Temporal Characteristics of Random Processes

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Random Variable Definition

A random variable

a real **function** over a **sample space** $S = \{s_1, s_2, s_3, ..., s_n\}$

$$s \to X(s)$$

$$x = X(s)$$

a random variable : a capital letter X
a particular value : a lowercase letter x

a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$
an element of S : s

Random Variable Example

Example

$$X(s_1) = x_1 \qquad s_1 \longrightarrow x_1$$
$$X(s_2) = x_2 \qquad s_2 \longrightarrow x_2$$
$$\cdots \qquad \cdots$$
$$X(s_n) = x_n \qquad s_n \longrightarrow x_n$$

$S = \{s_1, s_2, s_3, ..., s_n\}$ a sample space

$X = \{x_1, x_2, x_3, ..., x_n\}$ a random variable

Random Process

A random process

a function of both **outcome** s and **time** t

$$X(t,s)$$

assigning a time function to every **outcome** s_i

$$s_i \rightarrow x(t,s_i)$$

the family of such time functions is called a random process

$$x(t,s_i) = X(t,s_i)$$

$$x(t,s) = X(t,s)$$

Ensemble of time functions

Time functions

A random process $X(t, s)$ represents
a family or ensemble of time functions

$X(t, s)$ represents

- a single time function $x(t, s)$
- when t is a variable and s is fixed at an outcome

$x(t, s)$ represents

- a sample function,
- an ensemble member,
- a realization of the process

Short-form notation for time functions

The short-form notation $x(t)$

to represent a specific waveform of a random process $X(t)$ for a given **outcome** s_i

$$x(t) = x(t, s)$$

$$X(t) = X(t, s)$$

Random Process Example

Example

$X(t, s_1) = x_1(t)$ $\quad s_1 \longrightarrow x_1(t)$

$X(t, s_2) = x_2(t)$ $\quad s_2 \longrightarrow x_2(t)$

... $\quad$ ...

$X(t, s_n) = x_n(t)$ $\quad s_n \longrightarrow x_n(t)$

$S = \{s_1, s_2, s_3, \ldots, s_n\}$ a sample space

$X(t) = \{x_1(t), x_2(t), x_3(t), \ldots, x_n(t)\}$ a random process

Random variables with time

a **random process** $X(t,s)$ represents a single time function
when t is a variable and s is fixed at an outcome

a random process $X(t,s)$ represents a single **random variable**
when both t and s are fixed at a time and an outcome, respectively

$$X_i = X(t_i, s) = X(t_i)$$ ***random variable***

$$X(t,s) = X(t)$$ ***random process***

An alphabet

the alphabet of $X(t)$

the set of its possible values

- the values of **time** t for which a random process is defined
- the **alphabet** of the random variable $X = X(t)$ at time t

Classification of Random Processes (1)

N Gaussian random variables

- the values of **time** t for which a random process is defined
  - continuous time
  - discrete time
- the **alphabet** of the random variable $X = X(t)$ at time t
  - continuous alphabet
  - discrete alphabet

Classification of Random Processes(2)

N Gaussian random variables

- a continuous **alphabet** continuous **time** random process
  - $X(t)$ has continuous values and t has continuous values
- a discrete **alphabet** continuous **time** random process
  - $X(t)$ has discrete values and t has continuous values
- a continuous **alphabet** discrete **time** random process
  - $X(t)$ has continuous values and t has discrete values
- a discrete **alphabet** discrete **time** random process
  - $X(t)$ has discrete values and t has discrete values

Deterministic and Non-deterministic Processes

N Gaussian random variables

a sample function

- A process is **non-deterministic** if future values of any sample function cannot be predicted exactly from observed past values
- A process is **deterministic** if future values of any sample function can be predicted from observed past values

Deterministic Random Process Example

N Gaussian random variables

$$X(t) = A\cos(\omega_0 + \Theta)$$

A, Θ, or ω_0(or all) can be random variables.

Any one sample function corresponds to the above equation with particular values of these random variables.

Therefore the knowledge of the sample function
prior to any time instance fully allows
the prediction of the sample function's future values
because all the necessary information is known