Functor (1A)

Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Function Definition

Function Definition I.

```
square x = x * x
```

- function type is inferred → not efficient **Type Inference**

Function Definition II.

```
square :: Double -> Double

square x = x * x
```

– function type declaration

Function Type Declaration

- function **type declaration**
- **function definition**

Type Declaration

the declaration of an identifier's type

the identifier name :: the type name ...

type names in Haskell always begin with a capital letter,

identifier names (including function identifiers) must always begin with a lower-case letter

http://www.toves.org/books/hsfun/

Function Types and Type Classes

Function Definition I.

```
square x = x * x
```

Function Definition II.

```
square :: Double -> Double

square x = x * x
```

function definition

=

function definition

- function **type declaration**

=

type class – a set of types

- function **type** 1
- function **type** 2
-
- function **type** n

http://www.toves.org/books/hsfun/

Typeclasses and Instances

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

such *behavior* is defined by

- function **type declaration** only
- **function definition**

a function definition

```
(==) :: a -> a -> Bool
x == y = not (x /= y)
```

- **a type declaration**

a function type

```
(==) :: a -> a -> Bool
```

- **a type declaration**

A function definition can be **overloaded**

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Typeclasses and Type

Typeclasses are like interfaces

defines some behavior
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that **typeclass**
types possessing such behavior

a **type** is an **instance** of a **typeclass** implies

the function types declared by the **typeclass**
are defined (implemented) in the **instance**

so that we can use the functions
that the **typeclass** defines with that **type**

No relation with classes in Java or C++

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Car Type Example

the Eq typeclass

defines the functions **==** and ***/=***

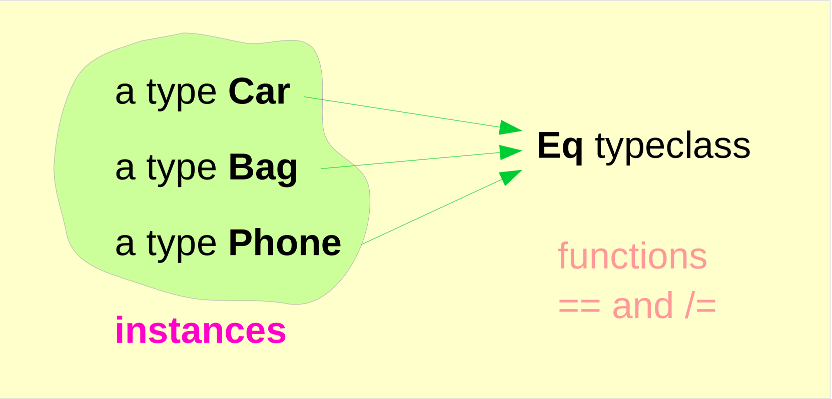
a type Car

comparing two cars c1 and c2 with the equality function ==

The Car type is an **instance** of Eq **typeclass**

Instances : various types

Typeclass : a group or a class of these similar types



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TrafficLight Type Example (1)

```
class Eq a where
    (==) :: a -> a -> Bool        - a type declaration
    (/=) :: a -> a -> Bool        - a type declaration
    x == y = not (x /= y)         - a function definition
    x /= y = not (x == y)         - a function definition
```

```
data TrafficLight = Red | Yellow | Green
```

```
instance Eq TrafficLight where
    Red    == Red     = True
    Green  == Green   = True
    Yellow == Yellow  = True
    _ == _            = False
```

```
ghci> Red == Red
True
ghci> Red == Yellow
False
ghci> Red `elem` [Red, Yellow, Green]
True
```

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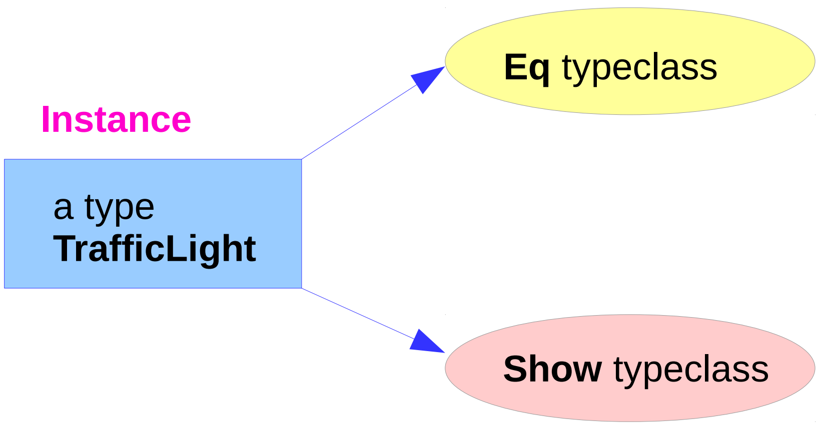
TrafficLight Type Example (2)

```
class Show a where
    show :: a -> String
    * * *
```

- a type declaration

```
data TrafficLight = Red | Yellow | Green
```

```
instance Show TrafficLight where
    show Red = "Red light"
    show Yellow = "Yellow light"
    show Green = "Green light"
```




```
ghci> [Red, Yellow, Green]
[Red light,Yellow light,Green light]
```

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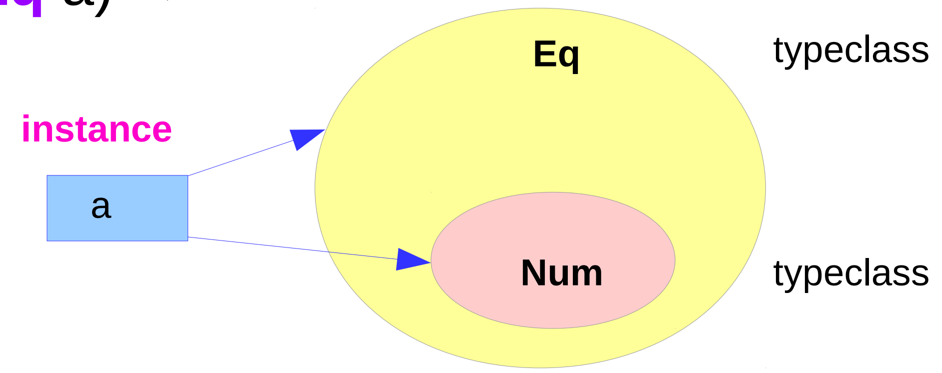
Class Constraints

```
class (Eq a) => Num a where
  ...
```

```
class Num a where
  ...
```

(Eq a) =>



Num : a subclass of **Eq**

class constraint on a class declaration

an instance of **Eq**
before being an instance of **Num**

the required function bodies can be defined in
- the class declaration
- an instance declarations,

we can safely use == because a is a part of **Eq**

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Class Constraints : class & instance declarations

class constraints in **class declarations**

to make a typeclass a **subclass** of another typeclass

subclass

```
class (Eq a) => Num a where
   …
```

class constraints in **instance declarations**

to express **requirements** about the contents of some type.

requirements

```
instance (Eq x, Eq y) => Eq (Pair x y) where
   Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
```

http://cmsc-16100.cs.uchicago.edu/2016/Lectures/07-type-classes.php

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Class constraints in instance declaration examples

```
instance (Eq m) => Eq (Maybe m) where
    Just x     ==  Just y           =     x == y     ⬅ Eq m
    Nothing  ==  Nothing           =     True
              _ == _                =     False
```

```
instance (Eq x, Eq y) => Eq (Pair x y) where
    Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
               ⬆                   ⬆           ⬆
         Eq (Pair x y)            Eq x        Eq y
```

Derived instance

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A Concrete Type and a Type Constructor

a : a concrete type

Maybe : not a concrete type

: a type constructor that takes one parameter produces a concrete type.

Maybe a : a concrete type

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Functor typeclass

the **Functor typeclass** is basically for things that can be *mapped over*

ex) mapping over lists

the list type is part of the Functor typeclass

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Functor typeclass

```
class Functor f where
   fmap :: (a -> b) -> f a -> f b
```

The **Functor typeclass**

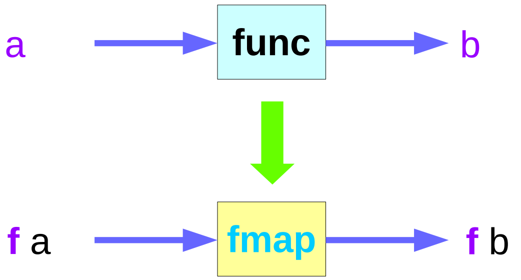
  defines one function, **fmap**

  no default implementation

the **type variable f**

  not a concrete type (a concrete type can hold a value)

  a **type constructor** taking one type parameter



function fmap
function func
type constructor f

Maybe Int : a concrete type

Maybe : a type constructor that takes one type as the parameter

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Function map & fmap

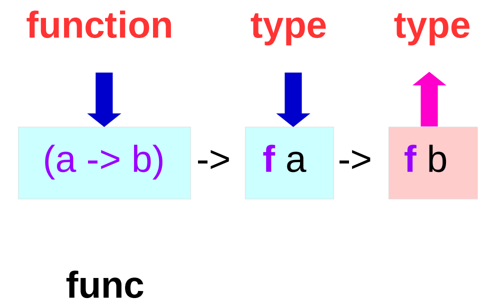
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

fmap takes

- a **function** from one type to another (a -> b)
- a **Functor f** applied with **one type** (f a)

fmap returns

- a **Functor f** applied with **another type** (f b)




```
map :: (a -> b) -> [a] -> [b]
```

map takes

- a function from one type to another (* 2)
- take a list of one type [1, 2, 3]
- returns a list of another type [2, 4, 6]

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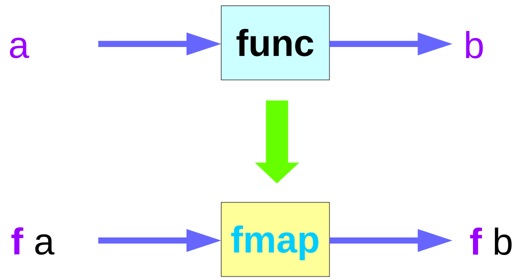
List : an instance of the Functor typeclass

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
map :: (a -> b) -> [a] -> [b]
```

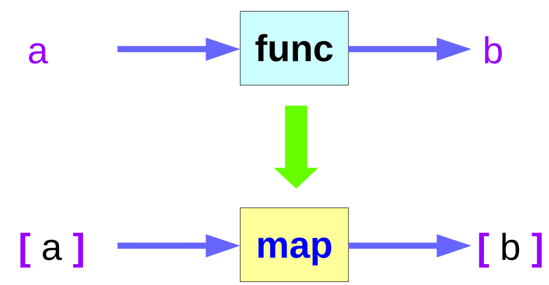


map is just a **fmap** that works only on **list**s

function fmap
function func
type constructor f

a **list** is an **instance** of the **Functor typeclass**.

```
instance Functor [ ] where
    fmap = map
```



f : a type constructor that takes one type
[ ] : a type constructor that takes one type
[a] : a concrete type ([Int], [String] or [[String]])

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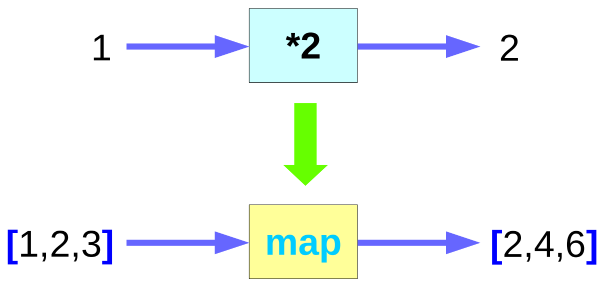
List Examples

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
map :: (a -> b) -> [a] -> [b]
```

```
instance Functor [ ] where
    fmap = map
```

```
map :: (a -> b) -> [a] -> [b]
ghci> fmap (*2) [1..3]
[2,4,6]
ghci> map (*2) [1..3]
[2,4,6]
```



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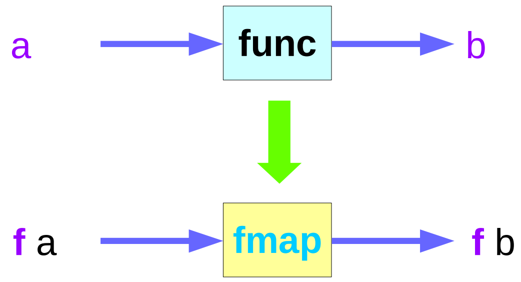
Maybe : an instance of the Functor typeclass

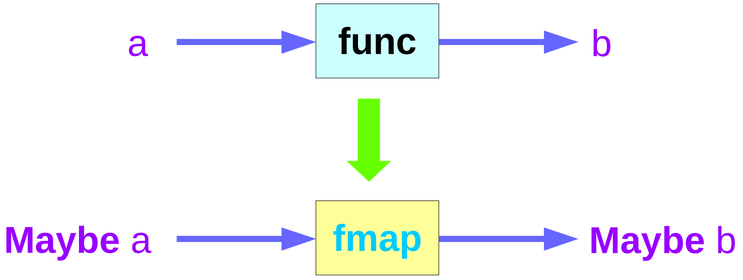
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```

f	⟺	Maybe
f a	⟺	Maybe a
f b	⟺	Maybe b

(a -> b)	⟺	func





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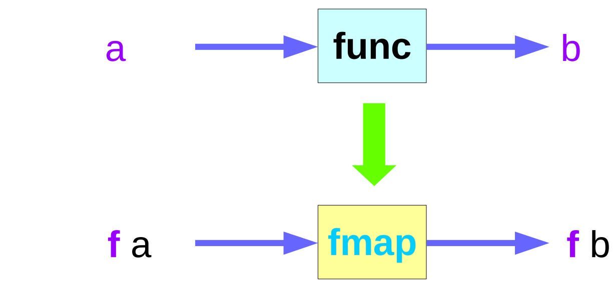
Maybe : a type constructor

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

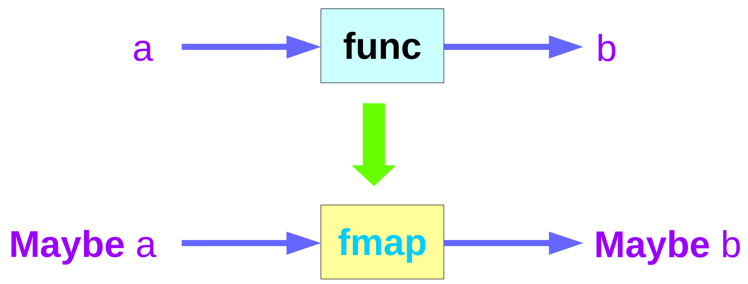
```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```



f : a **type variable**

f : a **type constructor** taking one type parameter

Maybe : an **instance** of **Functor** typeclass



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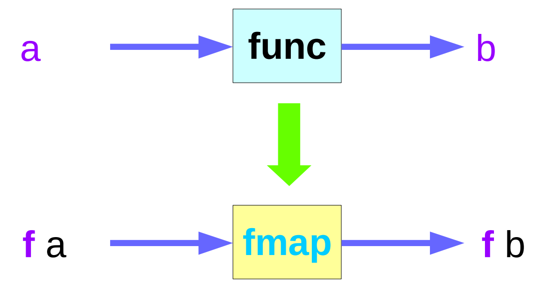
Maybe : an argument to fmap, together with a

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

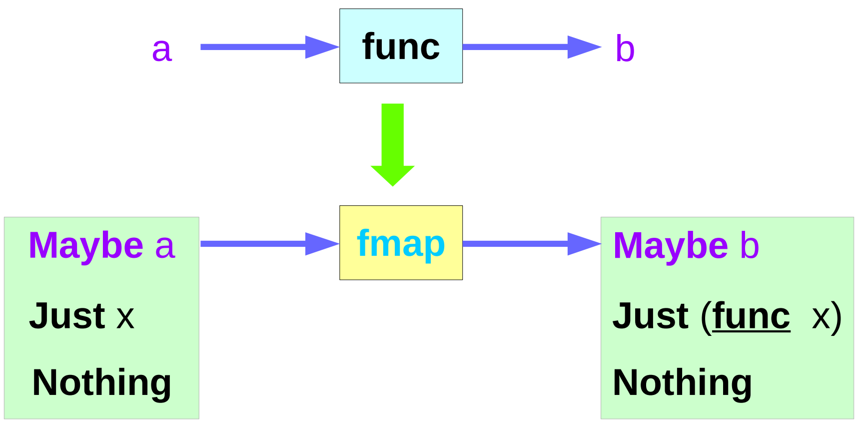
```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```




```
fmap :: (a -> b) -> f a -> f b

fmap func (Just x) = Just (func  x)

fmap func Nothing = Nothing
```



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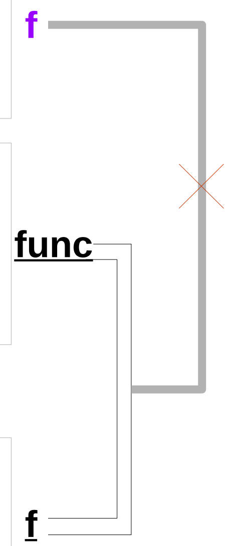
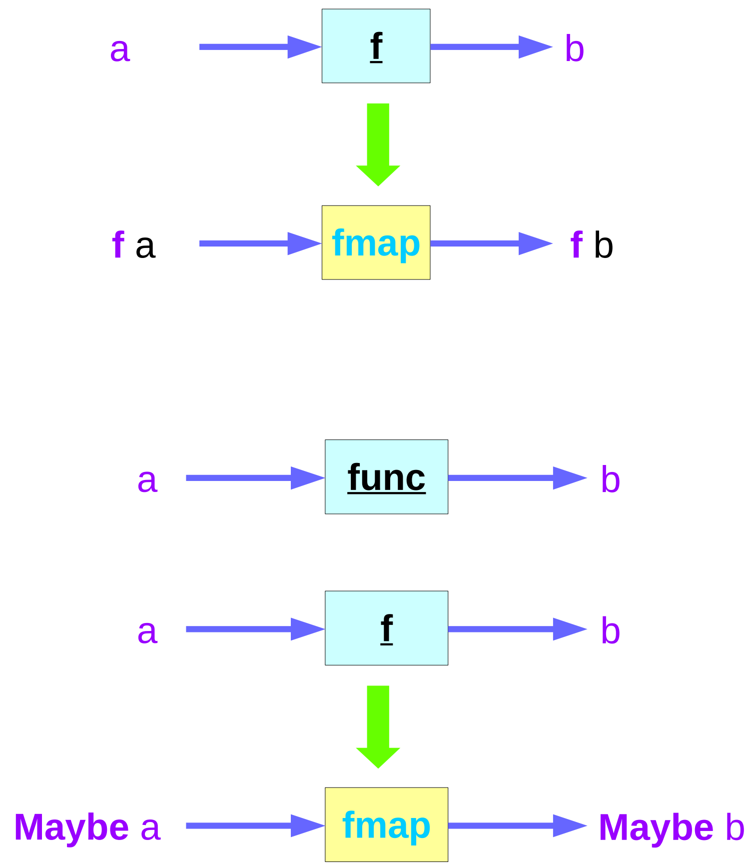
A function argument to fmap and a Functor f

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap func (Just x) = Just (func x)
    fmap func Nothing = Nothing
```




```
instance Functor Maybe where
    fmap f (Just x) = Just ( f x)
    fmap f Nothing = Nothing
```



f is different from the type constructor **f**

func : a -> b

f : a -> b

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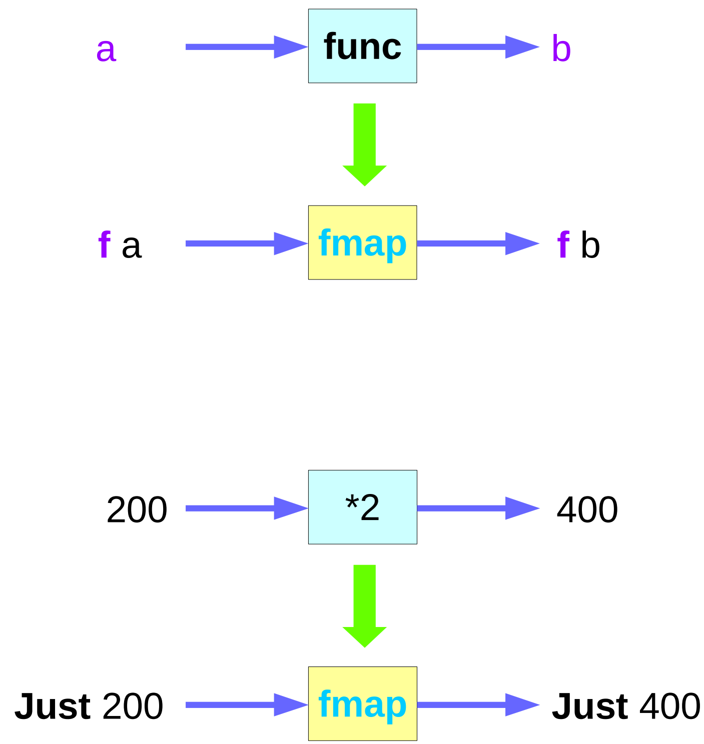
Maybe Examples (1)

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```




```
ghci> fmap (*2) (Just 200)
Just 400
ghci> fmap (*2) Nothing
Nothing
```

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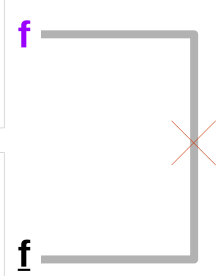
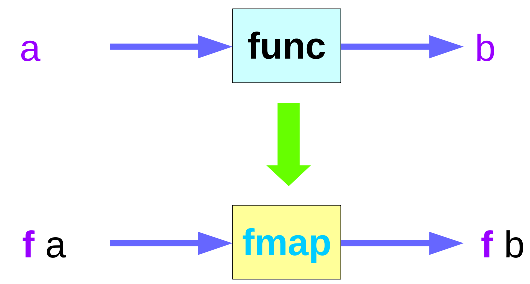
Maybe Examples (2)

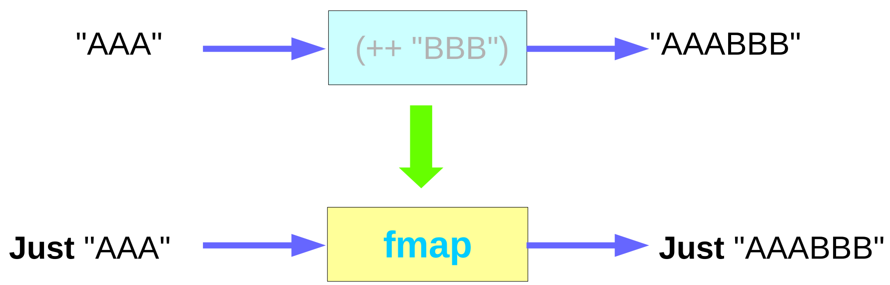
```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```






```
ghci> fmap (++ "BBB") (Just "AAA")
Just "AAABBB"
ghci> fmap (++ "BBB") Nothing
Nothing
```



http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

Maybe as a functor

Functor typeclass:

- transforming one type to another
- transforming operations of one type to those of another

Maybe a is an instance of a **functor** type class

Functor provides **fmap** method

maps functions of the base type (such as Integer)
to *functions* of the lifted type (such as Maybe Integer).

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

A *function* **f** transformed with **fmap**
can work on a **Maybe** value

```
case maybeVal of
  Nothing  -> Nothing          -- there is nothing, so just return Nothing
  Just val -> Just (f val)     -- there is a value, so apply the function to it
```

father :: Person -> **Maybe** Person
mother :: Person -> **Maybe** Person

```
       f :: Int           -> Int
fmap  f :: Maybe Integer -> Maybe Integer
```

a **Maybe** **Integer** value: **m_x**

fmap f **m_x**

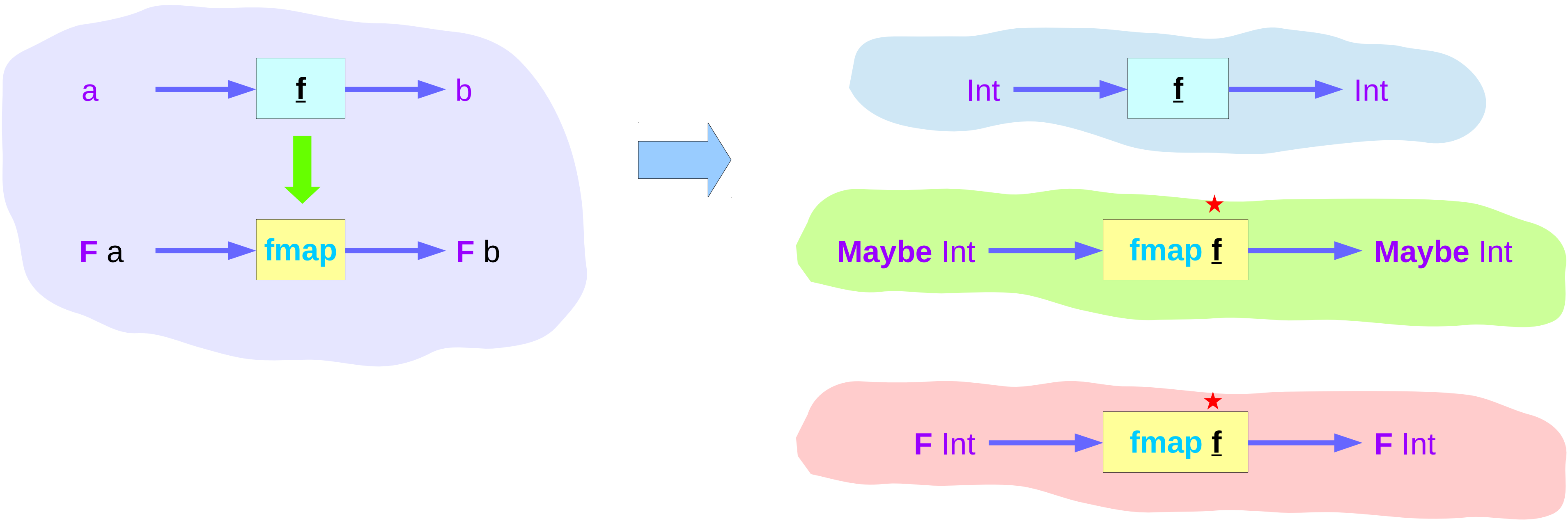
https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Transforming operations

Functor provides **fmap** method

maps functions of the base type (such as Integer)

to *functions* of the lifted type (such as Maybe Integer).



https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

m_x : a Maybe Integer value (Just 101, Nothing, …)
f :: Int -> Int

you can do **fmap** **f** m_x
to apply the function **f** directly to the Maybe Integer
without worrying whether it is Nothing or not

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

Function of (a -> b)

fmap f m_x

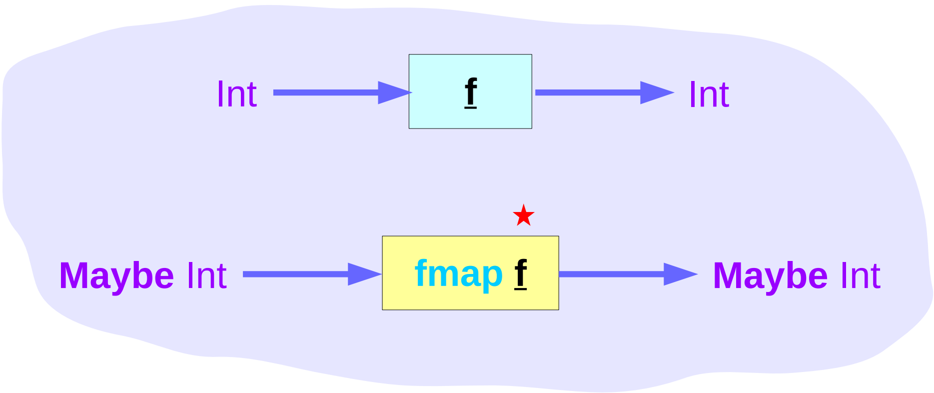
lifted type

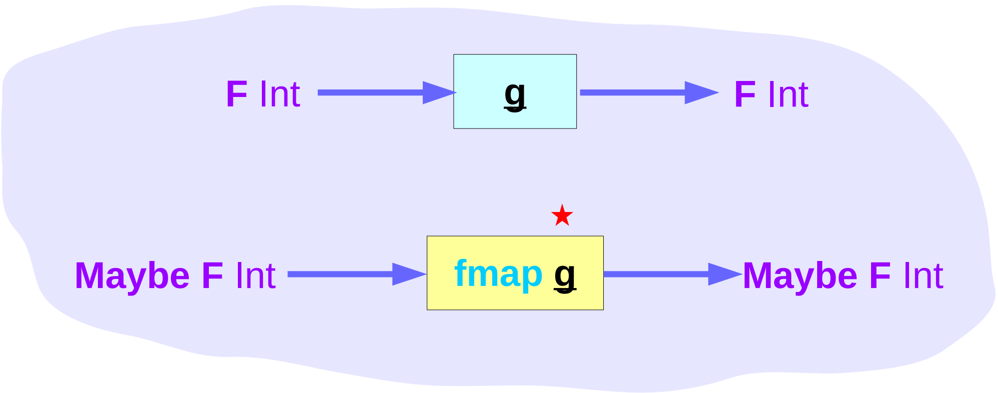
A Functor f applied with
one type **f** a or **f** b

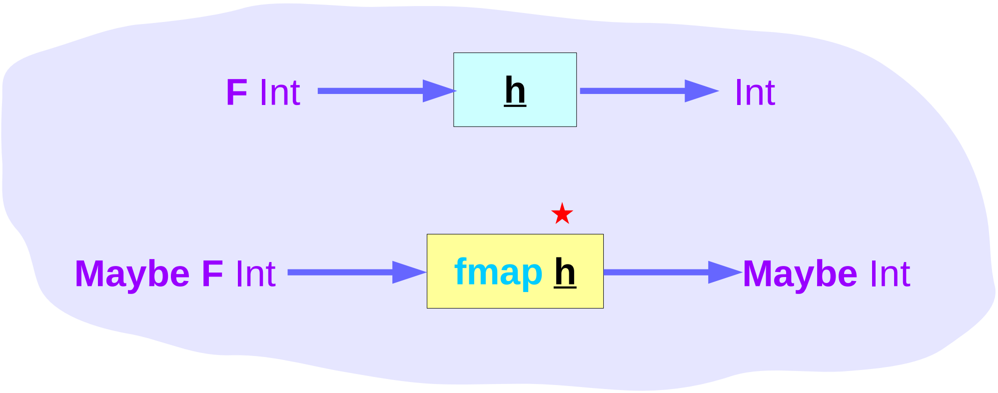
https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a functor

Can apply a whole chain of
lifted Integer -> Integer functions
to Maybe Integer values
and only have to worry about explicitly checking for Nothing
once when you're finished.







https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe Type Definition

The Maybe type definition

```
data Maybe a = Just a | Nothing
  deriving (Eq, Ord)
```

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

Maybe is

- an instance of **Eq** and **Ord** (as a base type)
- an instance of **Functor**
- an instance of **Monad**

For **Functor**, the fmap function f
moves inside the Just constructor and
is identity on the Nothing constructor.

For **Monad**,
the bind operation passes through Just, while
Nothing will force the result to always be Nothing.

https://wiki.haskell.org/Maybe

Maybe class

The Maybe type definition

```
data Maybe a = Just a | Nothing
    deriving (Eq, Ord)
```

Maybe is

an instance of **Eq** and **Ord** (as a base type)
an instance of **Functor**
an instance of **Monad**

For Functor, the fmap function f
moves inside the Just constructor and
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For Monad,
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https://wiki.haskell.org/Maybe

Monad

a **Monad** is just a special **Functor** with extra features

Monads like IO map *types* to new types
that represent "computations that result in values"

can *lift* regular functions into **Monad** types
via a **liftM** function (like a **fmap** function)

liftM transform a regular function
into a "computations that results in the value obtained by evaluating the function."

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a Monad

Maybe is also a **Monad**

represents “computations that could *fail* to return a value"

an immediate abort

a valueless return in the middle of a computation.

enable a whole bunch of computations

without explicit checking for errors in each step

a computation on **Maybe** values *stops*

as soon as a **Nothing** is encountered

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

Maybe as a Monad

```
f::Int -> Maybe Int
f 0 = Nothing
f x = Just x

g :: Int -> Maybe Int
g 100 = Nothing
g x     = Just x

h ::Int -> Maybe Int
h x = case f x of
          Just n -> g n
          Nothing -> Nothing

h' :: Int -> Maybe Int
h' x = do n <- f x
          g n
```

if x==0 then Nothing else Just x

if x==100 then Nothing else Just x

if f x==Nothing then Nothing else g n

g (f x)

h & h' give the same results
h 0 = h' 0 = h 100 = h' 100 = Nothing;
h x = h' x = Just x

https://wiki.haskell.org/Maybe

Maybe as a Library Function

When the module is imported **import Data.Maybe**

maybe :: b->(a->b) -> Maybe a -> b

Applies the second argument (a->b) to the third Maybe a,
when it is Just x, otherwise returns the first argument (b).

isJust, **isNothing**

Test the argument, returing a Bool based on the constructor.

ListToMaybe , **maybeToList**

Convert to/from a one element or empty list.

mapMaybe

A different way to filter a list.

https://wiki.haskell.org/Maybe

Maybe as Monad

maybe :: b->(a->b) -> Maybe a -> b
The maybe function takes
a default value (b),
a function (a->b), and
a Maybe value (Maybe a).

If the Maybe value is Nothing,
the function returns the default value.
Otherwise, it applies the function to the value inside the Just and returns the result.

```
>>> maybe False odd (Just 3)
True

>>> maybe False odd Nothing
False
```

https://hackage.haskell.org/package/base-4.10.0.0/docs/Data-Maybe.html

Then Operator (>>) and **do** Statements

```
putStr "Hello"  >>
putStr " "      >>
putStr "world!" >>
putStr "\n"
```

```
do { putStr "Hello"
   ; putStr " "
   ; putStr "world!"
   ; putStr "\n" }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

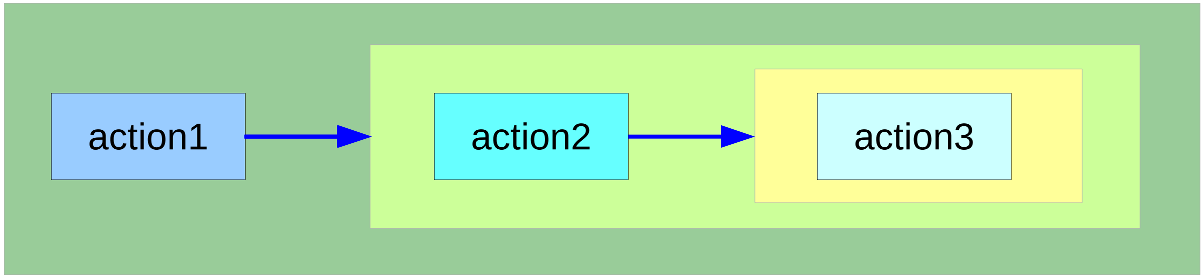
Translating in **do** notation

```
do { action1
   ; action2
   ; action3 }
```

```
action1 >>
do { action2
   ; action3 }
```

```
do { action1
   ; do { action2
        ; action3 } }
```

```
do { action1
   ; do { action2
        ; do { action3 } } }
```



can **chain** any actions
as long as all of them are
in **the same monad**

https://en.wikibooks.org/wiki/Haskell/do_notation

Bind Operator (>==) and **do** statements

The bind operator (**>>=**)

passes a value (the result of an action or function),

downstream in the binding sequence.

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```

anonymous function
(lambda expression)
is used

do notation assigns a variable name
to the passed value using the **<-**

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

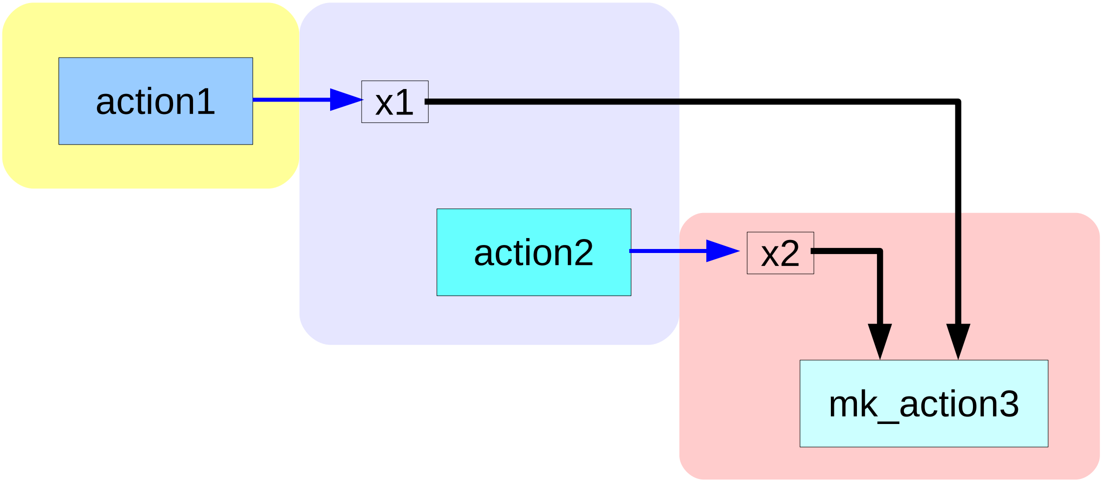
Translation using the bind operator (>>=)

```
do { x1 <- action1
     ; x2 <- action2
     ; mk_action3 x1 x2 }
```

```
action1 >>= (\ x1 -> action2 >>= (\ x2 -> mk_action3 x1 x2 ))
```

```
action1
  >>=
    (\ x1 -> action2
       >>=
         (\ x2 -> mk_action3 x1 x2 ))
```

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```



https://en.wikibooks.org/wiki/Haskell/do_notation

Anonymous Function

```
\x -> x + 1
```

```
(\x -> x + 1) 4
5 :: Integer
```

```
(\x y -> x + y) 3 5
8 :: Integer
```

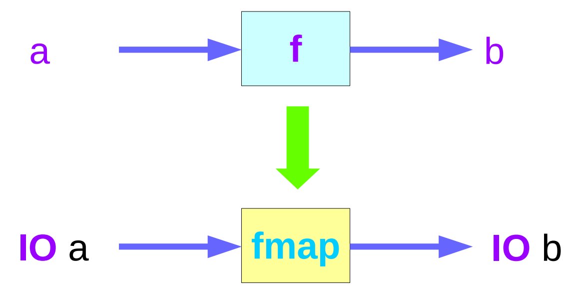
Lambda Expression

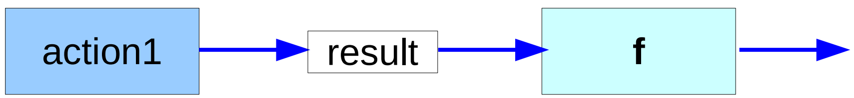
```
addOne = \x -> x + 1
```

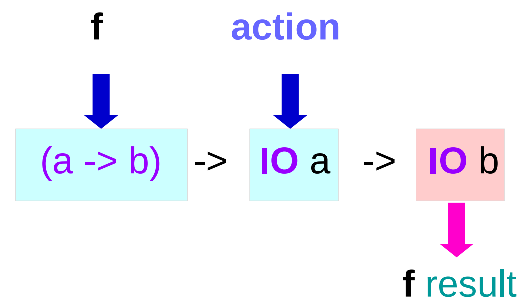
https://wiki.haskell.org/Anonymous_function

Functor Typeclass

```
instance Functor IO where
    fmap f action = do
        result <- action
        return (f result)
```








```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Typeclass

```
main = do line <- getLine
          let line' = reverse line
          putStrLn $ "You said " ++ line' ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line' ++ " backwards!"

main = do line <- fmap reverse getLine
          putStrLn $ "You said " ++ line ++ " backwards!"
          putStrLn $ "Yes, you really said" ++ line ++ " backwards!"
```

```
instance Functor IO where
  fmap f action = do
    result <- action
    return (f result)
```

```
fmap reverse getLine = do
  result <- getLine
  return (reverse result)
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

$ Operator

$ operator to avoid parentheses

Anything appearing after $

will take precedence over anything that comes before.

```
putStrLn (show (1 + 1))

putStrLn (show $ 1 + 1)
putStrLn $ show (1 + 1)
putStrLn $ show $ 1 + 1
```

https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign

. Operator

. operator to chain functions

putStrLn (**show** (1 + 1))

(1 + 1) is not a function, so the . operator cannot be applied
show can take an Int and return a String.
putStrLn can take a String and return an IO().

(**putStrLn . show**) (1 + 1)

putStrLn . show $ 1 + 1

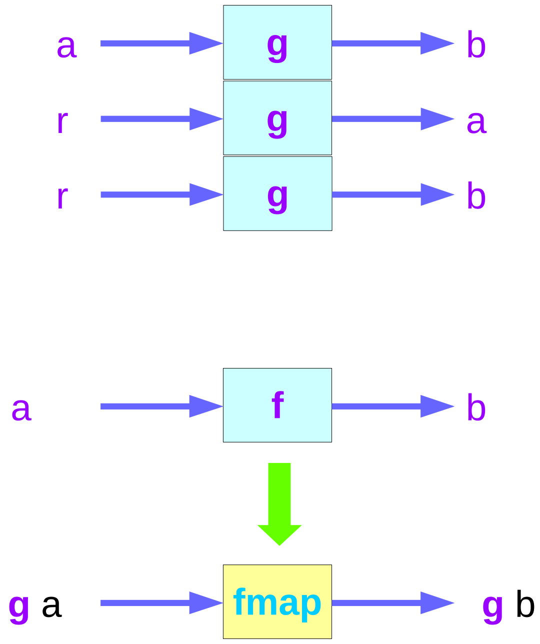
https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign

Functor Typeclass

instance Functor ((->) r) where
    fmap f g = (\x -> **f** (**g** x))

instance Functor Maybe where
    fmap f (**Just** x) = **Just** (**f** x)
    fmap f **Nothing** = **Nothing**

A function takes any thing and returns any thing
    g :: **a -> b**
    g :: **r -> a**



fmap :: (a -> b) -> f a -> f b
fmap :: (a -> b) -> ((->) r a) -> ((->) r b)
fmap :: (a -> b) -> (r -> a) -> (r -> b)

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

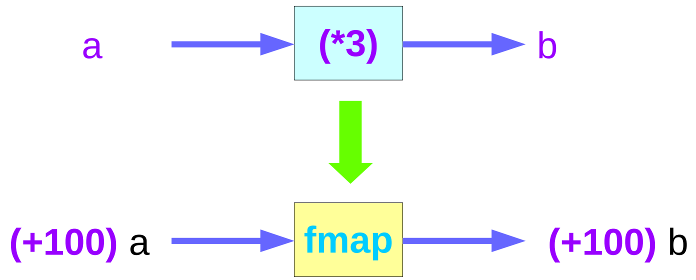
Functor Typeclass

```
instance Functor ((->) r) where
    fmap f g =  (\x -> f (g x))

instance Functor ((->) r) where
    fmap = ( . )
```

```
instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing
```

```
ghci> :t fmap (*3) (+100)
fmap (*3) (+100) :: (Num a) => a -> a
ghci> fmap (*3) (+100) 1
303
ghci> (*3) `fmap` (+100) $ 1
303
ghci> (*3) . (+100) $ 1
303
ghci> fmap (show . (*3)) (*100) 1
"300"
```

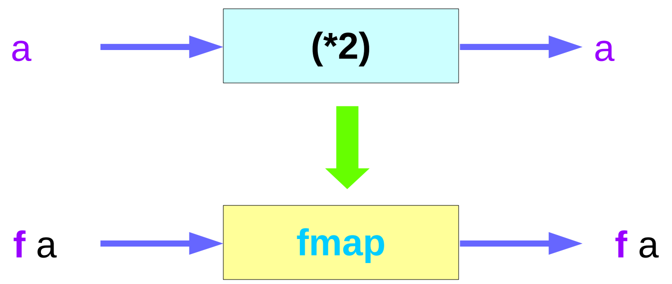


http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Typeclass

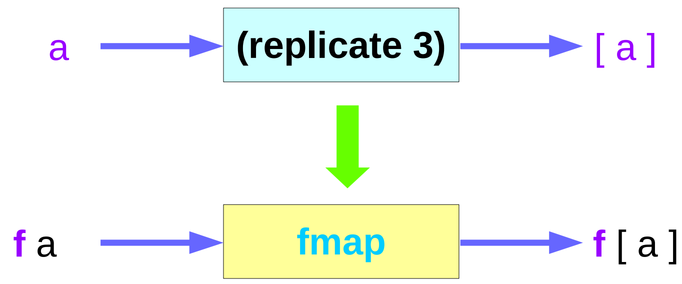
ghci> :t **fmap** (*2)

fmap (*2) :: (Num a, Functor **f**) => **f** a -> **f** a



ghci> :t **fmap** (replicate 3)

fmap (replicate 3) :: (Functor **f**) => **f** a -> **f** [a]



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Functor Typeclass

```
ghci> fmap (replicate 3) [1,2,3,4]
[[1,1,1],[2,2,2],[3,3,3],[4,4,4]]

ghci> fmap (replicate 3) (Just 4)
Just [4,4,4]

ghci> fmap (replicate 3) (Right "blah")
Right ["blah","blah","blah"]

ghci> fmap (replicate 3) Nothing
Nothing

ghci> fmap (replicate 3) (Left "foo")
Left "foo"
```

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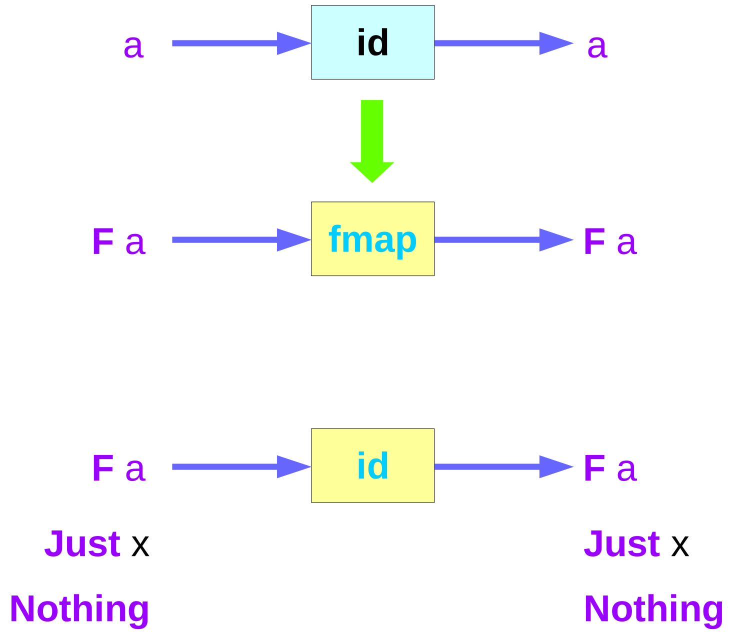
Functor Laws

```
fmap id = id

id :: a -> a
id    x = x
```




```
instance Functor Maybe where
    fmap func (Just x) = Just (func  x)
    fmap func Nothing = Nothing

instance Functor Maybe where
    fmap f (Just x) = Just (f x)
    fmap f Nothing = Nothing

instance Functor Maybe where
    fmap id (Just x) = Just (id x)
    fmap id Nothing = Nothing
```

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Functor Typeclass

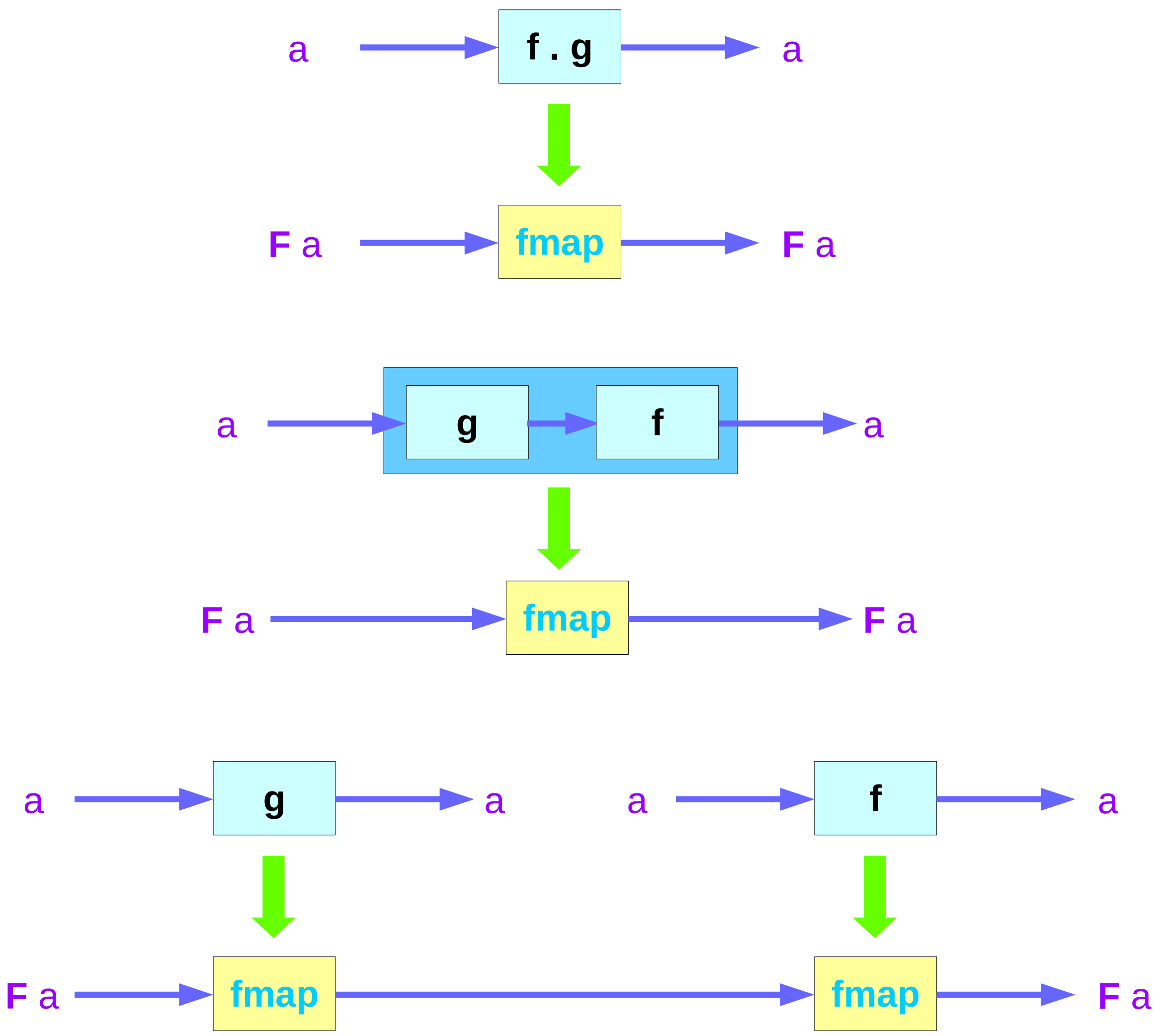
```
ghci> fmap id (Just 3)
Just 3
ghci> id (Just 3)
Just 3
ghci> fmap id [1..5]
[1,2,3,4,5]
ghci> id [1..5]
[1,2,3,4,5]
ghci> fmap id []
[]
ghci> fmap id Nothing
Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Functor Laws

fmap (**f . g**) = **fmap f** . **fmap g**

fmap (**f . g**) **F** = **fmap f** (**fmap g F**)



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Functor Laws

```
fmap (f . g) = fmap f . fmap g
fmap (f . g) F = fmap f (fmap g F)

    instance Functor Maybe where
        fmap f (Just x) = Just (f x)
        fmap f Nothing = Nothing



fmap (f . g) Nothing = Nothing
fmap f (fmap g Nothing) = Nothing

fmap (f . g) (Just x) = Just ((f . g) x) = Just (f (g x))
fmap f (fmap g (Just x)) = fmap f (Just (g x)) = Just (f (g x))
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf