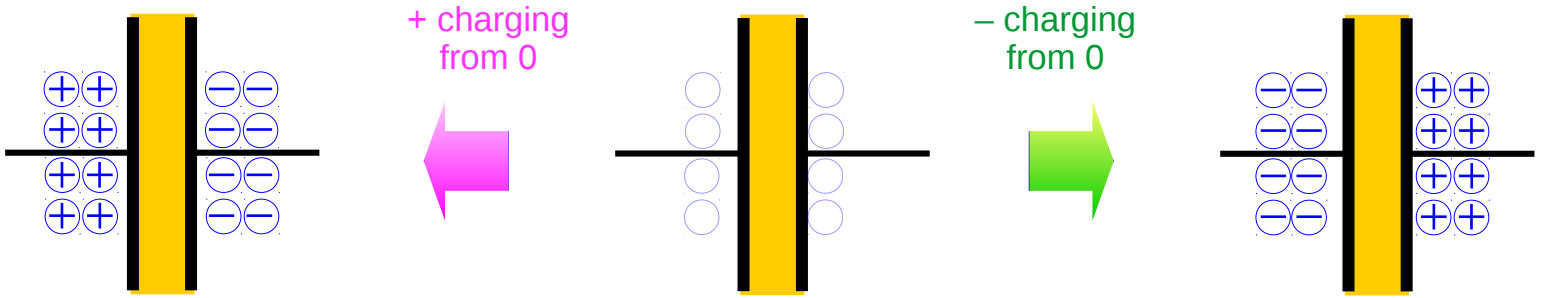


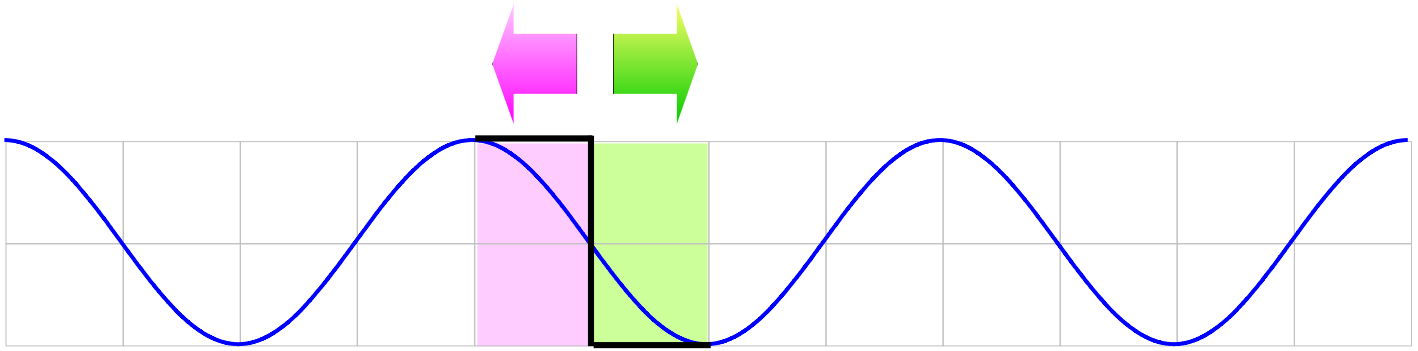




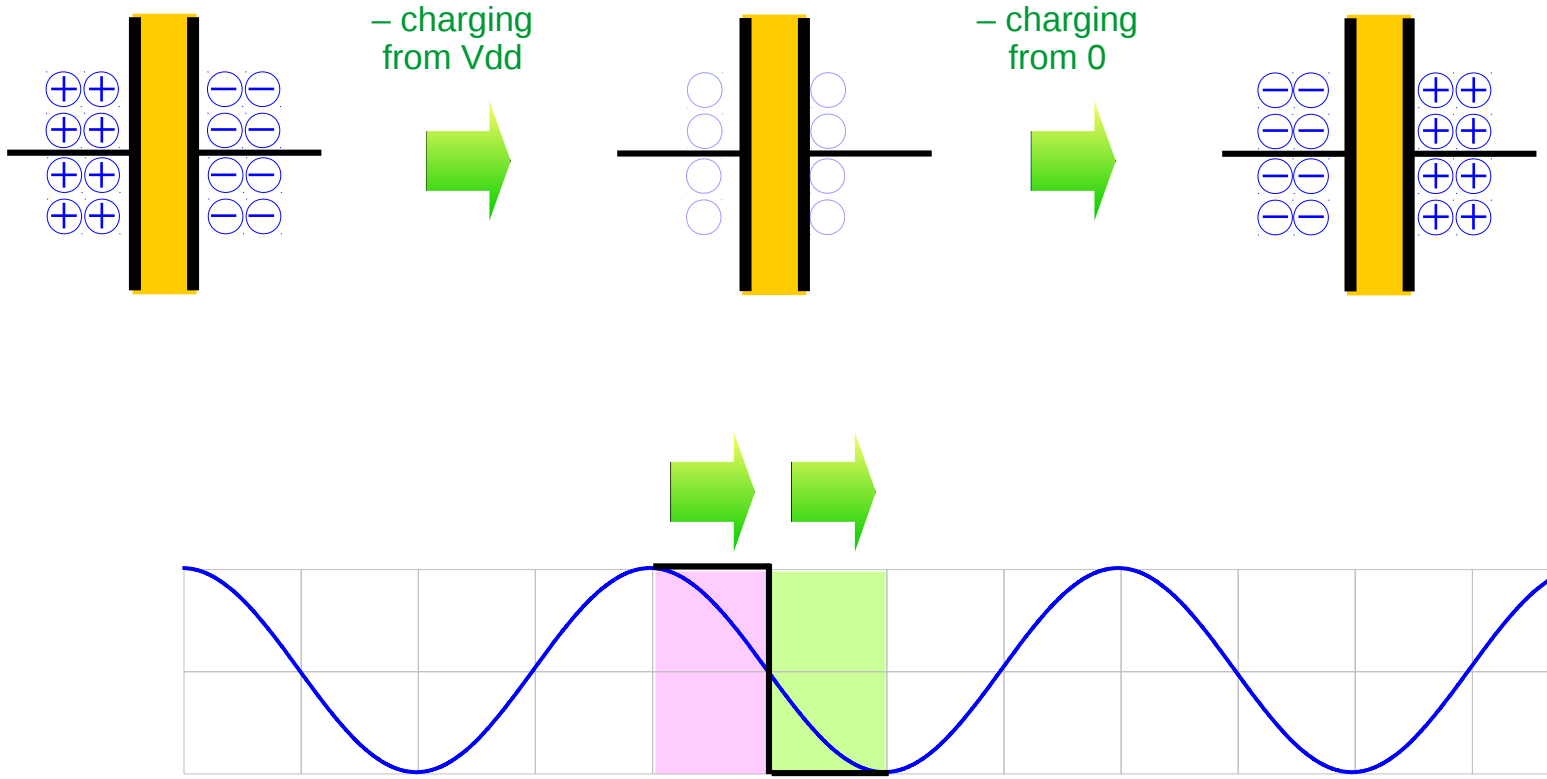


Three States

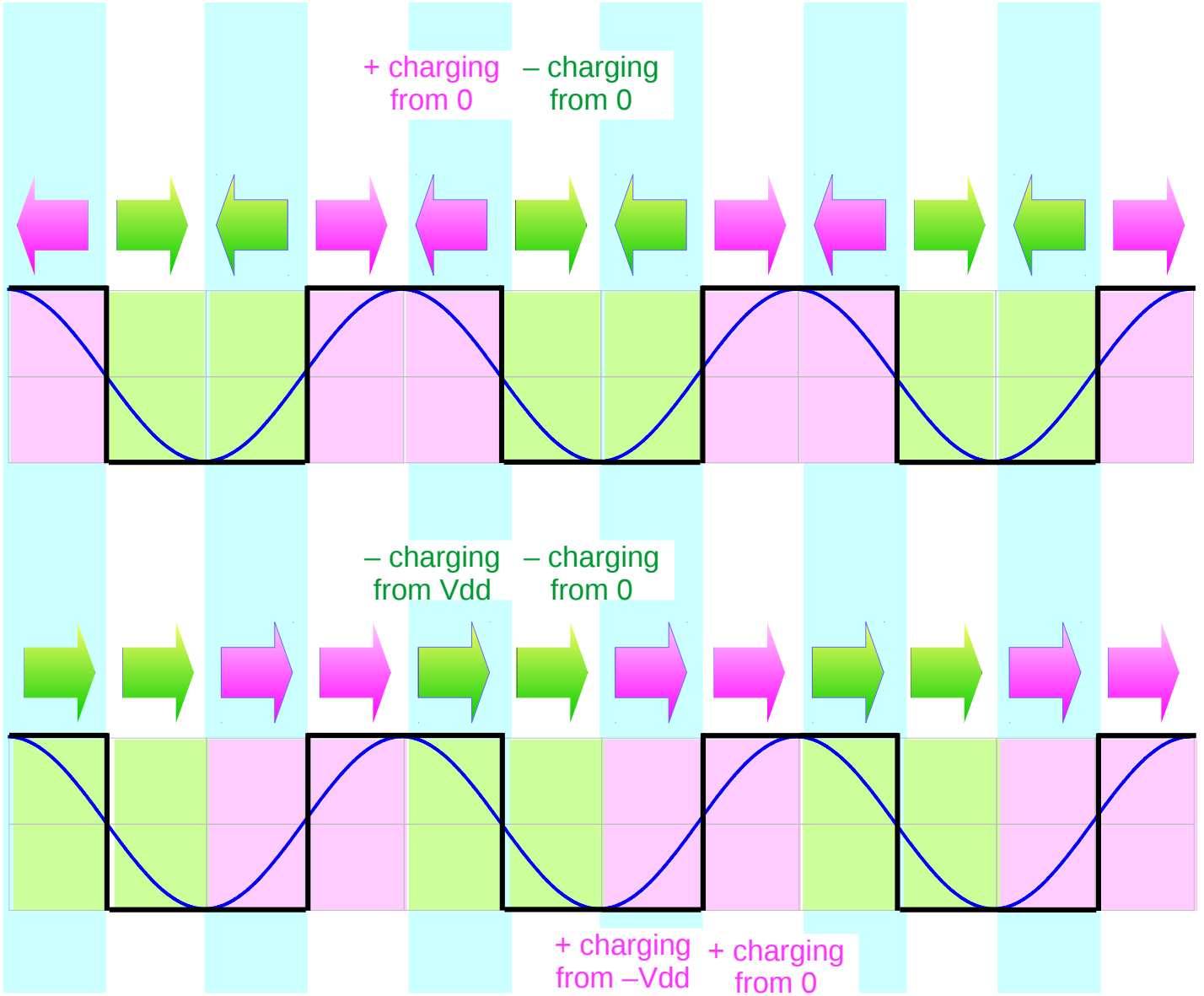




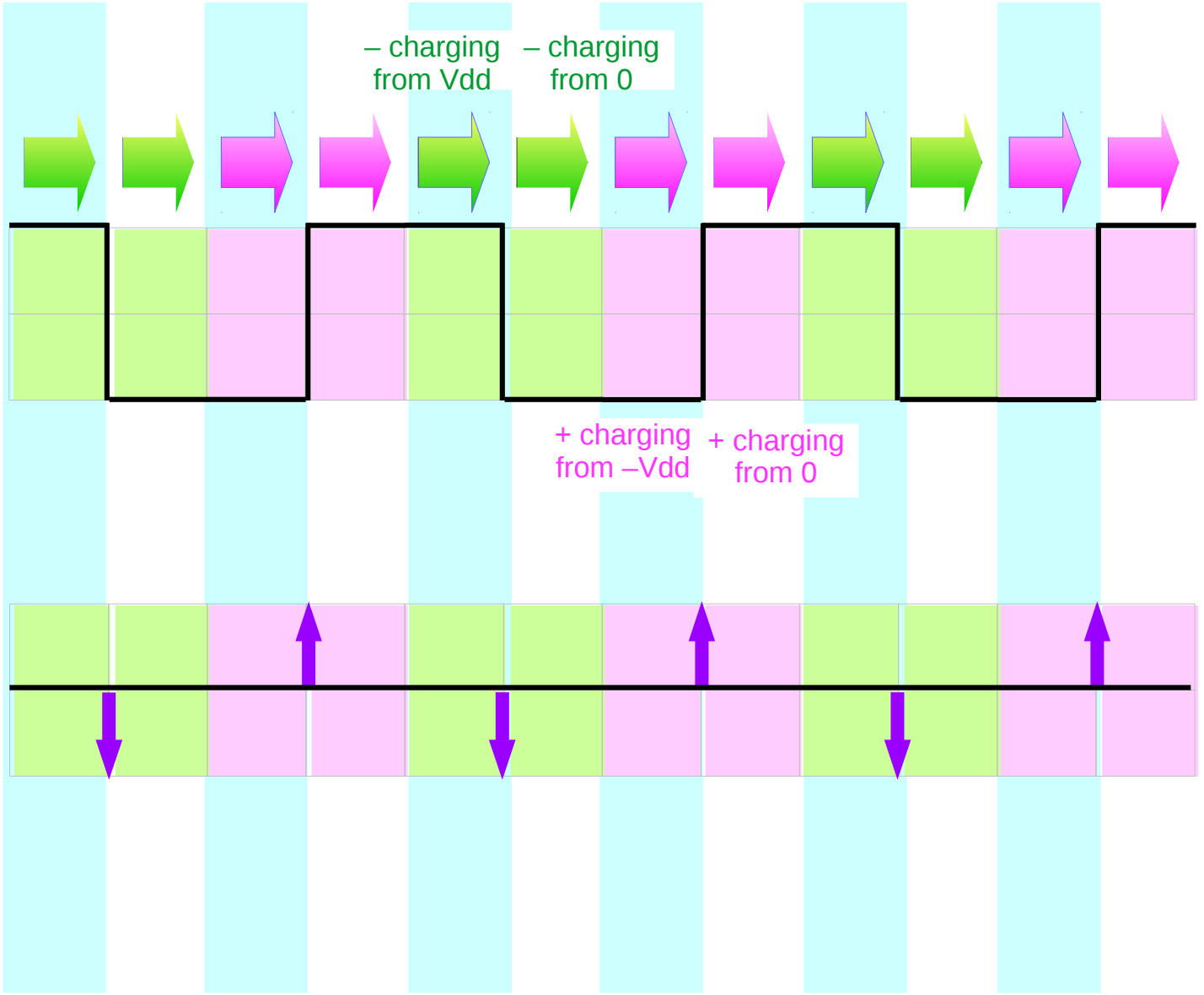
Three States



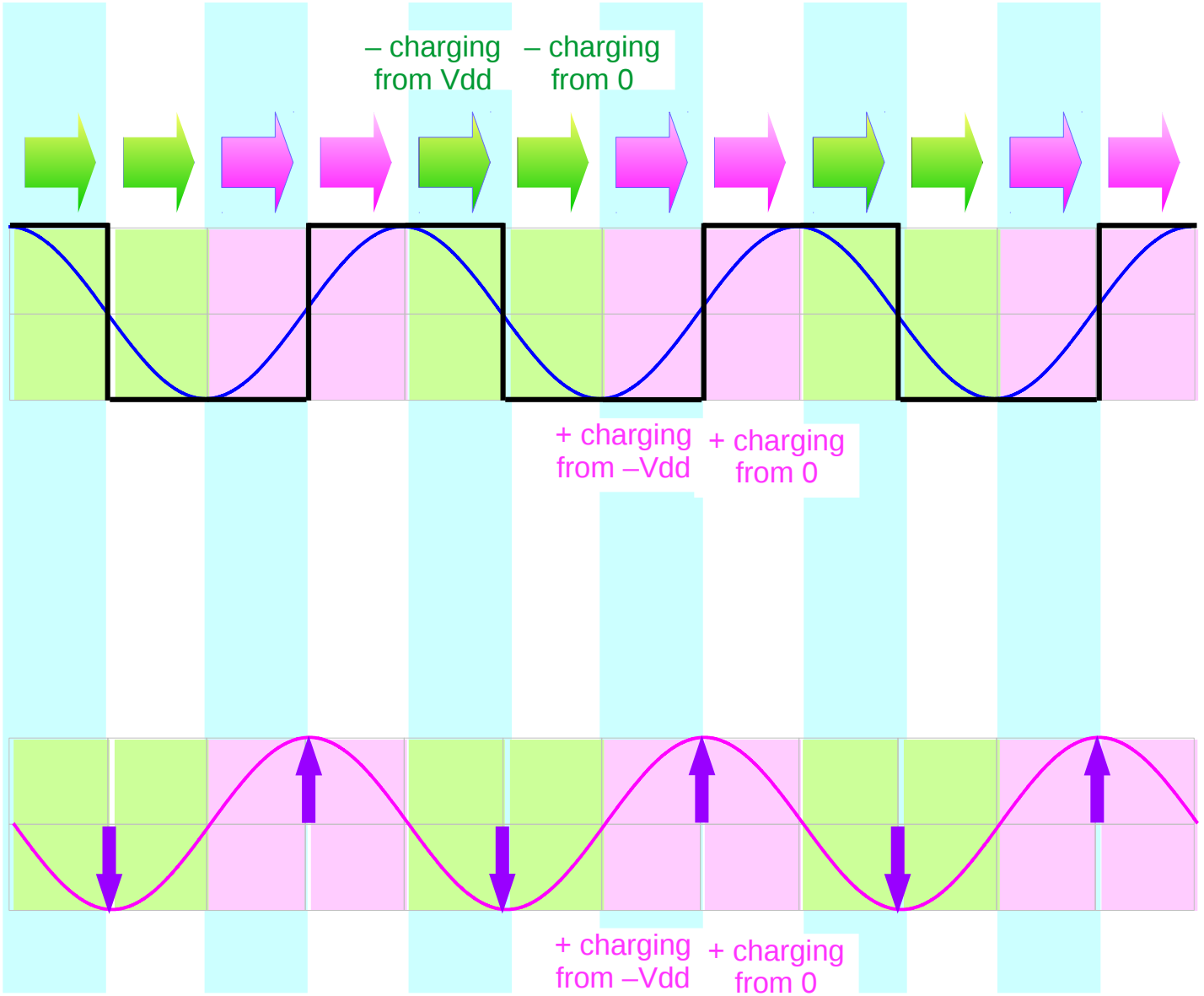
Three States



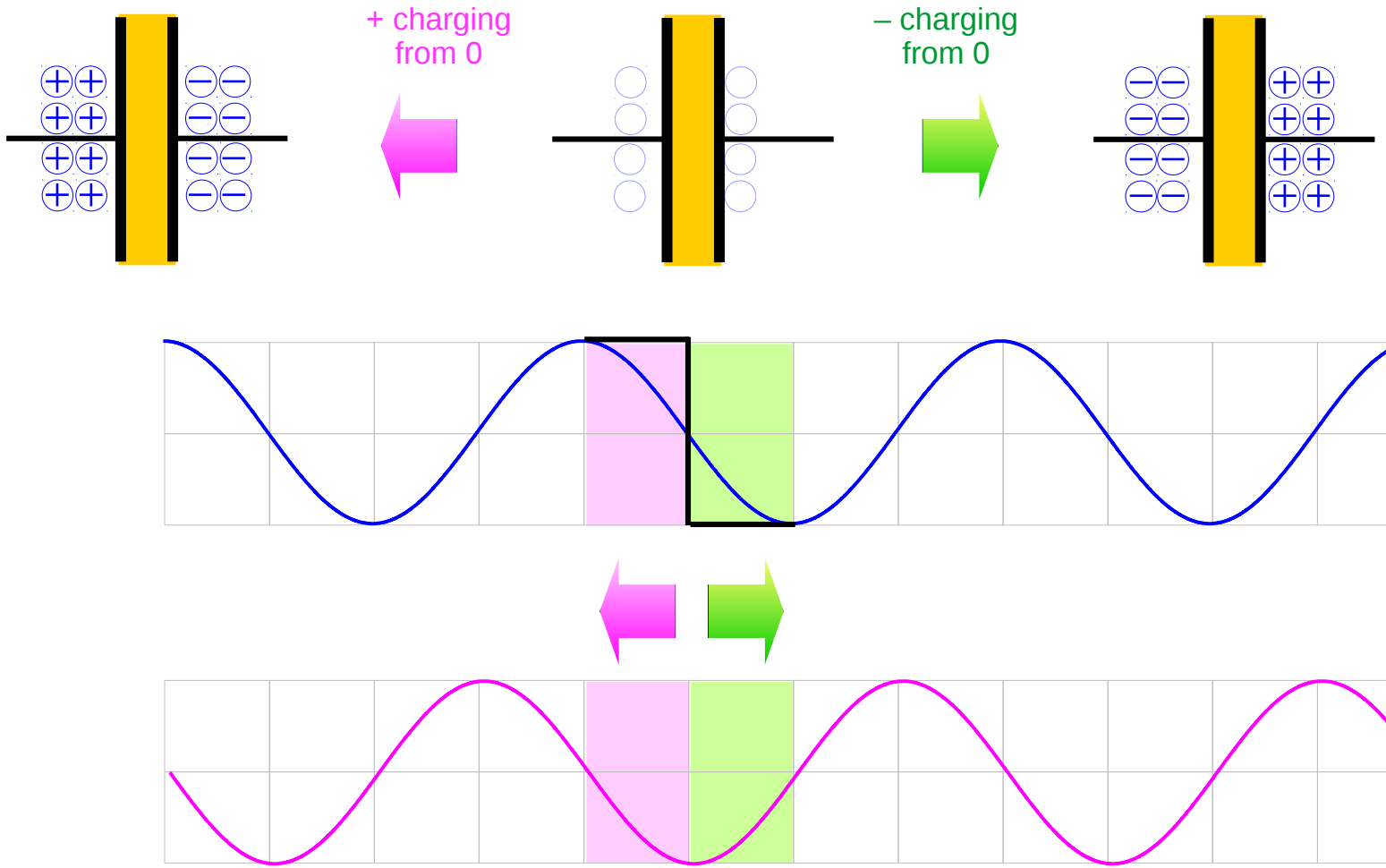
Three States



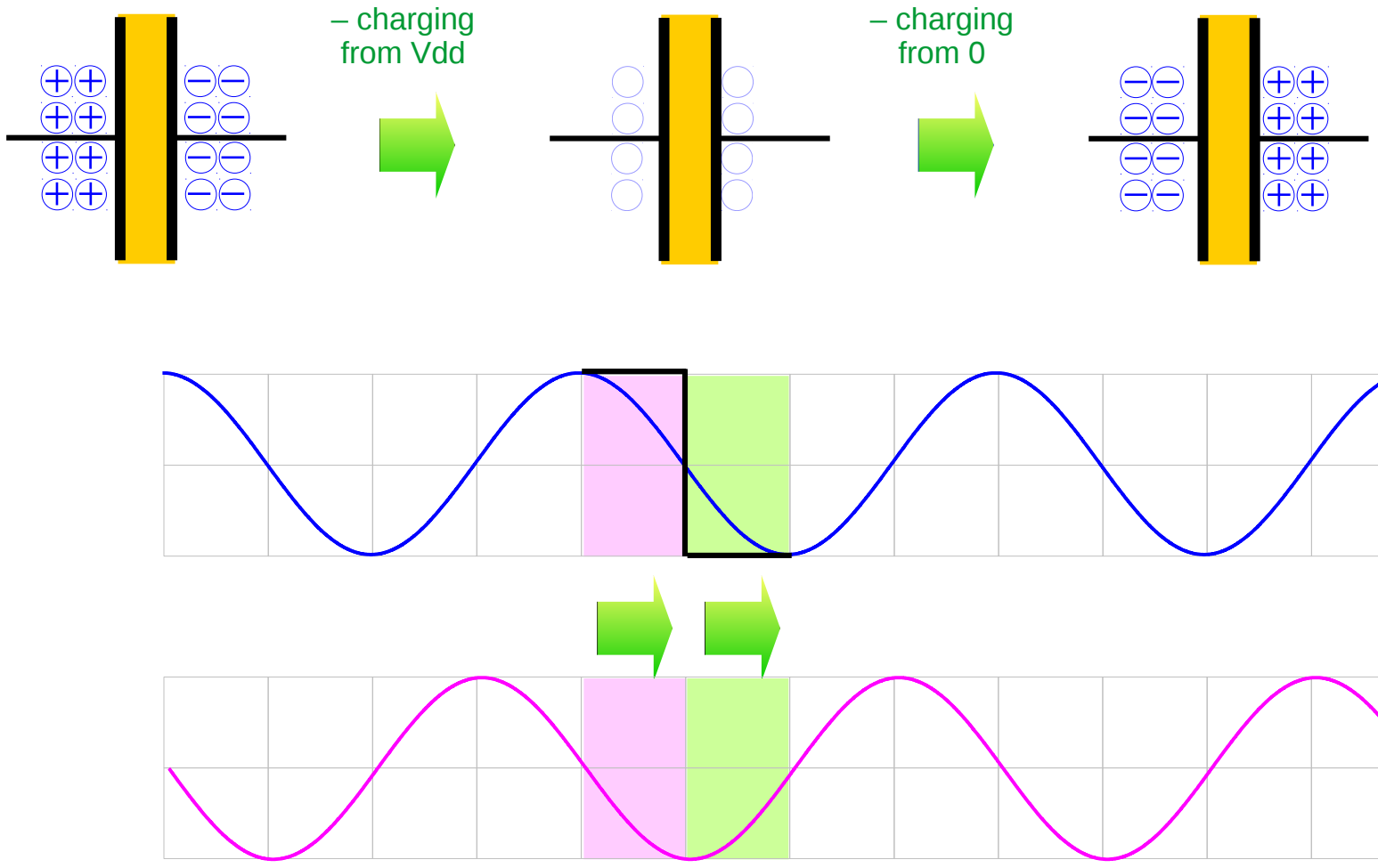
Three States



Three States

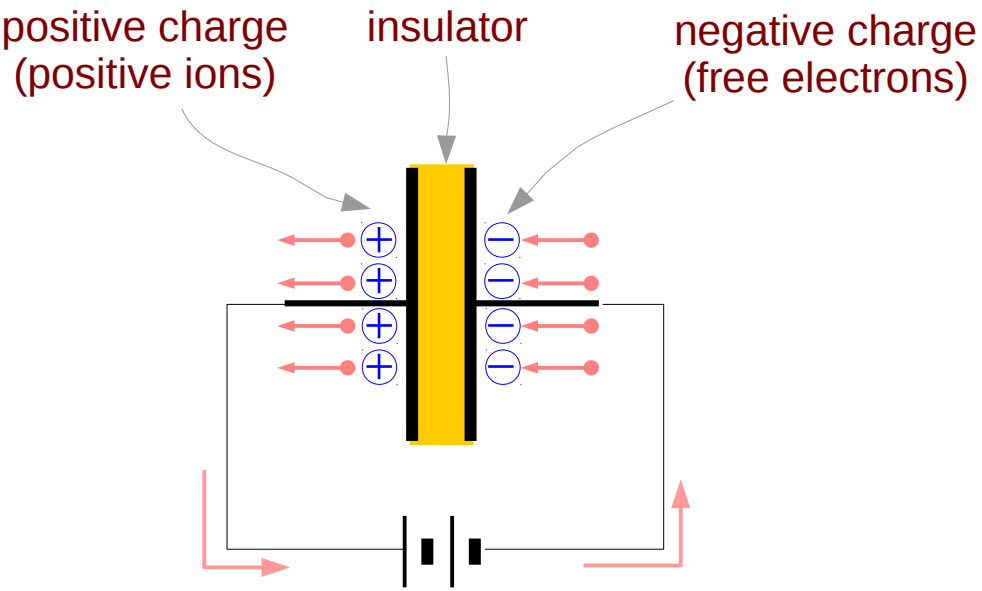


Three States

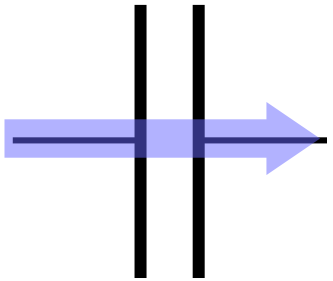


Three States

Capacitor Current



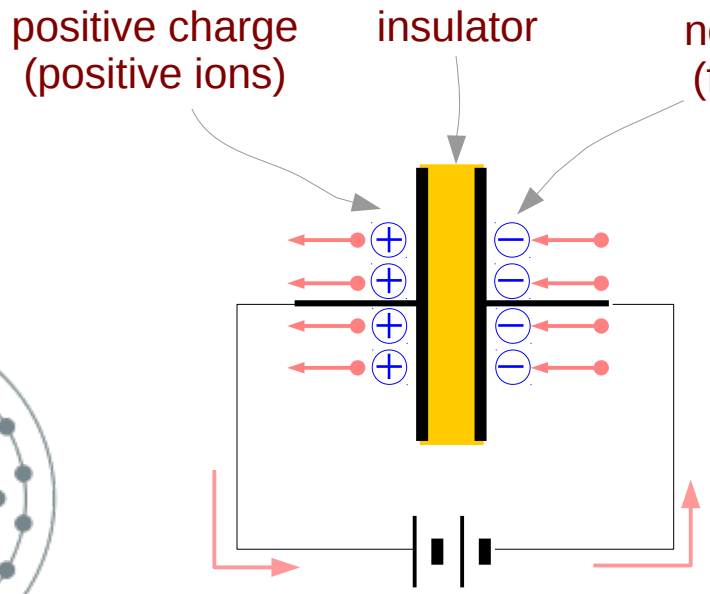
No actual electrons movement
across insulator materials

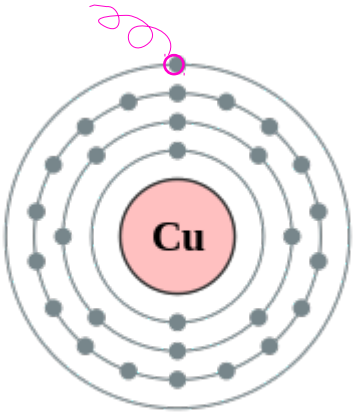


But, think as **Displacement Current**

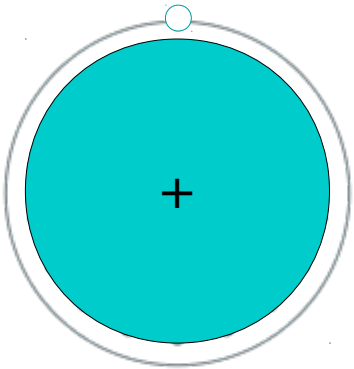
flows through the capacitor

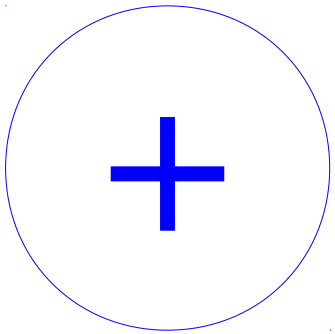
Positive ions and free electrons

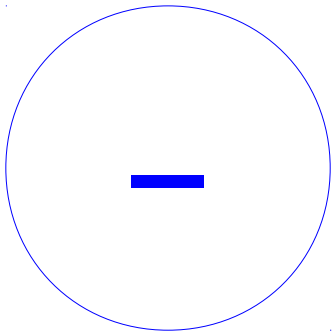




[[commons:User crap]] (original work by commons:User:Greg Robson) https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/Electron_shell_029_Copper_-_no_label.svg/200px-Electron_shell_029_Copper_-_no_label.svg.png





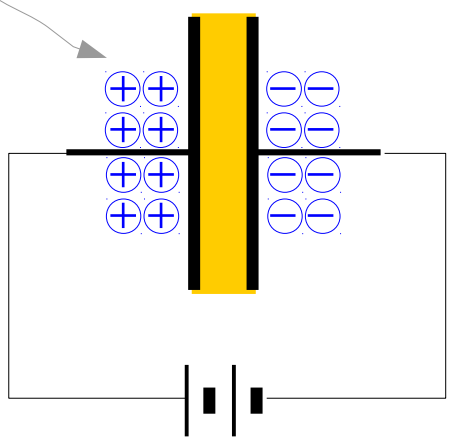


Three States

positive charge
(positive ions)

Positively Charged State

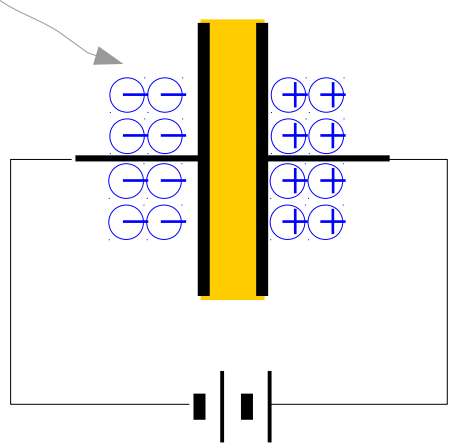
fully charged → no current

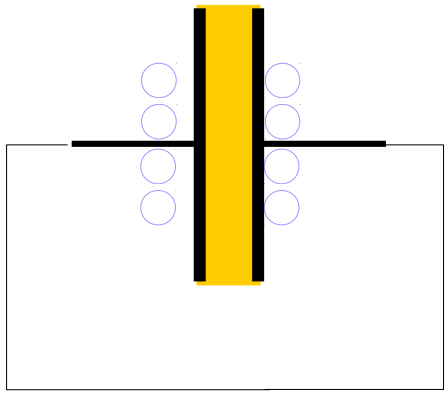


negative charge
(free electrons)

Negatively Charged State

fully charged → no current



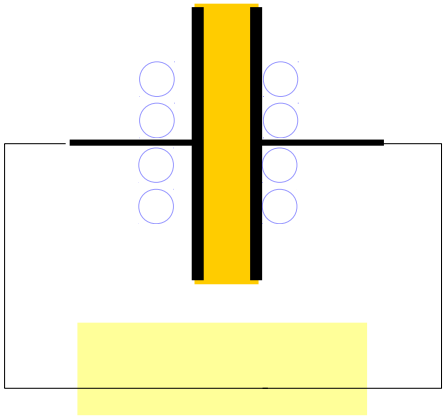


Fully Discharged State

possible large current

Currents in the Fully Discharged State

Initially no current

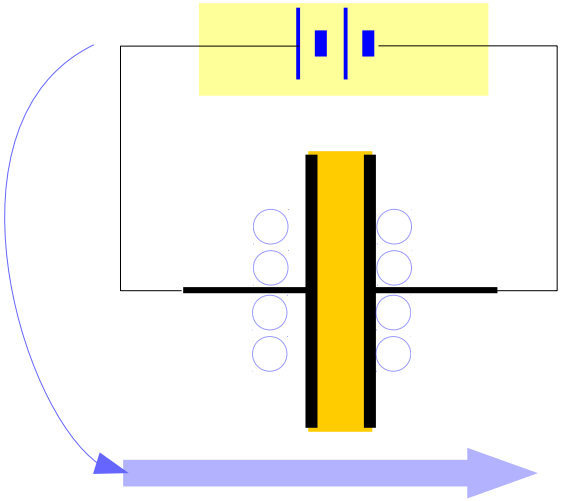


Fully Discharged State

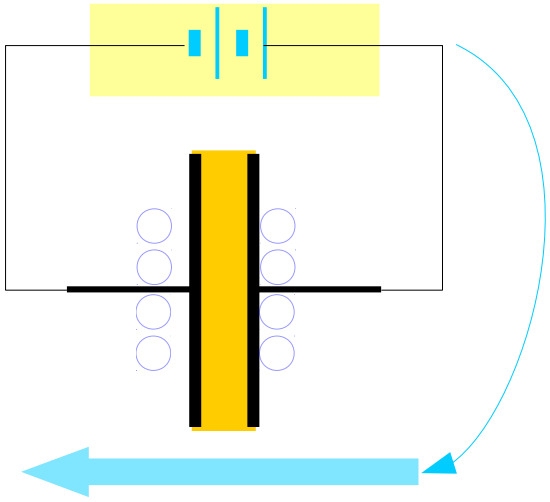
Fully Discharged State

This state can flow
large current
in either direction

Fully Discharged State



large current

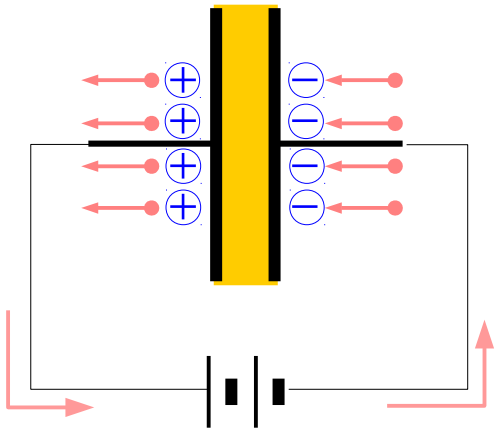


large current

Inter-State Current Flowing

Under Positively Charging

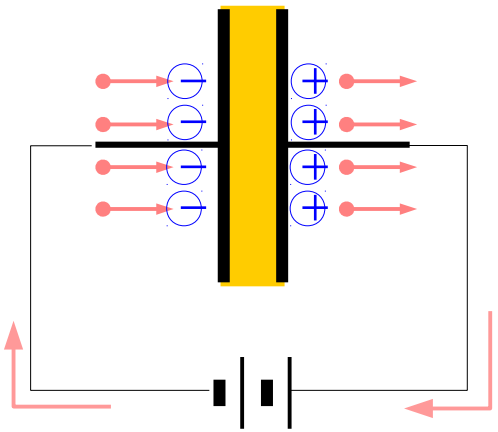
(+) current flow direction



electron flow direction

Under Negatively Charging

(–) current flow direction

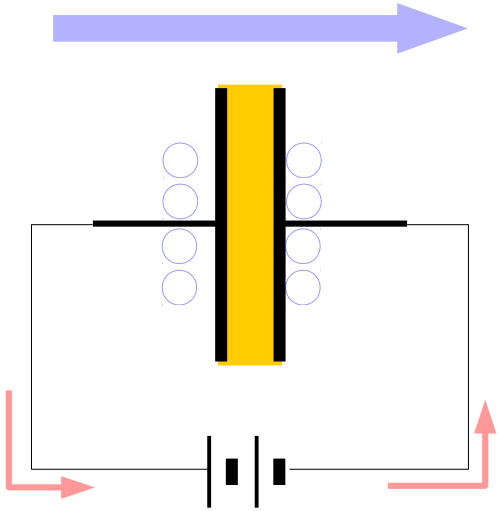


electron flow direction

Inter-State Current Flowing

Fully Discharged State

(+) current flow direction

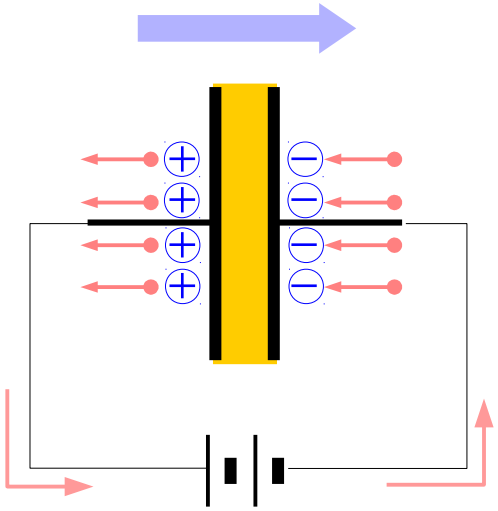


electron flow direction

Initial large current

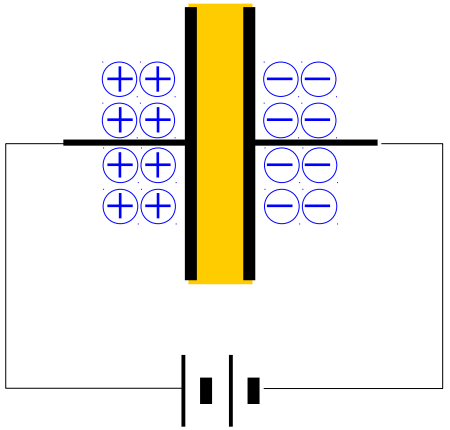
Under Positively Charging

(+) current flow direction



electron flow direction

Positively Charged State



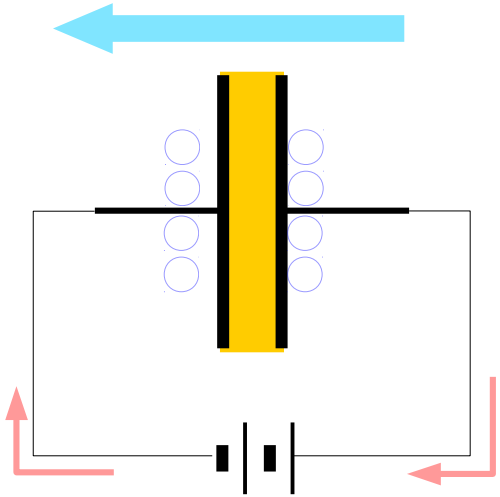
Crowded →
No more space

no current

Inter-State Current Flowing

Fully Discharged State

(–) current flow direction

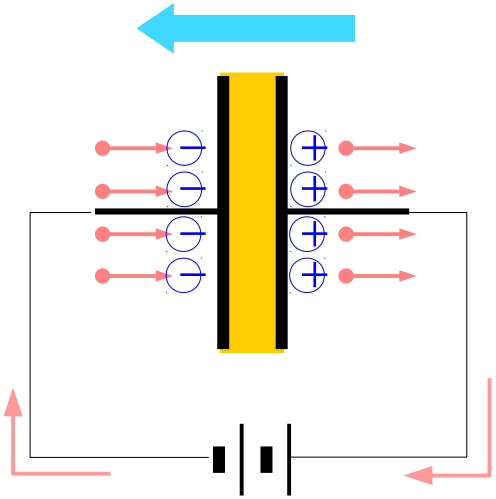


electron flow direction

Initial large current

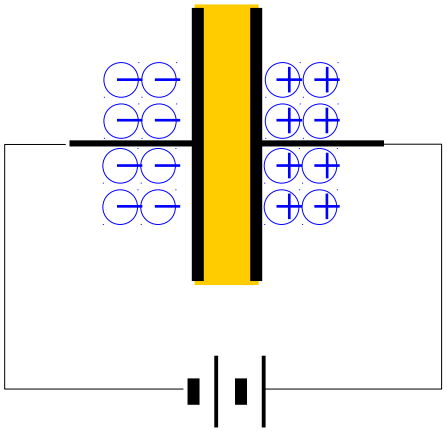
Under Negatively Charging

(–) current flow direction



electron flow direction

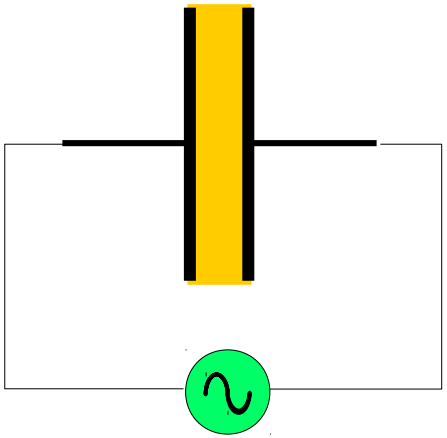
Negatively Charged State

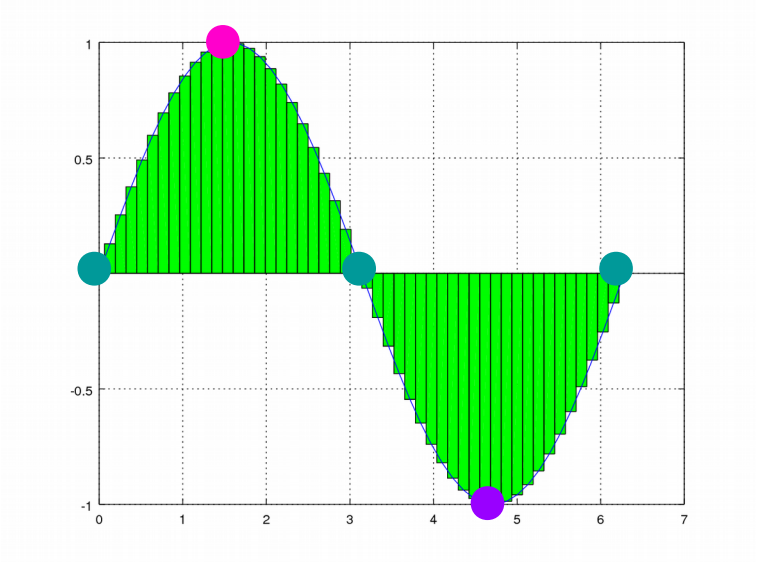


Crowded →
No more space

no current

An AC Voltage Source

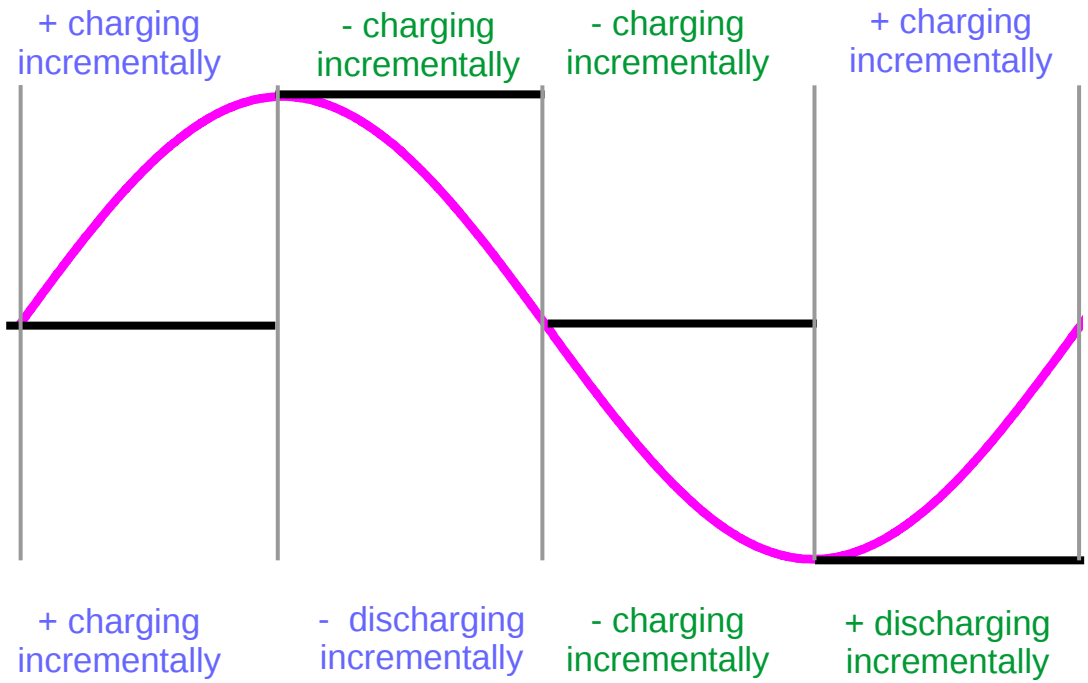




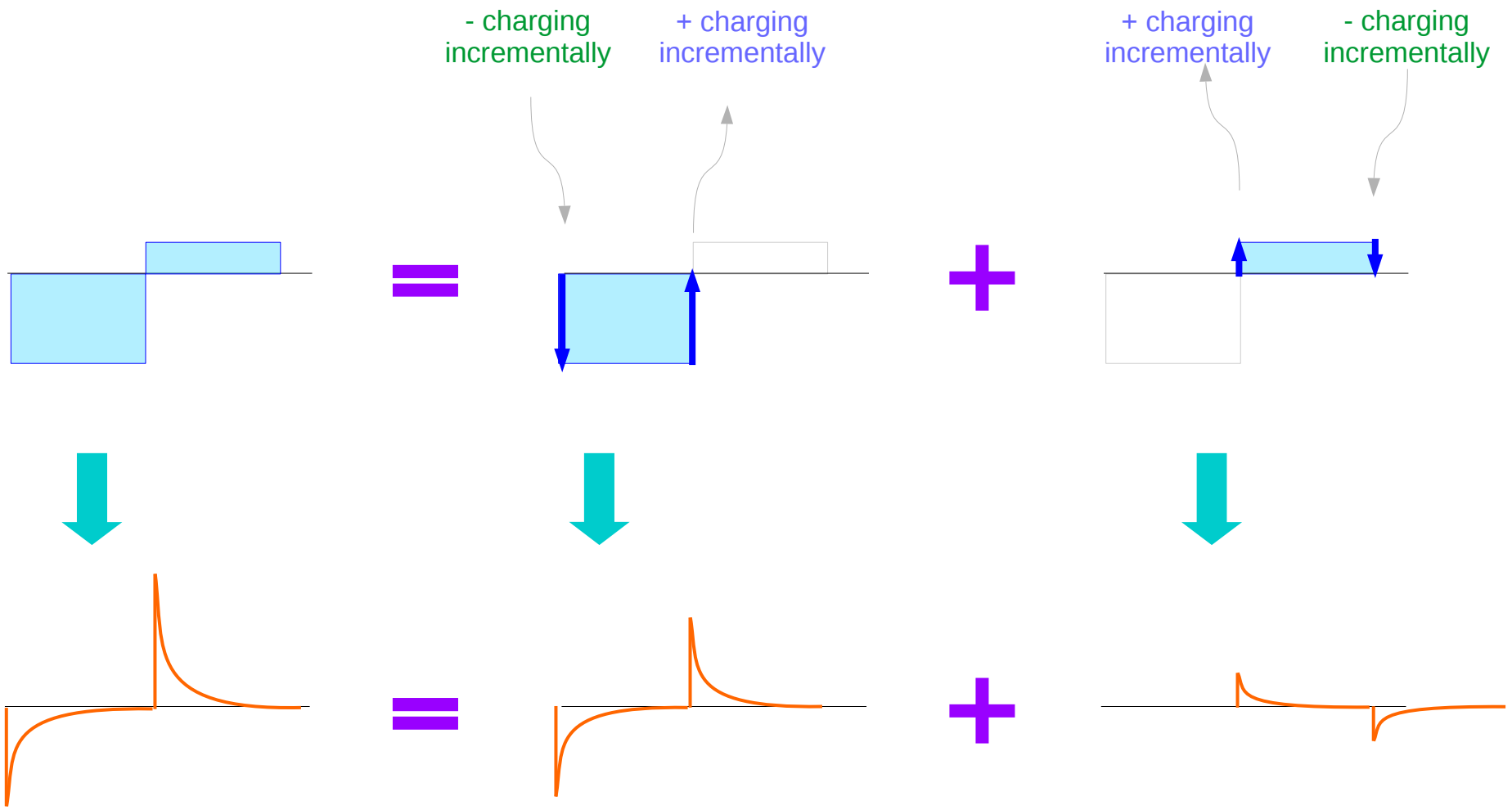
Continuous (Ever-) Charing Operations

Incremental Voltage Increment ➡ + Charging incrementally

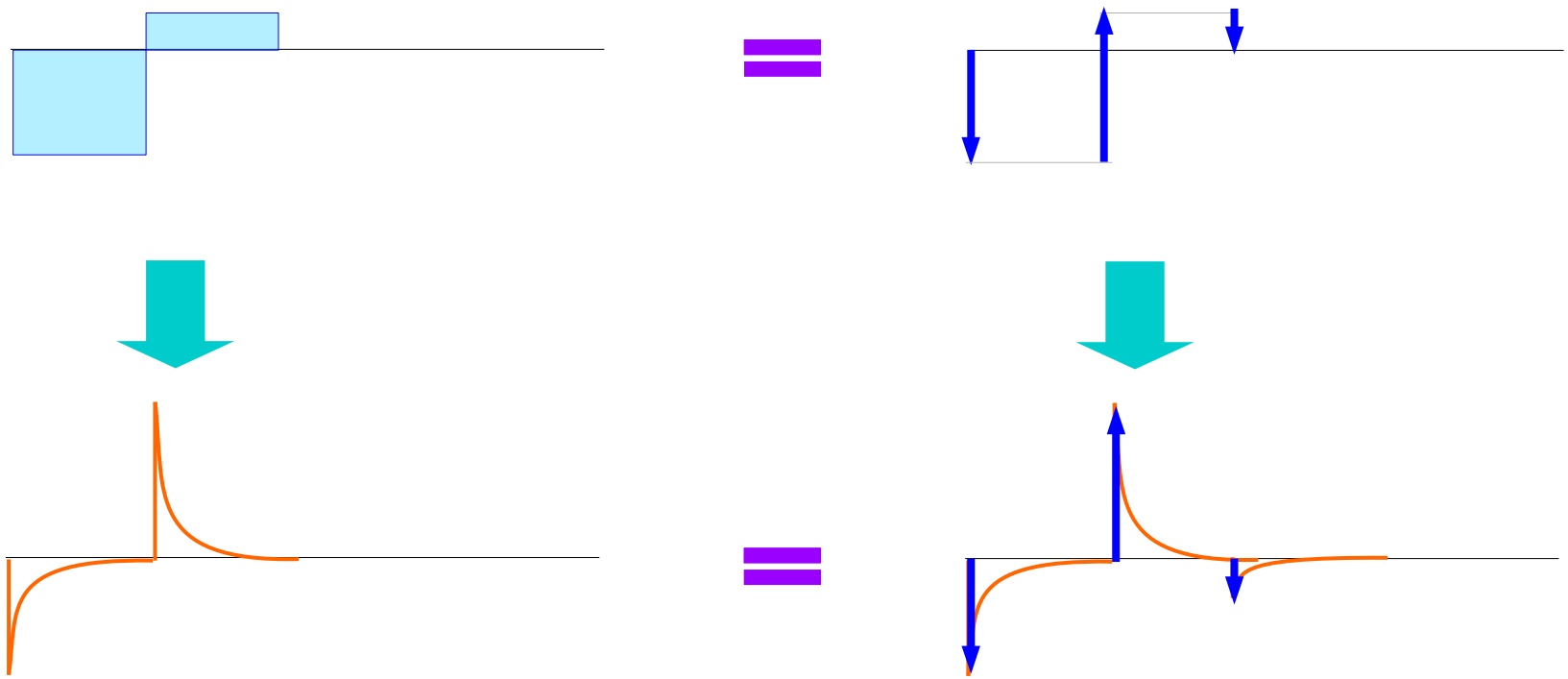
Incremental Voltage Decrement ➡ – Charging incrementally



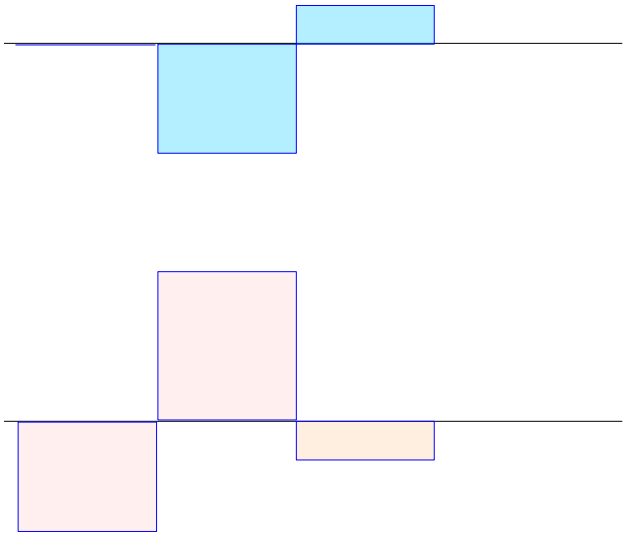
Superposition



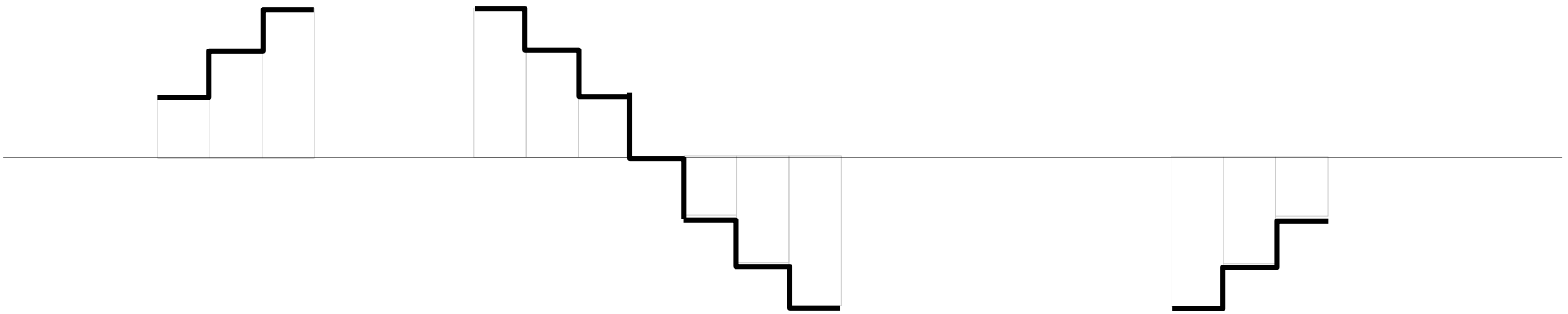
Superposition - Small Time Constant

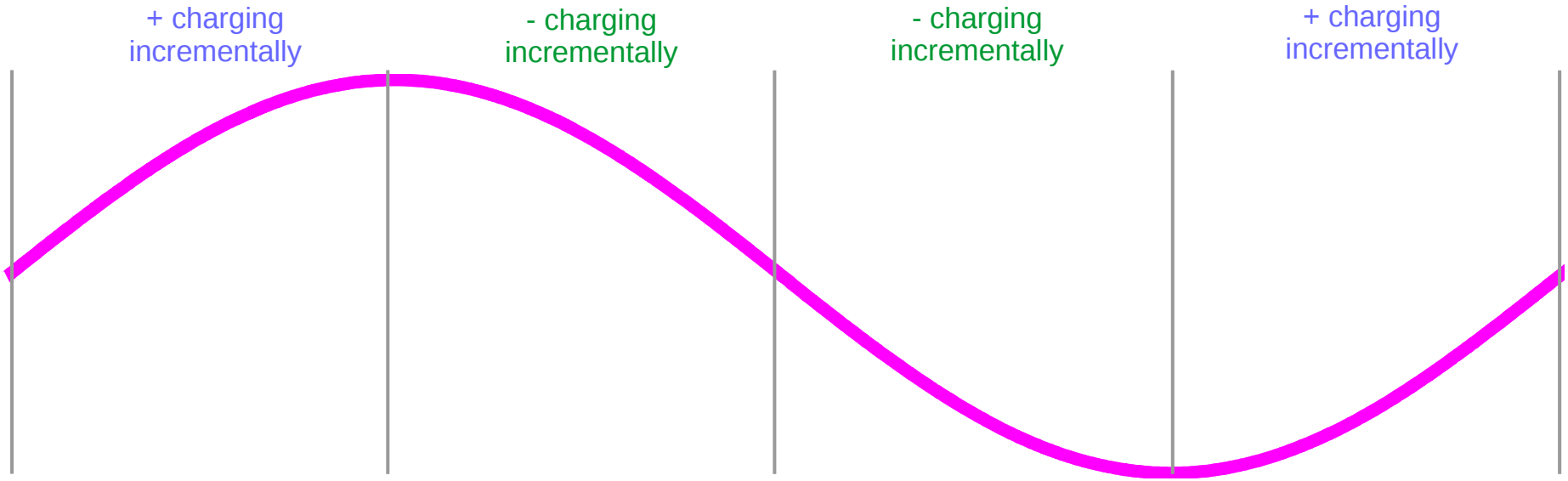


Difference, Differentiation

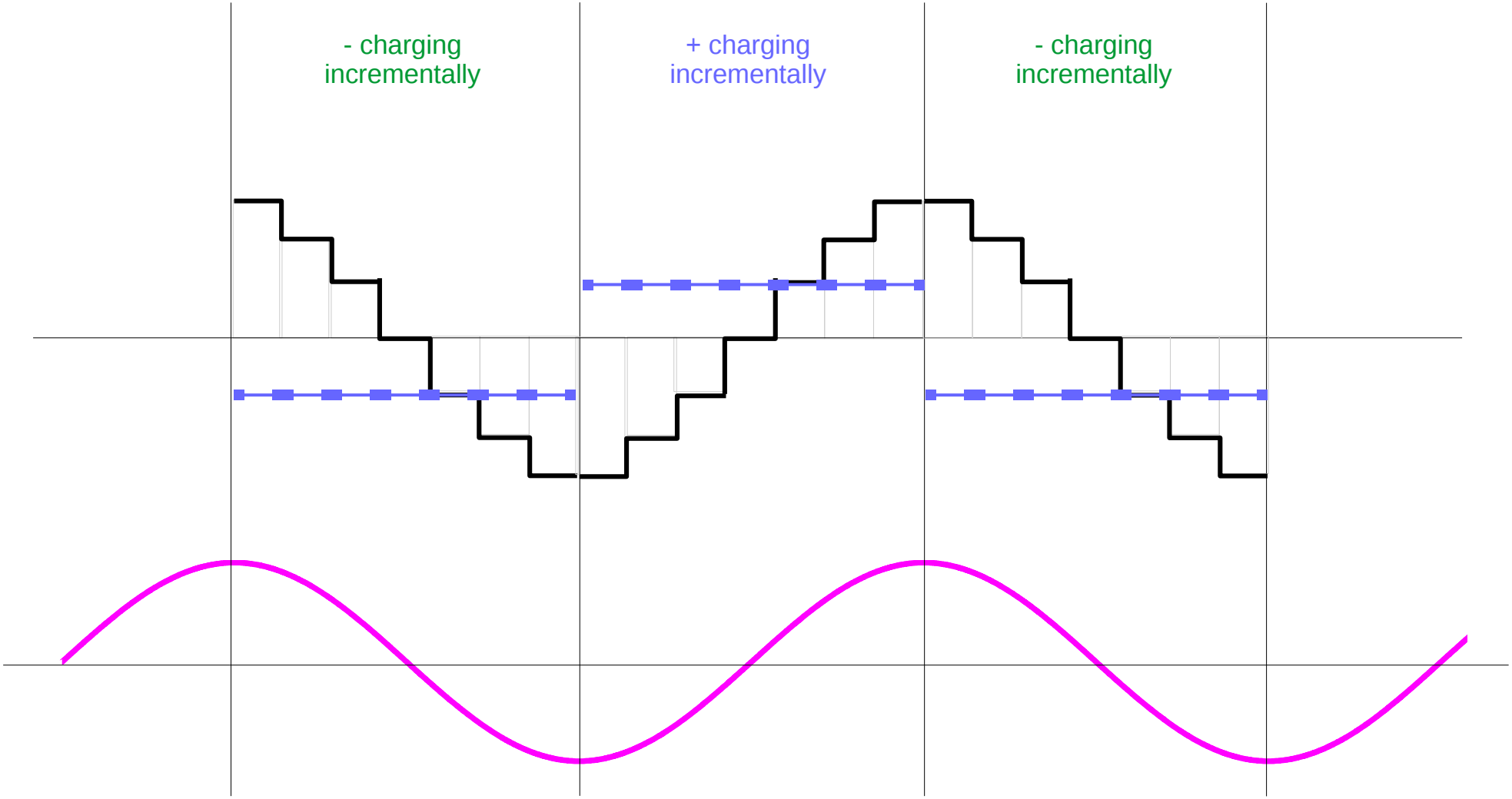


Continuous Charing and Discharging Operations





Incrementally Charging



An AC Voltage Source

Fully Discharged State

Under Positively Charging

Positively Charged State

Under Negatively Charging

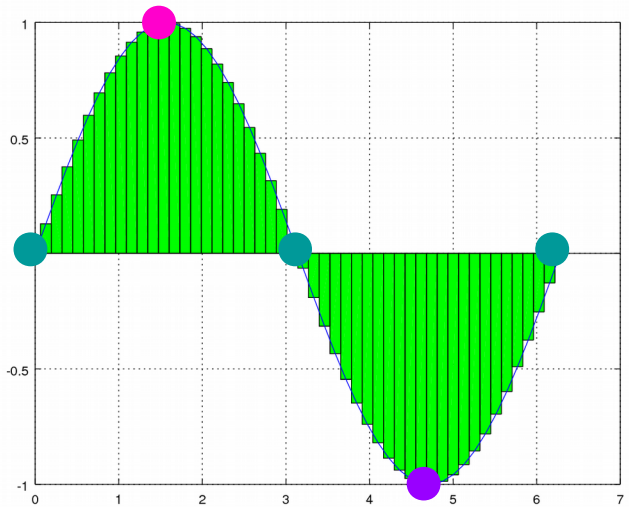
Fully Discharged State

Under Negatively Charging

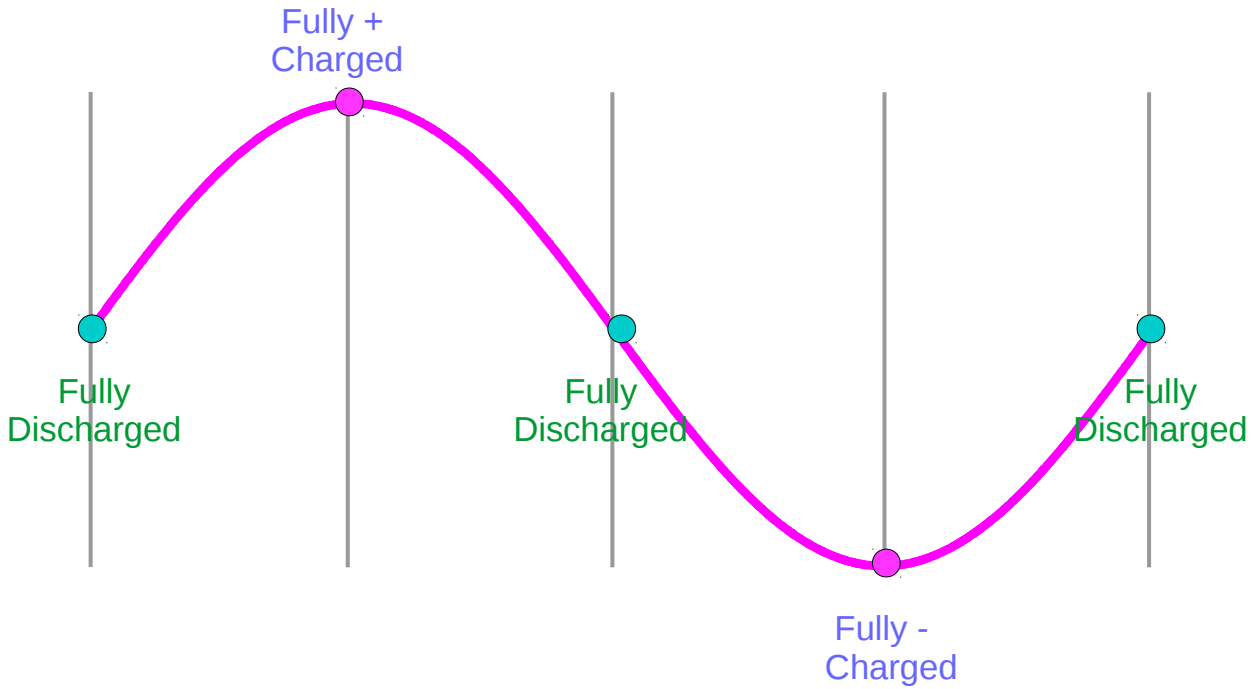
Negatively Charged State

Under Positively Charging

Fully Discharged State

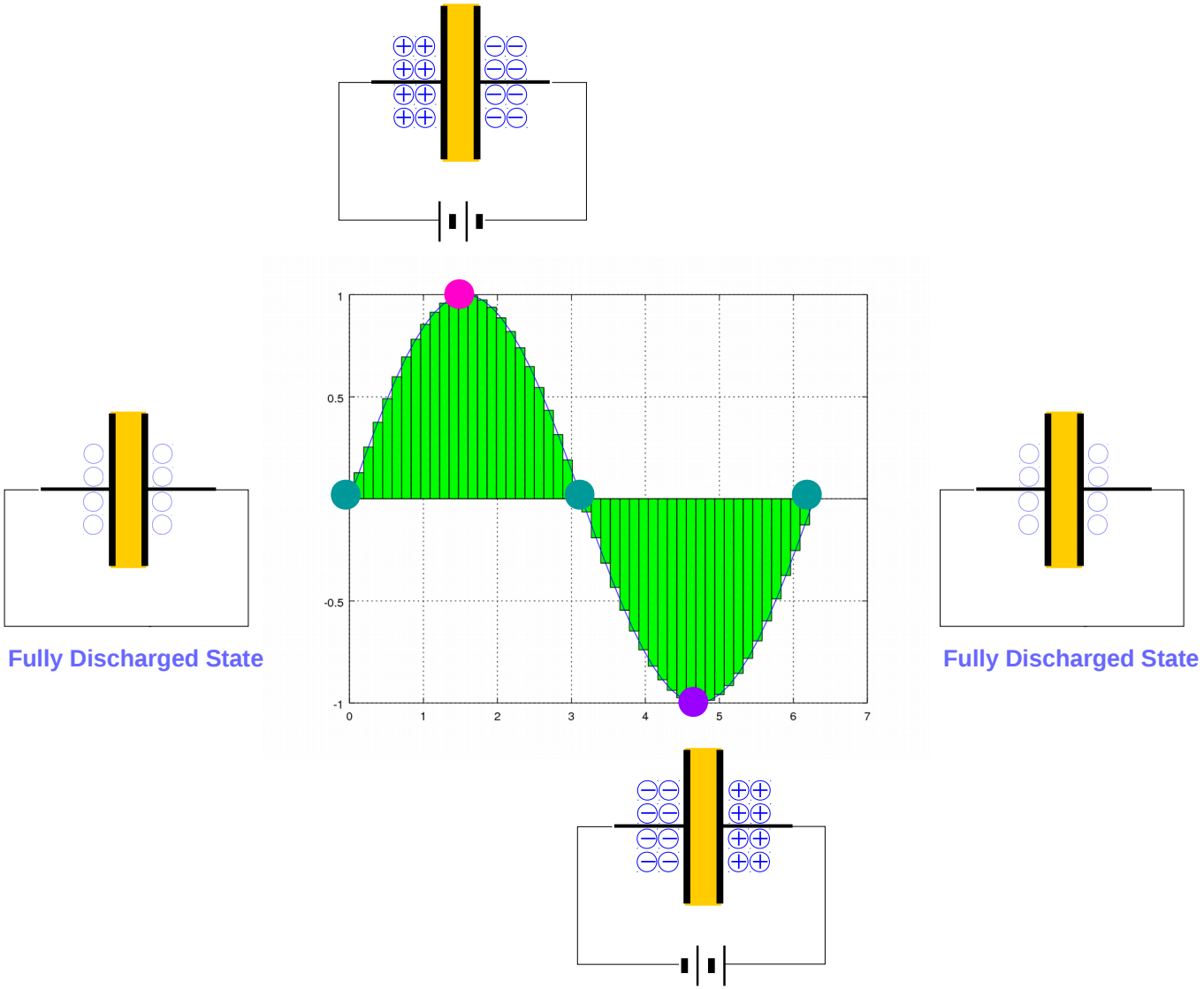


Fully Charged and Fully Discharged

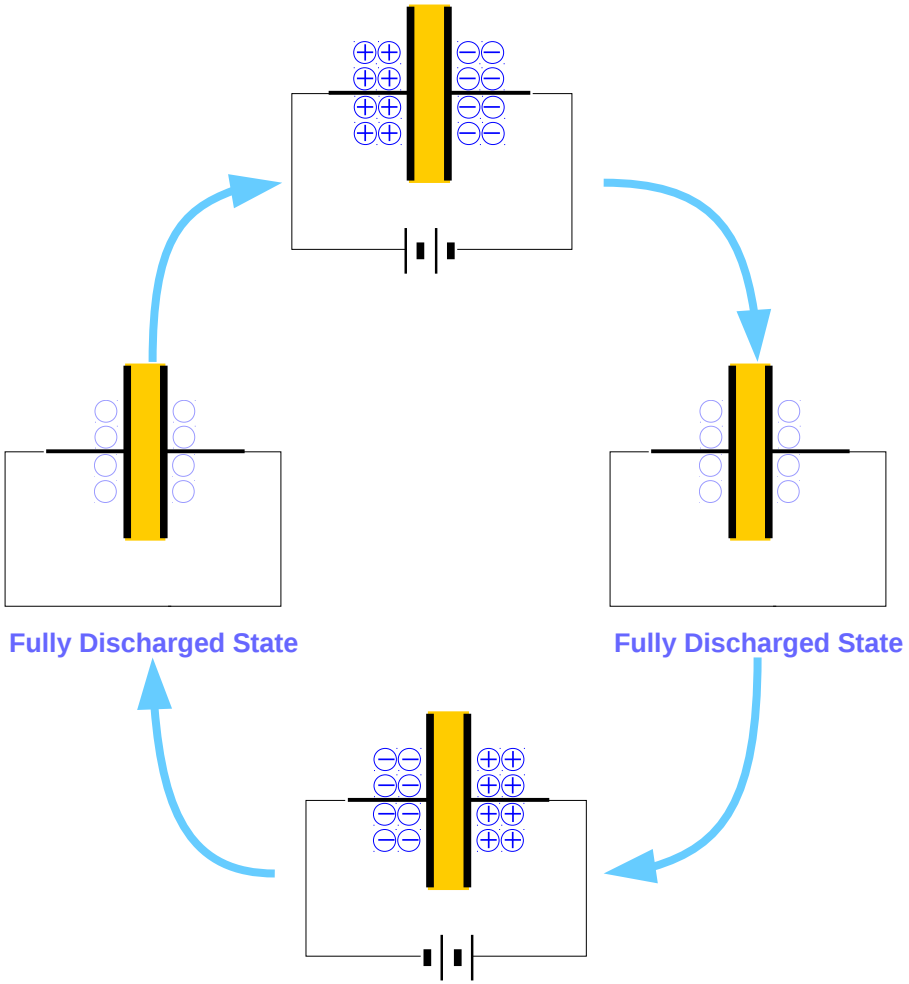


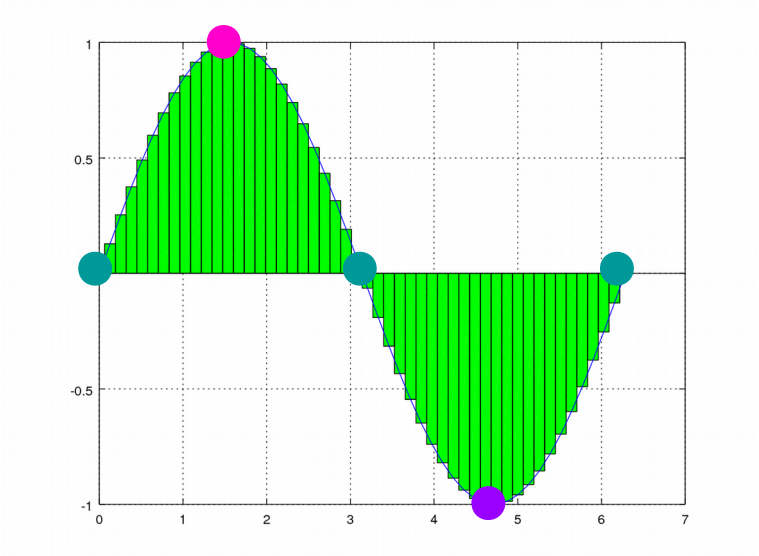
(+) Charging	(–) Charging	(–) Charging	(+) Charging
(+) Current	(–) Current	(–) Current	(+) Current
(+) Charging	(+) **Dis**charging	(–) Charging	(–) **Dis**charging

A Cycle

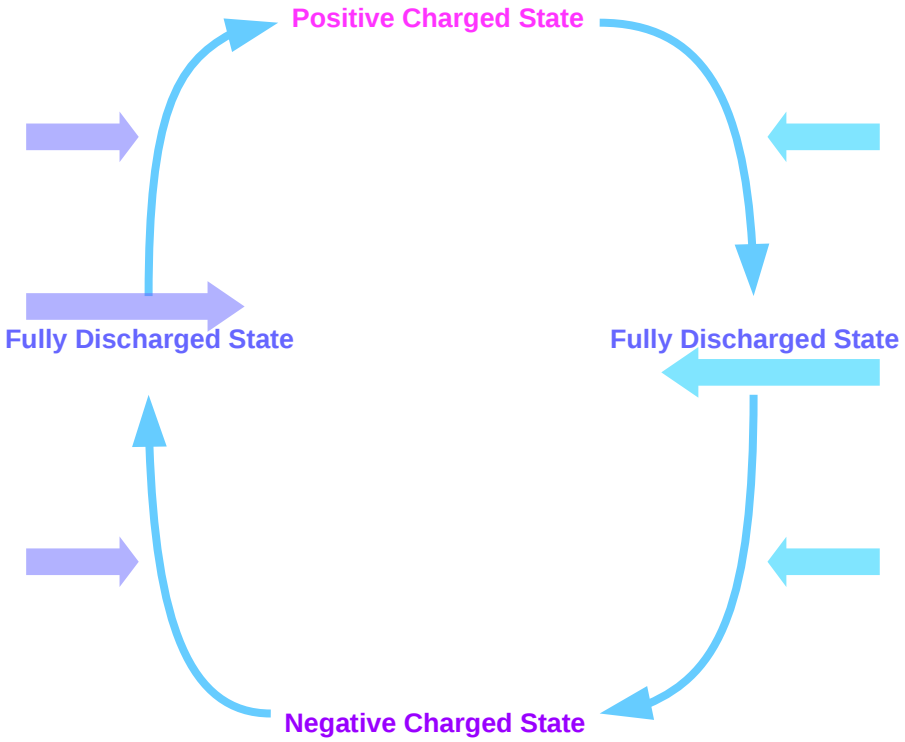


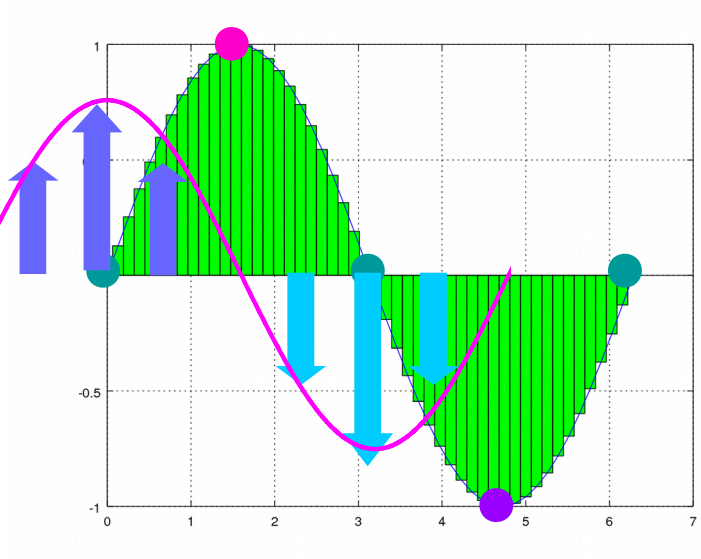
State Transition Diagram





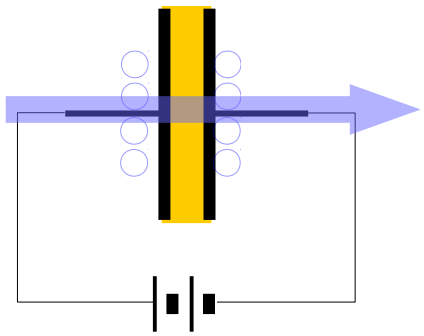
Current Flow



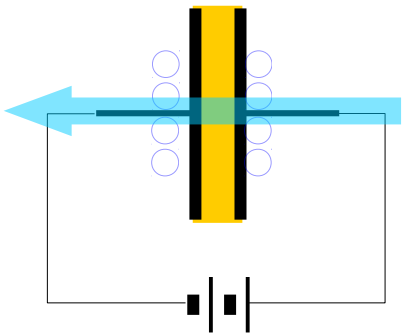


Fully Discharged : Large Current

Fully Discharged State
large current

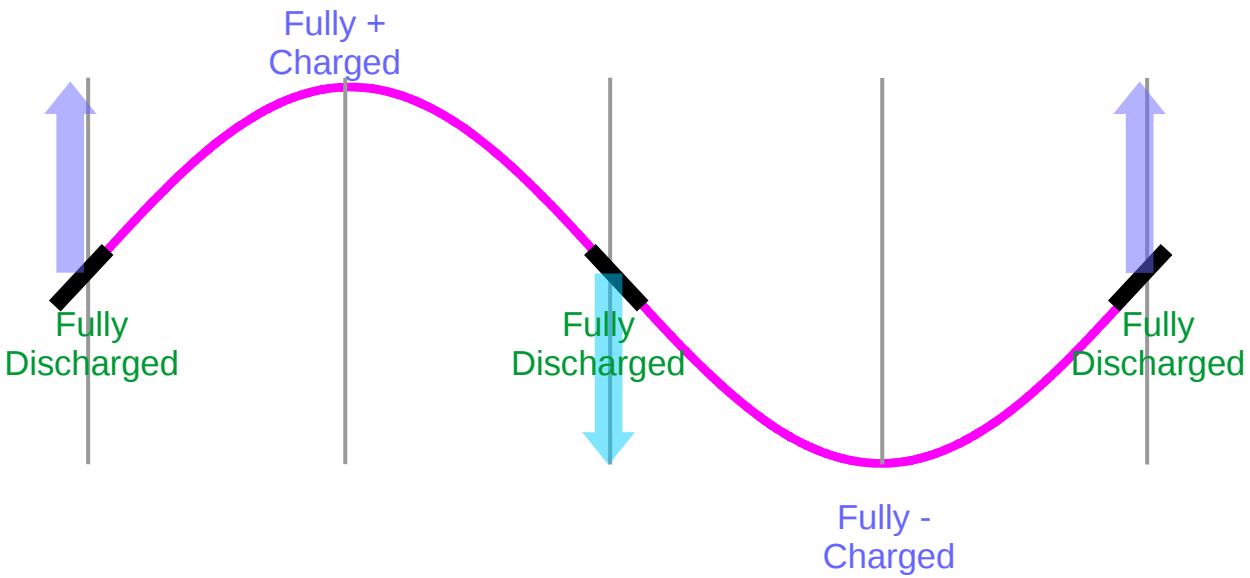


Fully Discharged State
large current



Enough space for
large movement of charges

This state can flow
large current
in either direction

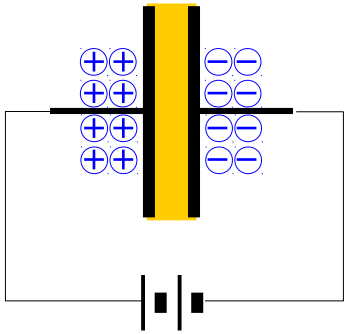


$$\left|\frac{dv_c}{dt}\right| = 1 \quad (max\ value)$$

Fully Charged : Zero Current

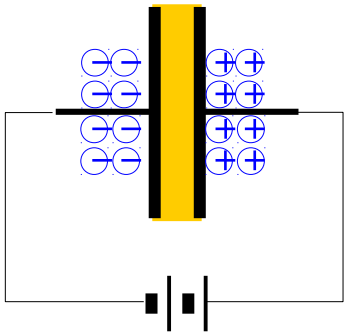
Positively Charged State

fully charged → no current



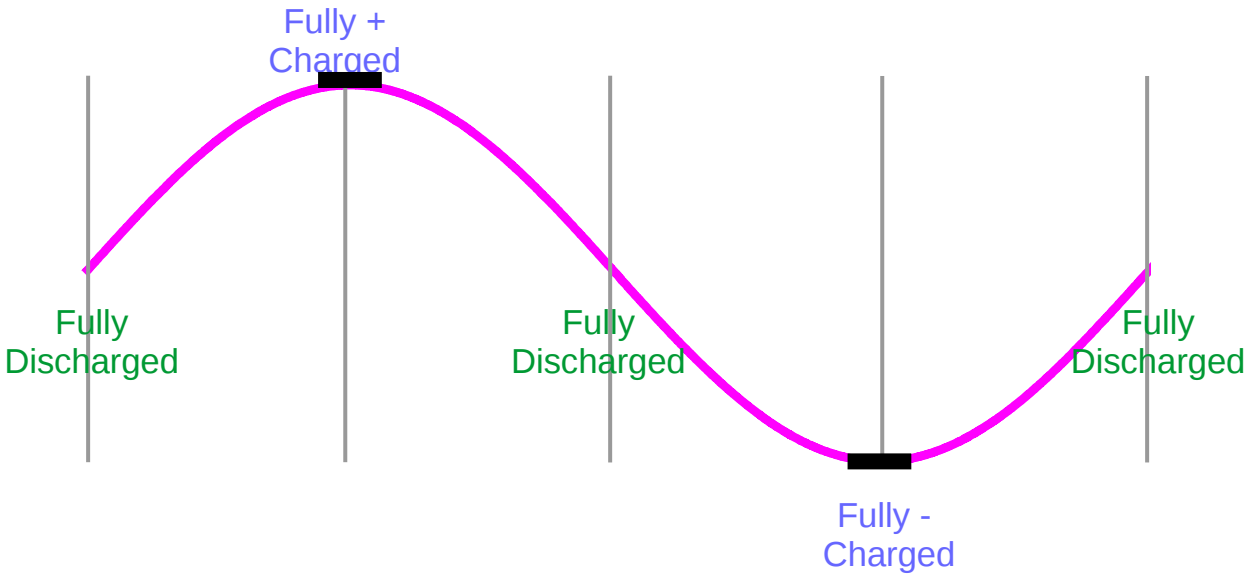
Negatively Charged State

fully charged → no current



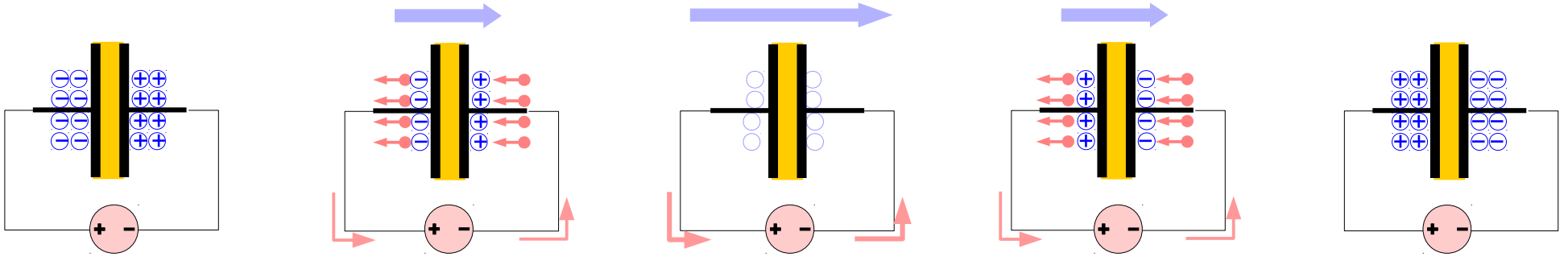
Crowded →
No more space

no current



$$\left|\frac{dv_c}{dt}\right| = 0 \quad (min\ value)$$

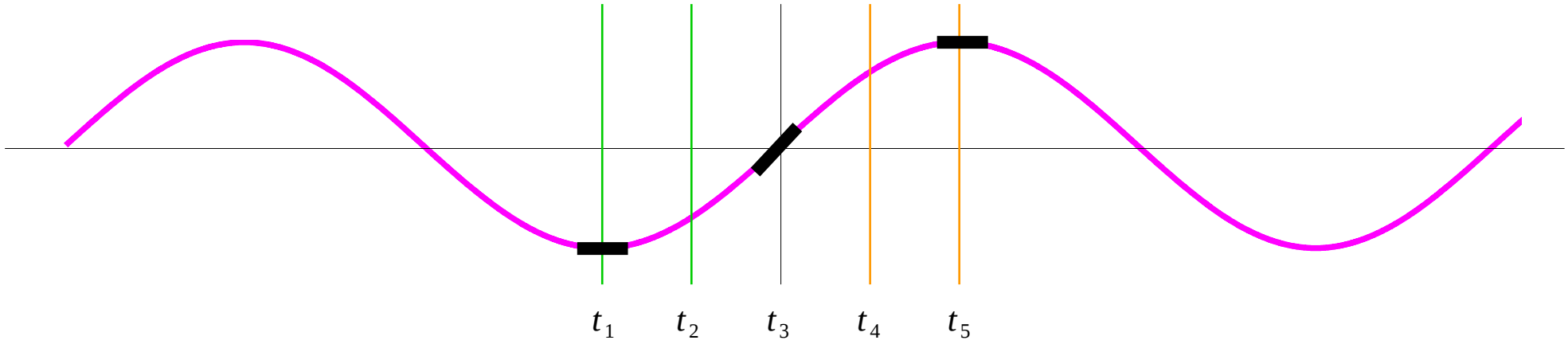
Incrementally, Charging Positively



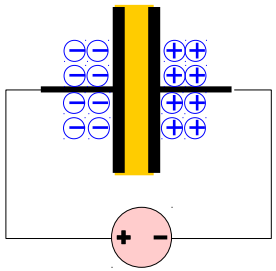
$v(t_1) < 0$ $v(t_2) < 0$ $v(t_3) < 0$ $v(t_4) > 0$ $v(t_5) > 0$

$v'(t_1) > 0$ $v'(t_2) > 0$ $v'(t_3) > 0$ $v'(t_4) > 0$ $v'(t_5) > 0$

$\Delta v(t_1) > 0$ $\Delta v(t_2) > 0$ $\Delta v(t_3) > 0$ $\Delta v(t_4) > 0$ $\Delta v(t_5) > 0$



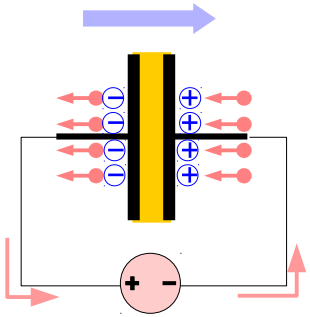
Incrementally, Charging Positively



$v(t_1) < 0$

$v'(t_1) > 0$

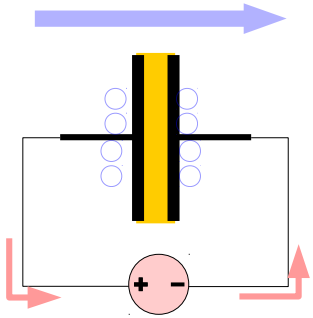
$\Delta v(t_1) > 0$



$v(t_2) < 0$

$v'(t_2) > 0$

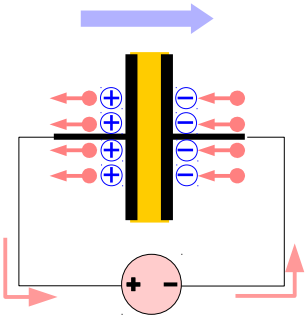
$\Delta v(t_2) > 0$



$v(t_3) < 0$

$v'(t_3) > 0$

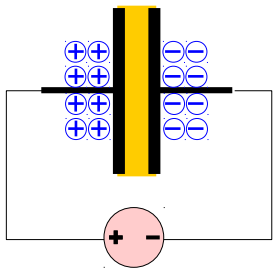
$\Delta v(t_3) > 0$



$v(t_4) > 0$

$v'(t_4) > 0$

$\Delta v(t_4) > 0$



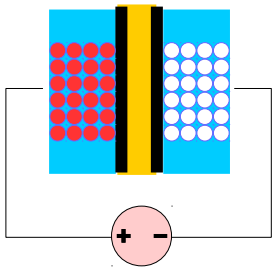
$v(t_5) > 0$

$v'(t_5) > 0$

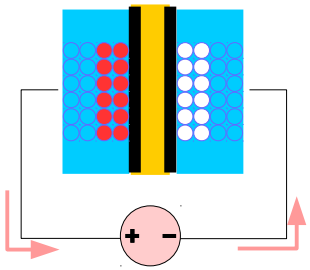
$\Delta v(t_5) > 0$

excess electrons

positive ions

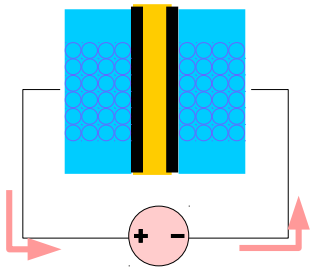


$\Delta v(t_1) > 0$

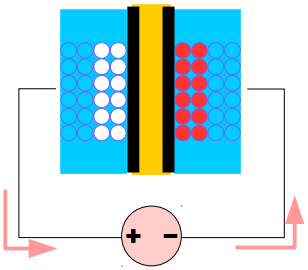


$\Delta v(t_2) > 0$

equilibrium



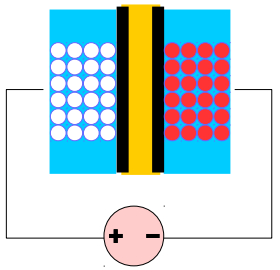
$\Delta v(t_3) > 0$



$\Delta v(t_4) > 0$

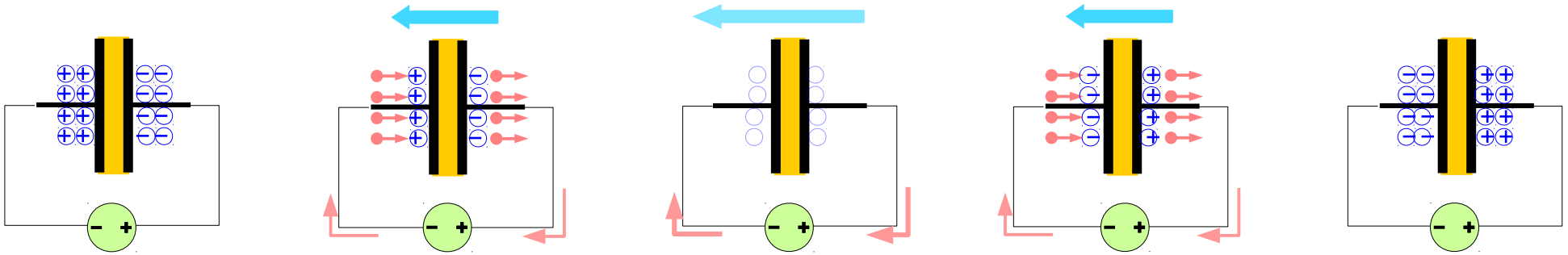
positive ions

excess electrons



$\Delta v(t_5) > 0$

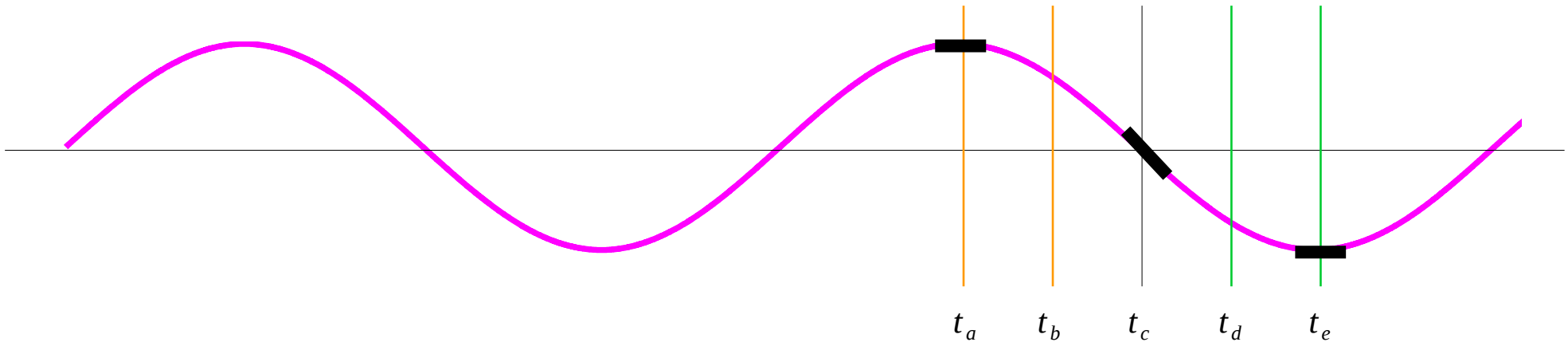
Incrementally, Charging Negatively



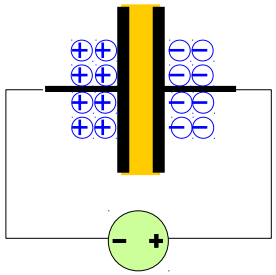
$v(t_a) > 0$ $v(t_b) > 0$ $v(t_c) = 0$ $v(t_d) < 0$ $v(t_e) < 0$

$v'(t_a) < 0$ $v'(t_b) < 0$ $v'(t_c) < 0$ $v'(t_d) < 0$ $v'(t_e) < 0$

$\Delta v(t_a) < 0$ $\Delta v(t_b) < 0$ $\Delta v(t_c) < 0$ $\Delta v(t_d) < 0$ $\Delta v(t_e) < 0$



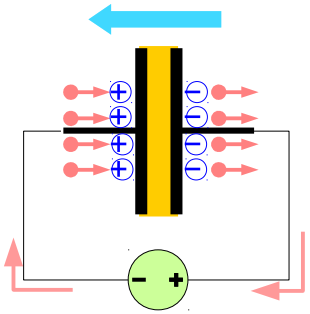
Incrementally, Charging Negatively



$v(t_a) > 0$

$v'(t_a) < 0$

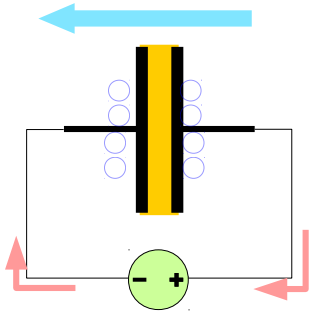
$\Delta v(t_a) < 0$



$v(t_b) > 0$

$v'(t_b) < 0$

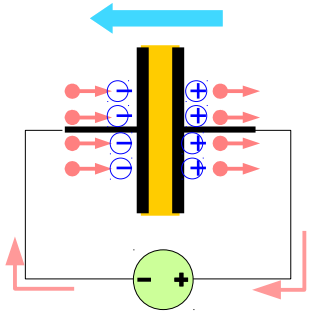
$\Delta v(t_b) < 0$



$v(t_c) = 0$

$v'(t_c) < 0$

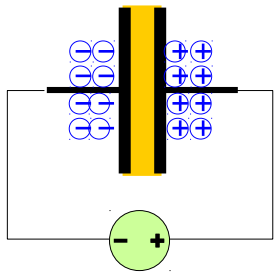
$\Delta v(t_c) < 0$



$v(t_d) < 0$

$v'(t_d) < 0$

$\Delta v(t_d) < 0$



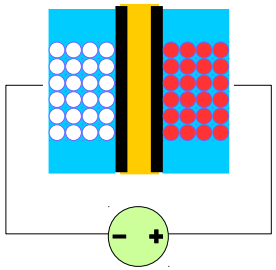
$v(t_e) < 0$

$v'(t_e) < 0$

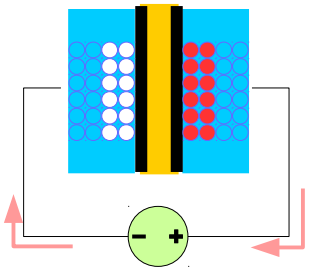
$\Delta v(t_e) < 0$

positive ions

excess electrons

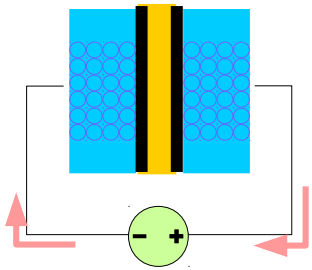


$\Delta v(t_a) < 0$

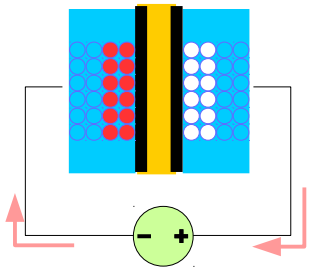


$\Delta v(t_b) < 0$

equilibrium



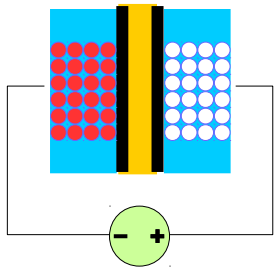
$\Delta v(t_c) < 0$



$\Delta v(t_d) < 0$

excess electrons

positive ions



$\Delta v(t_e) < 0$

Difference of Samples

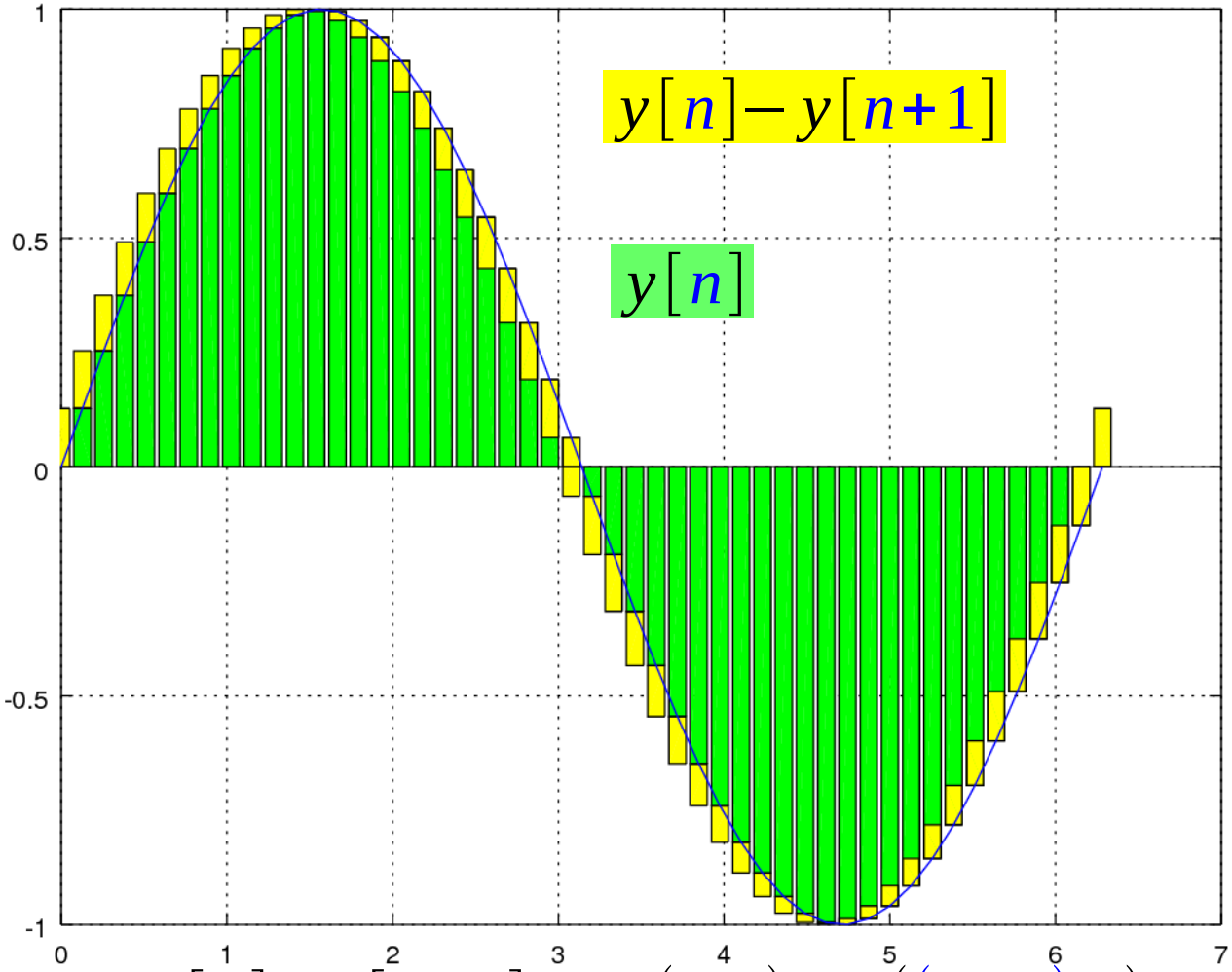
$$y(t) = \sin(t)$$

$$y[n] = \sin(nT)$$

$$y[n]-y[n+1] = \sin(nT)-\sin((n+1)T)$$

$$\frac{y[n]-y[n+1]}{T} \propto \frac{dy}{dt}$$

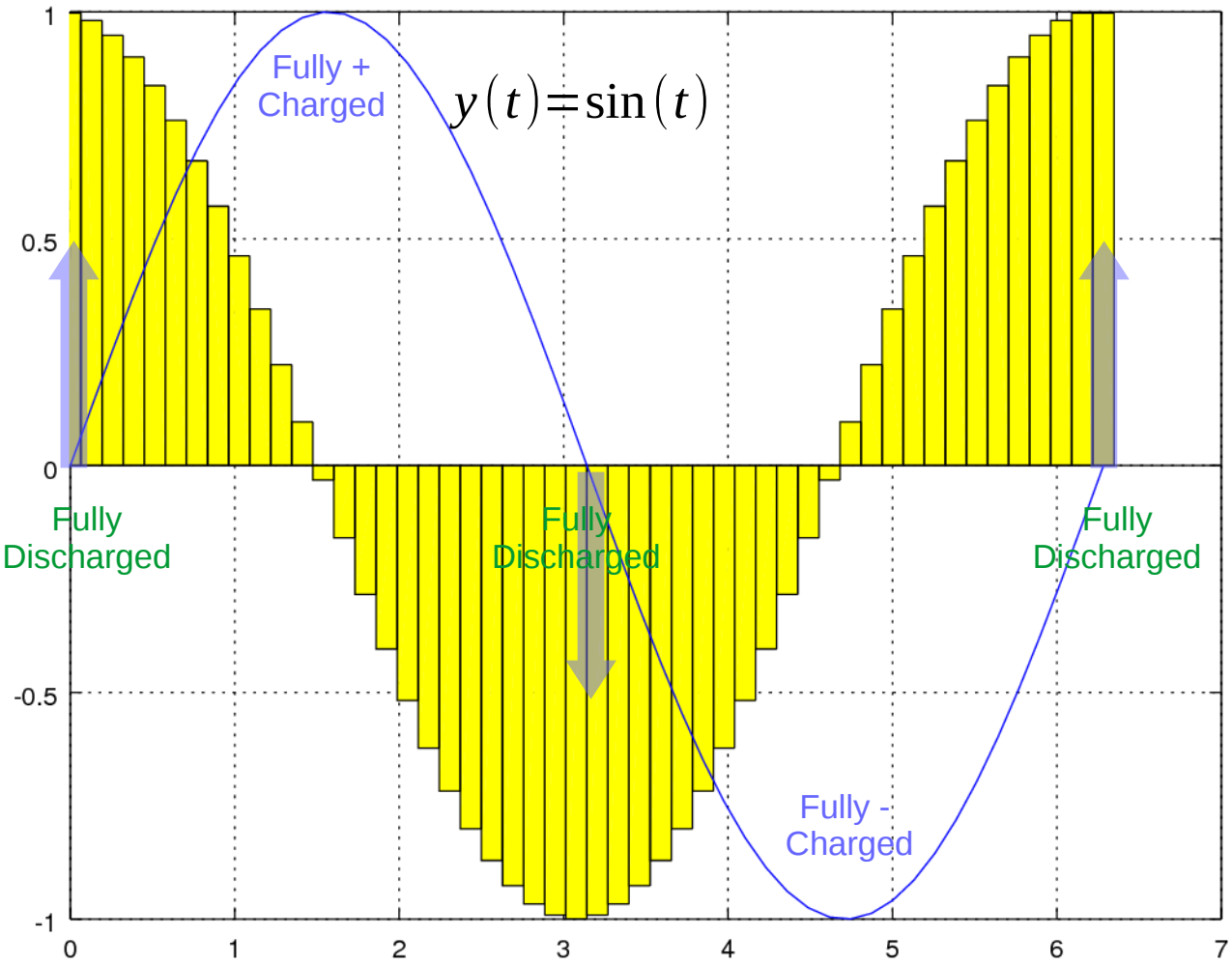
Fully Charged and Fully Discharged




```
 h = bar(t1, [y1' y2'], "stacked")
set(h(1), "facecolor", "g");
set(h(2), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 pi]);
```

$$y[n]-y[n+1] = y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

Fully Charged and Fully Discharged

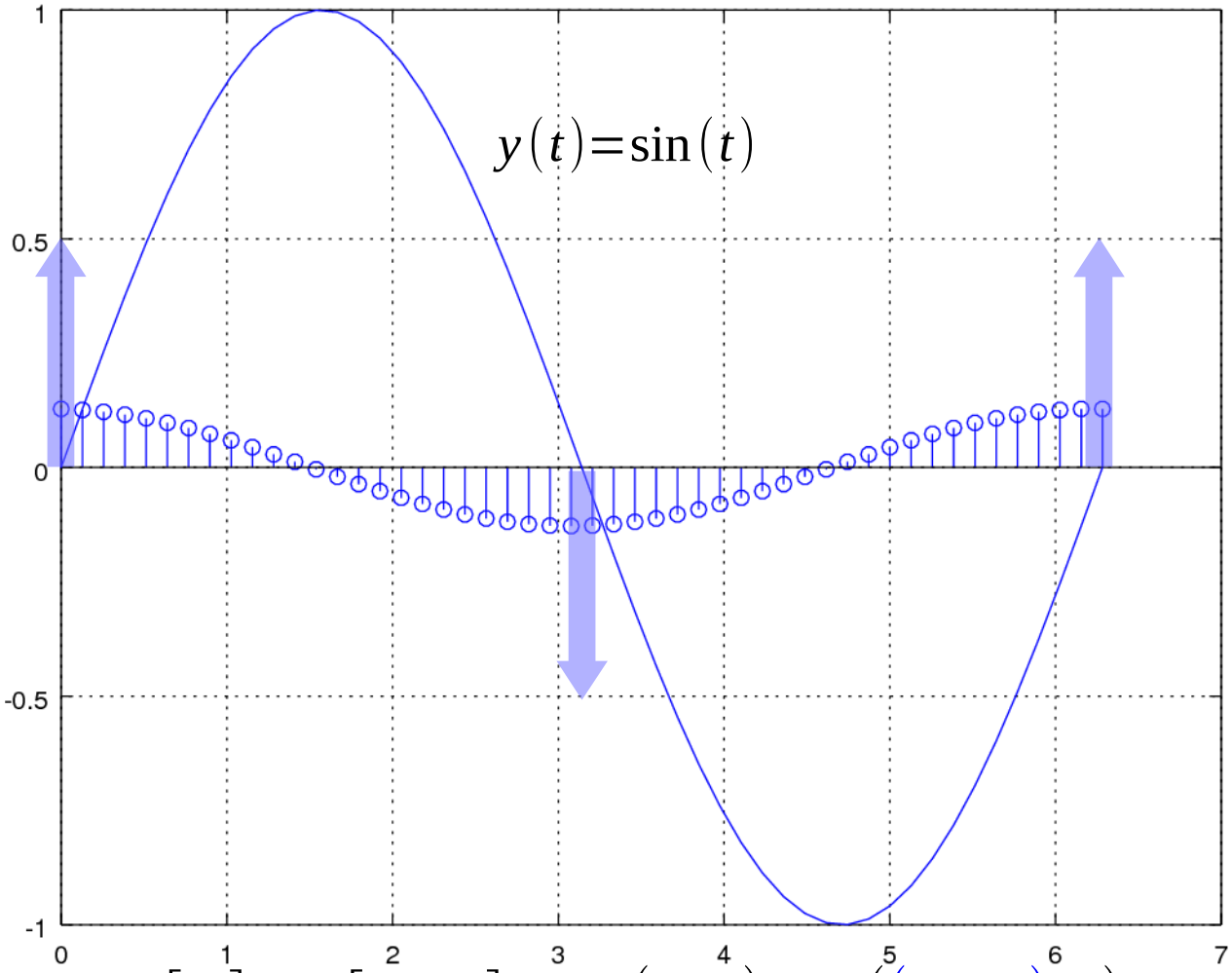



```
h = bar(t1, y2/t(2), "hist")
set(h(1), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$\frac{y[n]-y[n+1]}{T}$$

$$\propto \frac{dy}{dt}$$

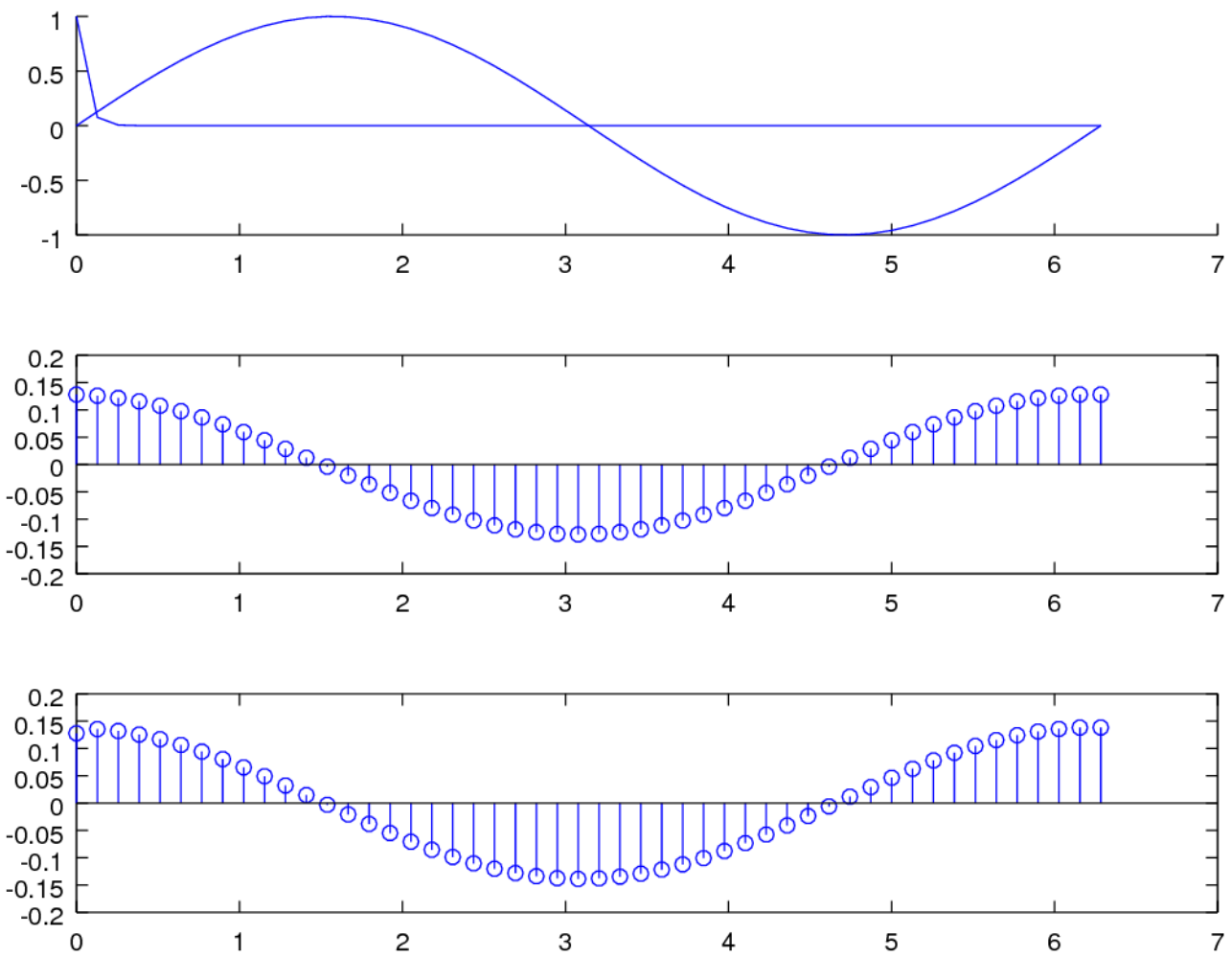
y[n+1] – y[n]




```
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
stem(t1, y2)
hold on
plot(t1, y1)
```

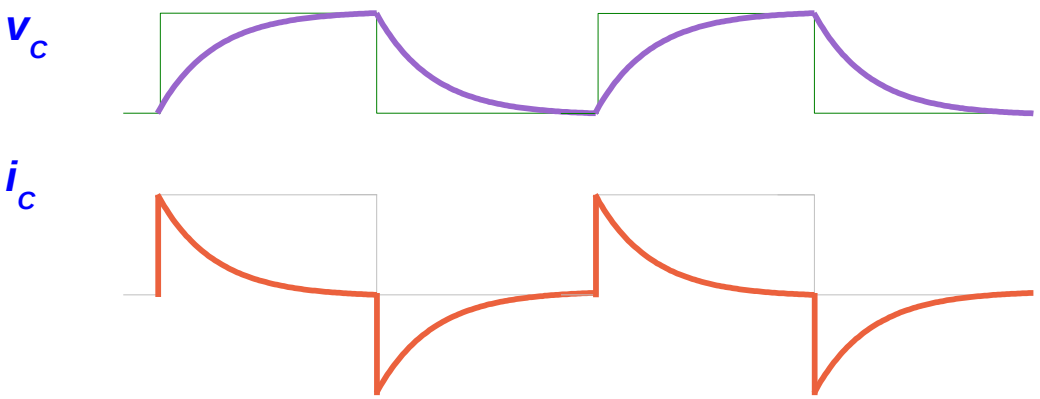
$$y[n]-y[n+1]= y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

Fully Charged and Fully Discharged



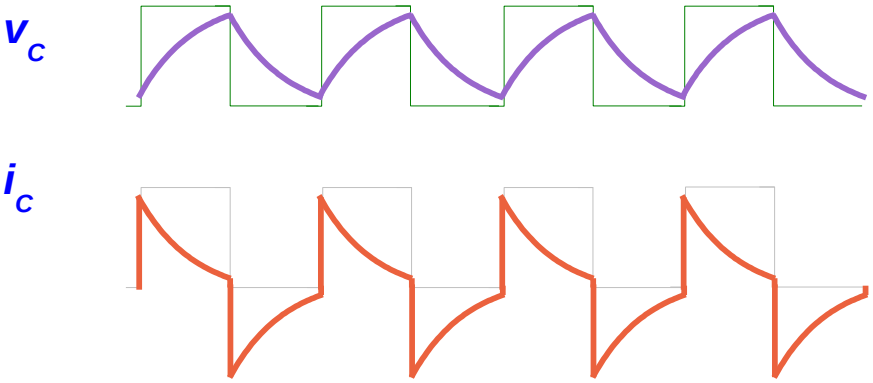

```
clf
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
y3 = e.^(-20*t);
y4 = conv(y2, y3);
y5 = y4([1:length(t1)]);

subplot(3, 1, 2);
stem(t1, y2)
subplot(3, 1, 1);
hold on
plot(t1, y1);
plot(t1, y3);
subplot(3, 1, 3);
stem(t1, y5);
```

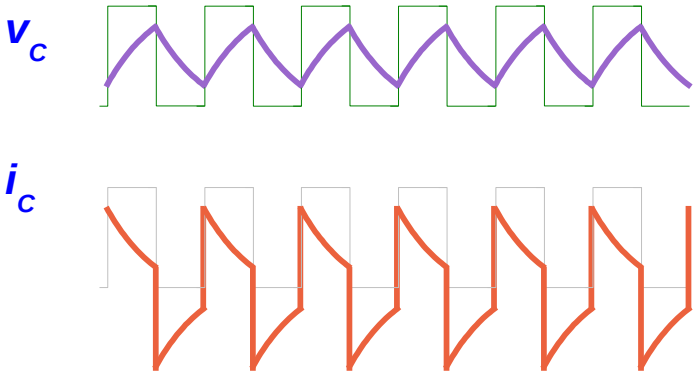
Pulse



$$i_C = C \frac{d v_C}{d t}$$

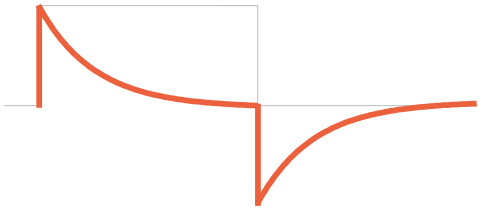
ω ↑ i_c ↑ x_c ↓



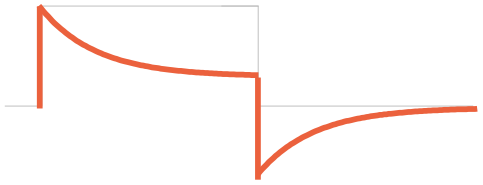


Time Constants

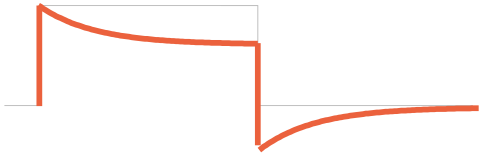
i_c



$\tau = RC$ *small time constant*



$\tau = RC$ *medium time constant*



$\tau = RC$ *large time constant*

Time Constants

i_c

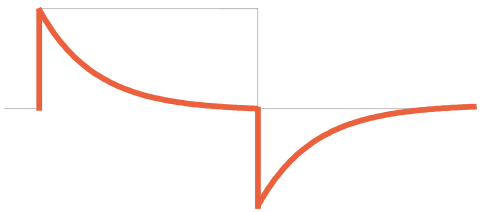
$$\tau_1 < \tau_2 < \tau_3$$
$$a_1 > a_2 > a_3$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}} = e^{-at}$$

$$\tau = RC = \frac{1}{a}$$

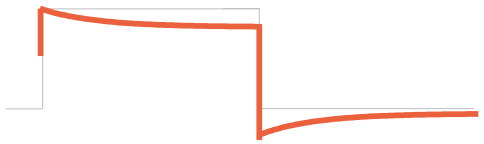
Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C} \gg R$

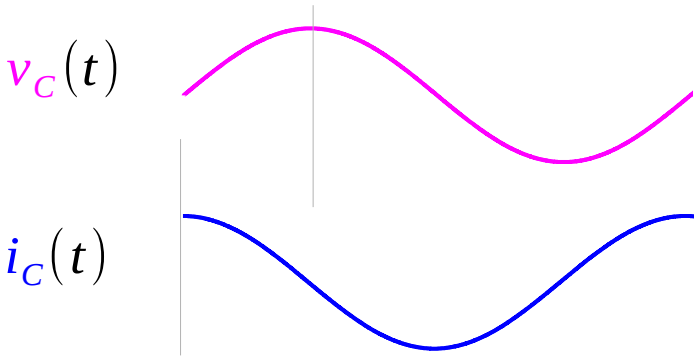
Fully Capacitative

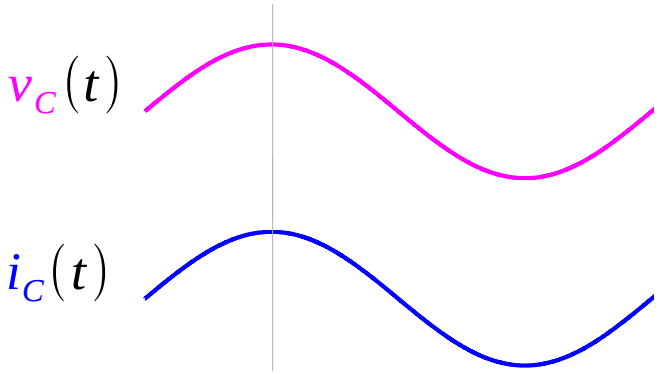
large τ

large C

small $\frac{1}{\omega C} \ll R$

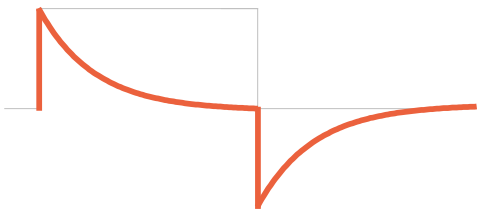
Fully Resistive





Time Constants

i_c



$$\tau = RC$$

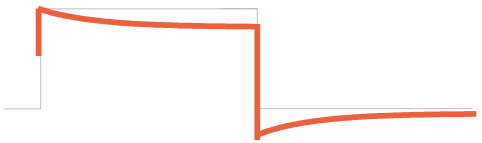
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C} \gg R$

Fully Capacitative



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

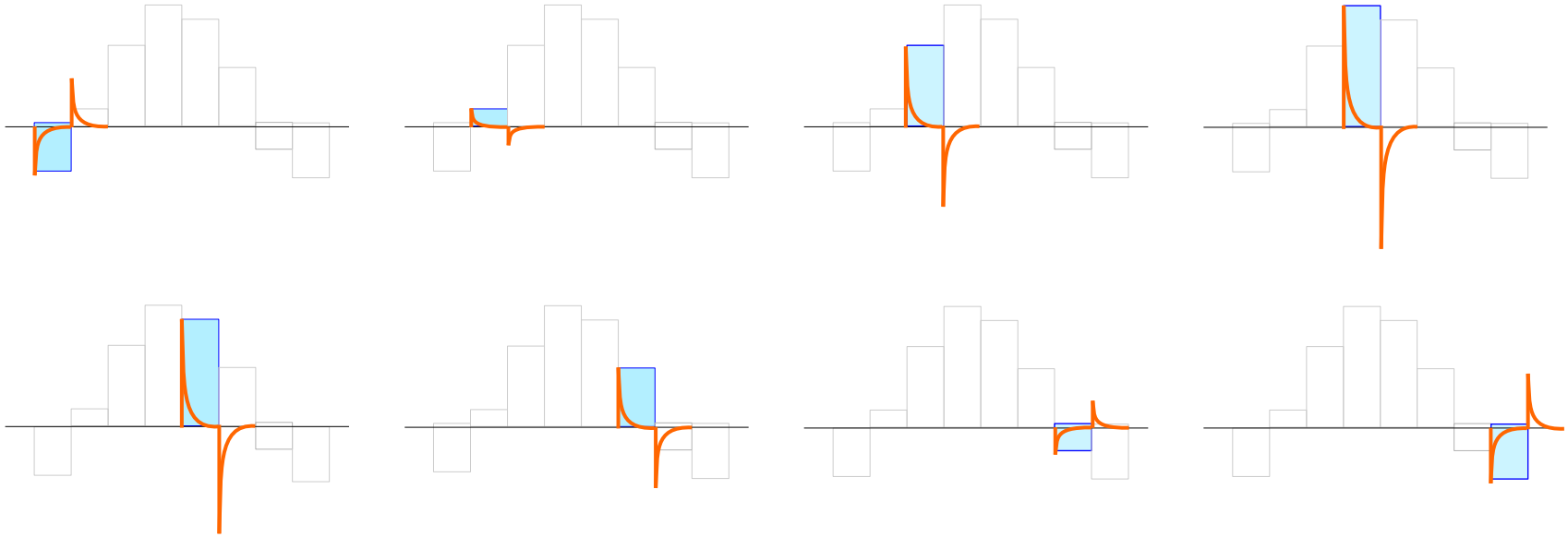
large τ

large C

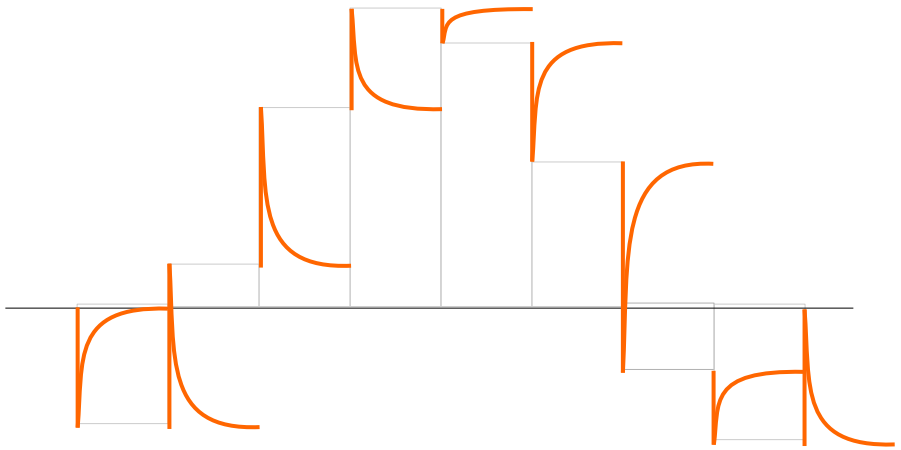
small $\frac{1}{\omega C} \ll R$

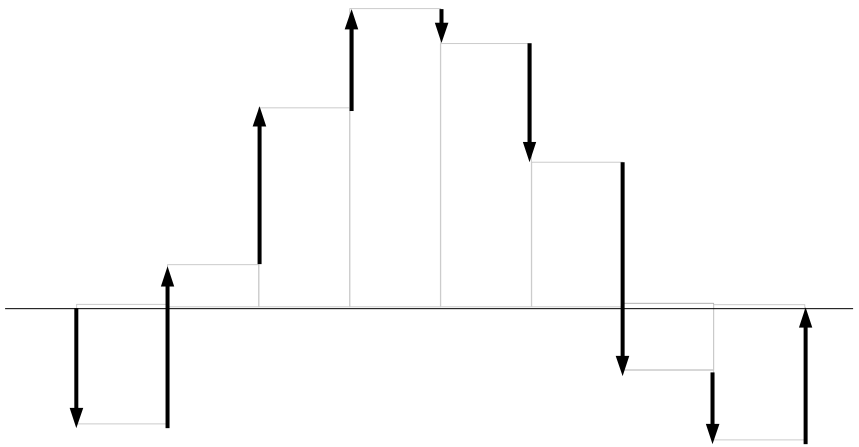
Fully Resistive

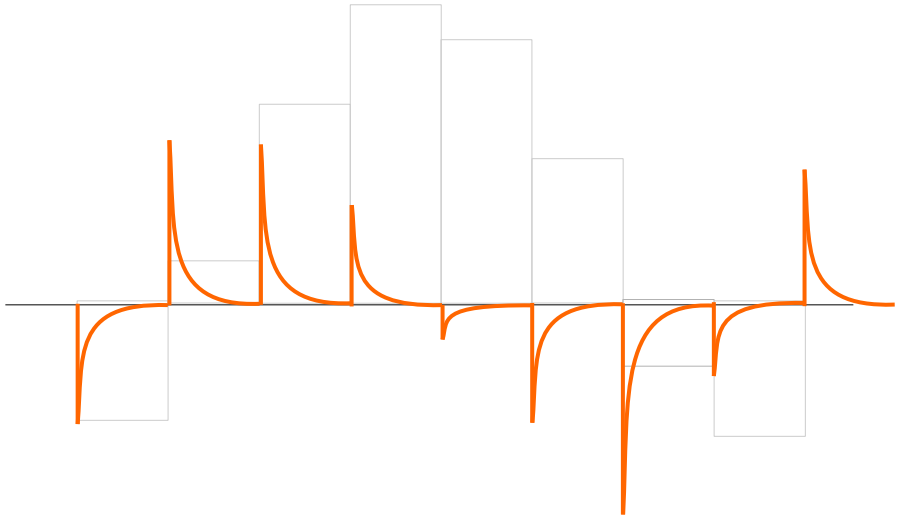
Superposition - Small Time Constant

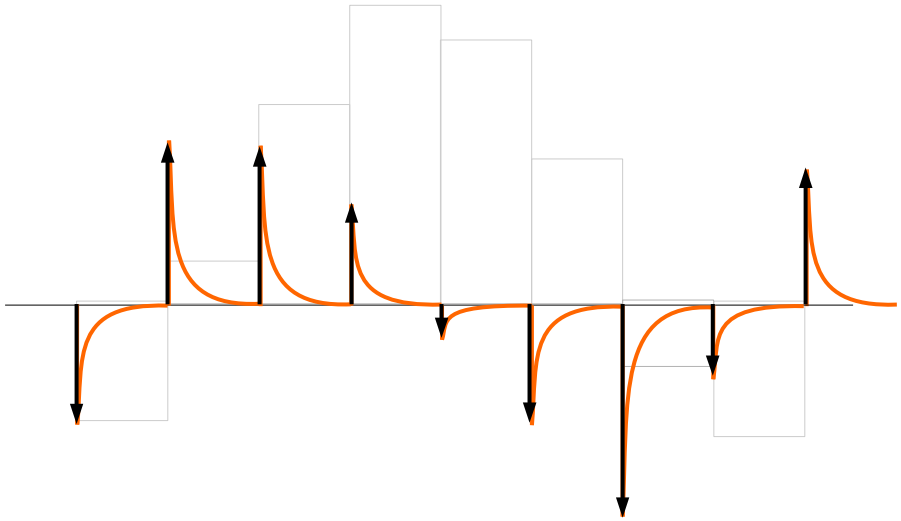


Small Time Constants

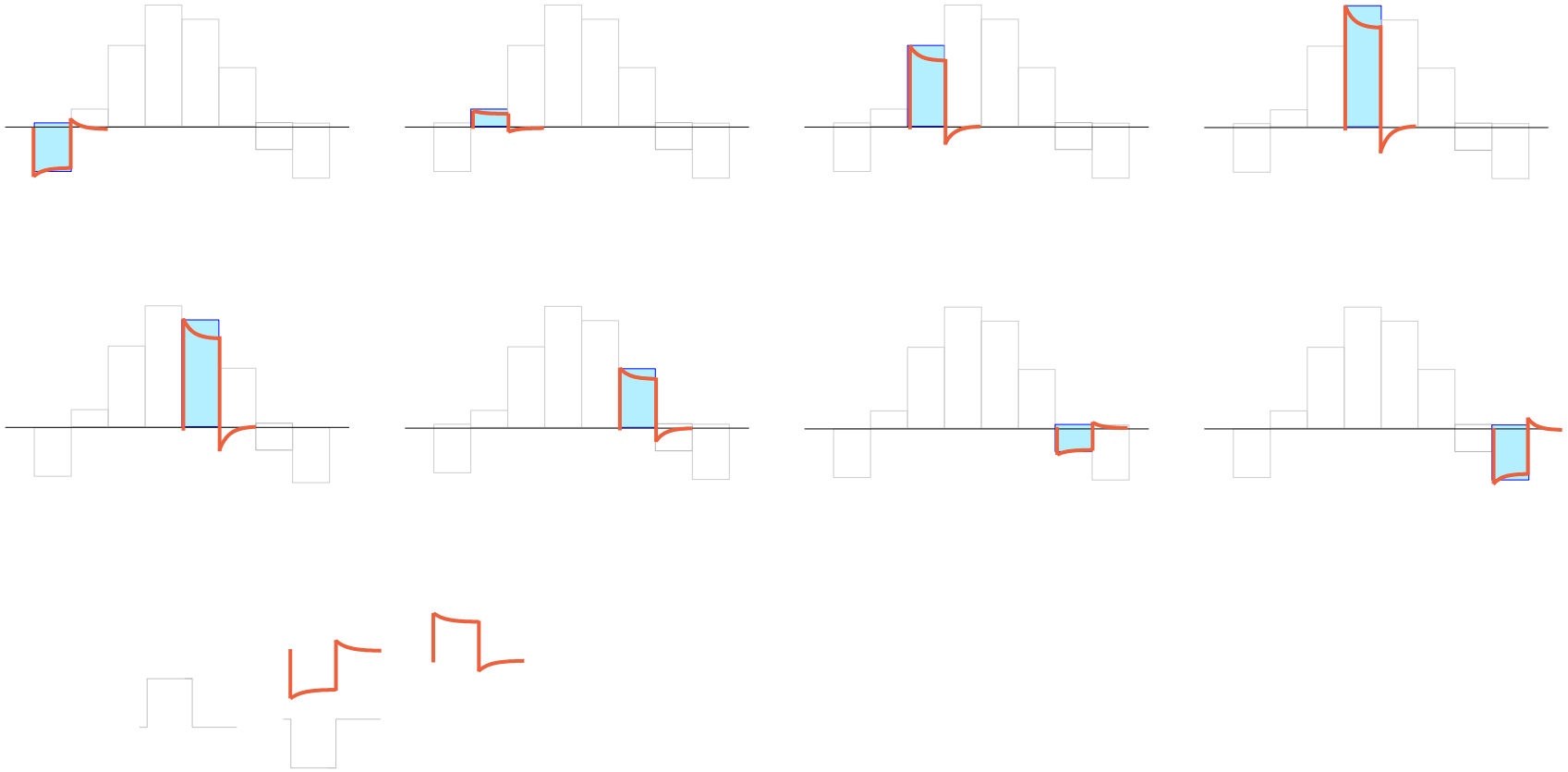




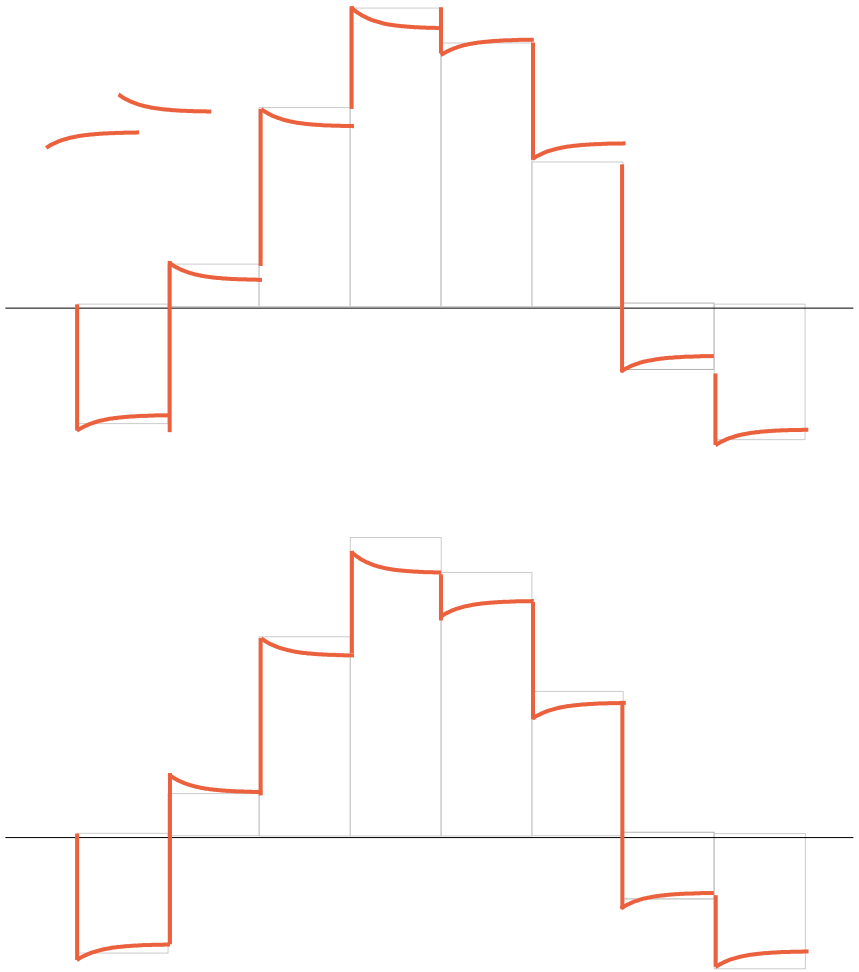




Superposition – Large Time Constant

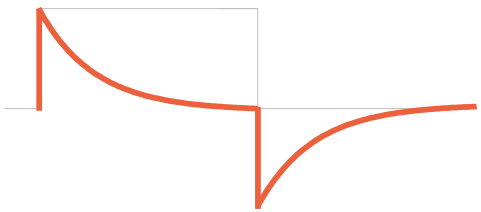


Large Time Constants



Time Constants

i_c



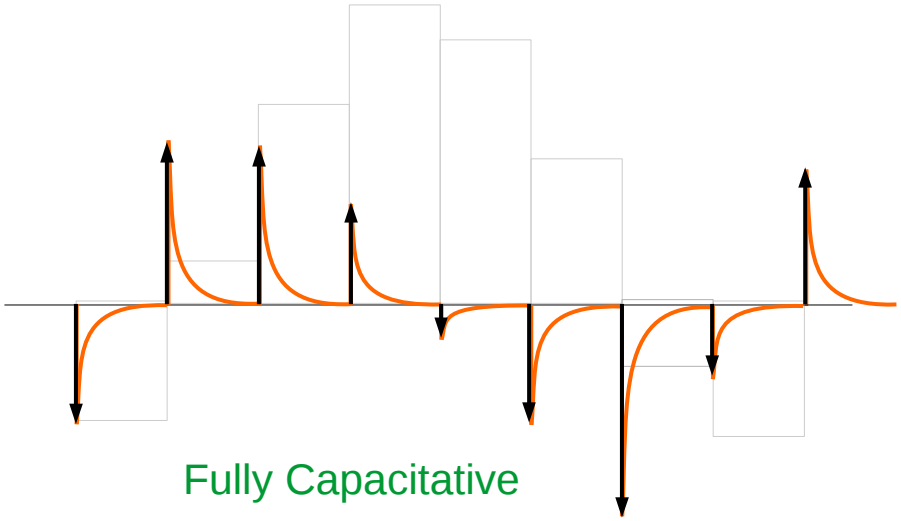
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

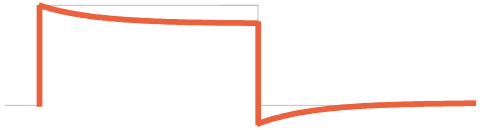
small τ

small C

large $\frac{1}{\omega C} \gg R$



Fully Capacitative



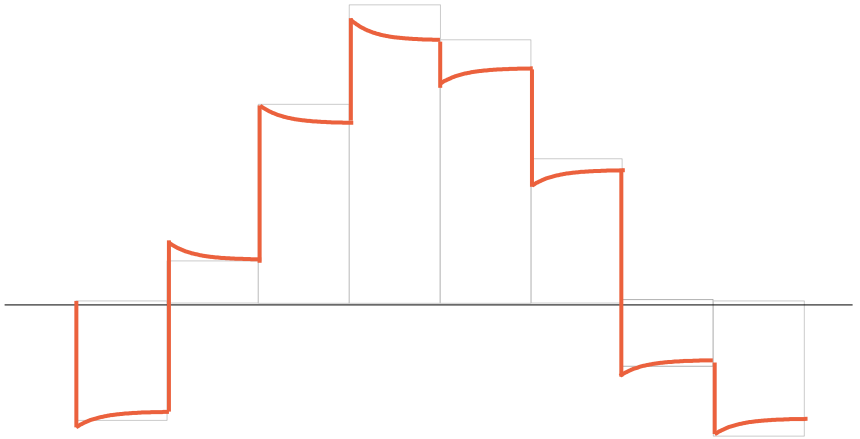
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ

large C

small $\frac{1}{\omega C} \ll R$



Fully Resistive

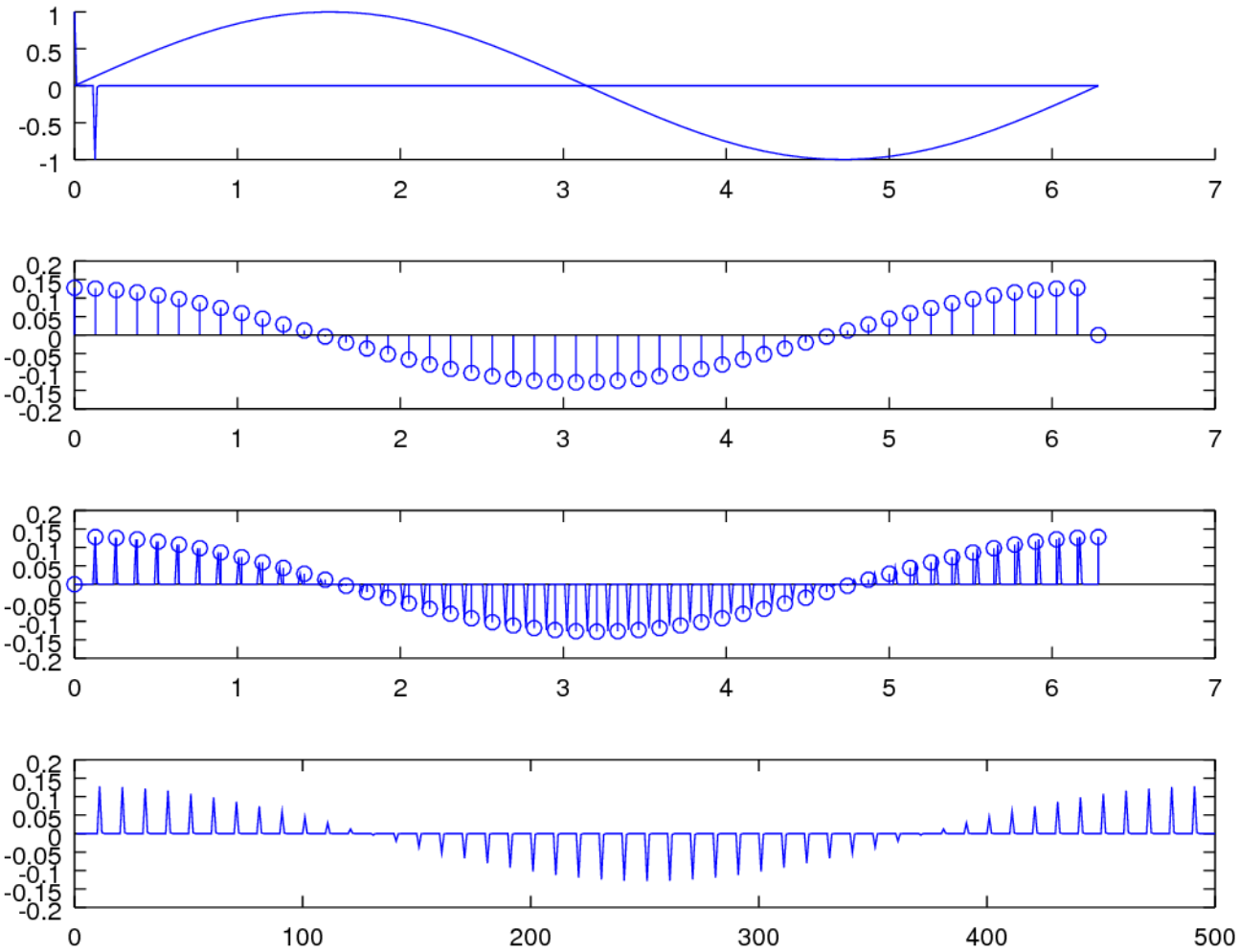
Plotting superposition results

```
clf
t = linspace(0, pi*2, 50);
tt= linspace(0, pi*2, 500);
N = length(t);
NN= length(tt);

t1 = t;
t2 = [t(2:N), t(N)];
y1 = sin(t1);
y2 = sin(t2) - sin(t1);

yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

```
svec = zeros(1, NN);
for i = 1:NN;
  tvec = zeros(1, NN);
  tvec = [zeros(1, i-1), yy3];
  tvec = yy2(i) * tvec(1:NN);
  svec = svec + tvec;
endfor
yy4 = svec;
% yy4= conv(yy2, yy3);
y5 = yy4([1:NN/N:NN]);
yy5= yy4([1:NN]);
```

```
subplot(4, 1, 2);
stem(t1, y2)
subplot(4, 1, 1);
hold on
plot(t1, y1);
plot(tt, yy3);
subplot(4, 1, 3);
stem(t1, y5); hold on
plot(tt, yy5)
subplot(4, 1, 4);
plot(yy4);
```

Small Time Constant




```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

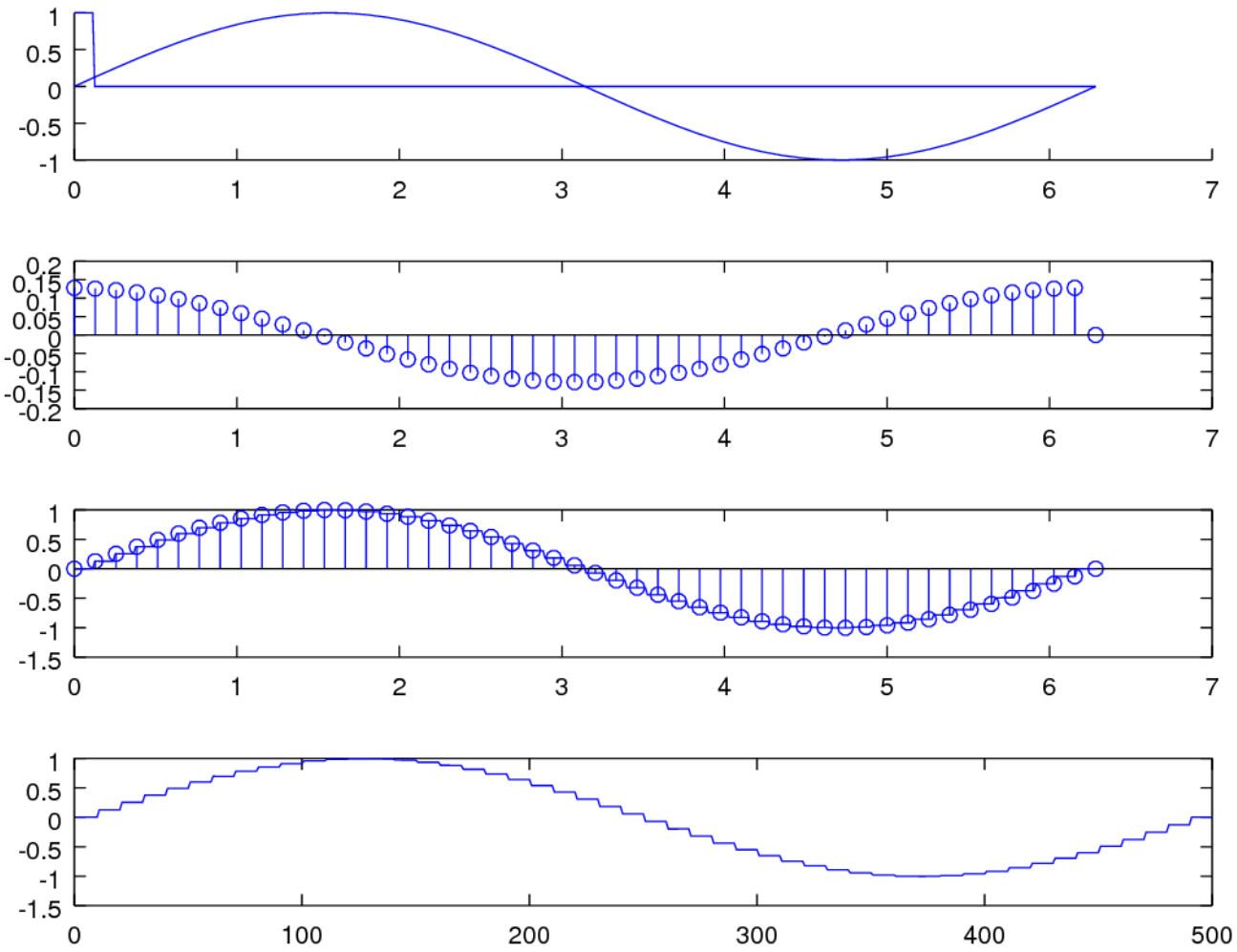
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

$$small\ \tau$$

$$small\ C$$

$$large\ \frac{1}{\omega C}$$

Large Time Constant




```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

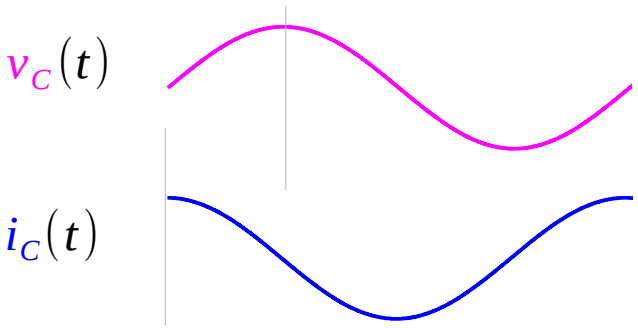
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

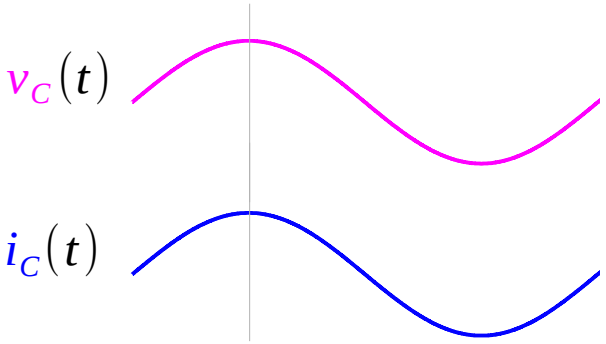
large τ

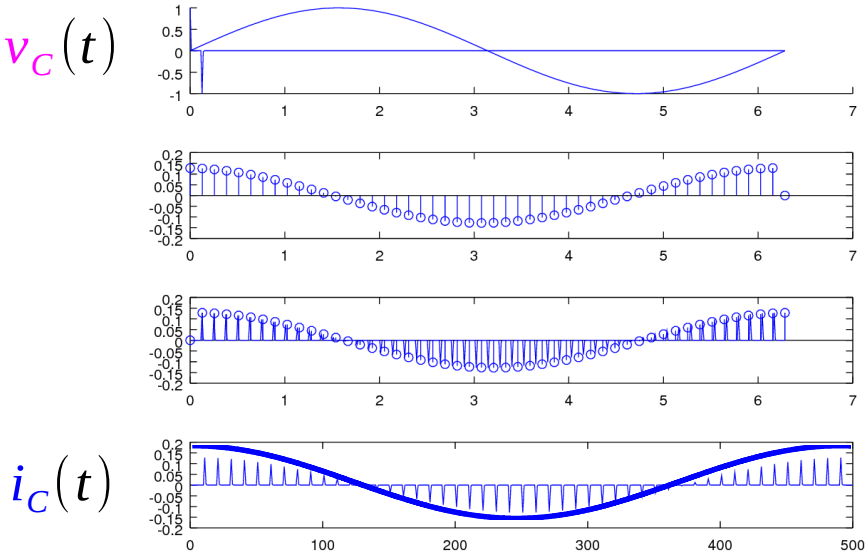
large C

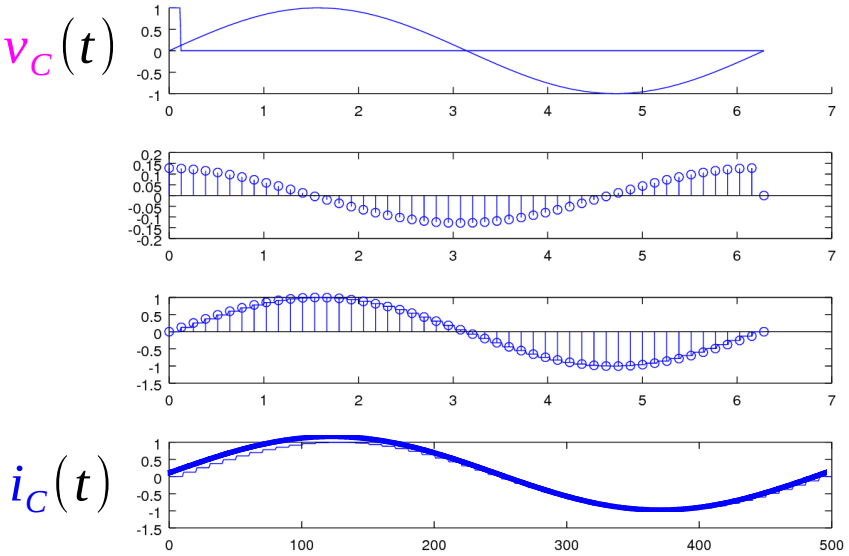
small $\frac{1}{\omega C}$

Envelope of the samples

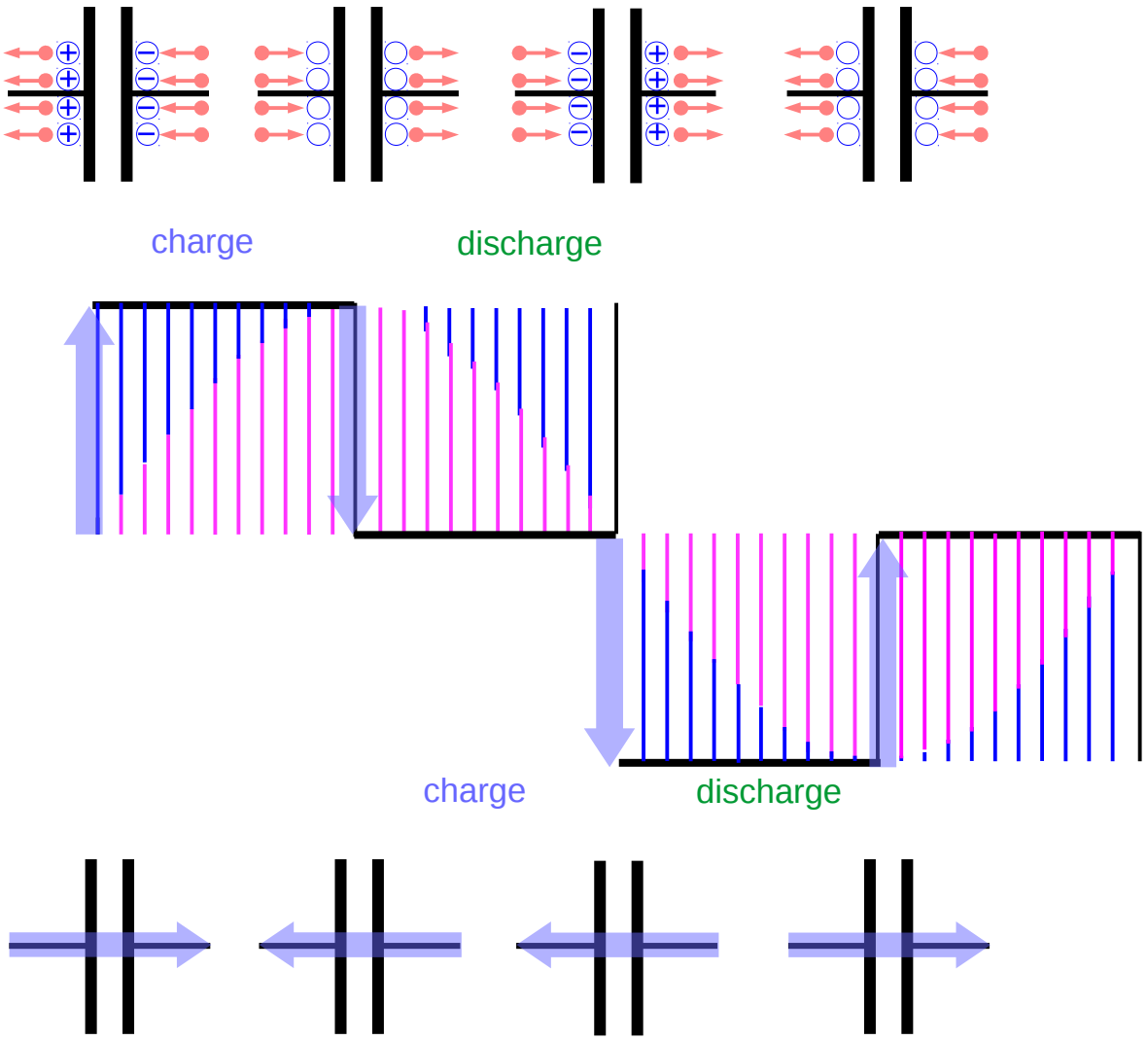






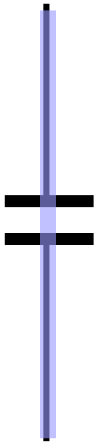


Evercharging signal pairs



I leads V by 90°

Initial charge

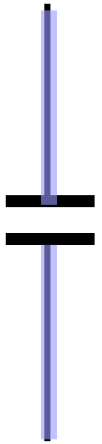


SHORT

V = 0

I : peak

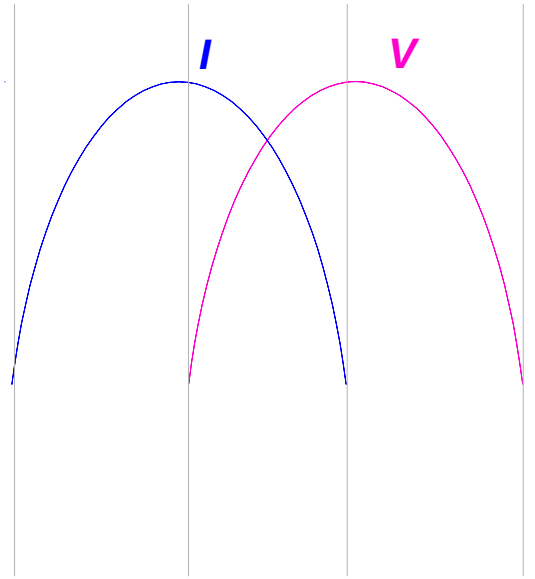
Full charge



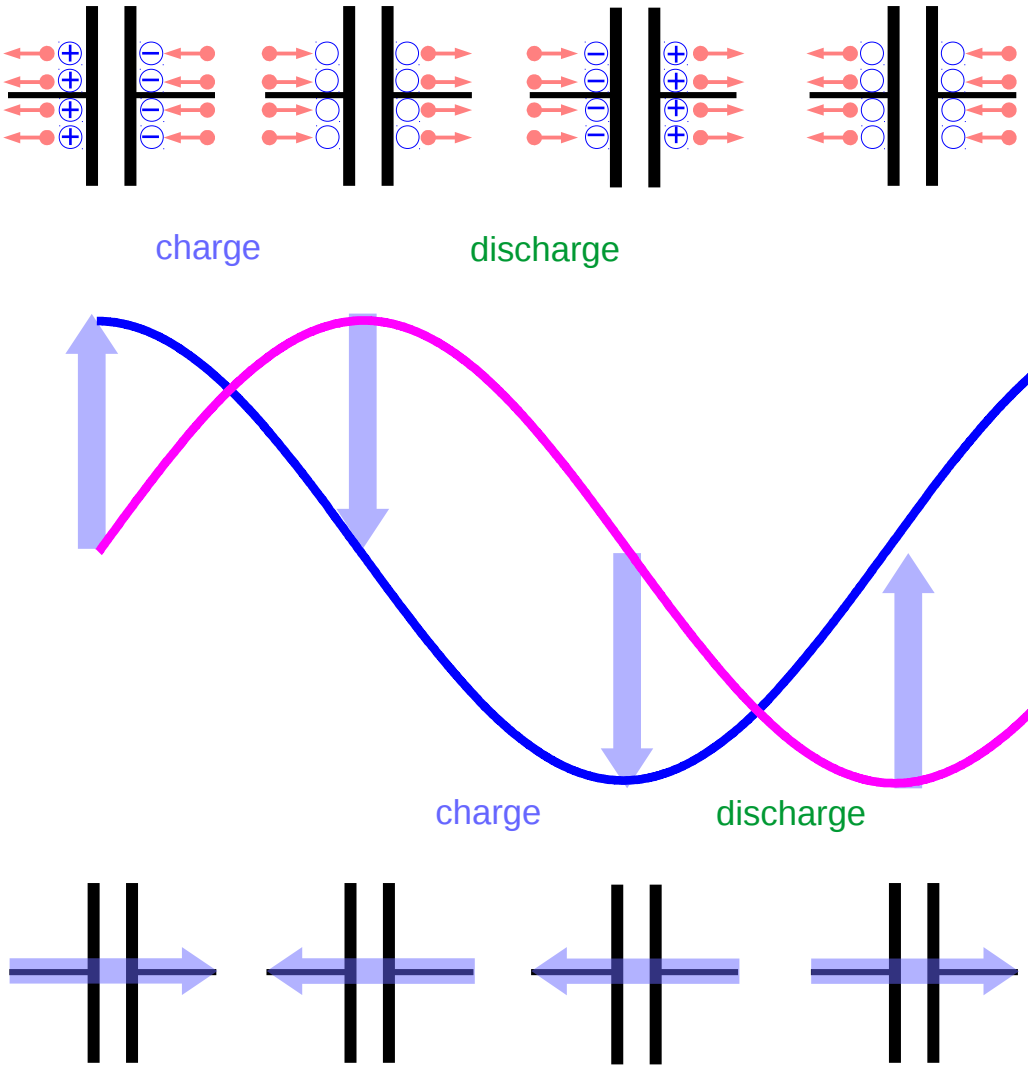
OPEN

I = 0

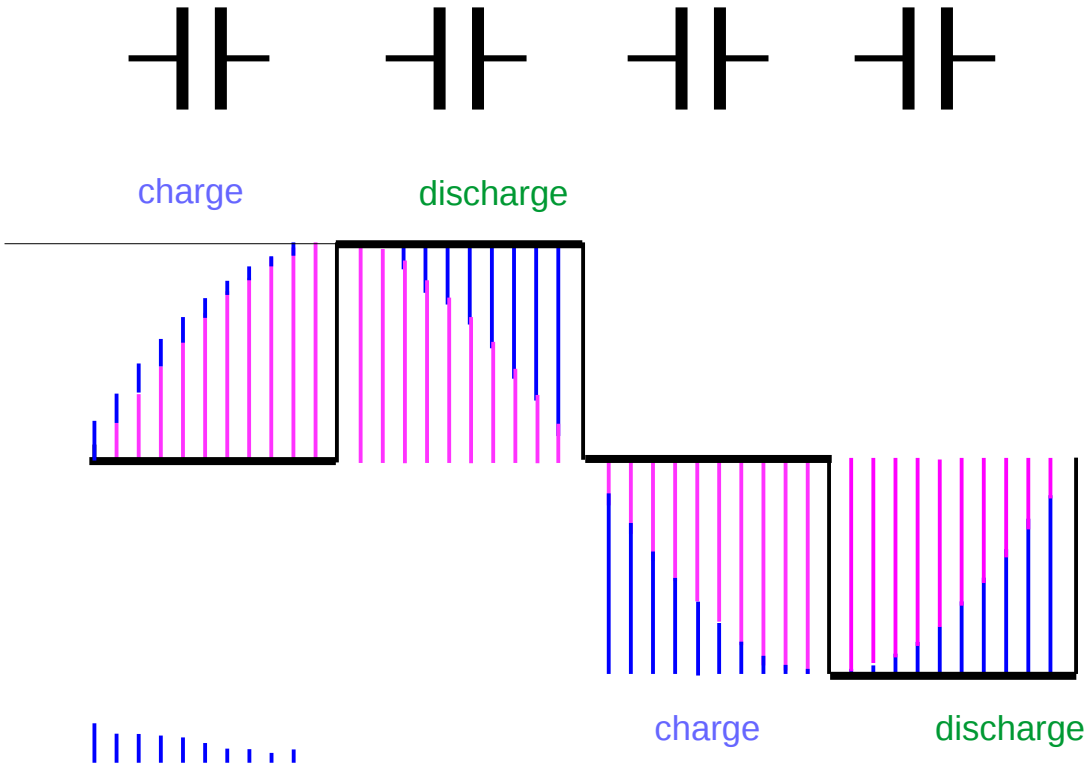
V : peak



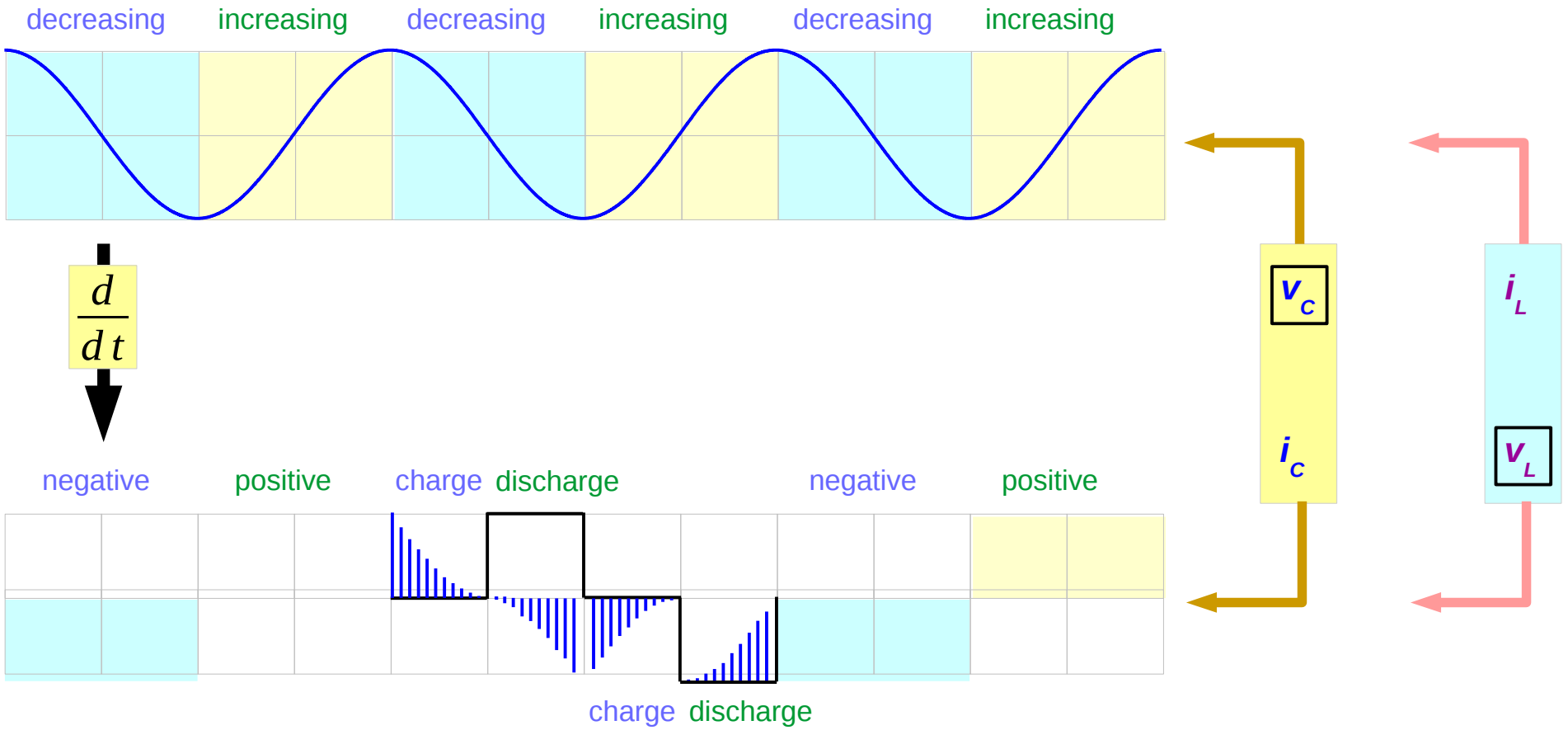
Evercharging signal pairs



Evercharging signal pairs



Evercharging signal pairs



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003