Laurent Series and z-Transform - Geometric Series Applications

B

20210504 Tue

Laurent Series $f(z)$ $g(z)$

z-transform $X(z)$ $Y(z)$

$\cdots$	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	$\cdots$
$\cdots$	z^{-3}	z^{-2}	z^{-1}	z^0	z^1	z^2	z^3	$\cdots$

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + a_{-3}z^{-3} + a_{-2}z^{-2} + a_{-1}z^{-1} \quad a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots$$

$\cdots$	a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	$\cdots$
$\cdots$	z^{3}	z^{2}	z^{1}	z^0	z^{-1}	z^{-2}	z^{-3}	$\cdots$

anti-causal z-transform $Y(z)$ causal z-transform $X(z)$

$$\cdots + a_{-3}z^{3} + a_{-2}z^{2} + a_{-1}z^{1} \quad a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$$

	anti-causal	causal
Laurent	$g(z)$	$f(z)$
z-transform	$Y(z)$	$X(z)$

Laurent Series $f(z^{-1})$ $g(z^{-1})$

$$\cdots \quad a_{-3} \quad a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots$$

$$\cdots \quad z^{-3} \quad z^{-2} \quad z^{-1} \quad z^0 \quad z^1 \quad z^2 \quad z^3 \quad \cdots$$

anti-causal Laurent $g(z)$ — causal Laurent $f(z)$

$$\underbrace{\cdots + a_{-3}z^{-3} + a_{-2}z^{-2} + a_{-1}z^{-1}}_{g(z)} \; \underbrace{a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots}_{f(z)}$$

$$\cdots \quad a_{3} \quad a_{2} \quad a_{1} \quad a_0 \quad a_{-1} \quad a_{-2} \quad a_{-3} \quad \cdots$$

$$\cdots \quad z^{-3} \quad z^{-2} \quad z^{-1} \quad z^0 \quad z^1 \quad z^2 \quad z^3 \quad \cdots$$

anti-causal Laurent $f(z^{-1})$ — causal Laurent $g(z^{-1})$

$$\underbrace{\cdots + a_{3}z^{-3} + a_{2}z^{-2} + a_{1}z^{-1} + a_0 z^0}_{f(z^{-1})} \; \underbrace{a_{-1} z^1 + a_{-2} z^2 + a_{-3} z^3 + \cdots}_{g(z^{-1})}$$

causal z-transform $X(z)$ — anti-causal z-transform $Y(z)$

anti-causal Laurent $g(z)$	causal Laurent $f(z)$
$\downarrow z^{-1}$	$\downarrow z^{-1}$
causal Laurent $g(z^{-1})$	anti-causal Laurent $f(z^{-1})$
$\Vert$	$\Vert$
anti-causal z-transform $Y(z)$	causal z-transform $X(z)$

z-transform $X(z^{-1})$ $Y(z^{-1})$

$$\cdots \quad a_{-3} \quad a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots$$

$$\cdots \quad z^{3} \quad z^{2} \quad z^{1} \quad z^{0} \quad z^{-1} \quad z^{-2} \quad z^{-3} \quad \cdots$$

anti- causal z-transform $Y(z)$ causal z-transform $X(z)$

$$\cdots + a_{-3}z^3 + a_{-2}z^2 + a_{-1}z^1 \quad a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$$

$$\cdots \quad a_{3} \quad a_{2} \quad a_{1} \quad a_0 \quad a_{-1} \quad a_{-2} \quad a_{-3} \quad \cdots$$

$$\cdots \quad z^{3} \quad z^{2} \quad z^{1} \quad z^{0} \quad z^{-1} \quad z^{-2} \quad z^{-3} \quad \cdots$$

anti- causal z-transform $X(z^{-1})$ causal z-transform $Y(z^{-1})$

$$\cdots + a_3 z^3 + a_2 z^2 + a_1 z^1 + a_0 z^0 \quad a_{-1} z^{-1} + a_{-2} z^{-2} + a_{-3} z^{-3} + \cdots$$

causal Laurent $f(z)$ anti- causal Laurent $g(z)$

anti- causal z-transform $Y(z)$ causal z-transform $X(z)$

$\downarrow z^{-1}$ $\downarrow z^{-1}$

causal z-transform $Y(z^{-1})$ anti- causal z-transform $X(z^{-1})$

$\|$ $\|$

anti- causal Laurent $g(z)$ causal Laurent $f(z)$

causal Laurent	anti-causal Laurent
$f(z)$	$g(z)$
$\parallel$	$\parallel$
$X(z^{-1})$	$Y(z^{-1})$
anti-causal z-transform	causal z-transform

causal z-transform	anti-causal z-transform
$X(z)$	$Y(z)$
$\parallel$	$\parallel$
$f(z^{-1})$	$g(z^{-1})$
anti-causal Laurent	causal Laurent

causal Laurent — anti-causal z-transform

$$f(z) = X(z^{-1})$$

$\parallel$ *different ROC* $\parallel$

$$g(z^{-1}) = Y(z)$$

anti-causal Laurent — causal z-transform

A unit starting Geometric Series unshifted

Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

(1) $\boxed{+\dfrac{1}{1-\boxed{az}}}$ $\boxed{|z|<a^{-1}}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

a^n u(n)	$(n \geq 0)$

(2) $\boxed{+\dfrac{1}{1-\boxed{a^{-1}z}}}$ $\boxed{|z|<a}$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(3) $\boxed{-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}}$ $\boxed{|z|>a^{-1}}$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-a^n$ u(-n)	$(n<1)$

(4) $\boxed{-\dfrac{1}{1-\boxed{az^{-1}}}}$ $\boxed{|z|>a}$

$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$-(\frac{1}{a})^n$ u(-n)	$(n<1)$

Geometric Series - a unit start term

z-Transform (n → -n)

(1) $+\dfrac{1}{1-\boxed{az}}$ $\quad$ $|z|<a^{-1}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^1 + (\frac{1}{a})^{-2}z^2 + \cdots)$

a^{-n} u((-n))	$(-n \geq 0)$
$(\frac{1}{a})^n$ u(-n)	$(n<1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\quad$ $|z|<a$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + \cdots)$

$(\frac{1}{a})^{-n}$ u((-n))	$(-n \geq 0)$
a^n u(-n)	$(n<1)$

(3) $-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad$ $|z|>a^{-1}$

$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + \cdots)$

$-a^{-n}$ u(-(-n))	$(-n<1)$
$-(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(4) $-\dfrac{1}{1-\boxed{az^{-1}}}$ $\quad$ $|z|>a$

$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + \cdots)$

$-(\frac{1}{a})^{-n}$ u(-(-n))	$(-n<1)$
$-a^n$ u(n)	$(n \geq 0)$

Geometric Series

Laurent Series vs. z-Transform $(n \to -n)$

(1) $+\dfrac{1}{1-\boxed{az}}$ $\quad |z| < a^{-1}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

Laurent	$a^n \; u(n)$	$(n \geq 0)$
z-Trans	$(\frac{1}{a})^n \; u(-n)$	$(n < 1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\quad |z| < a$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

Laurent	$(\frac{1}{a})^n \; u(n)$	$(n \geq 0)$
z-Trans	$a^n \; u(-n)$	$(n < 1)$

(3) $-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad |z| > a^{-1}$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \cdots)$

Laurent	$-a^n \; u(-n)$	$(n < 1)$
z-Trans	$-(\frac{1}{a})^n \; u(n)$	$(n \geq 0)$

(4) $-\dfrac{1}{1-\boxed{az^{-1}}}$ $\quad |z| > a$

$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

Laurent	$-(\frac{1}{a})^n \; u(-n)$	$(n < 1)$
z-Trans	$-a^n \; u(n)$	$(n \geq 0)$

Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

(5) $\boxed{-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\boxed{|z| > a^{-1}}$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-a^{n}\ u(-n-1)$	$(n<0)$

(6) $\boxed{-\dfrac{az^{-1}}{1-az^{-1}}}$ $\boxed{|z| > a}$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$-\left(\tfrac{1}{a}\right)^{n}\ u(-n-1)$	$(n<0)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\boxed{|z| < a^{-1}}$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$a^{n}\ u(n-1)$	$(n \geq 1)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\boxed{|z| < a}$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\left(\tfrac{1}{a}\right)^{n}\ u(n-1)$	$(n \geq 1)$

Geometric Series - a non-unit start term

z-Transform (n → -n)

(5) $-\dfrac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}$ $\quad\boxed{|z|>a^{-1}}$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$

$-a^{-n}$ u(-(-n)-1)	$(-n<0)$
$-\left(\tfrac{1}{a}\right)^{n}$ u(n-1)	$(n\geq 1)$

(6) $-\dfrac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}$ $\quad\boxed{|z|>a}$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$-\left(\tfrac{1}{a}\right)^{-n}$ u(-(-n)-1)	$(-n<0)$
$-a^{n}$ u(n-1)	$(n\geq 1)$

(7) $+\dfrac{\boxed{az}}{1-\boxed{az}}$ $\quad\boxed{|z|<a^{-1}}$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$

a^{-n} u((-n)-1)	$(-n\geq 1)$
$\left(\tfrac{1}{a}\right)^{n}$ u(-n-1)	$(n<0)$

(8) $+\dfrac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}$ $\quad\boxed{|z|<a}$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\left(\tfrac{1}{a}\right)^{-n}$ u((-n)-1)	$(-n\geq 1)$
a^{n} u(-n-1)	$(n<0)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform (n → -n)

(5)

$$-\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}} \qquad \boxed{|z|>a^{-1}}$$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$

Laurent	$-a^{n}$	$u(-n-1)$	$(n<0)$
z-Trans	$-\left(\frac{1}{a}\right)^{n}$	$u(n-1)$	$(n\geq 1)$

(6)

$$-\frac{\boxed{a z^{-1}}}{1-\boxed{a z^{-1}}} \qquad \boxed{|z|>a}$$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

Laurent	$-\left(\frac{1}{a}\right)^{n}$	$u(-n-1)$	$(n<0)$
z-Trans	$-a^{n}$	$u(n-1)$	$(n\geq 1)$

(7)

$$+\frac{\boxed{a z}}{1-\boxed{a z}} \qquad \boxed{|z|<a^{-1}}$$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$

Laurent	a^{n}	$u(n-1)$	$(n\geq 1)$
z-Trans	$\left(\frac{1}{a}\right)^{n}$	$u(-n-1)$	$(n<0)$

(8)

$$+\frac{\boxed{a^{-1} z}}{1-\boxed{a^{-1} z}} \qquad \boxed{|z|<a}$$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$

Laurent	$\left(\frac{1}{a}\right)^{n}$	$u(n-1)$	$(n\geq 1)$
z-Trans	a^{n}	$u(-n-1)$	$(n<0)$

4 cases of geometric series **Simple Pole Form**

- 2 representations for each case

	using z			using $1/z$			
	simple pole p		simple pole $1/p$		simple pole $1/p$		simple pole p
(A)	$\frac{1}{z-p}$	(B)	$\frac{1}{z-p^{-1}}$	(C)	$\frac{1}{z^{-1}-p}$	(D)	$\frac{1}{z^{-1}-p^{-1}}$
/p	$-\frac{p^{-1}}{1-p^{-1}z}$	*p	$-\frac{p}{1-pz}$	/p	$-\frac{p^{-1}}{1-p^{-1}z^{-1}}$	*p	$-\frac{p}{1-pz^{-1}}$
/z	$\frac{z^{-1}}{1-pz^{-1}}$	/z	$\frac{z^{-1}}{1-p^{-1}z^{-1}}$	*z	$\frac{z}{1-pz}$	*z	$\frac{z}{1-p^{-1}z}$

p^{-1} p^{-1}

z^{-1}

/p	$-p^{-n-1}$ u(n)	*p	$-p^{n+1}$ u(n)	/p	$-p^{n-1}$ u(-n)	*p	$-p^{-n+1}$ u(-n)
/z	p^{-n-1} u(-n-1)	/z	p^{n+1} u(-n-1)	*z	$-p^{n-1}$ u(n-1)	*z	$-p^{-n+1}$ u(n-1)

4 cases of geometric series **Simple Pole Form**
- 2 representations for each case

using p				using $1/p$			
	simple pole p		simple pole $1/p$		simple pole $1/p$		simple pole p
(A)	$\boxed{\dfrac{1}{z - p}}$	(C)	$\boxed{\dfrac{1}{z^{-1} - p}}$	(B)	$\boxed{\dfrac{1}{z - p^{-1}}}$	(D)	$\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$
/p	$\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$	/p	$\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$	*p	$\boxed{-\dfrac{p}{1 - p z}}$	*p	$\boxed{-\dfrac{p}{1 - p z^{-1}}}$
/z	$\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$	*z	$\boxed{\dfrac{z}{1 - p z}}$	/z	$\boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$	*z	$\boxed{\dfrac{z}{1 - p^{-1} z}}$

z^{-1} z^{-1}

p^{-1}

/p	$-p^{-n-1}$ u(n)	/p	$-p^{n-1}$ u(-n)	*p	$-p^{n+1}$ u(n)	*p	$-p^{-n+1}$ u(-n)
/z	p^{-n-1} u(-n-1)	*z	$-p^{n-1}$ u(n-1)	/z	p^{n+1} u(-n-1)	*z	$-p^{-n+1}$ u(n-1)

(A) $\boxed{\dfrac{1}{z-p}}$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

p^{2} p^{1} p^{0} p^{-1} p^{-2} p^{-3} p^{-4}

z^{-3} z^{-2} z^{-1} z^{0} z^{1} z^{2} z^{3}

$$\cdots + p^{2}z^{-3} + p^{1}z^{-2} + p^{0}z^{-1} \qquad p^{-1}z^{0} + p^{-2}z^{1} + p^{-3}z^{2} + p^{-4}z^{3} + \cdots$$

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$ $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{-\dfrac{p}{1-pz}}$

p^{-2} p^{-1} p^{0} p^{1} p^{2} p^{3} p^{4}

z^{-3} z^{-2} z^{-1} z^{0} z^{1} z^{2} z^{3}

$$\cdots + p^{-2}z^{-3} + p^{-1}z^{-2} + p^{0}z^{-1} \qquad p^{1}z^{0} + p^{2}z^{1} + p^{3}z^{2} + p^{4}z^{3} + \cdots$$

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\dfrac{z}{1-pz}}$

p^{-4} p^{-3} p^{-2} p^{-1} p^{0} p^{1} p^{2}

z^{-3} z^{-2} z^{-1} z^{0} z^{1} z^{2} z^{3}

$$\cdots + p^{-4}z^{-3} + p^{-3}z^{-2} + p^{-2}z^{-1} + p^{-1}z^{0} \qquad p^{0}z^{1} + p^{1}z^{2} + p^{2}z^{3} + \cdots$$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\boxed{\dfrac{z}{1-p^{-1}z}}$

p^{4} p^{3} p^{2} p^{1} p^{0} p^{-1} p^{-2}

z^{-3} z^{-2} z^{-1} z^{0} z^{1} z^{2} z^{3}

$$\cdots + p^{4}z^{-3} + p^{3}z^{-2} + p^{2}z^{-1} + p^{1}z^{0} \qquad p^{0}z^{1} + p^{-1}z^{2} + p^{-2}z^{3} + \cdots$$

(A) $\boxed{\dfrac{1}{z-p}}$ $\quad$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\quad$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

$p^{-n-1}u(-n-1)$ $\quad$ $p^{-n-1}u(n)$

$$\cdots + p^{2}z^{-3} + p^{1}z^{-2} + p^{0}z^{-1} \quad p^{-1}z^{0} + p^{-2}z^{1} + p^{-3}z^{2} + p^{-4}z^{3} + \cdots$$

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$ $\quad$ $\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ $\quad$ $\boxed{-\dfrac{p}{1-pz}}$

$p^{n+1}u(-n-1)$ $\quad$ $p^{n+1}u(n)$

$$\cdots + p^{-2}z^{-3} + p^{-1}z^{-2} + p^{0}z^{-1} \quad p^{1}z^{0} + p^{2}z^{1} + p^{3}z^{2} + p^{4}z^{3} + \cdots$$

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$ $\quad$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\quad$ $\boxed{\dfrac{z}{1-pz}}$

$p^{n-1}u(-n)$ $\quad$ $p^{n-1}u(n-1)$

$$\cdots + p^{-4}z^{-3} + p^{-3}z^{-2} + p^{-2}z^{-1} + p^{-1}z^{0} \quad p^{0}z^{1} + p^{1}z^{2} + p^{2}z^{3} + \cdots$$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ $\quad$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\quad$ $\boxed{\dfrac{z}{1-p^{-1}z}}$

$p^{-n+1}u(-n)$ $\quad$ $p^{-n+1}u(n-1)$

$$\cdots + p^{4}z^{-3} + p^{3}z^{-2} + p^{2}z^{-1} + p^{1}z^{0} \quad p^{0}z^{1} + p^{-1}z^{2} + p^{-2}z^{3} + \cdots$$

$g(z)$ $f(z)$

(A) $\boxed{\dfrac{1}{z-p}}$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$

$p^{-n-1}u(-n-1)$ $p^{-n-1}u(n)$

anti-causal Laurent $g(z)$ causal Laurent $f(z)$

$$\cdots + p^2z^{-3} + p^1z^{-2} + p^0z^{-1} \quad p^{-1}z^0 + p^{-2}z^1 + p^{-3}z^2 + p^{-4}z^3 + \cdots$$

causal z-transform $Y(z^{-1})$ anti-causal z-transform $X(z^{-1})$

$X(z)$ $Y(z)$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$ $\boxed{-\dfrac{p}{1-pz^{-1}}}$ $\boxed{\dfrac{z}{1-p^{-1}z}}$

$p^{-n+1}u(-n)$ $p^{-n+1}u(n-1)$

anti-causal Laurent $f(z^{-1})$ causal Laurent $g(z^{-1})$

$$\cdots + p^4z^{-3} + p^3z^{-2} + p^2z^{-1} + p^1z^0 \quad p^0z^1 + p^{-1}z^2 + p^{-2}z^3 + \cdots$$

causal z-transform $X(z)$ anti-causal z-transform $Y(z)$

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

$g(z)$: $\boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$ $\quad$ $f(z)$: $\boxed{-\dfrac{p}{1 - p z}}$

p^{n+1} u(-n-1) $\quad$ p^{n+1} u(n)

anti-causal Laurent $g(z)$ $\quad$ causal Laurent $f(z)$

$$\cdots + p^{-2} z^{-3} + p^{-1} z^{-2} + p^{0} z^{-1} \quad p^{1} z^{0} + p^{2} z^{1} + p^{3} z^{2} + p^{4} z^{3} + \cdots$$

causal z-transform $Y(z^{-1})$ $\quad$ anti-causal z-transform $X(z^{-1})$

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$

$X(z)$: $\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$ $\quad$ $Y(z)$: $\boxed{\dfrac{z}{1 - p z}}$

p^{n-1} u(-n) $\quad$ p^{n-1} u(n-1)

anti-causal Laurent $f(z^{-1})$ $\quad$ causal Laurent $g(z^{-1})$

$$\cdots + p^{-4} z^{-3} + p^{-3} z^{-2} + p^{-2} z^{-1} + p^{-1} z^{0} \quad p^{0} z^{1} + p^{1} z^{2} + p^{2} z^{3} + \cdots$$

causal z-transform $X(z)$ $\quad$ anti-causal z-transform $Y(z)$

(A) $\boxed{\dfrac{1}{z-p}}$

$g(z)\ \boxed{\dfrac{z^{-1}}{1-pz^{-1}}}\ Y(z^{-1})$ $\quad$ $f(z)\ \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}\ X(z^{-1})$

$p^{-n-1}u(-n-1)$ $\quad$ $p^{-n-1}u(n)$

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$

$Y(z)\ \boxed{\dfrac{z}{1-p^{-1}z}}\ f(z^{-1})$ $\quad$ $X(z)\ \boxed{-\dfrac{p}{1-pz^{-1}}}\ g(z^{-1})$

$p^{-n+1}u(n-1)$ $\quad$ $p^{-n+1}u(-n)$

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$

$g(z)\ \boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}\ Y(z^{-1})$ $\quad$ $f(z)\ \boxed{-\dfrac{p}{1-pz}}\ X(z^{-1})$

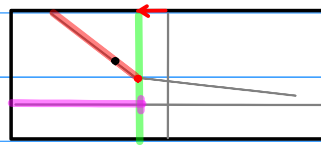
$p^{n+1}u(-n-1)$ $\quad$ $p^{n+1}u(n)$

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$

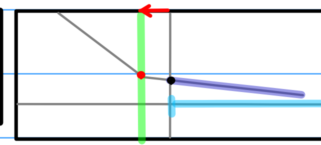
$Y(z)\ \boxed{\dfrac{z}{1-pz}}\ g(z^{-1})$ $\quad$ $X(z)\ \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}\ f(z^{-1})$

$p^{n-1}u(n-1)$ $\quad$ $p^{n-1}u(-n)$

(A) $\boxed{\dfrac{1}{z-p}}$

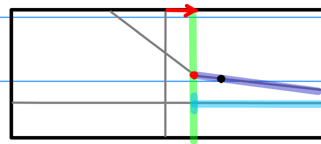
$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ 

p^{-n-1} u(-n-1)

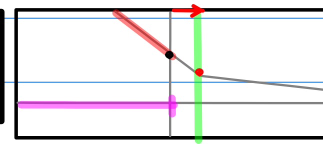
$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ 

p^{-n-1} u(n)

(D) $\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$

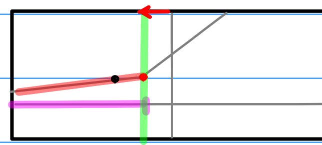
$\boxed{\dfrac{z}{1-p^{-1}z}}$ 

p^{-n+1} u(n-1)

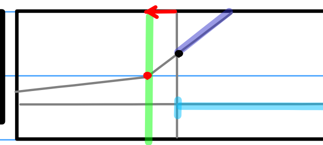
$\boxed{-\dfrac{p}{1-pz^{-1}}}$ 

p^{-n+1} u(-n)

(B) $\boxed{\dfrac{1}{z-p^{-1}}}$

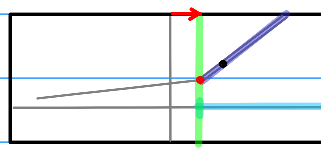
$\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$ 

p^{n+1} u(-n-1)

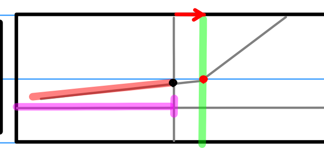
$\boxed{-\dfrac{p}{1-pz}}$ 

p^{n+1} u(n)

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$

$\boxed{\dfrac{z}{1-pz}}$ 

p^{n-1} u(n-1)

$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$ 

p^{n-1} u(-n)

when the pole is expressed as p

2 formulas **Simple Pole Form**

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p^{-1}}}$$

2 representations each **Shifted Geometric Series Form**

(A) $\boxed{\frac{1}{z-p}}$

causal Laurent — anti- causal z-transform

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1})$$

|| different ROC ||

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} \triangleq g(z^{-1}) = Y(z)$$

anti- causal Laurent — causal z-transform

(D) $\boxed{\frac{1}{z^{-1}-p^{-1}}}$

anti- causal Laurent — causal z-transform

$$\boxed{-\frac{p}{1-pz^{-1}}} \triangleq f(z^{-1}) = X(z)$$

|| different ROC ||

$$\boxed{\frac{z}{1-p^{-1}z}} \triangleq g(z) = Y(z^{-1})$$

causal Laurent — anti- causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

when the pole is expressed as $1/p$

2 formulas — **Simple Pole Form**

$$\boxed{\frac{1}{z - p^{-1}}} \qquad \boxed{\frac{1}{z^{-1} - p}}$$

2 representations each — **Shifted Geometric Series Form**

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

causal Laurent — anti- causal z-transform

$$\boxed{-\frac{p}{1 - pz}} \triangleq f(z) = X(z^{-1})$$

|| different ROC ||

$$\boxed{\frac{z^{-1}}{1 - p^{-1}z^{-1}}} \triangleq g(z^{-1}) = Y(z)$$

anti- causal Laurent — causal z-transform

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$

anti- causal Laurent — causal z-transform

$$\boxed{-\frac{p^{-1}}{1 - p^{-1}z^{-1}}} \triangleq f(z^{-1}) = X(z)$$

|| different ROC ||

$$\boxed{\frac{z}{1 - pz}} \triangleq g(z) = Y(z^{-1})$$

causal Laurent — anti- causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

(A) $\dfrac{1}{z - p}$

$-\dfrac{p^{-1}}{1 - p^{-1} z}$	$-\left(p^{-1} z^{0} + p^{-2} z^{1} + p^{-3} z^{2} + \cdots \right)$ $f(z)$ causal Laurent
	$-\left(\left(\tfrac{1}{p}\right)^{1} z^{0} + \left(\tfrac{1}{p}\right)^{2} z^{1} + \left(\tfrac{1}{p}\right)^{3} z^{2} + \cdots \right)$ $X(z^{-1})$ anti- causal z-transform
$\dfrac{z^{-1}}{1 - p z^{-1}}$	$\left(p^{0} z^{-1} + p^{1} z^{-2} + p^{2} z^{-3} + \cdots \right)$ $g(z^{-1})$ anti- causal Laurent
	$\left(\left(\tfrac{1}{p}\right)^{0} z^{-1} + \left(\tfrac{1}{p}\right)^{-1} z^{-2} + \left(\tfrac{1}{p}\right)^{-2} z^{-3} + \cdots \right)$ $Y(z)$ causal z-transform

(D) $\dfrac{1}{z^{-1} - p^{-1}}$

$-\dfrac{p}{1 - p z^{-1}}$	$-\left(p^{1} z^{0} + p^{2} z^{-1} + p^{3} z^{-2} + \cdots \right)$ $f(z^{-1})$ anti- causal Laurent
	$-\left(\left(\tfrac{1}{p}\right)^{-1} z^{0} + \left(\tfrac{1}{p}\right)^{-2} z^{-1} + \left(\tfrac{1}{p}\right)^{-3} z^{-2} + \cdots \right)$ $X(z)$ causal z-transform
$\dfrac{z}{1 - p^{-1} z}$	$\left(p^{0} z^{1} + p^{-1} z^{2} + p^{-2} z^{3} + \cdots \right)$ $g(z)$ causal Laurent
	$\left(\left(\tfrac{1}{p}\right)^{0} z^{1} + \left(\tfrac{1}{p}\right)^{1} z^{2} + \left(\tfrac{1}{p}\right)^{2} z^{3} + \cdots \right)$ $Y(z^{-1})$ anti- causal z-transform

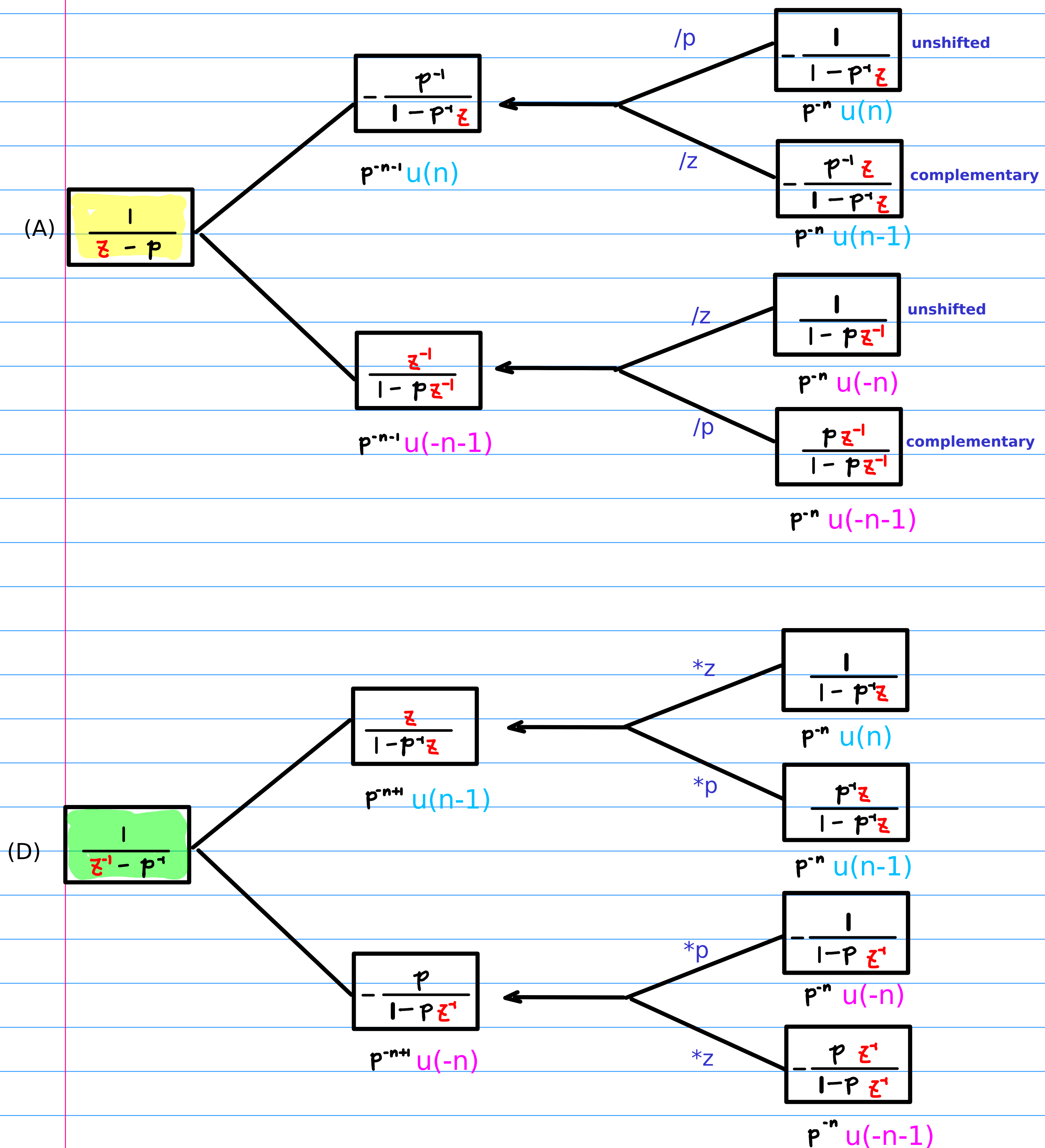
(B) $\dfrac{1}{z - p^{-1}}$

$-\dfrac{p}{1 - p z}$	$-\left(p^{1} z^{0} + p^{2} z^{1} + p^{3} z^{2} + \cdots \right)$ $f(z)$ causal Laurent
	$-\left(\left(\tfrac{1}{p}\right)^{-1} z^{0} + \left(\tfrac{1}{p}\right)^{-2} z^{1} + \left(\tfrac{1}{p}\right)^{-3} z^{2} + \cdots \right)$ $X(z^{-1})$ anti- causal z-transform
$\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}$	$\left(p^{0} z^{-1} + p^{-1} z^{-2} + p^{-2} z^{-3} + \cdots \right)$ $g(z^{-1})$ anti- causal Laurent
	$\left(\left(\tfrac{1}{p}\right)^{0} z^{-1} + \left(\tfrac{1}{p}\right)^{1} z^{-2} + \left(\tfrac{1}{p}\right)^{2} z^{-3} + \cdots \right)$ $Y(z)$ causal z-transform

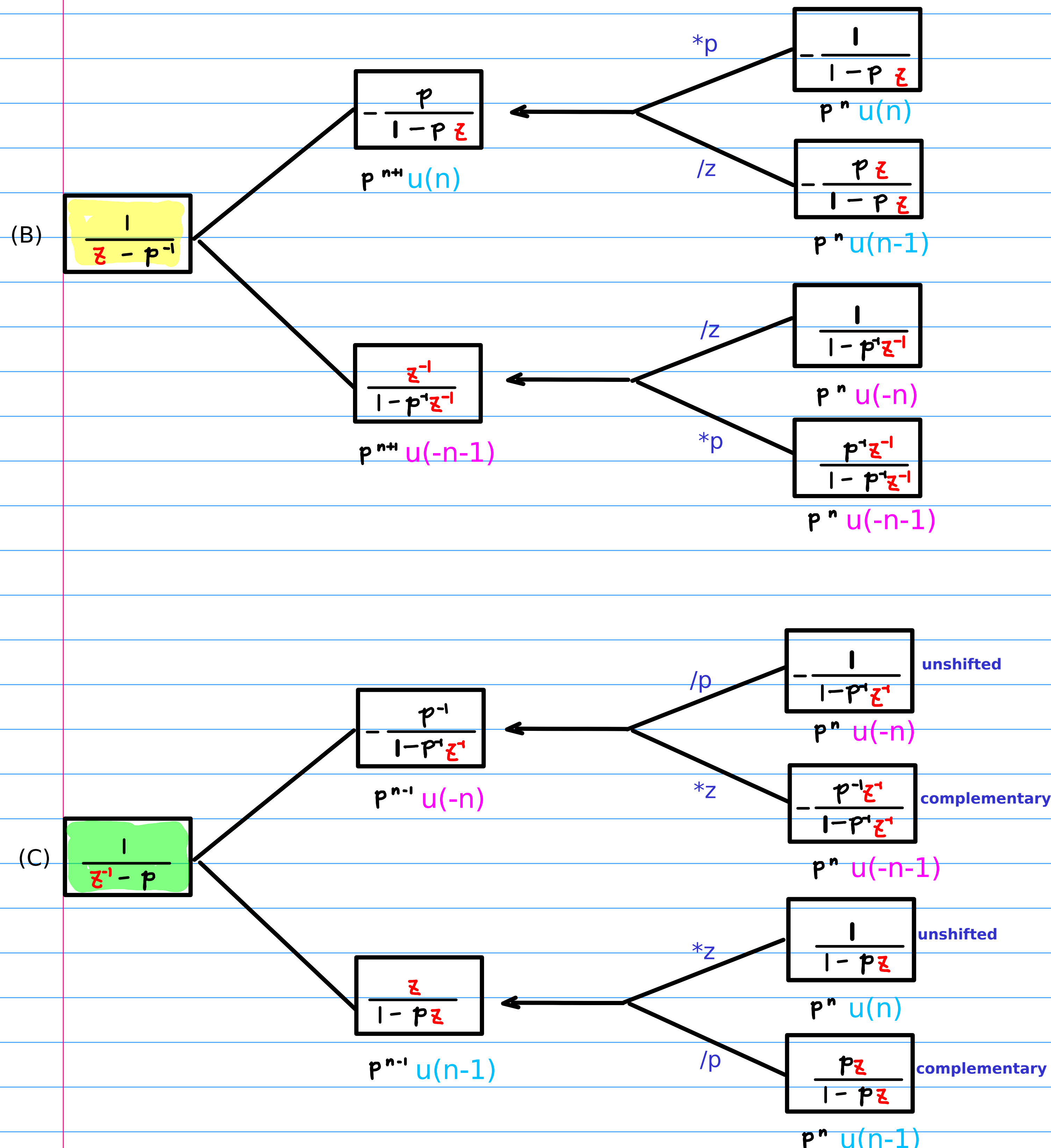
(C) $\dfrac{1}{z^{-1} - p}$

$-\dfrac{p}{1 - p z^{-1}}$	$-\left(p^{-1} z^{0} + p^{-2} z^{-1} + p^{-3} z^{-2} + \cdots \right)$ $f(z^{-1})$ anti- causal Laurent
	$-\left(\left(\tfrac{1}{p}\right)^{1} z^{0} + \left(\tfrac{1}{p}\right)^{2} z^{-1} + \left(\tfrac{1}{p}\right)^{3} z^{-2} + \cdots \right)$ $X(z)$ causal z-transform
$\dfrac{z}{1 - p z}$	$\left(p^{0} z^{1} + p^{1} z^{2} + p^{2} z^{3} + \cdots \right)$ $g(z)$ causal Laurent
	$\left(\left(\tfrac{1}{p}\right)^{0} z^{1} + \left(\tfrac{1}{p}\right)^{-1} z^{2} + \left(\tfrac{1}{p}\right)^{-2} z^{3} + \cdots \right)$ $Y(z^{-1})$ anti- causal z-transform

Shifted Geometric Series (1) p



Shifted Geometric Series (2) 1/p



Simple Pole Form

①

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$-a^n$	$(n \geq 0)$

②

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-\left(\frac{1}{a}\right)^n$	$(n < 1)$

③

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n	$(n < 1)$

④

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑤

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$-\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑥

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-a^n$	$(n < 1)$

⑦

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^n$	$(n < 1)$

⑧

$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n	$(n \geq 0)$

Geometric Series : $f(z)$, $g(z)$, $\bar{f}(z)$, $\bar{g}(z)$

$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n+1}$	$(n \geq 0)$

$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n-1}$	$(n \geq 1)$

Geometric Series :

$f(z) = g(z^{-1})$ the same algebraic formula but the complement ROC's

$g(z) = f(z^{-1})$ the same algebraic formula but the complement ROC's

$f(z) = f_a(z)$ two variable function

$g(z) = g_a(z)$ two variable function

$\bar{f}(z) = f_{a^{-1}}(z)$ inverse a

$\bar{g}(z) = g_{a^{-1}}(z)$ inverse a

associated simple pole forms

$a^{-1} f(z),\ z^{-1} g(z),\ a \bar{f}(z),\ z^{-1} \bar{g}(z)$

①

$a^{-1} f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^n$	$(n \geq 0)$

②

$a^{-1} f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$

③

$z\, g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

④

$z^{-1} g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑤

$a \bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑥

$a \bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

⑦

$z \bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

⑧

$z^{-1} \bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

(1)	(2)		(1)	(2)
(3)	(4)		(3)	(4)
(5)	(6)		(5)	(6)
(7)	(8)		(7)	(8)

$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$
$\bar{f}(z)$	$\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$	$\bar{g}(z)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) Sym(n) $\longleftrightarrow$ inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

① $\updownarrow$ ③: sign comp(z), sign comp(n)

② $\updownarrow$ ④: sign comp(z), sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

neg(n) Sym(n) $\longleftrightarrow$ inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

neg(n) Sym(n) $\longleftrightarrow$ inv(z) inv(z)

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

⑤ $\updownarrow$ ⑦: sign comp(z), sign comp(n)

⑥ $\updownarrow$ ⑧: sign comp(z), sign comp(n)

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

neg(n) Sym(n) $\longleftrightarrow$ inv(z) inv(z)

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$f(z)$ $f(z^{-1})$
$g(z^{-1})$ $g(z)$
$\bar{f}(z)$ $\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$ $\bar{g}(z)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -(\frac{1}{a})^{n+1}$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

③

$a_n = a^{n+1}$	$(n < 0)$
$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$
$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = (\frac{1}{a})^{n+1}$	$(n < 0)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$ $a^{-1}f(z^{-1})$

$z g(z^{-1})$ $z^{-1}g(z)$

$a\bar{f}(z)$ $a\bar{f}(z^{-1})$

$z\bar{g}(z^{-1})$ $z^{-1}\bar{g}(z)$

a unit nominator

① $a_n = -a^n$ $(n \geq 0)$

$a^{-1}f(z) = -\dfrac{1}{1-az}$ $|z| < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

② $a_n = -\left(\dfrac{1}{a}\right)^n$ $(n < 1)$

$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$ $|z| > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

③ $z g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

$a_n = a^n$ $(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

④ $z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$ $|z| < a$

$a_n = \left(\dfrac{1}{a}\right)^n$ $(n \geq 0)$

⑤ $a_n = -\left(\dfrac{1}{a}\right)^n$ $(n \geq 0)$

$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$ $|z| < a$

neg(n) sym(n)

inv(z) inv(z)

⑥ $a_n = -a^n$ $(n < 1)$

$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

⑦ $z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$ $|z| > a$

$a_n = \left(\dfrac{1}{a}\right)^n$ $(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

⑧ $z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$ $|z| < a^{-1}$

$a_n = a^n$ $(n \geq 0)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$	$a^{-1}f(z^{-1})$
$z\,g(z^{-1})$	$z^{-1}g(z)$
$a\bar{f}(z)$	$a\bar{f}(z^{-1})$
$z\,\bar{g}(z^{-1})$	$z^{-1}\bar{g}(z)$

a unit nominator

①

$a_n = -a^n$	$(n \geq 0)$
$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$
$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

③

$a_n = a^n$	$(n < 1)$
$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$
$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$z\,\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

Simple Pole Forms | Geometric Series Forms

#	Simple Pole Forms				Geometric Series Forms	
①	$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot a^{-1}$	id	$a_n = -a^{n}$	$(n \geq 0)$
	$f(z) = -\frac{a}{1-az}$	$\lvert z \rvert < a^{-1}$	$\cdot a^{1}$	id	$a^{-1} f(z) = -\frac{1}{1-az}$	$\lvert z \rvert < a^{-1}$
②	$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$	$\cdot a^{-1}$	id	$a_n = -(\frac{1}{a})^{n}$	$(n < 1)$
	$f(z^{-1}) = -\frac{a}{1-az^{-1}}$	$\lvert z \rvert > a$	$\cdot a^{-1}$	id	$a^{-1} f(z^{-1}) = -\frac{1}{1-az^{-1}}$	$\lvert z \rvert > a$
③	$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$	$\cdot z$	id	$z\, g(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$
	$a_n = a^{n+1}$	$(n < 0)$	$\cdot a^{-1}$	S(c(n))	$a_n = a^{n}$	$(n < 1)$
④	$g(z) = \frac{z}{1-a^{-1}z}$	$\lvert z \rvert < a$	$\cdot z^{-1}$	id	$z^{-1} g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z \rvert < a$
	$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$	$\cdot a^{-1}$	S(c(n))	$a_n = (\frac{1}{a})^{n}$	$(n \geq 0)$
⑤	$a_n = -(\frac{1}{a})^{n+1}$	$(n \geq 0)$	$\cdot a$	id	$a_n = -(\frac{1}{a})^{n}$	$(n \geq 0)$
	$\bar{f}(z) = -\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z \rvert < a$	$\cdot a$	id	$a \bar{f}(z) = -\frac{1}{1-a^{-1}z}$	$\lvert z \rvert < a$
⑥	$a_n = -a^{n-1}$	$(n < 1)$	$\cdot a$	id	$a_n = -a^{n}$	$(n < 1)$
	$\bar{f}(z^{-1}) = -\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$	$\cdot a$	id	$a \bar{f}(z^{-1}) = -\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$
⑦	$\bar{g}(z^{-1}) = \frac{z^{-1}}{1-az^{-1}}$	$\lvert z \rvert > a$	$\cdot z$	id	$z \bar{g}(z^{-1}) = \frac{1}{1-az^{-1}}$	$\lvert z \rvert > a$
	$a_n = (\frac{1}{a})^{n+1}$	$(n < 0)$	$\cdot a$	s(c(n))	$a_n = (\frac{1}{a})^{n}$	$(n < 1)$
⑧	$\bar{g}(z) = \frac{z}{1-az}$	$\lvert z \rvert < a^{-1}$	$\cdot z^{-1}$	id	$z^{-1} \bar{g}(z) = \frac{1}{1-az}$	$\lvert z \rvert < a^{-1}$
	$a_n = a^{n-1}$	$(n \geq 1)$	$\cdot a$	s(c(n))	$a_n = a^{n}$	$(n \geq 0)$

Simple Pole Forms			Geometric Series Forms	

①

$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot a$ id	$a_n = -a^n$	$(n \geq 0)$
$f(z) = -\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot a$ id	$a^{-1} f(z) = -\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

$-a\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$-\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

②

$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$	$\cdot a$ id	$a_n = -(\frac{1}{a})^n$	$(n < 1)$
$f(z^{-1}) = -\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot a$ id	$a^{-1} f(z^{-1}) = -\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

$-a\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-a\,((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$-\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-\,((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

③

$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot z^{-1}$ id	$z\,g(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$	$\cdot a$ S(c(n))	$a_n = a^n$	$(n < 1)$

$-\,z^{-1}(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-\,(a^0 z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + \cdots)$

$-\,(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

④

$g(z) = \frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot z$ id	$z^{-1} g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$	$\cdot a$ S(c(n))	$a_n = (\frac{1}{a})^n$	$(n \geq 0)$

$-\,z\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,((\frac{1}{a})^0 z^1 + (\frac{1}{a})^1 z^2 + (\frac{1}{a})^2 z^3 + \cdots)$

$-\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

Simple Pole Forms — Geometric Series Forms

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

$\cdot a^{-1}$ id

$\cdot a^{-1}$ id

$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$
$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

$-\left(\frac{1}{a}\right)\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^1 z^0 + \left(\frac{1}{a}\right)^2 z^1 + \left(\frac{1}{a}\right)^3 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

$\cdot a^{-1}$ id

$\cdot a^{-1}$ id

$a_n = -a^{n}$	$(n < 1)$
$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

$-a^{-1}(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-(a^{-1} z^0 + a^{-2} z^{-1} + a^{-3} z^{-2} + \cdots)$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\cdot z^{-1}$ id

$\cdot a^{-1}s(c(n))$

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 1)$

$z^{-1}\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^{-1} + \left(\frac{1}{a}\right)^{-1} z^{-2} + \left(\frac{1}{a}\right)^{-2} z^{-3} + \cdots\right)$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

$\cdot z$ id

$\cdot a^{-1}s(c(n))$

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n}$	$(n \geq 0)$

$z(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$(a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots)$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z \rvert < a^{-1}$

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z \rvert > a$

neg(n) sym(n)

inv(z) inv(z)

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z \rvert < a^{-1}$

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

neg(n) sym(n)

inv(z) inv(z)

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z \rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

· z id

· a s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z \rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z \rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

· z id

· a s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z \rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

$\cdot z^{-1}$ id

$\longleftarrow$

$\cdot a$ s(c(n))

$z\, g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n}$	$(n < 1)$

$$X(z) = \frac{1}{1-a^{-1}z^{-1}} \qquad \lvert z\rvert > a^{-1}$$

$$\Downarrow$$

$$a_n = a^{n} \qquad (n < 1)$$

$\bullet z^{-1}$ $\downarrow$

$$+\left(a^{0}z^{0} + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots\right) \qquad n = 0, -1, -2, \cdots$$

$$a^{0} \qquad a^{-1} \qquad a^{-2} \qquad \cdots \qquad \bullet a \downarrow$$

$$+\left(a^{0}z^{-1} + a^{-1}z^{-2} + a^{-2}z^{-3} + \cdots\right) \qquad n = -1, -2, -3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad \lvert z\rvert > a^{-1}$$

$$\Downarrow$$

$$a_{n-1} = a^{n+1} \qquad (n < 0)$$

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\xleftarrow[\cdot\, a^{-1} \quad s(c(n))]{\cdot\, z \quad \text{id}}$

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$$X(z) = \frac{1}{1-a^{-1}z} \qquad \lvert z\rvert < a$$

$$\Downarrow$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n \geq 0)$$

$$- \left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right) \qquad n = 0, 1, 2, \cdots$$

$$\bullet z \qquad a^0 \quad a^{-1} \quad a^{-2} \quad \cdots \qquad \bullet a^{-1}$$

$$- \left(a^0 z^1 + a^{-1} z^2 + a^{-2} z^3 + \cdots\right) \qquad n = 1, 2, 3, \cdots$$

$$z X(z) = \frac{z}{1-a^{-1}z} \qquad \lvert z\rvert < a$$

$$\Downarrow$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n-1} \qquad (n \geq 1)$$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n<0)$

$\cdot\, z$ id

$\cdot\, a^{-1}$ s(c(n))

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n<1)$

$$X(z) = \frac{1}{1-az^{-1}} \qquad |z|>a$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n<1)$$

$\bullet z$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right) \qquad n = 0, -1, -2, \cdots$$

$$a^0 \qquad a^1 \qquad a^2 \qquad \cdots \qquad \bullet a^{-1}$$

$$-\left(a^0 z^{-1} + a^1 z^{-2} + a^2 z^{-3} + \cdots\right) \qquad n = -1, -2, -3, \cdots$$

$$z\,X(z) = \frac{z^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n+1} \qquad (n<0)$$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

$\cdot z^{-1}$ id

$\cdot a^{-1}$ s(c(n))

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n}$	$(n \geq 0)$

$$X(z) = \frac{1}{1-az} \quad |z| < a^{-1}$$

$$a_n = a^n \quad (n \geq 0)$$

$\bullet z^{-1}$

$+\ (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} \quad)$ $\quad n = 0, 1, 2, \cdots$

$a^0 \quad a^1 \quad a^2 \quad \cdots$ $\quad \bullet a^{-1}$

$+\ (a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots)$ $\quad n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z}{1-az} \quad |z| < a^{-1}$$

$$a_{n-1} = a^{n-1} \quad (n \geq 1)$$

①

$a_n = -2^n$	$(n \geq 0)$
${}^{0.5}f(z) = -\frac{1}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^n$	$(n < 1)$
${}^{0.5}f(z^{-1}) = -\frac{1}{1-2z^{-1}}$	$\lvert z\rvert > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

③

$z\,g(z^{-1}) = \frac{1}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

④

$z^{-1}g(z) = \frac{1}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^n$	$(n \geq 0)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{2}{1-2z}$	$\lvert z \rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{2}{1-2z^{-1}}$	$\lvert z \rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z \rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-0.5z}$	$\lvert z \rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{2}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{2}{1-2z}$	$\lvert z\rvert < 0.5$

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$

neg(n) sym(n)

inv(z) inv(z)

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{2}{1-2z}$	$\lvert z\rvert < 0.5$

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) sym(n)

inv(z) inv(z)

$g(z) = \dfrac{z}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

- z id
- a s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

④

$g(z) = \dfrac{z}{1-2z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

- z id
- a s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n}$	$(n \geq 0)$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $\quad n=1$ $\quad n=2$ $\qquad n=0$ $\quad n=1$ $\quad n=2$

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $\quad n=-1$ $\quad n=-2$ $\qquad n=0$ $\quad n=-1$ $\quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \geq 0$ $\quad n \geq 1$ $\quad n < 0$ $\quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$

$-n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1}$ $\qquad n < 0$ $\quad n < 1$ $\quad n \geq 0$ $\quad n \geq 1$

$= \left(\frac{1}{a}\right)^{n-1}$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Big\updownarrow z^{-1} \qquad \Big\updownarrow z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad \Big\downarrow -n \qquad \Big\downarrow \quad \Big\downarrow \quad \Big\downarrow \quad \Big\downarrow$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\qquad \left(\frac{1}{a}\right)^{-n+1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$

① $\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$

② $\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$

③ $\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$

④ $\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$

$\boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}$ $(n\geq 0)$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}$$

$|z|<0.5$

$\boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}$ $(n<1)$

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}$$

$|z|>2$

$\boxed{z^{-1}\quad -n}$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}$$

$|z|<0.5$

$\boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}$$

$|z|>2$

$\boxed{z^{-1}\quad -n}$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}$$

$|z|<0.5$

$\boxed{\cdot z\quad n-1}$

$$\boxed{+\frac{1}{1-0.5z}-\frac{1}{1-2z}}$$

$|z|<0.5$

$\boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}$ $(n\geq 1)$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}$$

$|z|<0.5$

$\boxed{\cdot z\quad n-1}$

$$\boxed{+\frac{1}{1-z}-\frac{1}{1-2z}}$$

$|z|<0.5$

$z^{-1}X(z)$ Shifted Sequence

$$X(z) = \underset{|z|>1}{\frac{1}{1-z^{-1}}} - \underset{|z|>2}{\frac{1}{1-2z^{-1}}} \quad (|z|>2)$$

$$a_n = 1^n + 2^n \quad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots\right) + \left(2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

• z^{-1} $\qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \quad (n \geq 1)$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

$\cdot z$ $\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$\cdot z \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$z \qquad (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \qquad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$z^{-1} \qquad (1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \qquad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$

$|z| > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}$

$\{\,|z| < 0.5\,\} \cap \{\,|z| > 2\,\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a \quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0$

$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \Big\downarrow{-n}$

$|z| > a \quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1} \qquad n < 0 \quad n < 1 \quad n \geq 0$

$a^{n+1} z^n$

$a\,(az)^n$

$a \left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$zX(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\begin{aligned}
&(1^0z^0 + 1^1z^{-1} + 1^2z^{-2} + \cdots) + (2^0z^0 + 2^1z^{-1} + 2^2z^{-2}) && n && n = 0, 1, 2, \cdots \\
&\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \\
\cdot z^{-1} \downarrow \quad &(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots) + (2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots) && n-1 && n = 1, 2, 3, \cdots
\end{aligned}$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$\begin{aligned}
z \quad &(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots) + (2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots) && n && n = 1, 2, 3, \cdots \\
&\quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \\
z^{-1} \quad &(1^0z^{1} + 1^1z^{2} + 1^2z^{3} + \cdots) + (2^0z^{1} + 2^1z^{2} + 2^2z^{3} + \cdots) && -n && n = -1, -2, -3, \cdots
\end{aligned}$$

$$zX(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\qquad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\qquad f(z) = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$$

ROC $\qquad X(z) = \displaystyle\sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \qquad z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$

$\boxed{2z}$ $\boxed{2z^{-1}}$ $\boxed{2^{-1}z^{-1}}$ $\boxed{2^{-1}z}$

$\boxed{|z|<0.5}$ $\boxed{|z|>2}$ $\boxed{|z|>0.5}$ $\boxed{|z|<2}$

$$\begin{tikzcd}
-\dfrac{2}{1-2z} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, shift right=2, "\cdot\frac{(2z)^{-1}}{(2z)^{-1}}"'] & -\dfrac{2}{1-2z^{-1}} \arrow[d, shift right=2, "\cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}}"'] \\
+\dfrac{z^{-1}}{1-0.5z^{-1}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[u, shift right=2, "\cdot\frac{(2z)}{(2z)}"'] & +\dfrac{z}{1-0.5z} \arrow[u, shift right=2, "\cdot\frac{(2z^{-1})}{(2z^{-1})}"']
\end{tikzcd}$$

z-Transform $(n \to -n)$

(5) $\boxed{-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\quad\boxed{|z|>a^{-1}}$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$

$-a^{-n}\ u(-(-n)-1)$	$(-n<0)$
$-\left(\tfrac{1}{a}\right)^{n}\ u(n-1)$	$(n\geq 1)$

(6) $\boxed{-\dfrac{az^{-1}}{1-az^{-1}}}$ $\quad\boxed{|z|>a}$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$-\left(\tfrac{1}{a}\right)^{-n}\ u(-(-n)-1)$	$(-n<0)$
$-a^{n}\ u(n-1)$	$(n\geq 1)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\quad\boxed{|z|<a^{-1}}$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$

$a^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$\left(\tfrac{1}{a}\right)^{n}\ u(-n-1)$	$(n<0)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\quad\boxed{|z|<a}$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\left(\tfrac{1}{a}\right)^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$a^{n}\ u(-n-1)$	$(n<0)$

Laurent Series vs. z-Transform $(n \to -n)$

(5) $\boxed{-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\boxed{|z|>a^{-1}}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{1}z^{-1} + (\frac{1}{a})^{2}z^{-2} + (\frac{1}{a})^{3}z^{-3} + \cdots)$

Laurent	$-a^{n}$ $u(-n-1)$	$(n<0)$
z-Trans	$-(\frac{1}{a})^{n}$ $u(n-1)$	$(n\geq 1)$

(6) $\boxed{-\dfrac{az^{-1}}{1-az^{-1}}}$ $\boxed{|z|>a}$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

Laurent	$-(\frac{1}{a})^{n}$ $u(-n-1)$	$(n<0)$
z-Trans	$-a^{n}$ $u(n-1)$	$(n\geq 1)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\boxed{|z|<a^{-1}}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$((\frac{1}{a})^{-1}z^{1} + (\frac{1}{a})^{-2}z^{2} + (\frac{1}{a})^{-3}z^{3} + \cdots)$

Laurent	a^{n} $u(n-1)$	$(n\geq 1)$
z-Trans	$(\frac{1}{a})^{n}$ $u(-n-1)$	$(n<0)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\boxed{|z|<a}$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

Laurent	$(\frac{1}{a})^{n}$ $u(n-1)$	$(n\geq 1)$
z-Trans	a^{n} $u(-n-1)$	$(n<0)$