

# OP Amp

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# Operational amplifier

An **operational amplifier** (op-amp) is a DC-coupled high-gain electronic voltage amplifier with a differential input and, usually, a single-ended output.<sup>[1]</sup> In this configuration, an op-amp produces an output potential (relative to circuit ground) that is typically hundreds of thousands of times larger than the potential difference between its input terminals.<sup>[2]</sup>

Operational amplifiers had their origins in analog computers, where they were used to do mathematical operations in many linear, non-linear and frequency-dependent circuits. Characteristics of a circuit using an op-amp

are set by external components with little dependence on temperature changes or manufacturing variations in the op-amp itself, which makes op-amps popular building blocks for circuit design.

Op-amps are among the most widely used electronic devices today, being used in a vast array of consumer, industrial, and scientific devices. Many standard IC op-amps cost only a few cents in moderate production volume; however some integrated or hybrid operational amplifiers with special performance specifications may cost over \$100 US in small quantities. Op-amps may be packaged as components, or used as elements of more complex integrated circuits.

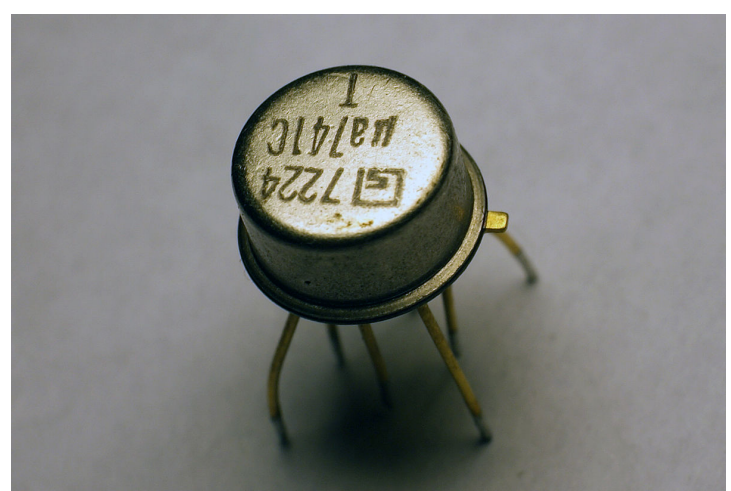
The op-amp is one type of differential amplifier. Other types of differential amplifier include the fully differential amplifier (similar to the op-amp, but with two outputs), the instrumentation amplifier (usually built from three op-amps), the isolation amplifier (similar to the instrumentation amplifier, but with tolerance to common-mode voltages that would destroy an ordinary op-amp), and negative feedback amplifier (usually built from one or more op-amps and a resistive feedback network).

## Circuit notation

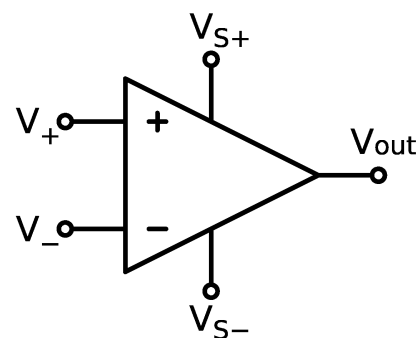
The circuit symbol for an op-amp is shown to the right, where:

- $V_+$ : non-inverting input
- $V_-$ : inverting input
- $V_{out}$ : output
- $V_{S+}$ : positive power supply
- $V_{S-}$ : negative power supply

The power supply pins ( $V_{S+}$  and  $V_{S-}$ ) can be labeled in different ways (See *IC power supply pins*). Often these pins are left out of the diagram for clarity, and the power configuration is described or assumed from the circuit.



A Signetics  $\mu$ a741 operational amplifier, one of the most successful op-amps.



Circuit diagram symbol for an op-amp

## Operation

The amplifier's differential inputs consist of a non-inverting input (+) with voltage  $V_+$  and an inverting input (-) with voltage  $V_-$ ; ideally the op-amp amplifies only the difference in voltage between the two, which is called the *differential input voltage*. The output voltage of the op-amp  $V_{out}$  is given by the equation:

$$V_{out} = A_{OL} (V_+ - V_-)$$

where  $A_{OL}$  is the open-loop gain of the amplifier (the term "open-loop" refers to the absence of a feedback loop from the output to the input).

## Open loop

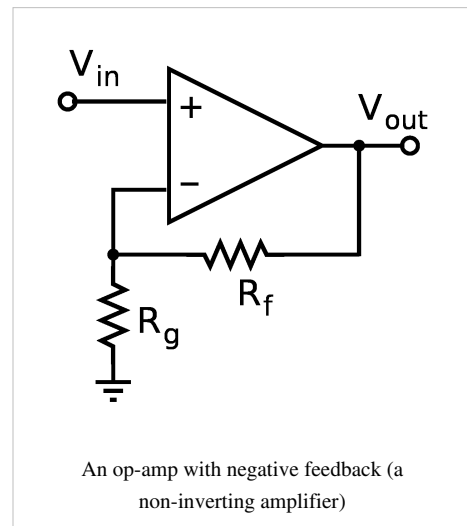
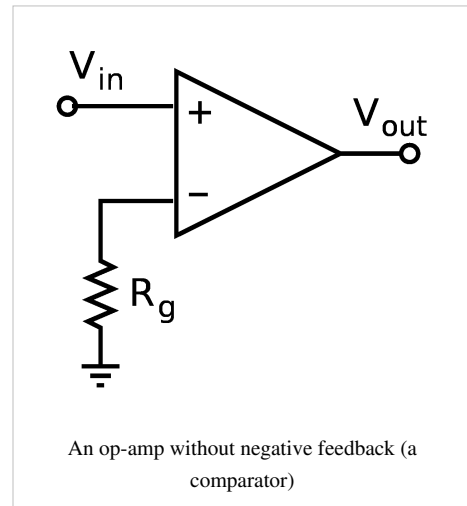
The magnitude of  $A_{OL}$  is typically very large—100,000 or more for integrated circuit op-amps—and therefore even a quite small difference between  $V_+$  and  $V_-$  drives the amplifier output nearly to the supply voltage. Situations in which the output voltage is equal to or greater than the supply voltage are referred to as *saturation* of the amplifier. The magnitude of  $A_{OL}$  is not well controlled by the manufacturing process, and so it is impractical to use an operational amplifier as a stand-alone differential amplifier.

Without negative feedback, and perhaps with positive feedback for regeneration, an op-amp acts as a comparator. If the inverting input is held at ground (0 V) directly or by a resistor  $R_g$ , and the input voltage  $V_{in}$  applied to the non-inverting input is positive, the output will be maximum positive; if  $V_{in}$  is negative, the output will be maximum negative. Since there is no feedback from the output to either input, this is an *open loop* circuit acting as a comparator. The circuit's gain is just the  $A_{OL}$  of the op-amp.

## Closed loop

If predictable operation is desired, negative feedback is used, by applying a portion of the output voltage to the inverting input. The *closed loop* feedback greatly reduces the gain of the circuit. When negative feedback is used, the circuit's overall gain and response becomes determined mostly by the feedback network, rather than by the op-amp characteristics. If the feedback network is made of components with values small relative to the op-amp's input impedance, the value of the op-amp's open loop response  $A_{OL}$  does not seriously affect the circuit's performance. The response of the op-amp circuit with its input, output, and feedback circuits to an input is characterized mathematically by a transfer function; designing an op-amp circuit to have a desired transfer function is in the realm of electrical engineering. The transfer functions are important in most applications of op-amps, such as in analog computers. High input impedance at the input terminals and low output impedance at the output terminal(s) are particularly useful features of an op-amp.

In the non-inverting amplifier on the right, the presence of negative feedback via the voltage divider  $R_f$ ,  $R_g$  determines the *closed-loop gain*  $A_{CL} = V_{out} / V_{in}$ . Equilibrium will be established when  $V_{out}$  is just sufficient to "reach around and pull" the inverting input to the same voltage as  $V_{in}$ . The voltage gain of the entire circuit is thus  $1 + R_f/R_g$ . As a simple example, if  $V_{in} = 1$  V and  $R_f = R_g$ ,  $V_{out}$  will be 2 V, exactly the amount required to keep  $V_-$  at



1 V. Because of the feedback provided by the  $R_f$   $R_g$  network, this is a *closed loop* circuit.

Another way to analyze this circuit proceeds by making the following (usually valid) assumptions:<sup>[3]</sup>

- When an op-amp operates in linear (i.e., not saturated) mode, the difference in voltage between the non-inverting (+) pin and the inverting (-) pin is negligibly small.
- The input impedance between (+) and (-) pins is much larger than other resistances in the circuit.

The input signal  $V_{in}$  appears at both (+) and (-) pins, resulting in a current  $i$  through  $R_g$  equal to  $V_{in}/R_g$ .

$$i = \frac{V_{in}}{R_g}$$

Since Kirchoff's current law states that the same current must leave a node as enter it, and since the impedance into the (-) pin is near infinity, we can assume practically all of the same current  $i$  flows through  $R_f$ , creating an output voltage

$$V_{out} = V_{in} + i \times R_f = V_{in} + \left( \frac{V_{in}}{R_g} \times R_f \right) = V_{in} + \frac{V_{in} \times R_f}{R_g} = V_{in} \left( 1 + \frac{R_f}{R_g} \right)$$

By combining terms, we determine the closed-loop gain  $A_{CL}$ :

$$A_{CL} = \frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_g}$$

## Op-amp characteristics

### Ideal op-amps

An ideal op-amp is usually considered to have the following properties:

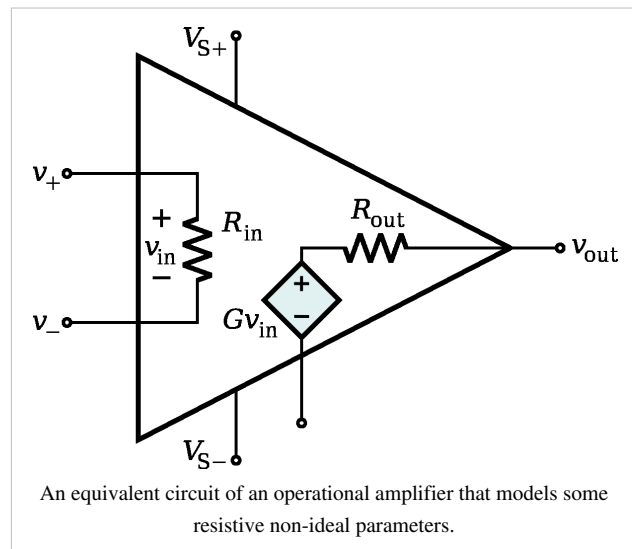
- Infinite open-loop gain  $G = v_{out} / v_{in}$
- Infinite input impedance  $R_{in}$ , and so zero input current
- Zero input offset voltage
- Infinite voltage range available at the output
- Infinite bandwidth with zero phase shift and infinite slew rate
- Zero output impedance  $R_{out}$
- Zero noise
- Infinite Common-mode rejection ratio (CMRR)
- Infinite Power supply rejection ratio.

These ideals can be summarized by the two "golden rules":

- I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.
- II. The inputs draw no current.<sup>:177</sup>

The first rule only applies in the usual case where the op-amp is used in a closed-loop design (negative feedback, where there is a signal path of some sort feeding back from the output to the inverting input). These rules are commonly used as a good first approximation for analyzing or designing op-amp circuits.<sup>:177</sup>

None of these ideals can be perfectly realized. A real op-amp may be modeled with non-infinite or non-zero parameters using equivalent resistors and capacitors in the op-amp model. The designer can then include these effects into the overall performance of the final circuit. Some parameters may turn out to have negligible effect on the final design while others represent actual limitations of the final performance that must be evaluated.



## Real op-amps

Real op-amps differ from the ideal model in various aspects.

### DC imperfections

Real operational amplifiers suffer from several non-ideal effects:

#### Finite gain

Open-loop gain is infinite in the ideal operational amplifier but finite in real operational amplifiers. Typical devices exhibit open-loop DC gain ranging from 100,000 to over 1 million. So long as the loop gain (i.e., the product of open-loop and feedback gains) is very large, the circuit gain will be determined entirely by the amount of negative feedback (i.e., it will be independent of open-loop gain). In cases where closed-loop gain must be very high, the feedback gain will be very low, and the low feedback gain causes low loop gain; in these cases, the operational amplifier will cease to behave ideally.

#### Finite input impedances

The *differential input impedance* of the operational amplifier is defined as the impedance *between* its two inputs; the *common-mode input impedance* is the impedance from each input to ground. MOSFET-input operational amplifiers often have protection circuits that effectively short circuit any input differences greater than a small threshold, so the input impedance can appear to be very low in some tests. However, as long as these operational amplifiers are used in a typical high-gain negative feedback application, these protection circuits will be inactive. The input bias and leakage currents described below are a more important design parameter for typical operational amplifier applications.

#### Non-zero output impedance

Low output impedance is important for low-impedance loads; for these loads, the voltage drop across the output impedance of the amplifier will be significant. Hence, the output impedance of the amplifier limits the maximum power that can be provided. In configurations with a voltage-sensing negative feedback, the output impedance of the amplifier is effectively lowered; thus, in linear applications, op-amps usually exhibit a very low output impedance indeed. Negative feedback can not, however, reduce the limitations that  $R_{load}$  in conjunction with  $R_{out}$  place on the maximum and minimum possible output voltages; it can only reduce output errors *within* that range.

Low-impedance outputs typically require high quiescent (i.e., idle) current in the output stage and will dissipate more power, so low-power designs may purposely sacrifice low output impedance.

#### Input current

Due to biasing requirements or leakage, a small amount of current (typically ~10 nanoamperes for bipolar op-amps, tens of picoamperes (pA) for JFET input stages, and only a few pA for MOSFET input stages) flows into the inputs. When large resistors or sources with high output impedances are used in the circuit, these small currents can produce large unmodeled voltage drops. If the input currents are matched, *and* the impedance looking *out* of *both* inputs are matched, then the voltages produced at each input will be equal. Because the operational amplifier operates on the *difference* between its inputs, these matched voltages will have no effect (unless the operational amplifier has poor CMRR, which is described below). It is more common for the input currents (or the impedances looking out of each input) to be slightly mismatched, and so a small *offset voltage* (different from the input offset voltage below) can be produced. This offset voltage can create offsets or drifting in the operational amplifier. It can often be nulled externally; however, many operational amplifiers include *offset null* or *balance* pins and some procedure for using them to remove this offset. Some operational amplifiers attempt to nullify this offset automatically

#### Input offset voltage

This voltage, which is what is required across the op-amp's input terminals to drive the output voltage to zero,<sup>[4][5]</sup> is related to the mismatches in input bias current. In the perfect amplifier, there would be no input offset voltage. However, it exists in actual op-amps because of imperfections in the differential amplifier that constitutes the input stage of the vast majority of these devices. Input offset voltage creates two problems: First, due to the amplifier's high voltage gain, it virtually assures that the amplifier output will go into saturation if it is operated without negative feedback, even when the input terminals are wired together. Second, in a closed loop, negative feedback configuration, the input offset voltage is amplified along with the signal and this may pose a problem if high precision DC amplification is required or if the input signal is very small.<sup>[6]</sup>

#### Common-mode gain

A perfect operational amplifier amplifies only the voltage difference between its two inputs, completely rejecting all voltages that are common to both. However, the differential input stage of an operational amplifier is never perfect, leading to the amplification of these common voltages to some degree. The standard measure of this defect is called the common-mode rejection ratio (denoted CMRR). Minimization of common mode gain is usually important in non-inverting amplifiers (described below) that operate at high amplification.

#### Power-supply rejection

The output of a perfect operational amplifier will be completely independent from ripples that arrive on its power supply inputs. Every real operational amplifier has a specified power supply rejection ratio (PSRR) that reflects how well the op-amp can reject changes in its supply voltage. Copious use of bypass capacitors can improve the PSRR of many devices, including the operational amplifier.

#### Temperature effects

All parameters change with temperature. Temperature drift of the input offset voltage is especially important.

#### Drift

Real op-amp parameters are subject to slow change over time and with changes in temperature, input conditions, etc.

#### Noise

Amplifiers generate random voltage at the output even when there is no signal applied. This can be due to thermal noise and flicker noise of the devices. For applications with high gain or high bandwidth, noise becomes a very important consideration.

### AC imperfections

The op-amp gain calculated at DC does not apply at higher frequencies. Thus, for high-speed operation, more sophisticated considerations must be used in an op-amp circuit design.

#### Finite bandwidth

All amplifiers have finite bandwidth. To a first approximation, the op-amp has the frequency response of an integrator with gain. That is, the gain of a typical op-amp is inversely proportional to frequency and is characterized by its gain–bandwidth product (GBWP). For example, an op-amp with a GBWP of 1 MHz would have a gain of 5 at 200 kHz, and a gain of 1 at 1 MHz. This dynamic response coupled with the very high DC gain of the op-amp gives it the characteristics of a first-order low-pass filter with very high DC gain and low cutoff frequency given by the GBWP divided by the DC gain.

The finite bandwidth of an op-amp can be the source of several problems, including:

- **Stability.** Associated with the bandwidth limitation is a phase difference between the input signal and the amplifier output that can lead to oscillation in some feedback circuits. For example, a sinusoidal output signal

meant to interfere destructively with an input signal of the same frequency will interfere constructively if delayed by 180 degrees forming positive feedback. In these cases, the feedback circuit can be stabilized by means of frequency compensation, which increases the gain or phase margin of the open-loop circuit. The circuit designer can implement this compensation externally with a separate circuit component. Alternatively, the compensation can be implemented within the operational amplifier with the addition of a dominant pole that sufficiently attenuates the high-frequency gain of the operational amplifier. The location of this pole may be fixed internally by the manufacturer or configured by the circuit designer using methods specific to the op-amp. In general, dominant-pole frequency compensation reduces the bandwidth of the op-amp even further. When the desired closed-loop gain is high, op-amp frequency compensation is often not needed because the requisite open-loop gain is sufficiently low; consequently, applications with high closed-loop gain can make use of op-amps with higher bandwidths.

- **Noise, Distortion, and Other Effects.** Reduced bandwidth also results in lower amounts of feedback at higher frequencies, producing higher distortion, noise, and output impedance and also reduced output phase linearity as the frequency increases.

Typical low-cost, general-purpose op-amps exhibit a GBWP of a few megahertz. Specialty and high-speed op-amps exist that can achieve a GBWP of hundreds of megahertz. For very high-frequency circuits, a current-feedback operational amplifier is often used.

#### Input capacitance

Most important for high frequency operation because it further reduces the open-loop bandwidth of the amplifier.

#### Common-mode gain

See DC imperfections, above.

### Non-linear imperfections

#### Saturation

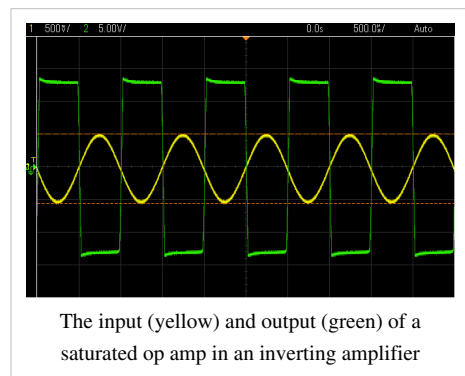
Output voltage is limited to a minimum and maximum value close to the power supply voltages.<sup>[7]</sup> Saturation occurs when the output of the amplifier reaches this value and is usually due to:

- In the case of an op-amp using a bipolar power supply, a voltage gain that produces an output that is more positive or more negative than that maximum or minimum; or
- In the case of an op-amp using a single supply voltage, either a voltage gain that produces an output that is more positive than that maximum, or a signal so close to ground that the amplifier's gain is not sufficient to raise it above the lower threshold.<sup>[8]</sup>

#### Slewing

The amplifier's output voltage reaches its maximum rate of change, the slew rate, usually specified in volts per microsecond. When slewing occurs, further increases in the input signal have no effect on the rate of change of the output. Slewing is usually caused by the input stage saturating; the result is a constant current  $i$  driving a capacitance  $C$  in the amplifier (especially those capacitances used to implement its frequency compensation); the slew rate is limited by  $dv/dt=i/C$ .

Slewing is associated with the *large-signal* performance of an op-amp. Consider for, example an op-amp configured for a gain of 10. Let the input be a 1 V, 100 kHz sawtooth wave. That is, the amplitude is 1 V and the period is 10 microseconds. Accordingly, the rate of change (i.e., the slope) of the input is 0.1 V per microsecond. After 10x amplification, the output should be a 10 V, 100 kHz sawtooth, with a corresponding





slew rate of 1 V per microsecond. However, the classic 741 op-amp has a 0.5 V per microsecond slew rate specification, so that its output can rise to no more than 5 V in the sawtooth's 10 microsecond period. Thus, if one were to measure the output, it would be a 5 V, 100 kHz sawtooth, rather than a 10 V, 100 kHz sawtooth.

Next consider the same amplifier and 100 kHz sawtooth, but now the input amplitude is 100 mV rather than 1 V. After 10x amplification the output is a 1 V, 100 kHz sawtooth with a corresponding slew rate of 0.1 V per microsecond. In this instance the 741 with its 0.5 V per microsecond slew rate will amplify the input properly.

Modern high speed op-amps can have slew rates in excess of 5,000 V per microsecond. However, it is more common for op-amps to have slew rates in the range 5-100 V per microsecond. For example, the general purpose TL081 op-amp has a slew rate of 13 V per microsecond. As a general rule, low power and small bandwidth op-amps have low slew rates. As an example, the LT1494 micropower op-amp consumes 1.5 microamp but has a 2.7 kHz gain-bandwidth product and a 0.001 V per microsecond slew rate.

#### Non-linear input-output relationship

The output voltage may not be accurately proportional to the difference between the input voltages. It is commonly called distortion when the input signal is a waveform. This effect will be very small in a practical circuit where substantial negative feedback is used.

#### Phase reversal

In some integrated op-amps, when the published common mode voltage is violated (e.g. by one of the inputs being driven to one of the supply voltages), the output may slew to the opposite polarity from what is expected in normal operation. Under such conditions, negative feedback becomes positive, likely causing the circuit to "lock up" in that state.

### **Power considerations**

#### Limited output current

The output current must be finite. In practice, most op-amps are designed to limit the output current so as not to exceed a specified level – around 25 mA for a type 741 IC op-amp – thus protecting the op-amp and associated circuitry from damage. Modern designs are electronically more rugged than earlier implementations and some can sustain direct short circuits on their outputs without damage.

#### Output sink current

The output sink current is the maximum current allowed to sink into the output stage. Some manufacturers show the output voltage vs. the output sink current plot, which gives an idea of the output voltage when it is sinking current from another source into the output pin.

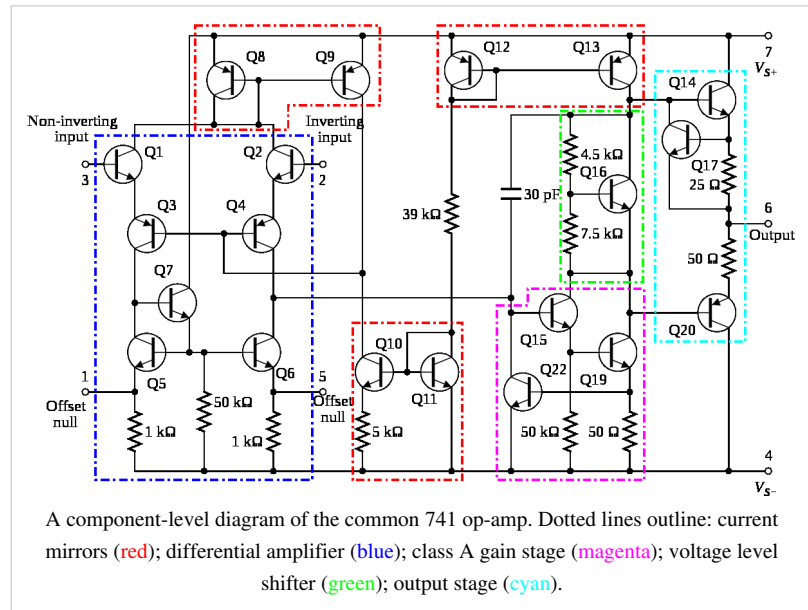
#### Limited dissipated power

The output current flows through the op-amp's internal output impedance, dissipating heat. If the op-amp dissipates too much power, then its temperature will increase above some safe limit. The op-amp may enter thermal shutdown, or it may be destroyed.

Modern integrated FET or MOSFET op-amps approximate more closely the ideal op-amp than bipolar ICs when it comes to input impedance and input bias currents. Bipolars are generally better when it comes to input *voltage* offset, and often have lower noise. Generally, at room temperature, with a fairly large signal, and limited bandwidth, FET and MOSFET op-amps now offer better performance.

## Internal circuitry of 741-type op-amp

Sourced by many manufacturers, and in multiple similar products, an example of a bipolar transistor operational amplifier is the 741 integrated circuit designed by Dave Fullagar at Fairchild Semiconductor after Bob Widlar's LM301 integrated circuit design. In this discussion, we use the parameters of the Hybrid- $\pi$  model to characterize the small-signal, grounded emitter characteristics of a transistor. In this model, the current gain of a transistor is denoted  $h_{fe}$ , more commonly called the  $\beta$ .



### Architecture

A small-scale integrated circuit, the 741 op-amp shares with most op-amps an internal structure consisting of three gain stages:

1. Differential amplifier (outlined blue) — provides high differential amplification (gain), with rejection of common-mode signal, low noise, high input impedance, and drives a
2. Voltage amplifier (outlined magenta) — provides high voltage gain, a single-pole frequency roll-off, and in turn drives the
3. Output amplifier (outlined cyan and green) — provides high current gain (low output impedance), along with output current limiting, and output short-circuit protection.

Additionally, it contains current mirror (outlined red) bias circuitry and a gain-stabilization capacitor (30 pF).

### Differential amplifier

A cascaded differential amplifier followed by a current-mirror active load, the input stage (outlined in blue) is a transconductance amplifier, turning a differential voltage signal at the bases of Q1, Q2 into a current signal into the base of Q15.

It entails two cascaded transistor pairs, satisfying conflicting requirements. The first stage consists of the matched NPN emitter follower pair Q1, Q2 that provide high input impedance. The second is the matched PNP common-base pair Q3, Q4 that eliminates the undesirable Miller effect; it drives an active load Q7 plus matched pair Q5, Q6.

That active load is implemented as a modified Wilson current mirror; its role is to convert the (differential) input current signal to a single-ended signal without the attendant 50% losses (increasing the op-amp's open-loop gain by 3dB).<sup>[9]</sup> Thus, a small-signal differential current in Q3 versus Q4 appears summed (doubled) at the base of Q15, the input of the voltage gain stage.

### Voltage amplifier

The (class-A) voltage gain stage (outlined in magenta) consists of the two NPN transistors Q15/Q19 connected in a Darlington configuration and uses the output side of current mirror Q12/Q13 as its collector (dynamic) load to achieve its high voltage gain. The output sink transistor Q20 receives its base drive from the common collectors of Q15 and Q19; the level-shifter Q16 provides base drive for the output source transistor Q14. Note the similarity between the transistors Q15 and Q7.

The transistor Q22 prevents this stage from saturating by diverting the excessive Q15 base current (it acts as a Baker clamp).

### Output amplifier

The output stage (Q14, Q20, outlined in cyan) is a Class AB push-pull emitter follower amplifier. It provides an output drive with impedance of  $\approx 50\Omega$ , in essence, current gain. Transistor Q16 (outlined in green) provides the quiescent current for the output transistors, and Q17 provides output current limiting.

### Biasing circuits

Provide appropriate quiescent current for each stage of the op-amp.

The resistor ( $39\text{ k}\Omega$ ) connecting the (diode-connected) Q11 and Q12, and the given supply voltage ( $V_{S+} - V_{S-}$ ), determine the current in the current mirrors, (matched pairs) Q10/Q11 and Q12/Q13. The collector current of Q11,  $i_{11} * 39\text{ k}\Omega = V_{S+} - V_{S-} - 2 V_{BE}$ . For the typical  $V_S = \pm 20\text{ V}$ , the standing current in Q11/Q12 (as well as in Q13) would be  $\approx 1\text{ mA}$ . A supply current for a typical 741 of about  $2\text{ mA}$  agrees with the notion that these two bias currents dominate the quiescent supply current.

Transistors Q11 and Q10 form a Widlar current mirror, with quiescent current in Q10  $i_{10}$  such that  $\ln(i_{11}/i_{10}) = i_{10} * 5\text{ k}\Omega / 28\text{ mV}$ , where  $5\text{ k}\Omega$  represents the emitter resistor of Q10, and  $28\text{ mV}$  is  $V_T$ , the thermal voltage at room temperature. In this case  $i_{10} \approx 20\text{ }\mu\text{A}$ .

### Differential amplifier

The biasing circuit of this stage is set by a feedback loop that forces the collector currents of Q10 and Q9 to (nearly) match. The small difference in these currents provides the drive for the common base of Q3/Q4 (note that the base drive for input transistors Q1/Q2 is the input bias current and must be sourced externally). The summed quiescent currents of Q1/Q3 plus Q2/Q4 is mirrored from Q8 into Q9, where it is summed with the collector current in Q10, the result being applied to the bases of Q3/Q4.

The quiescent currents of Q1/Q3 (resp., Q2/Q4)  $i_1$  will thus be half of  $i_{10}$ , of order  $\approx 10\text{ }\mu\text{A}$ . Input bias current for the base of Q1 (resp. Q2) will amount to  $i_1 / \beta$ ; typically  $\approx 50\text{ nA}$ , implying a current gain  $h_{fe} \approx 200$  for Q1(Q2).

This feedback circuit tends to draw the common base node of Q3/Q4 to a voltage  $V_{com} - 2 * V_{BE}$ , where  $V_{com}$  is the input common-mode voltage. At the same time, the magnitude of the quiescent current is relatively insensitive to the characteristics of the components Q1–Q4, such as  $h_{fe}$ , that would otherwise cause temperature dependence or part-to-part variations.

Transistor Q7 drives Q5 and Q6 into conduction until their (equal) collector currents match that of Q1/Q3 and Q2/Q4. The quiescent current in Q7 is  $V_{BE} / 50\text{ k}\Omega$ , about  $35\text{ }\mu\text{A}$ , as is the quiescent current in Q15, with its matching operating point. Thus, the quiescent currents are pairwise matched in Q1/Q2, Q3/Q4, Q5/Q6, and Q7/Q15.

### Voltage amplifier

Quiescent currents in Q16 and Q19 are set by the current mirror Q12/Q13, which is running at  $\approx 1\text{ mA}$ . Through some (?) mechanism, the collector current in Q19 tracks that standing current.

### Output amplifier

In the circuit involving Q16 (variously named rubber diode or  $V_{BE}$  multiplier), the  $4.5\text{ k}\Omega$  resistor must be conducting about  $100\text{ }\mu\text{A}$ , with the Q16  $V_{BE}$  roughly  $700\text{ mV}$ . Then the  $V_{CB}$  must be about  $0.45\text{ V}$  and  $V_{CE}$  at about  $1.0\text{ V}$ . Because the Q16 collector is driven by a current source and the Q16 emitter drives into the Q19 collector current sink, the Q16 transistor establishes a voltage difference between Q14 base and Q20 base of  $\approx 1\text{ V}$ , regardless of the common-mode voltage of Q14/Q20 base. The standing current in Q14/Q20 will be a factor  $\exp(100\text{ mV} / V_T) \approx 36$  smaller than the  $1\text{ mA}$  quiescent current in the class A portion of the op amp. This (small) standing current in

the output transistors establishes the output stage in class AB operation and reduces the crossover distortion of this stage.

### Small-signal differential mode

A small differential input voltage signal gives rise, through multiple stages of current amplification, to a much larger voltage signal on output.

#### Input impedance

Because Q1 and Q3 (resp. Q2 and Q4) form a Darlington pair, the small-signal differential input impedance is of order  $2h_{ie} h_{fe}$ , where  $h_{ie}$  is the small-signal input impedance (common emitter) of Q1 and Q3 (resp. Q2 and Q4) and where  $h_{fe}$  is the transistor small-signal current gain (or  $\beta$ ). This contrasts with what would be the case with a simpler emitter-coupled pair (long-tailed pair) input stage, where the differential input impedance is  $2h_{ie}$ , a factor of  $\beta$  lower. A typical 741 op amp has an input impedance 2–8 M $\Omega$ .

#### Differential amplifier

A differential voltage  $V_{In}$  at the op-amp inputs (pins 3 and 2, respectively) gives rise to a small differential current in the bases of Q1 and Q2  $i_{In} \approx V_{In} / (2 h_{ie} * h_{fe})$ . This differential base current causes a change in the differential collector current in each leg by  $i_{In} * h_{fe}$ . Introducing the transconductance of Q1,  $g_m = h_{fe} / h_{ie}$ , the (small-signal) current at the base of Q15 (the input of the voltage gain stage) is  $V_{In} * g_m / 2$ .

This portion of the op amp cleverly changes a differential signal at the op amp inputs to a single-ended signal at the base of Q15, and in a way that avoids wastefully discarding the signal in either leg. To see how, notice that a small negative change in voltage at the inverting input (Q2 base) drives it out of conduction, and this incremental decrease in current passes directly from Q4 collector to its emitter, resulting in an decrease in base drive for Q15. On the other hand, a small positive change in voltage at the non-inverting input (Q1 base) drives this transistor into conduction, reflected in an increase in current at the collector of Q3. This current drives Q7 further into conduction, which turns on current mirror Q5/Q6. Thus, the increase in Q3 emitter current is mirrored in an increase in Q6 collector current, resulting also in a decrease in base drive for Q15. Besides avoiding wasting 3dB of gain here, this technique decreases common-mode gain and feedthrough of power supply noise.

#### Voltage amplifier

A current signal  $i$  at Q15's base gives rise to a current in Q19 of order  $i * \beta^2$  (the product of the  $h_{fe}$  of each of Q15 and Q19, which are connected in a Darlington pair). This current signal develops a voltage at the bases of output transistors Q14/Q20 proportional to the  $h_{ie}$  of the respective transistor.

#### Output amplifier

Output transistors Q14 and Q20 are each configured as an emitter follower, so no voltage gain occurs there; instead, this stage provides current gain, equal to the  $h_{fe}$  of Q14 (resp. Q20).

The output impedance is not zero, as it would be in an ideal op-amp, but with negative feedback it approaches zero at low frequencies.

### Overall open-loop voltage gain

The net open-loop small-signal voltage gain of the op amp involves the product of the current gain  $h_{fe}$  of some 4 transistors. In practice, the voltage gain for a typical 741-style op amp is of order 200,000, and the current gain, the ratio of input impedance ( $\approx 2\text{--}6\text{ M}\Omega$ ) to output impedance ( $\approx 50\Omega$ ) provides yet more (power) gain.

### Other linear characteristics

#### Small-signal common mode gain

The ideal op amp has infinite common-mode rejection ratio, or zero common-mode gain.

In the present circuit, if the input voltages change in the same direction, the negative feedback makes Q3/Q4 base voltage follow (with  $2V_{BE}$  below) the input voltage variations. Now the output part (Q10) of Q10-Q11 current mirror keeps up the common current through Q9/Q8 constant in spite of varying voltage. Q3/Q4 collector currents, and accordingly the output current at the base of Q15, remain unchanged.

In the typical 741 op amp, the common-mode rejection ratio is 90dB, implying an open-loop common-mode voltage gain of about 6.

#### Frequency compensation

The innovation of the Fairchild  $\mu\text{A}741$  was the introduction of frequency compensation via an on-chip (monolithic) capacitor, simplifying application of the op amp by eliminating the need for external components for this function. The 30 pF capacitor stabilizes the amplifier via Miller compensation and functions in a manner similar to an op-amp integrator circuit. Also known as 'dominant pole compensation' because it introduces a pole that masks (dominates) the effects of other poles into the open loop frequency response; in a 741 op amp this pole can be as low as 10 Hz (where it causes a  $-3$  dB loss of open loop voltage gain).

This internal compensation is provided to achieve unconditional stability of the amplifier in negative feedback configurations where the feedback network is non-reactive and the closed loop gain is unity or higher. By contrast, amplifiers requiring external compensation, such as the  $\mu\text{A}748$ , may require external compensation or closed-loop gains significantly higher than unity.

#### Input offset voltage

The "offset null" pins may be used to place external resistors (typically in the form of the two ends of a potentiometer, with the slider connected to  $V_{s-}$ ) in parallel with the emitter resistors of Q5 and Q6, to adjust the balance of the Q5/Q6 current mirror. The potentiometer is adjusted such that the output is null (midrange) when the inputs are shorted together.

### Non-linear characteristics

#### Input breakdown voltage

The transistors Q3, Q4 help to increase the reverse  $V_{BE}$  rating: the base-emitter junctions of the NPN transistors Q1 and Q2 break down at around 7V, but the PNP transistors Q3 and Q4 have  $V_{BE}$  breakdown voltages around 50 V.<sup>[10]</sup>

### Output-stage voltage swing and current limiting

Variations in the quiescent current with temperature, or between parts with the same type number, are common, so crossover distortion and quiescent current may be subject to significant variation.

The output range of the amplifier is about one volt less than the supply voltage, owing in part to  $V_{BE}$  of the output transistors Q14 and Q20.

The 25  $\Omega$  resistor at the Q14 emitter, along with Q17, acts to limit Q14 current to about 25 mA; otherwise, Q17 conducts no current.

Current limiting for Q20 is performed in the voltage gain stage: Q22 senses the voltage across Q19's emitter resistor (50 $\Omega$ ); as it turns on, it diminishes the drive current to Q15 base.

Later versions of this amplifier schematic may show a somewhat different method of output current limiting.

### Applicability considerations

*Note: while the 741 was historically used in audio and other sensitive equipment, such use is now rare because of the improved noise performance of more modern op-amps. Apart from generating noticeable hiss, 741s and other older op-amps may have poor common-mode rejection ratios and so will often introduce cable-borne mains hum and other common-mode interference, such as switch 'clicks', into sensitive equipment.*

The "741" has come to often mean a generic op-amp IC (such as  $\mu$ A741, LM301, 558, LM324, TBA221 — or a more modern replacement such as the TL071). The description of the 741 output stage is qualitatively similar for many other designs (that may have quite different input stages), except:

- Some devices ( $\mu$ A748, LM301, LM308) are not internally compensated (require an external capacitor from output to some point within the operational amplifier, if used in low closed-loop gain applications).
- Some modern devices have "rail-to-rail output" capability, meaning that the output can range from within a few millivolts of the positive supply voltage to within a few millivolts of the negative supply voltage.

### Classification

Op-amps may be classified by their construction:

- discrete (built from individual transistors or tubes/valves)
- IC (fabricated in an Integrated circuit) — most common
- hybrid

IC op-amps may be classified in many ways, including:

- Military, Industrial, or Commercial grade (for example: the LM301 is the commercial grade version of the LM101, the LM201 is the industrial version). This may define operating temperature ranges and other environmental or quality factors.
- Classification by package type may also affect environmental hardiness, as well as manufacturing options; DIP, and other through-hole packages are tending to be replaced by surface-mount devices.
- Classification by internal compensation: op-amps may suffer from high frequency instability in some negative feedback circuits unless a small compensation capacitor modifies the phase and frequency responses. Op-amps with a built-in capacitor are termed "*compensated*", or perhaps compensated for closed-loop gains down to (say) 5. All others are considered uncompensated.
- Single, dual and quad versions of many commercial op-amp IC are available, meaning 1, 2 or 4 operational amplifiers are included in the same package.
- Rail-to-rail input (and/or output) op-amps can work with input (and/or output) signals very close to the power supply rails.

- CMOS op-amps (such as the CA3140E) provide extremely high input resistances, higher than JFET-input op-amps, which are normally higher than bipolar-input op-amps.
- other varieties of op-amp include programmable op-amps (simply meaning the quiescent current, gain, bandwidth and so on can be adjusted slightly by an external resistor).
- manufacturers often tabulate their op-amps according to purpose, such as low-noise pre-amplifiers, wide bandwidth amplifiers, and so on.

## Applications

Main article: Operational amplifier applications

### Use in electronics system design

The use of op-amps as circuit blocks is much easier and clearer than specifying all their individual circuit elements (transistors, resistors, etc.), whether the amplifiers used are integrated or discrete. In the first approximation op-amps can be used as if they were ideal differential gain blocks; at a later stage limits can be placed on the acceptable range of parameters for each op-amp.

Circuit design follows the same lines for all electronic circuits. A specification is drawn up governing what the circuit is required to do, with allowable limits. For example, the gain may be required to be 100 times, with a tolerance of 5% but drift of less than 1% in a specified temperature range; the input impedance not less than one megohm; etc.

A basic circuit is designed, often with the help of circuit modeling (on a computer). Specific commercially available op-amps and other components are then chosen that meet the design criteria within the specified tolerances at acceptable cost. If not all criteria can be met, the specification may need to be modified.

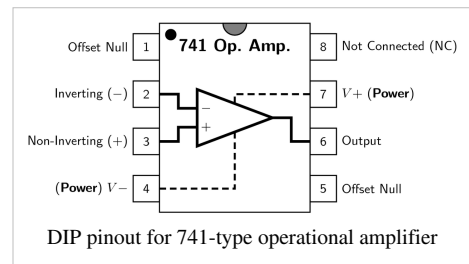
A prototype is then built and tested; changes to meet or improve the specification, alter functionality, or reduce the cost, may be made.

### Applications without using any feedback

That is, the op-amp is being used as a voltage comparator. Note that a device designed primarily as a comparator may be better if, for instance, speed is important or a wide range of input voltages may be found, since such devices can quickly recover from full on or full off ("saturated") states.

A *voltage level detector* can be obtained if a reference voltage  $V_{\text{ref}}$  is applied to one of the op-amp's inputs. This means that the op-amp is set up as a comparator to detect a positive voltage. If the voltage to be sensed,  $E_i$ , is applied to op amp's (+) input, the result is a noninverting positive-level detector: when  $E_i$  is above  $V_{\text{ref}}$ ,  $V_O$  equals  $+V_{\text{sat}}$ ; when  $E_i$  is below  $V_{\text{ref}}$ ,  $V_O$  equals  $-V_{\text{sat}}$ . If  $E_i$  is applied to the inverting input, the circuit is an inverting positive-level detector: When  $E_i$  is above  $V_{\text{ref}}$ ,  $V_O$  equals  $-V_{\text{sat}}$ .

A *zero voltage level detector* ( $E_i = 0$ ) can convert, for example, the output of a sine-wave from a function generator into a variable-frequency square wave. If  $E_i$  is a sine wave, triangular wave, or wave of any other shape that is symmetrical around zero, the zero-crossing detector's output will be square. Zero-crossing detection may also be useful in triggering TRIACs at the best time to reduce mains interference and current spikes.



## Positive feedback applications

Another typical configuration of op-amps is with positive feedback, which takes a fraction of the output signal back to the non-inverting input. An important application of it is the comparator with hysteresis, the Schmitt trigger. Some circuits may use *Positive* feedback and *Negative* feedback around the same amplifier, for example Triangle wave oscillators and active filters.

Because of the wide slew-range and lack of positive feedback, the response of all the open-loop level detectors described above will be relatively slow. External overall positive feedback may be applied but (unlike internal positive feedback that may be applied within the latter stages of a purpose-designed comparator) this markedly affects the accuracy of the zero-crossing detection point. Using a general-purpose op-amp, for example, the frequency of  $E_i$  for the sine to square wave converter should probably be below 100 Hz. Wikipedia:Citation needed

## Negative feedback applications

### Non-inverting amplifier

*In a non-inverting amplifier, the output voltage changes in the same direction as the input voltage.*

The gain equation for the op-amp is:

$$V_{\text{out}} = A_{OL} (V_+ - V_-)$$

However, in this circuit  $V_-$  is a function of  $V_{\text{out}}$  because of the negative feedback through the  $R_1$   $R_2$  network.  $R_1$  and  $R_2$  form a voltage divider, and as  $V_-$  is a high-impedance input, it does not load it appreciably.

Consequently:

$$V_- = \beta \cdot V_{\text{out}}$$

where

$$\beta = \frac{R_1}{R_1 + R_2}$$

Substituting this into the gain equation, we obtain:

$$V_{\text{out}} = A_{OL}(V_{\text{in}} - \beta \cdot V_{\text{out}})$$

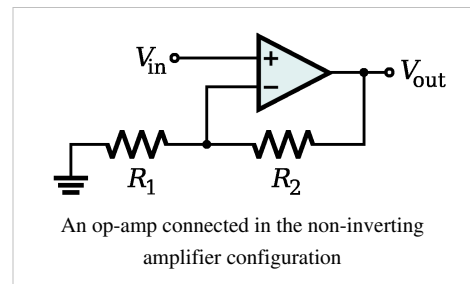
Solving for  $V_{\text{out}}$ :

$$V_{\text{out}} = V_{\text{in}} \left( \frac{1}{\beta + 1/A_{OL}} \right)$$

If  $A_{OL}$  is very large, this simplifies to

$$V_{\text{out}} \approx \frac{V_{\text{in}}}{\beta} = \frac{V_{\text{in}}}{\frac{R_1}{R_1 + R_2}} = V_{\text{in}} \left( 1 + \frac{R_2}{R_1} \right).$$

The non-inverting input of the operational amplifier needs a path for DC to ground; if the signal source does not supply a DC path, or if that source requires a given load impedance, then the circuit will require another resistor from the non-inverting input to ground. When the operational amplifier's input bias currents are significant, then the DC source resistances driving the inputs should be balanced.<sup>[11]</sup> The ideal value for the feedback resistors (to give minimum offset voltage) will be such that the two resistances in parallel roughly equal the resistance to ground at the non-inverting input pin. That ideal value assumes the bias currents are well-matched, which may not be true for all op-amps.<sup>[12]</sup>





### Inverting amplifier

In an inverting amplifier, the output voltage changes in an opposite direction to the input voltage.

As with the non-inverting amplifier, we start with the gain equation of the op-amp:

$$V_{\text{out}} = A_{OL} (V_+ - V_-)$$

This time,  $V_-$  is a function of both  $V_{\text{out}}$  and  $V_{\text{in}}$  due to the voltage divider formed by  $R_f$  and  $R_{\text{in}}$ . Again, the op-amp input does not apply an appreciable load, so:

$$V_- = \frac{1}{R_f + R_{\text{in}}} (R_f V_{\text{out}} + R_{\text{in}} V_{\text{in}})$$

Substituting this into the gain equation and solving for  $V_{\text{out}}$ :

$$V_{\text{out}} = -V_{\text{in}} \cdot \frac{A_{OL} R_f}{R_f + R_{\text{in}} + A_{OL} R_{\text{in}}}$$

If  $A_{OL}$  is very large, this simplifies to

$$V_{\text{out}} \approx -V_{\text{in}} \frac{R_f}{R_{\text{in}}}$$

A resistor is often inserted between the non-inverting input and ground (so both inputs "see" similar resistances), reducing the input offset voltage due to different voltage drops due to bias current, and may reduce distortion in some op-amps.

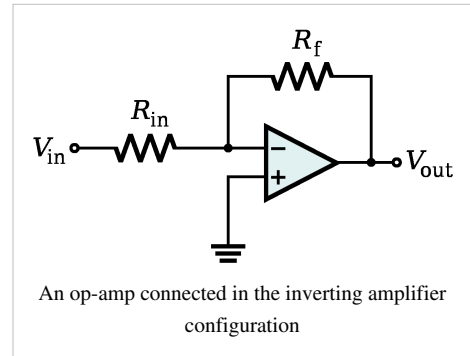
A DC-blocking capacitor may be inserted in series with the input resistor when a frequency response down to DC is not needed and any DC voltage on the input is unwanted. That is, the capacitive component of the input impedance inserts a DC zero and a low-frequency pole that gives the circuit a bandpass or high-pass characteristic.

The potentials at the operational amplifier inputs remain virtually constant (near ground) in the inverting configuration. The constant operating potential typically results in distortion levels that are lower than those attainable with the non-inverting topology.

### Other applications

- audio- and video-frequency pre-amplifiers and buffers
- differential amplifiers
- differentiators and integrators
- filters
- precision rectifiers
- precision peak detectors
- voltage and current regulators
- analog calculators
- analog-to-digital converters
- digital-to-analog converters
- Voltage clamping
- oscillators and waveform generators

Most single, dual and quad op-amps available have a standardized pin-out which permits one type to be substituted for another without wiring changes. A specific op-amp may be chosen for its open loop gain, bandwidth, noise performance, input impedance, power consumption, or a compromise between any of these factors.



## Historical timeline

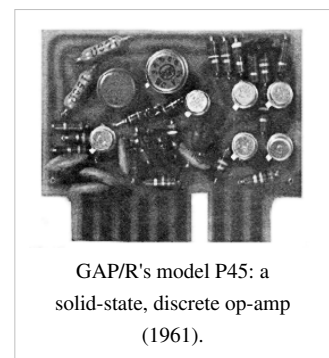
**1941: A vacuum tube op-amp.** An op-amp, defined as a general-purpose, DC-coupled, high gain, inverting feedback amplifier, is first found in U.S. Patent 2,401,779<sup>[13]</sup> "Summing Amplifier" filed by Karl D. Swartzel Jr. of Bell Labs in 1941. This design used three vacuum tubes to achieve a gain of 90 dB and operated on voltage rails of  $\pm 350$  V. It had a single inverting input rather than differential inverting and non-inverting inputs, as are common in today's op-amps. Throughout World War II, Swartzel's design proved its value by being liberally used in the M9 artillery director designed at Bell Labs. This artillery director worked with the SCR584 radar system to achieve extraordinary hit rates (near 90%) that would not have been possible otherwise.

**1947: An op-amp with an explicit non-inverting input.** In 1947, the operational amplifier was first formally defined and named in a paper by Professor John R. Ragazzini of Columbia University. In this same paper a footnote mentioned an op-amp design by a student that would turn out to be quite significant. This op-amp, designed by Loebe Julie, was superior in a variety of ways. It had two major innovations. Its input stage used a long-tailed triode pair with loads matched to reduce drift in the output and, far more importantly, it was the first op-amp design to have two inputs (one inverting, the other non-inverting). The differential input made a whole range of new functionality possible, but it would not be used for a long time due to the rise of the chopper-stabilized amplifier.

**1949: A chopper-stabilized op-amp.** In 1949, Edwin A. Goldberg designed a chopper-stabilized op-amp.<sup>[14]</sup> This set-up uses a normal op-amp with an additional AC amplifier that goes alongside the op-amp. The chopper gets an AC signal from DC by switching between the DC voltage and ground at a fast rate (60 Hz or 400 Hz). This signal is then amplified, rectified, filtered and fed into the op-amp's non-inverting input. This vastly improved the gain of the op-amp while significantly reducing the output drift and DC offset. Unfortunately, any design that used a chopper couldn't use their non-inverting input for any other purpose. Nevertheless, the much improved characteristics of the chopper-stabilized op-amp made it the dominant way to use op-amps. Techniques that used the non-inverting input regularly would not be very popular until the 1960s when op-amp ICs started to show up in the field.

**1953: A commercially available op-amp.** In 1953, vacuum tube op-amps became commercially available with the release of the model K2-W from George A. Philbrick Researches, Incorporated. The designation on the devices shown, GAP/R, is an acronym for the complete company name. Two nine-pin 12AX7 vacuum tubes were mounted in an octal package and had a model K2-P chopper add-on available that would effectively "use up" the non-inverting input. This op-amp was based on a descendant of Loebe Julie's 1947 design and, along with its successors, would start the widespread use of op-amps in industry.

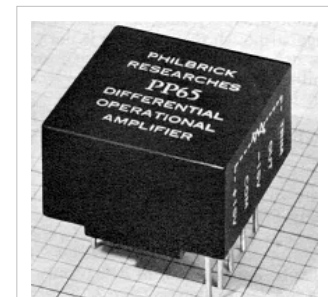
**1961: A discrete IC op-amp.** With the birth of the transistor in 1947, and the silicon transistor in 1954, the concept of ICs became a reality. The introduction of the planar process in 1959 made transistors and ICs stable enough to be commercially useful. By 1961, solid-state, discrete op-amps were being produced. These op-amps were effectively small circuit boards with packages such as edge connectors. They usually had hand-selected resistors in order to improve things such as voltage offset and drift. The P45 (1961) had a gain of 94 dB and ran on  $\pm 15$  V rails. It was intended to deal with signals in the range of  $\pm 10$  V.



**1961: A varactor bridge op-amp.** There have been many different directions taken in op-amp design. Varactor bridge op-amps started to be produced in the early 1960s.<sup>[15][16]</sup> They were designed to have extremely small input current and are still amongst the best op-amps available in terms of common-mode rejection with the ability to correctly deal with hundreds of volts at their inputs.

**1962: An op-amp in a potted module.** By 1962, several companies were producing modular potted packages that could be plugged into printed circuit boards. Wikipedia:Citation needed These packages were crucially important as they made the operational amplifier into a single black box which could be easily treated as a component in a larger circuit.

**1963: A monolithic IC op-amp.** In 1963, the first monolithic IC op-amp, the  $\mu$ A702 designed by Bob Widlar at Fairchild Semiconductor, was released. Monolithic ICs consist of a single chip as opposed to a chip and discrete parts (a discrete IC) or multiple chips bonded and connected on a circuit board (a hybrid IC). Almost all modern op-amps are monolithic ICs; however, this first IC did not meet with much success. Issues such as an uneven supply voltage, low gain and a small dynamic range held off the dominance of monolithic op-amps until 1965 when the  $\mu$ A709<sup>[17]</sup> (also designed by Bob Widlar) was released.



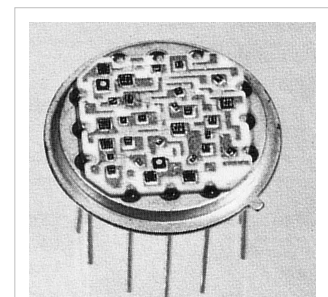
GAP/R's model PP65: a solid-state op-amp in a potted module (1962)

**1968: Release of the  $\mu$ A741.** The popularity of monolithic op-amps was further improved upon the release of the LM101 in 1967, which solved a variety of issues, and the subsequent release of the  $\mu$ A741 in 1968. The  $\mu$ A741 was extremely similar to the LM101 except that Fairchild's facilities allowed them to include a 30 pF compensation capacitor inside the chip instead of requiring external compensation. This simple difference has made the 741 *the* canonical op-amp and many modern amps base their pinout on the 741s. The  $\mu$ A741 is still in production, and has become ubiquitous in electronics—many manufacturers produce a version of this classic chip, recognizable by part numbers containing 741. The same part is manufactured by several companies.

**1970: First high-speed, low-input current FET design.** In the 1970s high speed, low-input current designs started to be made by using FETs. These would be largely replaced by op-amps made with MOSFETs in the 1980s. During the 1970s single sided supply op-amps also became available.

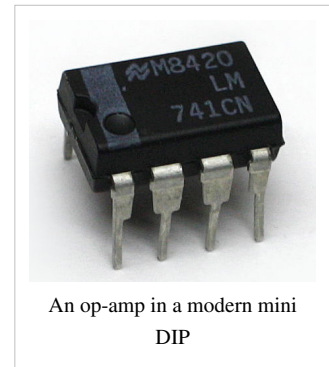
**1972: Single sided supply op-amps being produced.** A single sided supply op-amp is one where the input and output voltages can be as low as the negative power supply voltage instead of needing to be at least two volts above it. The result is that it can operate in many applications with the negative supply pin on the op-amp being connected to the signal ground, thus eliminating the need for a separate negative power supply.

The LM324 (released in 1972) was one such op-amp that came in a quad package (four separate op-amps in one package) and became an industry standard. In addition to packaging multiple op-amps in a single package, the 1970s also saw the birth of op-amps in hybrid packages. These op-amps were generally improved versions of existing monolithic op-amps. As the properties of monolithic op-amps improved, the more complex hybrid ICs were quickly relegated to systems that are required to have extremely long service lives or other specialty systems.



ADI's HOS-050: a high speed hybrid IC op-amp (1979)

**Recent trends.** Recently supply voltages in analog circuits have decreased (as they have in digital logic) and low-voltage op-amps have been introduced reflecting this. Supplies of  $\pm 5$  V and increasingly 3.3 V (sometimes as low as 1.8 V) are common. To maximize the signal range modern op-amps commonly have rail-to-rail output (the output signal can range from the lowest supply voltage to the highest) and sometimes rail-to-rail inputs.



## Notes

- [1] Maxim Application Note 1108: Understanding Single-Ended, Pseudo-Differential and Differential ADC Inputs ([http://www.maxim-ic.com/appnotes.cfm/an\\_pk/1108](http://www.maxim-ic.com/appnotes.cfm/an_pk/1108)) – Retrieved November 10, 2007
- [2] Analog devices MT-044 Tutorial (<http://www.analog.com/static/imported-files/tutorials/MT-044.pdf>)
- [3] Jacob Millman, *Microelectronics: Digital and Analog Circuits and Systems*, McGraw-Hill, 1979, ISBN 0-07-042327-X, pp. 523-527
- [4] D.F. Stout *Handbook of Operational Amplifier Circuit Design* (McGraw-Hill, 1976, ISBN 0-07-061797-X ) pp. 1–11.
- [5] This definition hews to the convention of measuring op-amp parameters with respect to the zero voltage point in the circuit, which is usually half the total voltage between the amplifier's positive and negative power rails.
- [6] Many older designs of operational amplifiers have offset null inputs to allow the offset to be manually adjusted away. Modern precision op-amps can have internal circuits that automatically cancel this offset using choppers or other circuits that measure the offset voltage periodically and subtract it from the input voltage.
- [7] That the output cannot reach the power supply voltages is usually the result of limitations of the amplifier's output stage transistors. See Output stage.
- [8] The output of older op-amps can reach to within one or two volts of the supply rails. The output of newer so-called "rail to rail" op-amps can reach to within millivolts of the supply rails when providing low output currents.
- [9] Widlar used this same trick in  $\mu A702$  and  $\mu A709$
- [10] The  $\mu A741$  Operational Amplifier (<http://ecow.engr.wisc.edu/cgi-bin/get/ece/342/schowalter/notes/chapter10/theua741operationalamplifier.ppt>)
- [11] An input bias current of  $1 \mu A$  through a DC source resistance of  $10 \text{ k}\Omega$  produces a  $10 \text{ mV}$  offset voltage. If the other input bias current is the same and sees the same source resistance, then the two input offset voltages will cancel out. Balancing the DC source resistances may not be necessary if the input bias current and source resistance product is small.
- [12] <http://www.analog.com/static/imported-files/tutorials/MT-038.pdf>
- [13] <http://www.google.com/patents/US2401779>
- [14] [http://www.analog.com/library/analogDialogue/archives/39-05/Web\\_ChH\\_final.pdf](http://www.analog.com/library/analogDialogue/archives/39-05/Web_ChH_final.pdf)
- [15] <http://www.philbrickarchive.org/>
- [16] June 1961 advertisement for Philbrick P2, <http://www.philbrickarchive.org/p2%20and%206033%20ad%20rsi%20vol32%20no6%20june1961.pdf>
- [17] A.P. Malvino, *Electronic Principles* (2nd Ed. 1979. ISBN 0-07-039867-4) p. 476.

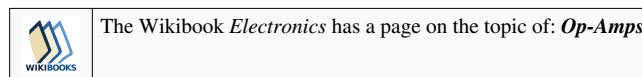
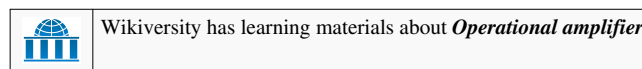
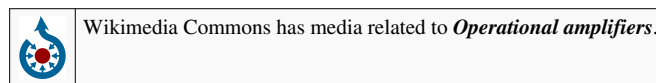
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### Further reading

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- *Small Signal Audio Design*; Douglas Self; Focal Press; 556 pages; 2010; ISBN 978-0-240-52177-0.
- *Op Amp Applications Handbook*; Walt G. Jung; Newnes; 896 pages; 2004; ISBN 978-0750678445. (Free PDF download) ([http://www.analog.com/library/analogDialogue/archives/39-05/op\\_amp\\_applications\\_handbook.html](http://www.analog.com/library/analogDialogue/archives/39-05/op_amp_applications_handbook.html))
- *Op Amps and Linear Integrated Circuits*; James M. Fiore; Cengage Learning; 616 pages; 2000; ISBN 978-0766817937.

- *Operational Amplifiers and Linear Integrated Circuits*; 6th Ed; Robert F Coughlin; Prentice Hall; 529 pages; 2000; ISBN 978-0130149916.
- *Op-Amps and Linear Integrated Circuits*; 4th Ed; Ram Gayakwad; Prentice Hall; 543 pages; 1999; ISBN 978-0132808682.
- *Basic Operational Amplifiers and Linear Integrated Circuits*; 2nd Ed; Thomas L Floyd and David Buchla; Prentice Hall; 593 pages; 1998; ISBN 978-0130829870.
- *Troubleshooting Analog Circuits*; Bob Pease; Newnes; 217 pages; 1991; ISBN 978-0750694995.
- *IC Op-Amp Cookbook*; 3rd Ed; Walter G. Jung; Prentice Hall; 433 pages; 1986; ISBN 978-0138896010.
- *Engineer's Mini-Notebook – OpAmp IC Circuits*; Forrest Mims III; Radio Shack; 49 pages; 1985; ASIN B000DZG196.

## External links



- Simple Op Amp Measurements ([http://www.analog.com/library/analogDialogue/archives/45-04/op\\_amp\\_measurements.html](http://www.analog.com/library/analogDialogue/archives/45-04/op_amp_measurements.html)) How to measure offset voltage, offset and bias current, gain, CMRR, and PSRR.
- Introduction to op-amp circuit stages, second order filters, single op-amp bandpass filters, and a simple intercom (<http://www.bowdenshobbycircuits.info/opamp.htm>)
- *MOS op amp design: A tutorial overview* ([http://lyle.smu.edu/ee/7321/MOS\\_op-amp\\_design.pdf](http://lyle.smu.edu/ee/7321/MOS_op-amp_design.pdf))
- Operational Amplifier Noise Prediction (All Op Amps) (<http://www.intersil.com/data/an/an519.pdf>) using spot noise
- Operational Amplifier Basics ([http://www.williamson-labs.com/480\\_opam.htm](http://www.williamson-labs.com/480_opam.htm))
- History of the Op-amp ([http://www.analog.com/library/analogDialogue/archives/39-05/Web\\_ChH\\_final.pdf](http://www.analog.com/library/analogDialogue/archives/39-05/Web_ChH_final.pdf)) from vacuum tubes to about 2002. Lots of detail, with schematics. IC part is somewhat ADI-centric.
- Loebe Julie historical OpAmp interview (<http://electronicdesign.com/article/analog-and-mixed-signal/what-s-all-this-julie-stuff-anyhow-6071.aspx>) by Bob Pease
- [www.PhilbrickArchive.org](http://www.PhilbrickArchive.org) (<http://www.PhilbrickArchive.org>) – A free repository of materials from George A Philbrick / Researches - Operational Amplifier Pioneer
- What's The Difference Between Operational Amplifiers And Instrumentation Amplifiers? (<http://electronicdesign.com/print/power/what-s-difference-between-operational-amplifiers-and-instrumentation-amplifiers>), Electronic Design Magazine

### IC Datasheets

- LM301, Single BJT OpAmp, Texas Instruments (<http://www.ti.com/lit/gpn/lm301a-n>)
- LM324, Quad BJT OpAmp, Texas Instruments (<http://www.ti.com/lit/gpn/lm324>)
- LM741, Single BJT OpAmp, Texas Instruments (<http://www.ti.com/lit/gpn/lm741>)
- NE5532, Dual BJT OpAmp, Texas Instruments (<http://www.ti.com/lit/gpn/ne5532>) (NE5534 is Quad)
- TL072, Dual JFET OpAmp, Texas Instruments (<http://www.ti.com/lit/gpn/tl072>) (TL074 is Quad)

# Operational amplifier applications

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This article illustrates some typical applications of operational amplifiers. A simplified schematic notation is used, and the reader is reminded that many details such as device selection and power supply connections are not shown. Operational amplifiers are optimised for use with negative feedback, and this article discusses only negative-feedback applications. When positive feedback is required, a comparator is usually more appropriate. See Comparator applications for further information.

## Practical considerations

### Operational amplifiers parameter requirements

In order for a particular device to be used in an application, it must satisfy certain requirements. The operational amplifier must

- have large open-loop signal gain (voltage gain of 200,000 is obtained in early integrated circuit exemplars), and
- have input impedance large with respect to values present in the feedback network.

With these requirements satisfied, the op amp is considered ideal, and one can use the method of virtual ground to quickly and intuitively grasp the 'behavior' of any of the op amp circuits below.

Those interested in construction of any of these circuits for practical use should consult a more detailed reference. See the External links section.

### Component specification

Resistors used in practical solid-state op-amp circuits are typically in the  $k\Omega$  range. Resistors much greater than  $1 M\Omega$  cause excessive thermal noise and make the circuit operation susceptible to significant errors due to bias or leakage currents.

### Input bias currents and input offset

Practical operational amplifiers draw a small current from each of their inputs due to bias requirements (in the case of bipolar junction transistor-based inputs) or leakage (in the case of MOSFET-based inputs).

These currents flow through the resistances connected to the inputs and produce small voltage drops across those resistances. Appropriate design of the feedback network can alleviate problems associated with input bias currents and common-mode gain, as explained below. The heuristic rule is to ensure that the impedance "looking out" of each input terminal is identical.

To the extent that the input bias currents do not match, there will be an effective input offset voltage present, which can lead to problems in circuit performance. Many commercial op amp offerings provide a method for tuning the operational amplifier to balance the inputs (e.g., "offset null" or "balance" pins that can interact with an external voltage source attached to a potentiometer). Alternatively, a tunable external voltage can be added to one of the inputs in order to balance out the offset effect. In cases where a design calls for one input to be short-circuited to ground, that short circuit can be replaced with a variable resistance that can be tuned to mitigate the offset problem.

Operational amplifiers using MOSFET-based input stages have input leakage currents that will be, in many designs, negligible.

## Power supply effects

Although power supplies are not indicated in the (simplified) operational amplifier designs below, they are nonetheless present and can be critical in operational amplifier circuit design.

### Supply noise

Power supply imperfections (e.g., power signal ripple, non-zero source impedance) may lead to noticeable deviations from ideal operational amplifier behavior. For example, operational amplifiers have a specified power supply rejection ratio that indicates how well the output can reject signals that appear on the power supply inputs. Power supply inputs are often noisy in large designs because the power supply is used by nearly every component in the design, and inductance effects prevent current from being instantaneously delivered to every component at once. As a consequence, when a component requires large injections of current (e.g., a digital component that is frequently switching from one state to another), nearby components can experience sagging at their connection to the power supply. This problem can be mitigated with appropriate use of bypass capacitors connected across each power supply pin and ground. When bursts of current are required by a component, the component can *bypass* the power supply by receiving the current directly from the nearby capacitor (which is then slowly recharged by the power supply).

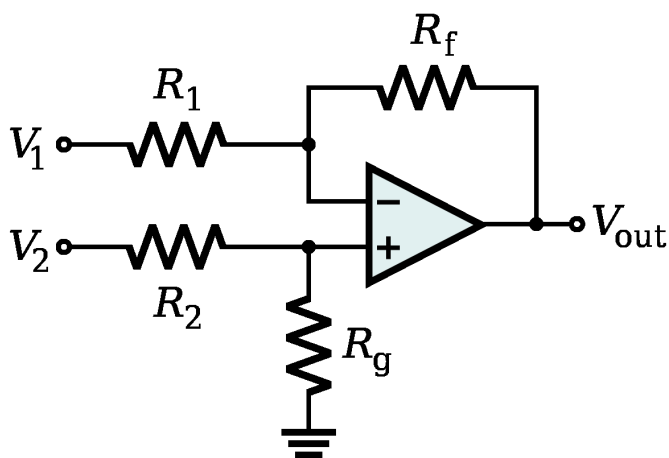
### Using power supply currents in the signal path

Additionally, current drawn into the operational amplifier from the power supply can be used as inputs to external circuitry that augment the capabilities of the operational amplifier. For example, an operational amplifier may not be fit for a particular high-gain application because its output would be required to generate signals outside of the safe range generated by the amplifier. In this case, an external push–pull amplifier can be controlled by the current into and out of the operational amplifier. Thus, the operational amplifier may itself operate within its factory specified bounds while still allowing the negative feedback path to include a large output signal well outside of those bounds.<sup>[1]</sup>

## Amplifiers

We begin these examples with that of the differential amplifier, from which can be derived many of the other applications, including the inverting, non-inverting, and summing amplifier, the voltage follower, integrator, differentiator, and gyrator.

### Differential amplifier (difference amplifier)



Main article: Differential amplifier

Amplifies the difference in voltage between its inputs.

The name "differential amplifier" must not be confused with the "differentiator," which is also shown on this page.

The "instrumentation amplifier," which is also shown on this page, is a modification of the differential amplifier that also provides high input impedance.

The circuit shown computes the difference of two voltages, multiplied by some gain factor. The output voltage:

$$V_{\text{out}} = \frac{(R_f + R_1) R_g}{(R_g + R_2) R_1} V_2 - \frac{R_f}{R_1} V_1 = \left( \frac{R_1 + R_f}{R_1} \right) \cdot \left( \frac{R_g}{R_g + R_2} \right) V_2 - \frac{R_f}{R_1} V_1.$$

Or, expressed as a function of the common mode input  $V_{\text{com}}$  and difference input  $V_{\text{dif}}$

$$V_{\text{com}} = (V_1 + V_2)/2; V_{\text{dif}} = V_1 - V_2,$$

the output voltage is

$$V_{\text{out}} \frac{R_1}{R_f} = V_{\text{com}} \frac{R_1/R_f - R_2/R_g}{1 + R_2/R_g} + V_{\text{dif}} \frac{1 + (R_2/R_g + R_1/R_f)/2}{1 + R_2/R_g}.$$

In order for this circuit to produce a signal proportional to the voltage difference of the input terminals, the coefficient of the  $V_{\text{com}}$  term (the common-mode gain) must be zero, or

$$R_1/R_f = R_2/R_g$$

With this constraint<sup>[2]</sup> in place, the common-mode rejection ratio of this circuit is infinitely large, and the output

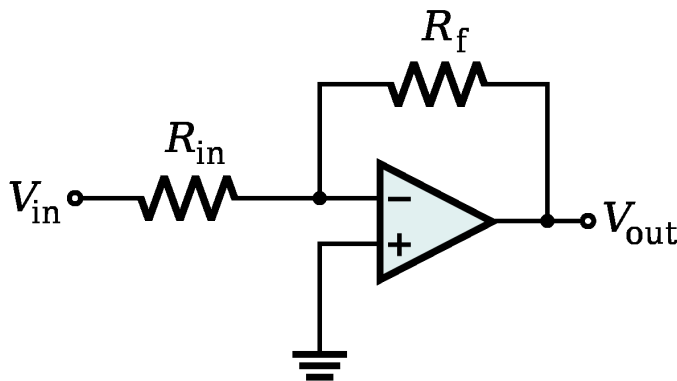
$$V_{\text{out}} = \frac{R_f}{R_1} V_{\text{dif}} = \frac{R_f}{R_1} (V_2 - V_1).$$

where the simple expression  $R_f/R_1$  represents the closed-loop gain of the differential amplifier.

The special case when the closed-loop gain is unity is a differential follower, with:

$$V_{\text{out}} = V_2 - V_1.$$

### Inverting amplifier



An inverting amplifier is a special case of the differential amplifier in which that circuit's non-inverting input  $V_2$  is grounded, and inverting input  $V_1$  is identified with  $V_{\text{in}}$  above. The closed-loop gain is  $R_f/R_{\text{in}}$ , hence

$$V_{\text{out}} = -\frac{R_f}{R_{\text{in}}} V_{\text{in}}.$$

The simplified circuit above is like the differential amplifier in the limit of  $R_2$  and  $R_g$  very small. In this case, though, the circuit will be susceptible to input bias current drift because of the mismatch between  $R_f$  and  $R_{\text{in}}$ .

To intuitively see the gain equation above, calculate the current in  $R_{\text{in}}$ :

$$i_{\text{in}} = \frac{V_{\text{in}}}{R_{\text{in}}}$$

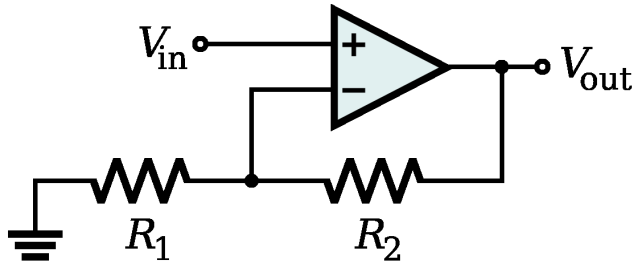
then recall that this same current must be passing through  $R_f$ , therefore (because  $V_- = V_+ = 0$ ):



$$V_{\text{out}} = -i_{\text{in}}R_f = -V_{\text{in}}\frac{R_f}{R_{\text{in}}}$$

A mechanical analogy is a seesaw, with the  $V_-$  node (between  $R_{\text{in}}$  and  $R_f$ ) as the fulcrum, at ground potential.  $V_{\text{in}}$  is at a length  $R_{\text{in}}$  from the fulcrum;  $V_{\text{out}}$  is at a length  $R_f$ . When  $V_{\text{in}}$  descends "below ground", the output  $V_{\text{out}}$  rises proportionately to balance the seesaw, and *vice versa*.<sup>[3]</sup>

### Non-inverting amplifier



A non-inverting amplifier is a special case of the differential amplifier in which that circuit's inverting input  $V_-$  is grounded, and non-inverting input  $V_+$  is identified with  $V_{\text{in}}$  above, with  $R_g \gg R_2$ . Referring to the circuit immediately above,

$$V_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right) V_{\text{in}}$$

To intuitively see this gain equation, use the virtual ground technique to calculate the current in resistor  $R_1$ :

$$i_1 = \frac{V_{\text{in}}}{R_1},$$

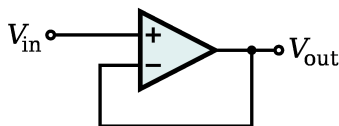
then recall that this same current must be passing through  $R_2$ , therefore:

$$V_{\text{out}} = V_{\text{in}} + i_{\text{in}}R_2 = V_{\text{in}} \left(1 + \frac{R_2}{R_1}\right)$$

A mechanical analogy is a class-2 machine, a lever, with one terminal of  $R_1$  as the fulcrum, at ground potential.  $V_{\text{in}}$  is at a length  $R_1$  from the fulcrum;  $V_{\text{out}}$  is at a length  $R_2$  further along. When  $V_{\text{in}}$  ascends "above ground", the output  $V_{\text{out}}$  rises proportionately with the lever.

The input impedance of the simplified non-inverting amplifier is high, of order  $R_{\text{dif}} \times A_{\text{OL}}$  times the closed-loop gain, where  $R_{\text{dif}}$  is the op amp's input impedance to differential signals, and  $A_{\text{OL}}$  is the open-loop voltage gain of the op amp; in the case of the ideal op amp, with  $A_{\text{OL}}$  infinite and  $R_{\text{dif}}$  infinite, the input impedance is infinite. In this case, though, the circuit will be susceptible to input bias current drift because of the mismatch between the impedances driving the  $V_+$  and  $V_-$  op amp inputs.

### Voltage follower (unity buffer amplifier)



Used as a buffer amplifier to eliminate loading effects (e.g., connecting a device with a high source impedance to a device with a low input impedance).

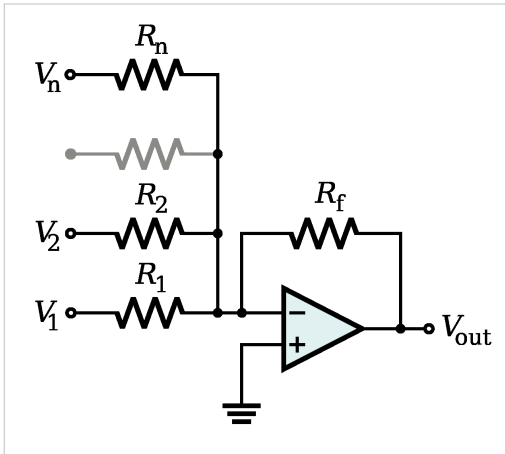
$$V_{\text{out}} = V_{\text{in}}$$

$$Z_{\text{in}} = \infty \text{ (realistically, the differential input impedance of the op-amp itself, } 1 \text{ M}\Omega \text{ to } 1 \text{ T}\Omega)$$

Due to the strong (i.e., unity gain) feedback and certain non-ideal characteristics of real operational amplifiers, this feedback system is prone to have poor stability margins. Consequently, the system may be unstable when connected

to sufficiently capacitive loads. In these cases, a lag compensation network (e.g., connecting the load to the voltage follower through a resistor) can be used to restore stability. The manufacturer data sheet for the operational amplifier may provide guidance for the selection of components in external compensation networks. Alternatively, another operational amplifier can be chosen that has more appropriate internal compensation.

### Summing amplifier



A summing amplifier sums several (weighted) voltages:

$$V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} \right)$$

- When  $R_1 = R_2 = \dots = R_n$ , and  $R_f$  independent

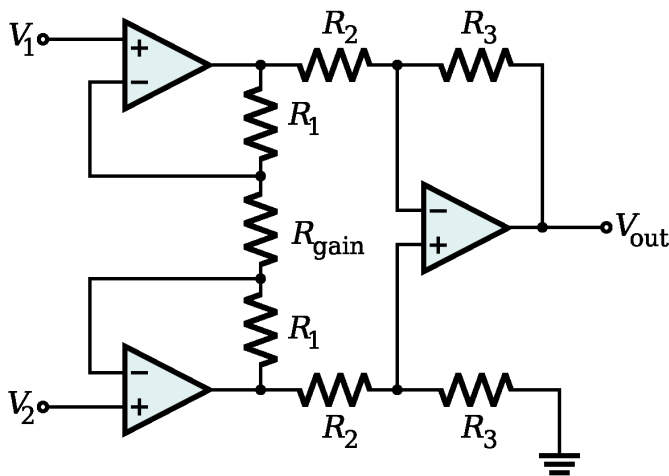
$$V_{\text{out}} = -\frac{R_f}{R_1} (V_1 + V_2 + \dots + V_n)$$

- When  $R_1 = R_2 = \dots = R_n = R_f$

$$V_{\text{out}} = -(V_1 + V_2 + \dots + V_n)$$

- Output is inverted
- Input impedance of the  $n$ th input is  $Z_n = R_n$  ( $V_-$  is a virtual ground)

### Instrumentation amplifier



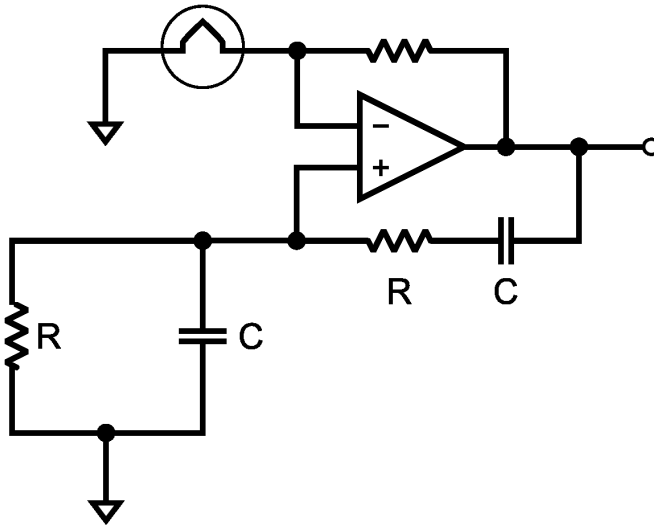
Main article: Instrumentation amplifier

Combines very high input impedance, high common-mode rejection, low DC offset, and other properties used in making very accurate, low-noise measurements

- Is made by adding a non-inverting buffer to each input of the differential amplifier to increase the input impedance.

## Oscillators

### Wien bridge oscillator



Main article: Wien bridge oscillator

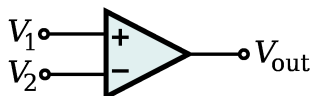
Produces a very low distortion sine wave. Uses negative temperature compensation in the form of a light bulb or diode.

## Filters

Main article: Active filter

Operational amplifiers can be used in construction of active filters, providing high-pass, low-pass, band-pass, reject and delay functions. The high input impedance and gain of an op-amp allow straightforward calculation of element values, allowing accurate implementation of any desired filter topology with little concern for the loading effects of stages in the filter or of subsequent stages. However, the frequencies at which active filters can be implemented is limited; when the behavior of the amplifiers departs significantly from the ideal behavior assumed in elementary design of the filters, filter performance is degraded.

## Comparator



Main article: Comparator

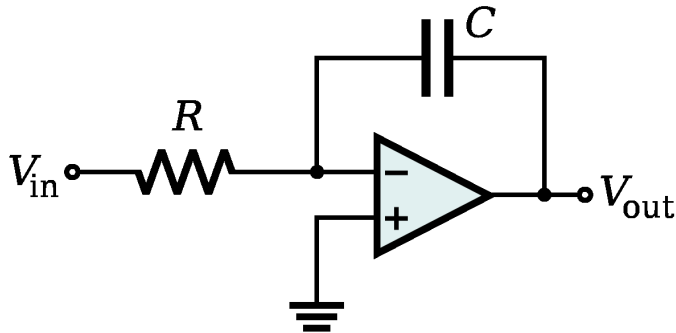
Main article: Comparator applications

An operational amplifier can, if necessary, be forced to act as a comparator. The smallest difference between the input voltages will be amplified enormously, causing the output to swing to nearly the supply voltage. However, it is usually better to use a dedicated comparator for this purpose, as its output has a higher slew rate and can reach either power supply rail. Some op-amps have clamping diodes on the input that prevent use as a comparator.<sup>[4]</sup>

## Integration and differentiation

### Inverting integrator

The integrator is mostly used in analog computers, analog-to-digital converters and wave-shaping circuits.



Integrates (and inverts) the input signal  $V_{in}(t)$  over a time interval  $t$ ,  $t_0 < t < t_1$ , yielding an output voltage at time  $t = t_1$  of

$$V_{out}(t_1) = V_{out}(t_0) - \frac{1}{RC} \int_{t_0}^{t_1} V_{in}(t) dt$$

where  $V_{out}(t_0)$  represents the output voltage of the circuit at time  $t = t_0$ . This is the same as saying that the output voltage changes over time  $t_0 < t < t_1$  by an amount proportional to the time integral of the input voltage:

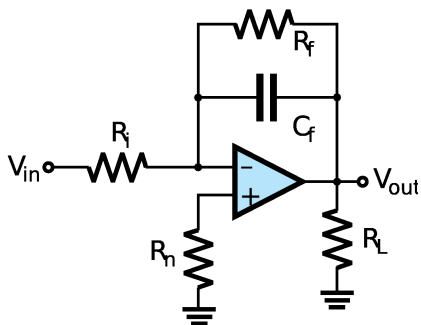
$$-\frac{1}{RC} \int_{t_0}^{t_1} V_{in}(t) dt$$

This circuit can be viewed as a low-pass electronic filter, one with a single pole at DC (i.e., where  $\omega = 0$ ) and with gain.

In a practical application one encounters a significant difficulty: unless the capacitor  $C$  is periodically discharged, the output voltage will eventually drift outside of the operational amplifier's operating range. This can be due to any combination of:

- The input  $V_{in}$  has a non-zero DC component,
- Input bias current is non-zero,
- Input offset voltage is non-zero.

A slightly more complex circuit can ameliorate the second two problems, and in some cases, the first as well.



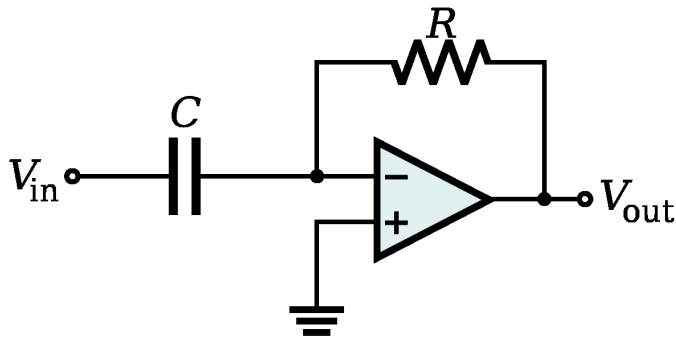
Here, the feedback resistor  $R_f$  provides a discharge path for capacitor  $C_f$ , while the series resistor at the non-inverting input  $R_n$ , when of the correct value, alleviates input bias current and common-mode problems. That value is the parallel resistance of  $R_i$  and  $R_f$ , or using the shorthand notation ||:

$$R_n = \frac{1}{\frac{1}{R_i} + \frac{1}{R_f}} = R_i || R_f$$

The relationship between input signal and output signal is now:

$$V_{\text{out}}(t_1) = V_{\text{out}}(t_0) - \frac{1}{R_i C_f} \int_{t_0}^{t_1} V_{\text{in}}(t) dt$$

### Inverting differentiator



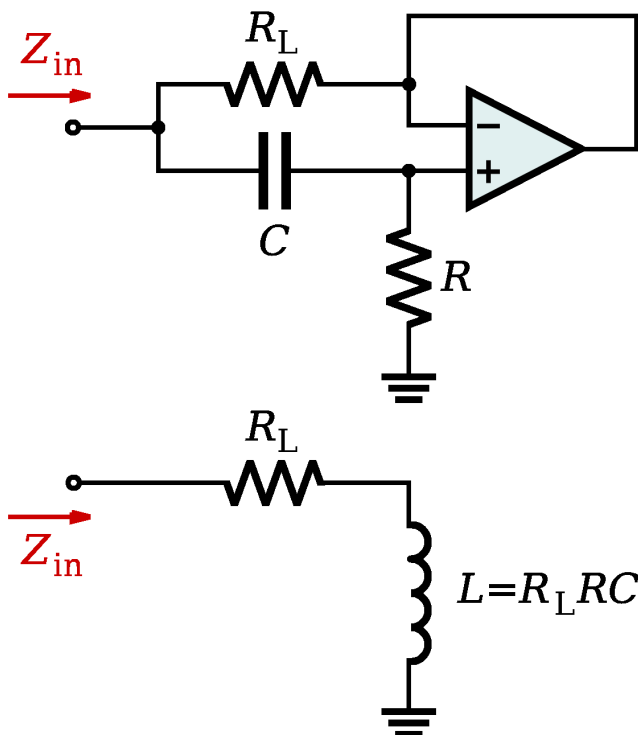
Differentiates the (inverted) signal over time.

$$V_{\text{out}} = -RC \frac{dV_{\text{in}}}{dt} \quad \text{where } V_{\text{in}} \text{ and } V_{\text{out}} \text{ are functions of time.}$$

- This can also be viewed as a high-pass electronic filter. It is a filter with a single zero at DC (i.e., where angular frequency  $\omega = 0$  radians) and gain. The high-pass characteristics of a differentiating amplifier (i.e., the low-frequency zero) can lead to stability challenges when the circuit is used in an analog servo loop (e.g., in a PID controller with a significant derivative gain). In particular, as a root locus analysis would show, increasing feedback gain will drive a closed-loop pole toward marginal stability at the DC zero introduced by the differentiator.

### Synthetic elements

#### Inductance gyrator

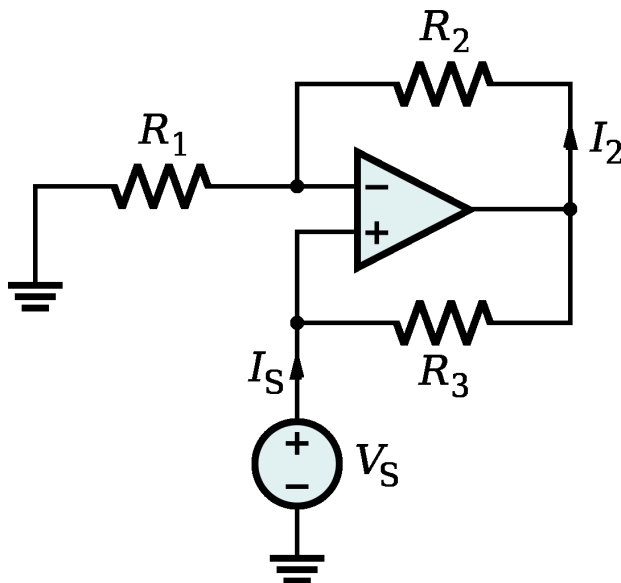


Main article: Gyrator

Simulates an inductor (i.e., provides inductance without the use of a possibly costly inductor). The circuit exploits the fact that the current flowing through a capacitor behaves through time as the voltage across an inductor. The capacitor used in this circuit is smaller than the inductor it simulates and its capacitance is less subject to changes in value due to environmental changes.

This circuit is unsuitable for applications relying on the back EMF property of an inductor as this will be limited in a gyrator circuit to the voltage supplies of the op-amp.

### Negative impedance converter (NIC)



Main article: Negative impedance converter

Creates a resistor having a negative value for any signal generator

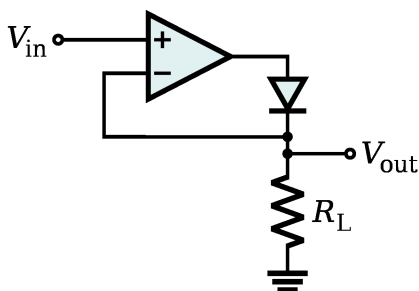
- In this case, the ratio between the input voltage and the input current (thus the input resistance) is given by:

$$R_{in} = -R_3 \frac{R_1}{R_2}$$

In general, the components  $R_1$ ,  $R_2$ , and  $R_3$  need not be resistors; they can be any component that can be described with an impedance.

## Non-linear

### Precision rectifier



Main article: Precision rectifier

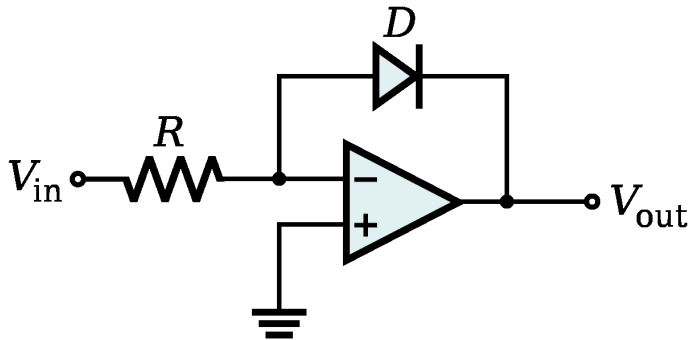
The voltage drop  $V_F$  across the forward biased diode in the circuit of a passive rectifier is undesired. In this active version, the problem is solved by connecting the diode in the negative feedback loop. The op-amp compares the output voltage across the load with the input voltage and increases its own output voltage with the value of  $V_F$ . As a

result, the voltage drop  $V_F$  is compensated and the circuit behaves very nearly as an ideal (*super*) diode with  $V_F = 0$  V.

The circuit has speed limitations at high frequency because of the slow negative feedback and due to the low slew rate of many non-ideal op-amps.

### Logarithmic output

See also: Log amplifier



- The relationship between the input voltage  $v_{in}$  and the output voltage  $v_{out}$  is given by:

$$v_{out} = -V_T \ln \left( \frac{v_{in}}{I_S R} \right)$$

where  $I_S$  is the saturation current and  $V_T$  is the thermal voltage.

- If the operational amplifier is considered ideal, the inverting input pin is virtually grounded, so the current flowing into the resistor from the source (and thus through the diode to the output, since the op-amp inputs draw no current) is:

$$\frac{v_{in}}{R} = I_R = I_D$$

where  $I_D$  is the current through the diode. As known, the relationship between the current and the voltage for a diode is:

$$I_D = I_S \left( e^{\frac{v_D}{V_T}} - 1 \right).$$

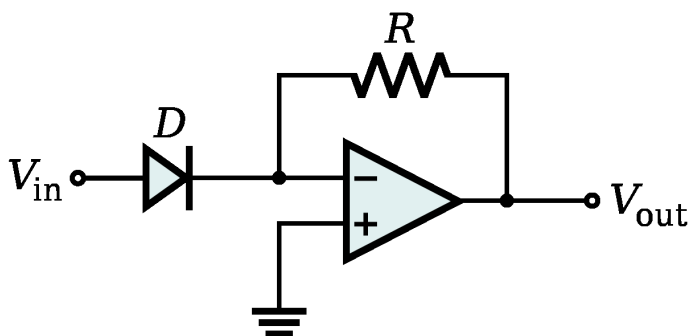
This, when the voltage is greater than zero, can be approximated by:

$$I_D \simeq I_S e^{\frac{v_D}{V_T}}.$$

Putting these two formulae together and considering that the output voltage is the negative of the voltage across the diode ( $V_{out} = -V_D$ ), the relationship is proven.

This implementation does not consider temperature stability and other non-ideal effects.

### Exponential output



- The relationship between the input voltage  $v_{in}$  and the output voltage  $v_{out}$  is given by:

$$v_{out} = -RI_S e^{\frac{v_{in}}{V_T}}$$

where  $I_S$  is the saturation current and  $V_T$  is the thermal voltage.

- Considering the operational amplifier ideal, then the negative pin is virtually grounded, so the current through the diode is given by:

$$I_D = I_S \left( e^{\frac{V_D}{V_T}} - 1 \right)$$

when the voltage is greater than zero, it can be approximated by:

$$I_D \simeq I_S e^{\frac{V_D}{V_T}}.$$

The output voltage is given by:

$$v_{out} = -RI_D.$$

## Other applications

- audio and video preamplifiers and buffers
- filters
- voltage regulator and current regulator
- analog-to-digital converter
- digital-to-analog converter
- voltage clamps
- oscillators and waveform generators
- Analog computer
- Capacitance multiplier
- Charge amplifier

## Notes

- [1] Paul Horowitz and Winfield Hill, *The Art of Electronics*. 2nd ed. Cambridge University Press, Cambridge, 1989 ISBN 0-521-37095-7
- [2] If you think of the left-hand side of the relation as the closed-loop gain of the inverting input, and the right-hand side as the gain of the non-inverting input, then matching these two quantities provides an output insensitive to the common-mode voltage of  $V_{-1}$  and  $V_{-2}$ .
- [3] Basic Electronics Theory, Delton T. Horn, 4th ed. McGraw-Hill Professional, 1994, p. 342–343.
- [4] [http://e2e.ti.com/blogs\\_/archives/b/thesignal/archive/2012/03/14/op-amps-used-as-comparators-is-it-okay.aspx](http://e2e.ti.com/blogs_/archives/b/thesignal/archive/2012/03/14/op-amps-used-as-comparators-is-it-okay.aspx)

## References

### Further reading

- *Basic Operational Amplifiers and Linear Integrated Circuits*; 2nd Ed; Thomas L Floyd; David Buchla; 593 pages; 1998; ISBN 978-0-13-082987-0.
- *Design with Operational Amplifiers and Analog Integrated Circuits*; 3rd Ed; Sergio Franco; 672 pages; 2002; ISBN 978-0-07-232084-8. (book website) (<http://www.mhhe.com/engcs/electrical/franco3/>)
- *Operational Amplifiers and Linear Integrated Circuits*; 6th Ed; Robert F Coughlin; 529 pages; 2000; ISBN 978-0-13-014991-6.
- *Op-Amps and Linear Integrated Circuits*; 4th Ed; Ram Gayakwad; 543 pages; 1999; ISBN 978-0-13-280868-2.
- *Op Amps and Linear Integrated Circuits*; 1st Ed; James M Fiore; 640 pages; 2001; ISBN 978-0766817937.
- *Op Amps For Everyone*; 1st Ed; Ron Mancini; 464 pages; 2002; Texas Instruments SLOD006B. PDF (<http://focus.ti.com/lit/an/slod006b/slod006b.pdf>)
- *Small Signal Audio Design*; 1st Ed; Douglas Self; 556 pages; 2010; ISBN 978-0-240-52177-0.



## External links



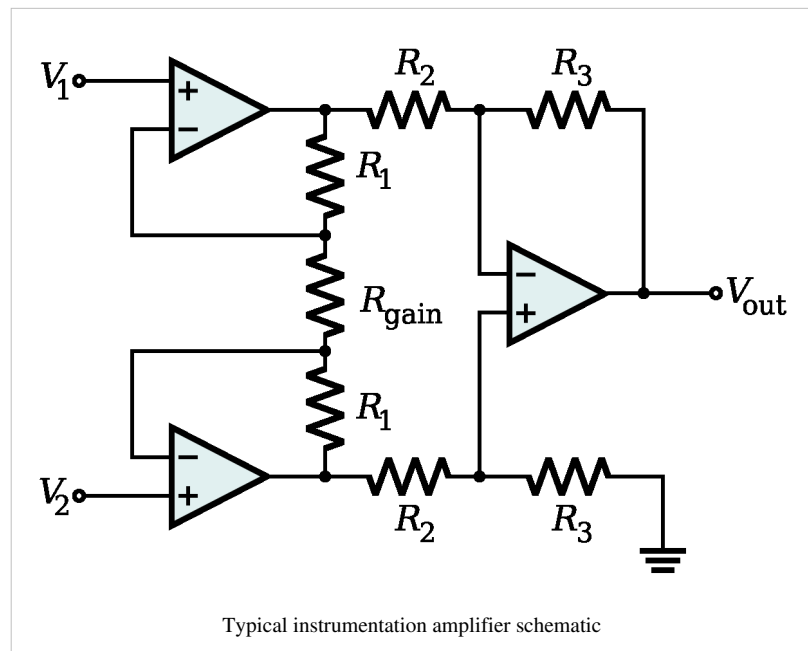
The Wikibook *Electronics* has a page on the topic of: *Op-Amps*

- Op Amps for Everyone (<http://focus.ti.com/lit/an/slod006b/slod006b.pdf>) PDF (1.96 MiB)
- Single supply op-amp circuit collection (<http://instruct1.cit.cornell.edu/courses/bionb440/datasheets/SingleSupply.pdf>) PDF (163 KiB)
- Op-amp circuit collection (<http://www.national.com/an/AN/AN-31.pdf>) PDF (962 KiB)
- A Collection of Amp Applications ([http://www.analog.com/static/imported-files/application\\_notes/28080533AN106.pdf](http://www.analog.com/static/imported-files/application_notes/28080533AN106.pdf)) PDF (1.06 MiB) – Analog Devices Application note
- Basic OpAmp Applications (<http://www.ligo.caltech.edu/~vsanni/ph5/pdf/BasicOpAmpApplications.pdf>) PDF (173 KiB)
- Handbook of operational amplifier applications (<http://focus.ti.com/lit/an/sboa092a/sboa092a.pdf>) PDF (2.00 MiB) – Texas Instruments Application note
- Low Side Current Sensing Using Operational Amplifiers (<http://focus.ti.com/analog/docs/gencontent.tsp?familyId=57&genContentId=28017>)
- Log/anti-log generators, cube generator, multiply/divide amp (<http://www.national.com/an/AN/AN-30.pdf>) PDF (165 KiB)
- Logarithmically variable gain from a linear variable component (<http://www.edn.com/archives/1994/030394/05di7.htm>)
- Impedance and admittance transformations using operational amplifiers ([http://www.philbrickarchive.org/1964-1\\_v12\\_no1\\_the\\_lightning\\_empiricist.htm](http://www.philbrickarchive.org/1964-1_v12_no1_the_lightning_empiricist.htm)) by D. H. Sheingold
- *High Speed Amplifier Techniques* (<http://www.linear.com/docs/4138>) very practical and readable – with photos and real waveforms
- Single supply op-amp circuit collection (<http://instruct1.cit.cornell.edu/courses/bionb440/datasheets/SingleSupply.pdf>)
- Properly terminating an unused op amp ([http://www.electronicproducts.com/Analog\\_Mixed\\_Signal\\_ICs/Amplifiers/Properly\\_terminating\\_an\\_unused\\_op\\_amp.aspx](http://www.electronicproducts.com/Analog_Mixed_Signal_ICs/Amplifiers/Properly_terminating_an_unused_op_amp.aspx))

# Instrumentation amplifier

This article is about amplifiers for measurement and electronic test equipment. For amplifiers for musical instruments, see instrument amplifier.

An **instrumentation amplifier** (or **instrumentational amplifier**) is a type of differential amplifier that has been outfitted with input buffer amplifiers, which eliminate the need for input impedance matching and thus make the amplifier particularly suitable for use in measurement and test equipment. Additional characteristics include very low DC offset, low drift, low noise, very high open-loop gain, very high common-mode rejection ratio, and very high input impedances. Instrumentation amplifiers are used where great accuracy and stability of the circuit both short and long-term are required.



Typical instrumentation amplifier schematic

Although the instrumentation amplifier is usually shown schematically identical to a standard operational amplifier (op-amp), the electronic instrumentation amp is almost always internally composed of 3 op-amps. These are arranged so that there is one op-amp to buffer each input (+,-), and one to produce the desired output with adequate impedance matching for the function.<sup>[1][2]</sup>

The most commonly used instrumentation amplifier circuit is shown in the figure. The gain of the circuit is

$$\frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}}\right) \frac{R_3}{R_2}$$

The rightmost amplifier, along with the resistors labelled  $R_2$  and  $R_3$  is just the standard differential amplifier circuit, with gain =  $R_3/R_2$  and differential input resistance =  $2 \cdot R_2$ . The two amplifiers on the left are the buffers. With  $R_{gain}$  removed (open circuited), they are simple unity gain buffers; the circuit will work in that state, with gain simply equal to  $R_3/R_2$  and high input impedance because of the buffers. The buffer gain could be increased by putting resistors between the buffer inverting inputs and ground to shunt away some of the negative feedback; however, the single resistor  $R_{gain}$  between the two inverting inputs is a much more elegant method: it increases the differential-mode gain of the buffer pair while leaving the common-mode gain equal to 1. This increases the common-mode rejection ratio (CMRR) of the circuit and also enables the buffers to handle much larger common-mode signals without clipping than would be the case if they were separate and had the same gain. Another benefit of the method is that it boosts the gain using a single resistor rather than a pair, thus avoiding a resistor-matching problem (although the two  $R_1$ s need to be matched), and very conveniently allowing the gain of the circuit to be changed by changing the value of a single resistor. A set of switch-selectable resistors or even a potentiometer can be used for  $R_{gain}$ , providing easy changes to the gain of the circuit, without the complexity of having to switch matched pairs of resistors.

The ideal common-mode gain of an instrumentation amplifier is zero. In the circuit shown, common-mode gain is caused by mismatches in the values of the equally numbered resistors and by the mis-match in common mode gains of the two input op-amps. Obtaining very closely matched resistors is a significant difficulty in fabricating these

circuits, as is optimizing the common mode performance of the input op-amps.<sup>[3]</sup>

An instrumentation amp can also be built with two op-amps to save on cost and increase CMRR, but the gain must be higher than two (+6 dB).

Instrumentation amplifiers can be built with individual op-amps and precision resistors, but are also available in integrated circuit form from several manufacturers (including Texas Instruments, National Semiconductor, Analog Devices, Linear Technology and Maxim Integrated Products). An IC instrumentation amplifier typically contains closely matched laser-trimmed resistors, and therefore offers excellent common-mode rejection. Examples include AD8221<sup>[4]</sup>, MAX4194<sup>[5]</sup>, LT1167<sup>[6]</sup> and INA128<sup>[7]</sup>.

Instrumentation Amplifiers can also be designed using "Indirect Current-feedback Architecture", which extend the operating range of these amplifiers to the negative power supply rail, and in some cases the positive power supply rail. This can be particularly useful in single-supply systems, where the negative power rail is simply the circuit ground (GND). Examples of parts utilizing this architecture are MAX4208/MAX4209<sup>[8]</sup> and AD8129/AD8130<sup>[9]</sup>.

**Feedback-free instrumentation amplifier** is the high input impedance differential amplifier designed without the external feedback network. This allows reduction in the number of amplifiers (one instead of three), reduced noise (no thermal noise is brought on by the feedback resistors) and increased bandwidth (no frequency compensation is needed). Chopper stabilized (or zero drift) instrumentation amplifiers such as the LTC2053<sup>[10]</sup> use a switching input front end to eliminate DC offset errors and drift.

## References

- [1] R.F. Coughlin, F.F. Driscoll *Operational Amplifiers and Linear Integrated Circuits* (2nd Ed.1982. ISBN 0-13-637785-8) p.161.
- [2] Moore, Davis, Coplan *Building Scientific Apparatus* (2nd Ed. 1989 ISBN 0-201-13189-7)p.407.
- [3] Smither, Pugh and Woolard: 'CMRR Analysis of the 3-op-amp instrumentation amplifier', Electronics letters, 2 February 1989.
- [4] <http://www.analog.com/en/amplifiers-and-comparators/instrumentation-amplifiers/ad8221/products/product.html>
- [5] [http://www.maxim-ic.com/quick\\_view2.cfm/qv\\_pk/2006](http://www.maxim-ic.com/quick_view2.cfm/qv_pk/2006)
- [6] <http://www.linear.com/1167>
- [7] <http://focus.ti.com/docs/prod/folders/print/ina128.html>
- [8] [http://www.maxim-ic.com/quick\\_view2.cfm/qv\\_pk/4925](http://www.maxim-ic.com/quick_view2.cfm/qv_pk/4925)
- [9] <http://www.analog.com/en/audiovideo-products/video-ampsbuffersfilters/ad8129/products/product.html>
- [10] <http://www.linear.com/product/LTC2053>

## External links



- Opamp Instrumentation Amplifier (<http://www.ecircuitcenter.com/Circuits/instamp1/instamp1.htm>)
- The instrumentation amplifier ([http://www.allaboutcircuits.com/vol\\_3/chpt\\_8/10.html](http://www.allaboutcircuits.com/vol_3/chpt_8/10.html))
- Lessons In Electric Circuits — Volume III — The instrumentation amplifier ([http://www.ibiblio.org/kuphaldt/electricCircuits/Semi/SEMI\\_8.html#xtocid100169](http://www.ibiblio.org/kuphaldt/electricCircuits/Semi/SEMI_8.html#xtocid100169))
- A Practical Review of Common Mode and Instrumentation Amplifiers ([http://www.analog.com/static/imported-files/tech\\_articles/25406877Common.pdf](http://www.analog.com/static/imported-files/tech_articles/25406877Common.pdf)) (PDF)
- The Instrumentation Amplifier (<http://www.univie.ac.at/cga/courses/BE513/Instrumentation/>)
- Instrumentation Amplifiers Tutorial (<http://www.educyclopedia.be/electronics/Instrumentation.htm>)
- A Designer's Guide to Instrumentation Amplifiers (3rd Edition) ([http://www.analog.com/analog\\_root/static/technology/amplifiersLinear/InstrumentationAmplifiers/designersGuide.html](http://www.analog.com/analog_root/static/technology/amplifiersLinear/InstrumentationAmplifiers/designersGuide.html))
- Three is a Crowd for Instrumentation Amplifiers ([http://www.maxim-ic.com/appnotes.cfm/appnote\\_number/4034/](http://www.maxim-ic.com/appnotes.cfm/appnote_number/4034/))

- Instrumentation Amplifier Solutions, Circuits and Applications ([http://www.linear.com/products/instrumentation\\_amplifiers](http://www.linear.com/products/instrumentation_amplifiers))
- Fixed-gain CMOS differential amplifiers with no external feedback for a wide temperature range (Cryogenics) (<http://dx.doi.org/10.1016/j.cryogenics.2008.12.014>)

## Isolation amplifier

**Isolation amplifiers** are a form of differential amplifier that allow measurement of small signals in the presence of a high common mode voltage by providing electrical isolation and an electrical safety barrier. They protect data acquisition components from common mode voltages, which are potential differences between instrument ground and signal ground. Instruments that are applied in the presence of a common mode voltage without an isolation barrier allow ground currents to circulate, leading in the best case to a noisy representation of the signal under investigation. In the worst case, assuming that the magnitude of common mode voltage or current is sufficient, instrument destruction is likely. Isolation amplifiers are used in medical instruments to ensure isolation of a patient from power supply leakage current.

Amplifiers with an isolation barrier allow the front-end of the amplifier to float with respect to common mode voltage to the limit of the barrier's breakdown voltage, which is often 1,000 volts or more. This action protects the amplifier and the instrument connected to it, while still allowing a reasonably accurate measurement.

These amplifiers are also used for amplifying low-level signals in multi-channel applications. They can also eliminate measurement errors caused by ground loops. Amplifiers with internal transformers eliminate external isolated power supply. They are usually used as analogue interfaces between systems with separated grounds.

Isolation amplifiers may include isolated power supplies for both the input and output stages, or may use external power supplies on each isolated portion.<sup>[1]</sup>

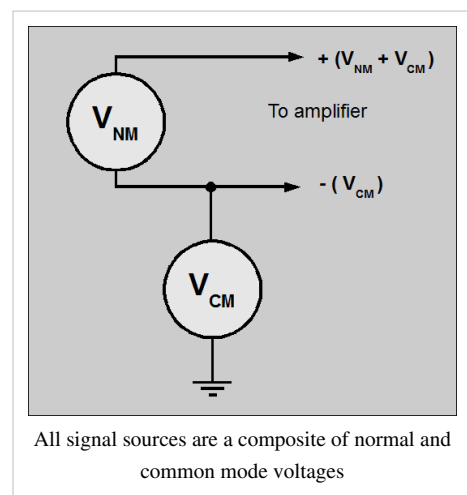
## Concepts

### Signal source components

All signal sources are a composite of two major components. The normal mode component ( $V_{NM}$ ) represents the signal of interest and is the voltage that is applied directly across the inputs of the amplifier. The common mode component ( $V_{CM}$ ) represents the difference in potential between the low side of the normal mode component and the ground of the amplifier that is used to measure the signal of interest (the normal mode voltage).

In many measurement situations the common mode component is irrelevantly low, but rarely zero. Common mode components of only a few millivolts are frequently encountered and largely and successfully ignored, especially when the normal mode component is orders of magnitude larger.

The first indicator that common mode voltage magnitude is competing with the normal mode component is a noisy reproduction of the latter at the amplifier's output. Such a situation does not usually define the need for an isolation amplifier, but rather a differential amplifier. Since the common mode component appears simultaneously and in phase on both amplifier inputs, the differential amplifier, within the limits of the amplifier's design, can reject it.



However, if the sum of the normal mode and common mode voltages exceeds either the differential amplifier's common mode range, or maximum range without damage then the need for an isolation amplifier is firmly established.

## Operating principles

Isolation amplifiers are commercially available as hybrid integrated circuits made by several manufacturers. There are three methods of providing isolation.

A transformer-isolated amplifier relies on transformer coupling of a high-frequency carrier signal between input and output. Some models also include a transformer-isolated power supply, that may also be used to power external signal processing devices on the isolated side of the system. The bandwidth available depends on the model and may range from 2 to 20 kHz. The isolation amplifier contains a voltage-to-frequency converter connected through a transformer to a frequency-to-voltage converter. The isolation between input and output is provided by the insulation on the transformer windings.

An optically-isolated amplifier modulates current through an LED optocoupler. The linearity is improved by using a second optocoupler within a feedback loop. Some devices provide up to 60 kHz bandwidth. Isolation is provided by the layer of transparent glass or plastic between the LED and detector.

A third strategy is to use small capacitors to couple a modulated high-frequency carrier; the capacitors can stand off large DC or power frequency AC voltages but provide coupling for the much higher frequency carrier signal. Some models on this principle can stand off 3.5 kilovolts and provide up to 70 kHz bandwidth.

## Isolation amplifier usage

Isolation amplifiers are used to allow measurement of small signals in the presence of a high common mode voltage. The capacity of an isolation amplifier is a function of two key isolation amplifier specifications:

- The amplifier's isolation breakdown voltage, which defines the absolute maximum common mode voltage that it will tolerate without damage. Specifications of 1,000 volts and more are common.
- The amplifier's common mode rejection ratio (CMRR). The CMRR specification defines the degree to which the common mode voltage will disrupt the normal mode component measurement, and therefore affect measurement accuracy.

The frequency of the common mode voltage can adversely affect performance. Higher frequency common mode voltages create difficulty for many isolation amplifiers due to the parasitic capacitance of the isolation barrier. This capacitance appears as a low impedance to higher frequency signals, and allows the common mode voltage to essentially blow past the barrier and interfere with measurements, or even damage the amplifier. However, most common mode voltages are a composite of line voltages, so frequencies generally remain in the 50 to 60 Hz region with some harmonic content, well within the rejection range of most isolation amplifiers.

## Differential amplifiers

A non-isolated differential amplifier does not provide isolation between input and output circuits. They share a power supply and a DC path can exist between input and output. A non-isolated differential amplifier can only withstand common-mode voltages up to the power supply voltage.

Similar to the instrumentation amplifier, isolation amplifiers have fixed differential gain over a wide range of frequencies, high input impedance and low output impedance.

## Amplifier selection guidelines

Instrumentation amplifiers can be classified into four broad categories, organized from least to most costly:

- **Single-ended.** An unbalanced input, non-isolated. Suitable for measurements where common mode voltages are zero, or extremely small. Very inexpensive.
- **Differential.** A balanced input, non-isolated. Suitable for measurements where the sum of common mode and normal mode voltages remains within the measurement range of the amplifier.
- **Single-ended, floating common.** An isolated and quasi-balanced input (the floating common is typically connected to the (-) input of a differential amplifier). Suitable for off-ground measurements up to the breakdown voltage of the isolation barrier, and exhibits very good common mode rejection (100 db typical).
- **Differential, floating common.** An, isolated and balanced input. Suitable for off-ground measurements to the breakdown voltage of the isolation barrier, and exhibits superb common mode rejection (>120 db).

For most industrial applications that require isolation, the single-ended floating design provides the best price/performance.

There are also two broad classifications of isolation amplifiers that should be considered in tandem with the application:

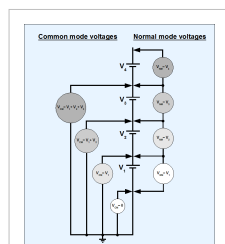
- **Amplifiers providing input-to-output isolation without channel-to-channel isolation.** This is a less expensive form of isolation that offers only one isolation barrier for a multi-channel instrument. Although the commons of each channel are isolated from power ground by the input-to-output isolation barrier, they are not isolated from each other. Therefore a common mode voltage on one will attempt to float all the others, sometimes with disastrous results. This form of isolation is suitable only when it is certain that there is only one common mode voltage that is equally applied to all channels.
- **Amplifiers providing both input-to-output and channel-to-channel isolation.** This is the purest form of isolation, and the option that should be considered for nearly all applications. Multi-channel instruments that employ it are immune to inconsistent common mode voltages on any combination of channels within the limits of the amplifiers.

## Typical application

### Stacked voltage cell measurements

Stacked voltage cell measurements are common with the growing popularity of solar cells and fuel cells. In this application the technician wants to profile the performance of individual series-connected voltages cells, but the need for an isolated amplifier is often overlooked. Each voltage cell (the normal mode voltage) is removed from ground by an amount equal to the sum of the voltage cells below it (the common mode voltage). Unless the amplifiers used to measure individual cell voltages are allowed to float at a level equal to the common mode voltage, measurements are not likely to be accurate for any but the first cell in the string where the common mode voltage is zero.

A non-isolated differential amplifier can be used but it will have a rated maximum common mode voltage that cannot be exceeded while maintaining accuracy.



Stacked voltage cell measurements illustrate the need for an isolation amplifier.

## References

### External links

- Learn the importance of isolation (<http://www.dataq.com/applicat/articles/isolation.htm>)
- A guide to isolation amplifier selection (<http://ad.usno.navy.mil/edboard/070518p.pdf>)

## Active filter

An **active filter** is a type of analog electronic filter that uses active components such as an amplifier. Amplifiers included in a filter design can be used to improve the performance and predictability of a filter, while avoiding the need for inductors (which are typically expensive compared to other components). An amplifier prevents the load impedance of the following stage from affecting the characteristics of the filter. An active filter can have complex poles and zeros without using a bulky or expensive inductor. The shape of the response, the Q (quality factor), and the tuned frequency can often be set with inexpensive variable resistors. In some active filter circuits, one parameter can be adjusted without affecting the others. <sup>[1]</sup>

Using active elements has some limitations. Basic filter design equations neglect the finite bandwidth of amplifiers. Available active devices have limited bandwidth, so they are often impractical at high frequencies. Amplifiers consume power and inject noise into a system. Certain circuit topologies may be impractical if no DC path is provided for bias current to the amplifier elements. Power handling capability is limited by the amplifier stages. <sup>[1]</sup>

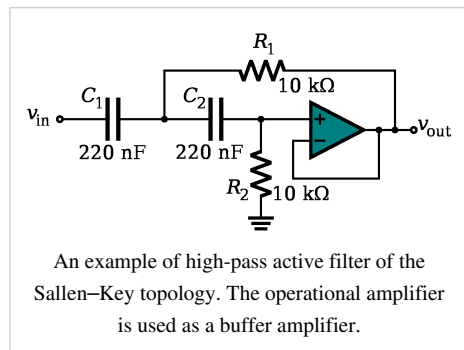
Active filter circuit configurations (electronic filter topology) include:

- Sallen-Key, and VCVS filters (low sensitivity to component tolerance)
- State variable filters and biquadratic or biquad filters
- Dual Amplifier Bandpass (DABP)
- Wien notch
- Multiple Feedback Filters
- Fliege (lowest component count for 2 opamp but with good controllability over frequency and type)
- Akerberg Mossberg (one of the topologies that offer complete and independent control over gain, frequency, and type)

Active filters can implement the same transfer functions as passive filters. Common transfer functions are:

- High-pass filter – attenuation of frequencies below their cut-off points.
- Low-pass filter – attenuation of frequencies above their cut-off points.
- Band-pass filter – attenuation of frequencies both above and below those they allow to pass.
- Notch filter – attenuation of certain frequencies while allowing all others to pass.

Combinations are possible, such as notch and high-pass (in a rumble filter where most of the offending rumble comes from a particular frequency). Another example is an elliptic filter.



## Design of active filters

To design filters, the specifications that need to be established include:

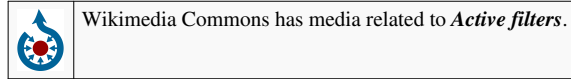
- The range of desired frequencies (the passband) together with the shape of the frequency response. This indicates the variety of filter (see above) and the center or corner frequencies.
- Input and output impedance requirements. These limit the circuit topologies available; for example, most, but not all active filter topologies provide a buffered (low impedance) output. However, remember that the internal output impedance of operational amplifiers, if used, may rise markedly at high frequencies and reduce the attenuation from that expected. Be aware that some high-pass filter topologies present the input with almost a short circuit to high frequencies.
- Dynamic range of the active elements. The amplifier should not saturate (run into the power supply rails) at expected input signals, nor should it be operated at such low amplitudes that noise dominates.
- The degree to which unwanted signals should be rejected.
  - In the case of narrow-band bandpass filters, the  $Q$  determines the  $-3\text{dB}$  bandwidth but also the degree of rejection of frequencies far removed from the center frequency; if these two requirements are in conflict then a staggered-tuning bandpass filter may be needed.
  - For notch filters, the degree to which unwanted signals at the notch frequency must be rejected determines the accuracy of the components, but not the  $Q$ , which is governed by desired steepness of the notch, i.e. the bandwidth around the notch before attenuation becomes small.
  - For high-pass and low-pass (as well as band-pass filters far from the center frequency), the required rejection may determine the slope of attenuation needed, and thus the "order" of the filter. A second-order all-pole filter gives an ultimate slope of about 12 dB per octave (40dB/decade), but the slope close to the corner frequency is much less, sometimes necessitating a notch be added to the filter.
- The allowable "ripple" (variation from a flat response, in decibels) within the passband of high-pass and low-pass filters, along with the shape of the frequency response curve near the corner frequency, determine the damping ratio or damping factor ( $= 1/(2Q)$ ). This also affects the phase response, and the time response to a square-wave input. Several important response shapes (damping ratios) have well-known names:
  - Chebyshev filter – peaking/ripple in the passband before the corner;  $Q > 0.7071$  for 2nd-order filters
  - Butterworth filter – maximally flat amplitude response;  $Q = 0.7071$  for 2nd-order filters
  - Linkwitz–Riley filter – desirable properties for audio crossover applications, fastest rise time with no overshoot;  $Q = 0.5$  (critically damped)
  - Paynter or transitional Thompson-Butterworth or "compromise" filter – faster fall-off than Bessel;  $Q = 0.639$  for 2nd-order filters
  - Bessel filter – maximally flat group delay;  $Q = 0.577$  for 2nd-order filters
  - Elliptic filter or Cauer filter – add a notch (or "zero") just outside the passband, to give a much greater slope in this region than the combination of order and damping ratio *without* the notch.



## References

[1] Muhammad H. Rashid, *Microelectronic Circuits: Analysis and Design*, Cengage Learning, 2010 ISBN 0-495-66772-2, page 804

## External links



- Split-Supply Analog Filter Expert (<http://www-k.ext.ti.com/SRV/S/Data/ti/KnowledgeBases/analog/document/faqs/spexpert.htm>)
- Single-Supply Analog Filter Expert (<http://www-k.ext.ti.com/SRV/S/Data/ti/KnowledgeBases/analog/document/faqs/ssexpert.htm>)
- Introduction to active filters (<http://www.swarthmore.edu/NatSci/echeeve1/Ref/FilterBkgrnd/Filter.html>)
- Lacanette, Kerry (April 21, 2010), *A Basic Introduction to Filters—Active, Passive, and Switched-Capacitor* (<http://www.national.com/an/AN/AN-779.pdf>), Application Note, National Semiconductor, AN-779
- Active filter design - related articles (<http://www.postreh.com/vmichal/articles/articles.htm>)
- Analog Filter Wizard (<http://www.analog.com/filterwizard>): Design tool for active filters

## High-pass filter

This article is about an electronic component. For the Australian band, see High Pass Filter (band).

A **high-pass filter** (HPF) is an electronic filter that passes high-frequency signals but attenuates (reduces the amplitude of) signals with frequencies lower than the cutoff frequency. The actual amount of attenuation for each frequency varies from filter to filter. A high-pass filter is usually modeled as a linear time-invariant system. It is sometimes called a **low-cut filter** or **bass-cut filter**. High-pass filters have many uses, such as blocking DC from circuitry sensitive to non-zero average voltages or RF devices. They can also be used in conjunction with a low-pass filter to make a bandpass filter.

### First-order continuous-time implementation

The simple first-order electronic high-pass filter shown in Figure 1 is implemented by placing an input voltage across the series combination of a capacitor and a resistor and using the voltage across the resistor as an output. The product of the resistance and capacitance ( $R \times C$ ) is the time constant ( $\tau$ ); it is inversely proportional to the cutoff frequency  $f_c$ , that is,

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC},$$

where  $f_c$  is in hertz,  $\tau$  is in seconds,  $R$  is in ohms, and  $C$  is in farads.

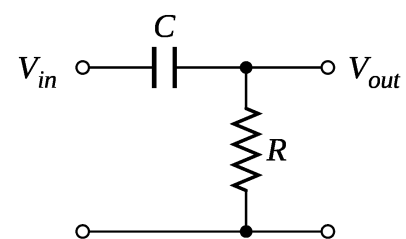


Figure 1: A passive, analog, first-order high-pass filter, realized by an RC circuit

Figure 2 shows an active electronic implementation of a first-order high-pass filter using an operational amplifier. In this case, the filter has a passband gain of  $-R_2/R_1$  and has a corner frequency of

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi R_1 C},$$

Because this filter is active, it may have non-unity passband gain. That is, high-frequency signals are inverted and amplified by  $R_2/R_1$ .

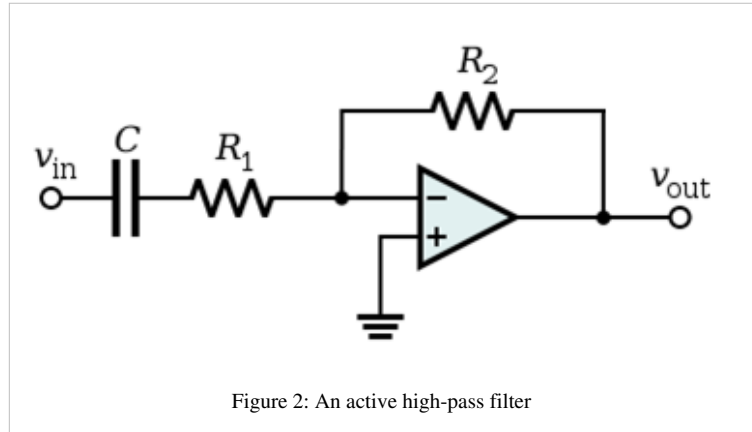


Figure 2: An active high-pass filter

## Discrete-time realization

For another method of conversion from continuous- to discrete-time, see Bilinear transform.

Discrete-time high-pass filters can also be designed. Discrete-time filter design is beyond the scope of this article; however, a simple example comes from the conversion of the continuous-time high-pass filter above to a discrete-time realization. That is, the continuous-time behavior can be discretized.

From the circuit in Figure 1 above, according to Kirchoff's Laws and the definition of capacitance:

$$\begin{cases} V_{\text{out}}(t) = I(t) R & \text{(V)} \\ Q_c(t) = C (V_{\text{in}}(t) - V_{\text{out}}(t)) & \text{(Q)} \\ I(t) = \frac{dQ_c}{dt} & \text{(I)} \end{cases}$$

where  $Q_c(t)$  is the charge stored in the capacitor at time  $t$ . Substituting Equation (Q) into Equation (I) and then Equation (I) into Equation (V) gives:

$$V_{\text{out}}(t) = C \overbrace{\left( \frac{dV_{\text{in}}}{dt} - \frac{dV_{\text{out}}}{dt} \right)}^{I(t)} R = RC \left( \frac{dV_{\text{in}}}{dt} - \frac{dV_{\text{out}}}{dt} \right)$$

This equation can be discretized. For simplicity, assume that samples of the input and output are taken at evenly-spaced points in time separated by  $\Delta_T$  time. Let the samples of  $V_{\text{in}}$  be represented by the sequence  $(x_1, x_2, \dots, x_n)$ , and let  $V_{\text{out}}$  be represented by the sequence  $(y_1, y_2, \dots, y_n)$  which correspond to the same points in time. Making these substitutions:

$$y_i = RC \left( \frac{x_i - x_{i-1}}{\Delta_T} - \frac{y_i - y_{i-1}}{\Delta_T} \right)$$

And rearranging terms gives the recurrence relation

$$y_i = \underbrace{\frac{RC}{RC + \Delta_T}}_{\text{Decaying contribution from prior inputs}} y_{i-1} + \underbrace{\frac{RC}{RC + \Delta_T}}_{\text{Contribution from change in input}} (x_i - x_{i-1})$$

That is, this discrete-time implementation of a simple continuous-time RC high-pass filter is

$$y_i = \alpha y_{i-1} + \alpha (x_i - x_{i-1}) \quad \text{where} \quad \alpha \triangleq \frac{RC}{RC + \Delta_T}$$

By definition,  $0 \leq \alpha \leq 1$ . The expression for parameter  $\alpha$  yields the equivalent time constant  $RC$  in terms of the sampling period  $\Delta_T$  and  $\alpha$ :

$$RC = \Delta_T \left( \frac{\alpha}{1 - \alpha} \right)$$

If  $\alpha = 0.5$ , then the  $RC$  time constant equal to the sampling period. If  $\alpha \ll 0.5$ , then  $RC$  is significantly smaller than the sampling interval, and  $RC \approx \alpha \Delta_T$ .

### Algorithmic implementation

The filter recurrence relation provides a way to determine the output samples in terms of the input samples and the preceding output. The following pseudocode algorithm will simulate the effect of a high-pass filter on a series of digital samples:

```
// Return RC high-pass filter output samples, given input samples,
// time interval dt, and time constant RC
function highpass(real[0..n] x, real dt, real RC)
  var real[0..n] y
  var real  $\alpha := RC / (RC + dt)$ 
  y[0] := x[0]
  for i from 1 to n
    y[i] :=  $\alpha * y[i-1] + \alpha * (x[i] - x[i-1])$ 
  return y
```

The loop which calculates each of the  $n$  outputs can be refactored into the equivalent:

```
for i from 1 to n
  y[i] :=  $\alpha * (y[i-1] + x[i] - x[i-1])$ 
```

However, the earlier form shows how the parameter  $\alpha$  changes the impact of the prior output  $y[i-1]$  and current change in input  $(x[i] - x[i-1])$ . In particular,

- A large  $\alpha$  implies that the output will decay very slowly but will also be strongly influenced by even small changes in input. By the relationship between parameter  $\alpha$  and time constant  $RC$  above, a large  $\alpha$  corresponds to a large  $RC$  and therefore a low corner frequency of the filter. Hence, this case corresponds to a high-pass filter with a very narrow stop band. Because it is excited by small changes and tends to hold its prior output values for a long time, it can pass relatively low frequencies. However, a constant input (i.e., an input with  $(x[i] - x[i-1]) = 0$ ) will always decay to zero, as would be expected with a high-pass filter with a large  $RC$ .
- A small  $\alpha$  implies that the output will decay quickly and will require large changes in the input (i.e.,  $(x[i] - x[i-1])$  is large) to cause the output to change much. By the relationship between parameter  $\alpha$  and time constant  $RC$  above, a small  $\alpha$  corresponds to a small  $RC$  and therefore a high corner frequency of the filter. Hence, this case corresponds to a high-pass filter with a very wide stop band. Because it requires large (i.e., fast) changes and tends to quickly forget its prior output values, it can only pass relatively high frequencies, as would be expected with a high-pass filter with a small  $RC$ .

## Applications

### Audio

High-pass filters have many applications. They are used as part of an audio crossover to direct high frequencies to a tweeter while attenuating bass signals which could interfere with, or damage, the speaker. When such a filter is built into a loudspeaker cabinet it is normally a passive filter that also includes a low-pass filter for the woofer and so often employs both a capacitor and inductor (although very simple high-pass filters for tweeters can consist of a series capacitor and nothing else). As an example, the formula above, applied to a tweeter with  $R=10$  Ohm, will determine the capacitor value for a cut-off frequency of 5 kHz.

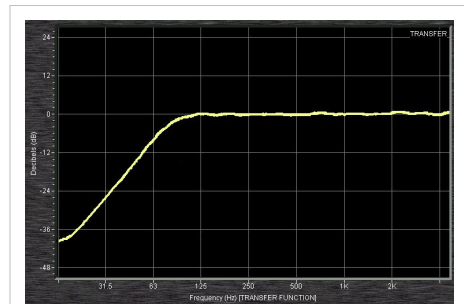
$$C = \frac{1}{2\pi f R} = \frac{1}{6.28 \times 5000 \times 10} = 3.18 \times 10^{-6}, \text{ or approx } 3.2 \mu\text{F}.$$

An alternative, which provides good quality sound without inductors (which are prone to parasitic coupling, are expensive, and may have significant internal resistance) is to employ bi-amplification with active RC filters or active digital filters with separate power amplifiers for each loudspeaker. Such low-current and low-voltage line level crossovers are called active crossovers.

Rumble filters are high-pass filters applied to the removal of unwanted sounds near to the lower end of the audible range or below. For example, noises (e.g., footsteps, or motor noises from record players and tape decks) may be removed because they are undesired or may overload the RIAA equalization circuit of the preamp.

High-pass filters are also used for AC coupling at the inputs of many audio power amplifiers, for preventing the amplification of DC currents which may harm the amplifier, rob the amplifier of headroom, and generate waste heat at the loudspeakers voice coil. One amplifier, the professional audio model DC300 made by Crown International beginning in the 1960s, did not have high-pass filtering at all, and could be used to amplify the DC signal of a common 9-volt battery at the input to supply 18 volts DC in an emergency for mixing console power. However, that model's basic design has been superseded by newer designs such as the Crown Macro-Tech series developed in the late 1980s which included 10 Hz high-pass filtering on the inputs and switchable 35 Hz high-pass filtering on the outputs. Another example is the QSC Audio PLX amplifier series which includes an internal 5 Hz high-pass filter which is applied to the inputs whenever the optional 50 and 30 Hz high-pass filters are turned off.

Mixing consoles often include high-pass filtering at each channel strip. Some models have fixed-slope, fixed-frequency high-pass filters at 80 or 100 Hz that can be engaged; other models have 'sweepable HPF'—a high-pass filter of fixed slope that can be set within a specified frequency range, such as from 20 to 400 Hz on the Midas Heritage 3000, or 20 to 20,000 Hz on the Yamaha M7CL digital mixing console. Veteran systems engineer and live sound mixer Bruce Main recommends that high-pass filters be engaged for most mixer input sources, except for those such as kick drum, bass guitar and piano, sources which will have useful low frequency sounds. Main writes that DI unit inputs (as opposed to microphone inputs) do not need high-pass filtering as they are not subject to modulation by low-frequency stage wash—low frequency sounds coming from the subwoofers or the public address system and wrapping around to the stage. Main indicates that high-pass filters are commonly used for directional microphones which have a proximity effect—a low-frequency boost for very close sources. This low frequency boost commonly causes problems up to 200 or 300 Hz, but Main notes that he has seen microphones that benefit from a 500 Hz HPF setting on the console.



A 75 Hz "low cut" filter from an input channel of a Mackie 1402 mixing console as measured by Smaart software. This high-pass filter has a slope of 18 dB per octave.

## Image

High-pass and low-pass filters are also used in digital image processing to perform image modifications, enhancements, noise reduction, etc., using designs done in either the spatial domain or the frequency domain.

A high-pass filter, if the imaging software does not have one, can be done by duplicating the layer, putting a gaussian blur, inverting, and then blending with the original layer using an opacity (say 50%) with the original layer.

The unsharp masking, or sharpening, operation used in image editing software is a high-boost filter, a generalization of high-pass.



Example of high-pass filter applied to the right half of a photograph. Left side is unmodified, Right side is with a high-pass filter applied (in this case, with a radius of 4.9)

## References

## External links



Wikimedia Commons has media related to *Highpass filters*.

- Common Impulse Responses (<http://www.dspguide.com/ch7/1.htm>)
- ECE 209: Review of Circuits as LTI Systems ([http://www.tedpavlic.com/teaching/osu/ece209/support/circuits\\_sys\\_review.pdf](http://www.tedpavlic.com/teaching/osu/ece209/support/circuits_sys_review.pdf)), a short primer on the mathematical analysis of (electrical) LTI systems.
- ECE 209: Sources of Phase Shift ([http://www.tedpavlic.com/teaching/osu/ece209/lab3\\_opamp\\_FO/lab3\\_opamp\\_FO\\_phase\\_shift.pdf](http://www.tedpavlic.com/teaching/osu/ece209/lab3_opamp_FO/lab3_opamp_FO_phase_shift.pdf)), an intuitive explanation of the source of phase shift in a high-pass filter. Also verifies simple passive LPF transfer function by means of trigonometric identity.

# Low-pass filter

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A **low-pass filter** is a filter that passes signals with a frequency lower than a certain cutoff frequency and attenuates signals with frequencies higher than the cutoff frequency. The amount of attenuation for each frequency depends on the filter design. The filter is sometimes called a **high-cut filter**, or **treble cut filter** in audio applications. A low-pass filter is the opposite of a high-pass filter. A band-pass filter is a combination of a low-pass and a high-pass filter.

Low-pass filters exist in many different forms, including electronic circuits (such as a *hiss filter* used in audio), anti-aliasing filters for conditioning signals prior to analog-to-digital conversion, digital filters for smoothing sets of data, acoustic barriers, blurring of images, and so on. The moving average operation used in fields such as finance is a particular kind of low-pass filter, and can be analyzed with the same signal processing techniques as are used for other low-pass filters. Low-pass filters provide a smoother form of a signal, removing the short-term fluctuations, and leaving the longer-term trend.

An optical filter can correctly be called a low-pass filter, but conventionally is called a *longpass* filter (low frequency is long wavelength), to avoid confusion.

## Examples

### Acoustics

A stiff physical barrier tends to reflect higher sound frequencies, and so acts as a low-pass filter for transmitting sound. When music is playing in another room, the low notes are easily heard, while the high notes are attenuated.

### Electronics

In an electronic low-pass RC filter for voltage signals, high frequencies in the input signal are attenuated, but the filter has little attenuation below the cutoff frequency determined by its RC time constant. For current signals, a similar circuit, using a resistor and capacitor in parallel, works in a similar manner. (See current divider discussed in more detail below.)

Electronic low-pass filters are used on inputs to subwoofers and other types of loudspeakers, to block high pitches that they can't efficiently reproduce. Radio transmitters use low-pass filters to block harmonic emissions that might interfere with other communications. The tone knob on many electric guitars is a low-pass filter used to reduce the amount of treble in the sound. An integrator is another time constant low-pass filter.

Telephone lines fitted with DSL splitters use low-pass and high-pass filters to separate DSL and POTS signals sharing the same pair of wires.

Low-pass filters also play a significant role in the sculpting of sound created by analogue and virtual analogue synthesisers. *See subtractive synthesis.*

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## Ideal and real filters

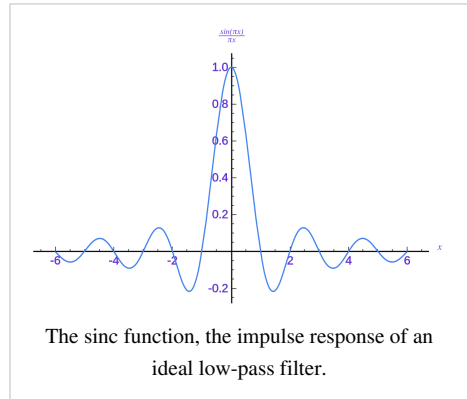
An ideal low-pass filter completely eliminates all frequencies above the cutoff frequency while passing those below unchanged; its frequency response is a rectangular function and is a brick-wall filter. The transition region present in practical filters does not exist in an ideal filter. An ideal low-pass filter can be realized mathematically (theoretically) by multiplying a signal by the rectangular function in the frequency domain or, equivalently, convolution with its impulse response, a sinc function, in the time domain.

However, the ideal filter is impossible to realize without also having signals of infinite extent in time, and so generally needs to be approximated for real ongoing signals, because the sinc function's support region extends to all past and future times. The filter would therefore need to have infinite delay, or knowledge of the infinite future and past, in order to perform the convolution. It is effectively realizable for pre-recorded digital signals by assuming extensions of zero into the past and future, or more typically by making the signal repetitive and using Fourier analysis.

Real filters for real-time applications approximate the ideal filter by truncating and windowing the infinite impulse response to make a finite impulse response; applying that filter requires delaying the signal for a moderate period of time, allowing the computation to "see" a little bit into the future. This delay is manifested as phase shift. Greater accuracy in approximation requires a longer delay.

An ideal low-pass filter results in ringing artifacts via the Gibbs phenomenon. These can be reduced or worsened by choice of windowing function, and the design and choice of real filters involves understanding and minimizing these artifacts. For example, "simple truncation [of sinc] causes severe ringing artifacts," in signal reconstruction, and to reduce these artifacts one uses window functions "which drop off more smoothly at the edges."<sup>[1]</sup>

The Whittaker–Shannon interpolation formula describes how to use a perfect low-pass filter to reconstruct a continuous signal from a sampled digital signal. Real digital-to-analog converters use real filter approximations.



## Continuous-time low-pass filters

There are many different types of filter circuits, with different responses to changing frequency. The frequency response of a filter is generally represented using a Bode plot, and the filter is characterized by its cutoff frequency and rate of frequency rolloff. In all cases, at the *cutoff frequency*, the filter attenuates the input power by half or 3 dB. So the **order** of the filter determines the amount of additional attenuation for frequencies higher than the cutoff frequency.

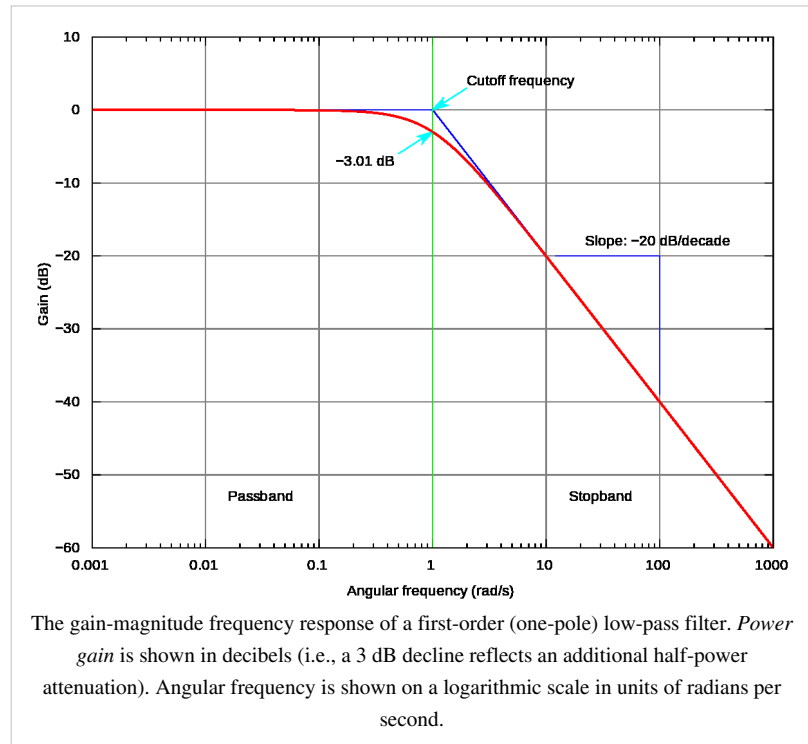
- A **first-order filter**, for example, reduces the signal amplitude by half (so power reduces by a factor of 4), or 6 dB, every time the frequency doubles (goes up one octave); more

precisely, the power rolloff approaches 20 dB per decade in the limit of high frequency. The magnitude Bode plot for a first-order filter looks like a horizontal line below the cutoff frequency, and a diagonal line above the cutoff frequency. There is also a "knee curve" at the boundary between the two, which smoothly transitions between the two straight line regions. If the transfer function of a first-order low-pass filter has a zero as well as a pole, the Bode plot flattens out again, at some maximum attenuation of high frequencies; such an effect is caused for example by a little bit of the input leaking around the one-pole filter; this one-pole–one-zero filter is still a first-order low-pass. See *Pole–zero plot and RC circuit*.

- A **second-order filter** attenuates higher frequencies more steeply. The Bode plot for this type of filter resembles that of a first-order filter, except that it falls off more quickly. For example, a second-order Butterworth filter reduces the signal amplitude to one fourth its original level every time the frequency doubles (so power decreases by 12 dB per octave, or 40 dB per decade). Other all-pole second-order filters may roll off at different rates initially depending on their Q factor, but approach the same final rate of 12 dB per octave; as with the first-order filters, zeroes in the transfer function can change the high-frequency asymptote. See RLC circuit.
- Third- and higher-order filters are defined similarly. In general, the final rate of power rolloff for an order-  $n$  all-pole filter is  $6n$  dB per octave (i.e.,  $20n$  dB per decade).

On any Butterworth filter, if one extends the horizontal line to the right and the diagonal line to the upper-left (the asymptotes of the function), they intersect at exactly the *cutoff frequency*. The frequency response at the cutoff frequency in a first-order filter is 3 dB below the horizontal line. The various types of filters (Butterworth filter, Chebyshev filter, Bessel filter, etc.) all have different-looking *knee curves*. Many second-order filters have "peaking" or resonance that puts their frequency response at the cutoff frequency *above* the horizontal line. Furthermore, the actual frequency where this peaking occurs can be predicted without calculus, as shown by Cartwright<sup>[2]</sup> et al. For third-order filters, the peaking and its frequency of occurrence can too be predicted without calculus as recently shown by Cartwright et al. See *electronic filter for other types*.

The meanings of 'low' and 'high'—that is, the cutoff frequency—depend on the characteristics of the filter. The term "low-pass filter" merely refers to the shape of the filter's response; a high-pass filter could be built that cuts off at a





lower frequency than any low-pass filter—it is their responses that set them apart. Electronic circuits can be devised for any desired frequency range, right up through microwave frequencies (above 1 GHz) and higher.

## Laplace notation

Continuous-time filters can also be described in terms of the Laplace transform of their impulse response, in a way that lets all characteristics of the filter be easily analyzed by considering the pattern of poles and zeros of the Laplace transform in the complex plane. (In discrete time, one can similarly consider the Z-transform of the impulse response.)

For example, a first-order low-pass filter can be described in Laplace notation as:

$$\frac{\text{Output}}{\text{Input}} = K \frac{1}{1 + s\tau}$$

where  $s$  is the Laplace transform variable,  $\tau$  is the filter time constant, and  $K$  is the filter passband gain.

## Electronic low-pass filters

### Passive electronic realization

One simple low-pass filter circuit consists of a resistor in series with a load, and a capacitor in parallel with the load. The capacitor exhibits reactance, and blocks low-frequency signals, forcing them through the load instead. At higher frequencies the reactance drops, and the capacitor effectively functions as a short circuit. The combination of resistance and capacitance gives the time constant of the filter  $\tau = RC$  (represented by the Greek letter tau). The break frequency, also called the turnover frequency or cutoff frequency (in hertz), is determined by the time constant:

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$$

or equivalently (in radians per second):

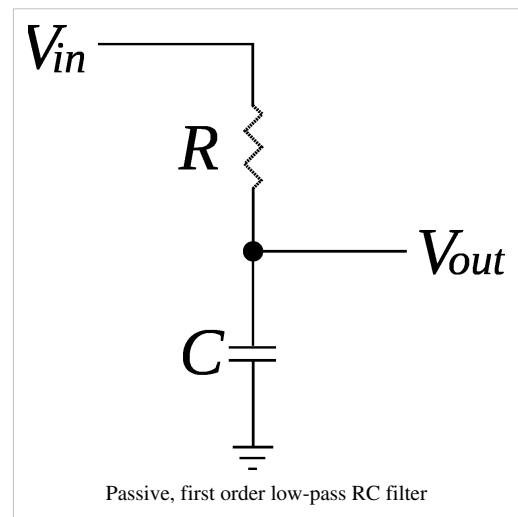
$$\omega_c = \frac{1}{\tau} = \frac{1}{RC}$$

This circuit may be understood by considering the time the capacitor needs to charge or discharge through the resistor:

- At low frequencies, there is plenty of time for the capacitor to charge up to practically the same voltage as the input voltage.
- At high frequencies, the capacitor only has time to charge up a small amount before the input switches direction. The output goes up and down only a small fraction of the amount the input goes up and down. At double the frequency, there's only time for it to charge up half the amount.

Another way to understand this circuit is through the concept of reactance at a particular frequency:

- Since direct current (DC) cannot flow through the capacitor, DC input must flow out the path marked  $V_{out}$  (analogous to removing the capacitor).
- Since alternating current (AC) flows very well through the capacitor, almost as well as it flows through solid wire, AC input flows out through the capacitor, effectively short circuiting to ground (analogous to replacing the capacitor with just a wire).



The capacitor is not an "on/off" object (like the block or pass fluidic explanation above). The capacitor variably acts between these two extremes. It is the Bode plot and frequency response that show this variability.

**Active electronic realization**

Another type of electrical circuit is an *active* low-pass filter.

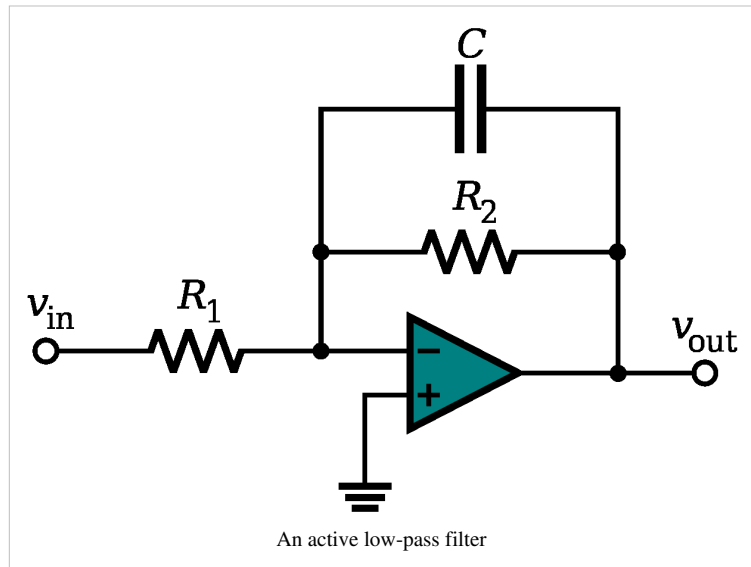
In the operational amplifier circuit shown in the figure, the cutoff frequency (in hertz) is defined as:

$$f_c = \frac{1}{2\pi R_2 C}$$

or equivalently (in radians per second):

$$\omega_c = \frac{1}{R_2 C}$$

The gain in the passband is  $-R_2/R_1$ , and the stopband drops off at  $-6$  dB per octave (that is  $-20$  dB per decade) as it is a first-order filter.



**Discrete-time realization**

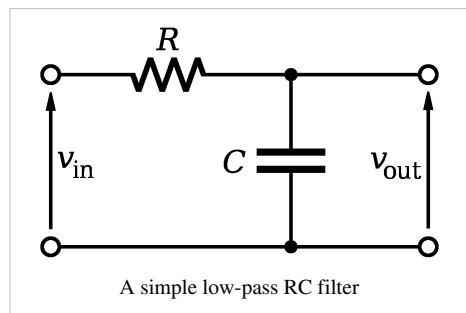
For another method of conversion from continuous- to discrete-time, see Bilinear transform.

Many digital filters are designed to give low-pass characteristics. Both infinite impulse response and finite impulse response low pass filters as well as filters using fourier transforms are widely used.

**Simple infinite impulse response filter**

The effect of an infinite impulse response low-pass filter can be simulated on a computer by analyzing an RC filter's behavior in the time domain, and then discretizing the model.

From the circuit diagram to the right, according to Kirchoff's Laws and the definition of capacitance:



$$v_{in}(t) - v_{out}(t) = R i(t) \quad (V)$$

$$Q_c(t) = C v_{out}(t) \quad (Q)$$

$$i(t) = \frac{dQ_c}{dt} \quad (I)$$

where  $Q_c(t)$  is the charge stored in the capacitor at time  $t$ . Substituting equation **Q** into equation **I** gives  $i(t) = C \frac{dv_{\text{out}}}{dt}$ , which can be substituted into equation **V** so that:

$$v_{\text{in}}(t) - v_{\text{out}}(t) = RC \frac{dv_{\text{out}}}{dt}$$

This equation can be discretized. For simplicity, assume that samples of the input and output are taken at evenly-spaced points in time separated by  $\Delta_T$  time. Let the samples of  $v_{\text{in}}$  be represented by the sequence  $(x_1, x_2, \dots, x_n)$ , and let  $v_{\text{out}}$  be represented by the sequence  $(y_1, y_2, \dots, y_n)$ , which correspond to the same points in time. Making these substitutions:

$$x_i - y_i = RC \frac{y_i - y_{i-1}}{\Delta_T}$$

And rearranging terms gives the recurrence relation

$$y_i = \underbrace{x_i \left( \frac{\Delta_T}{RC + \Delta_T} \right)}_{\text{Input contribution}} + \underbrace{y_{i-1} \left( \frac{RC}{RC + \Delta_T} \right)}_{\text{Inertia from previous output}}.$$

That is, this discrete-time implementation of a simple RC low-pass filter is the exponentially-weighted moving average

$$y_i = \alpha x_i + (1 - \alpha) y_{i-1} \quad \text{where} \quad \alpha \triangleq \frac{\Delta_T}{RC + \Delta_T}$$

By definition, the *smoothing factor*  $0 \leq \alpha \leq 1$ . The expression for  $\alpha$  yields the equivalent time constant  $RC$  in terms of the sampling period  $\Delta_T$  and smoothing factor  $\alpha$ :

$$RC = \Delta_T \left( \frac{1 - \alpha}{\alpha} \right)$$

If  $\alpha = 0.5$ , then the  $RC$  time constant is equal to the sampling period. If  $\alpha \ll 0.5$ , then  $RC$  is significantly larger than the sampling interval, and  $\Delta_T \approx \alpha RC$ .

The filter recurrence relation provides a way to determine the output samples in terms of the input samples and the preceding output. The following pseudocode algorithm simulates the effect of a low-pass filter on a series of digital samples:

```
// Return RC low-pass filter output samples, given input samples,
// time interval dt, and time constant RC
function lowpass(real[0..n] x, real dt, real RC)
  var real[0..n] y
  var real  $\alpha := dt / (RC + dt)$ 
  y[0] := x[0]
  for i from 1 to n
    y[i] :=  $\alpha * x[i] + (1-\alpha) * y[i-1]$ 
  return y
```

The loop that calculates each of the  $n$  outputs can be refactored into the equivalent:

```
for i from 1 to n
  y[i] := y[i-1] +  $\alpha * (x[i] - y[i-1])$ 
```

That is, the change from one filter output to the next is proportional to the difference between the previous output and the next input. This exponential smoothing property matches the exponential decay seen in the continuous-time system. As expected, as the time constant  $RC$  increases, the discrete-time smoothing parameter  $\alpha$  decreases, and the output samples  $(y_1, y_2, \dots, y_n)$  respond more slowly to a change in the input samples  $(x_1, x_2, \dots, x_n)$ ; the system has more *inertia*. This filter is an infinite-impulse-response (IIR) single-pole low-pass filter.

### Finite impulse response

Finite-impulse-response filters can be built that approximate to the sinc function time-domain response of an ideal sharp-cutoff low-pass filter. In practice, the time-domain response must be time truncated and is often of a simplified shape; in the simplest case, a running average can be used, giving a square time response.<sup>[3]</sup>

## References

- [1] Mastering Windows: Improving Reconstruction (<http://www.cg.tuwien.ac.at/research/vis/vismed/Windows/MasteringWindows.pdf>)
- [2] K. V. Cartwright, P. Russell and E. J. Kaminsky, "Finding the maximum magnitude response (gain) of second-order filters without calculus ([http://www.lajpe.org/dec2012/8\\_LAJPE\\_703\\_Kenneth\\_Cartwright\\_preprint\\_corr\\_f.pdf](http://www.lajpe.org/dec2012/8_LAJPE_703_Kenneth_Cartwright_preprint_corr_f.pdf))," Lat. Am. J. Phys. Educ. Vol. 6, No. 4, pp. 559-565, 2012.
- [3] Signal recovery from noise in electronic instrumentation – T H Whilmshurst

## External links

- Low-pass filter ([http://www.allaboutcircuits.com/vol\\_2/chpt\\_8/2.html](http://www.allaboutcircuits.com/vol_2/chpt_8/2.html))
- Low Pass Filter java simulator ([http://www.st-andrews.ac.uk/~www\\_pa/Scots\\_Guide/experiment/lowpass/lpf.html](http://www.st-andrews.ac.uk/~www_pa/Scots_Guide/experiment/lowpass/lpf.html))
- ECE 209: Review of Circuits as LTI Systems ([http://www.tedpavlic.com/teaching/osu/ece209/support/circuits\\_sys\\_review.pdf](http://www.tedpavlic.com/teaching/osu/ece209/support/circuits_sys_review.pdf)), a short primer on the mathematical analysis of (electrical) LTI systems.
- ECE 209: Sources of Phase Shift ([http://www.tedpavlic.com/teaching/osu/ece209/lab3\\_opamp\\_FO/lab3\\_opamp\\_FO\\_phase\\_shift.pdf](http://www.tedpavlic.com/teaching/osu/ece209/lab3_opamp_FO/lab3_opamp_FO_phase_shift.pdf)), an intuitive explanation of the source of phase shift in a low-pass filter. Also verifies simple passive LPF transfer function by means of trigonometric identity.

# Band-pass filter

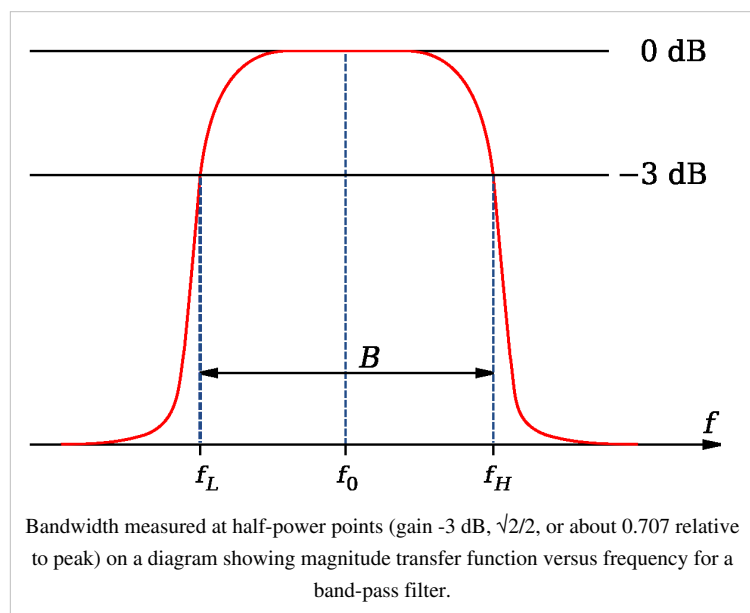
A **band-pass filter** is a device that passes frequencies within a certain range and rejects (attenuates) frequencies outside that range.

## Description

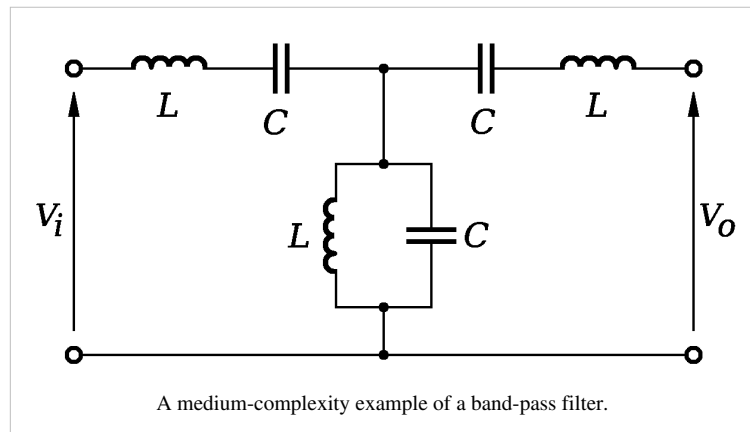
An example of an analogue electronic band-pass filter is an RLC circuit (a resistor–inductor–capacitor circuit). These filters can also be created by combining a low-pass filter with a high-pass filter.

*Bandpass* is an adjective that describes a type of filter or filtering process; it is to be distinguished from passband, which refers to the actual portion of affected spectrum.

Hence, one might say "A dual bandpass filter has two passbands." A *bandpass signal* is a signal containing a band of frequencies not adjacent to zero frequency, such as a signal that comes out of a bandpass filter.



An ideal bandpass filter would have a completely flat passband (e.g. with no gain/attenuation throughout) and would completely attenuate all frequencies outside the passband. Additionally, the transition out of the passband would be instantaneous in frequency. In practice, no bandpass filter is ideal. The filter does not attenuate all frequencies outside the desired frequency range completely; in particular, there is a region just outside the intended passband where frequencies are attenuated, but not rejected. This is known as the filter roll-off, and it is usually expressed in dB of attenuation per octave or decade of frequency. Generally, the design of a filter seeks to make the roll-off as narrow as possible, thus allowing the filter to perform as close as possible to its intended design. Often, this is achieved at the expense of pass-band or stop-band *ripple*.



The bandwidth of the filter is simply the difference between the upper and lower cutoff frequencies. The shape factor is the ratio of bandwidths measured using two different attenuation values to determine the cutoff frequency, e.g., a shape factor of 2:1 at 30/3 dB means the bandwidth measured between frequencies at 30 dB attenuation is twice that measured between frequencies at 3 dB attenuation.

Optical band-pass filters are common in photography and theatre lighting work. These filters take the form of a transparent coloured film or sheet.

## Q-factor

A band-pass filter can be characterised by its Q-factor. The Q-factor is the inverse of the fractional bandwidth. A high-Q filter will have a narrow passband and a low-Q filter will have a wide passband. These are respectively referred to as narrow-band and wide-band filters.

## Applications

Bandpass filters are widely used in wireless transmitters and receivers. The main function of such a filter in a transmitter is to limit the bandwidth of the output signal to the band allocated for the transmission. This prevents the transmitter from interfering with other stations. In a receiver, a bandpass filter allows signals within a selected range of frequencies to be heard or decoded, while preventing signals at unwanted frequencies from getting through. A bandpass filter also optimizes the signal-to-noise ratio and sensitivity of a receiver.

In both transmitting and receiving applications, well-designed bandpass filters, having the optimum bandwidth for the mode and speed of communication being used, maximize the number of signal transmitters that can exist in a system, while minimizing the interference or competition among signals.

Outside of electronics and signal processing, one example of the use of band-pass filters is in the atmospheric sciences. It is common to band-pass filter recent meteorological data with a period range of, for example, 3 to 10 days, so that only cyclones remain as fluctuations in the data fields.

In neuroscience, visual cortical simple cells were first shown by David Hubel and Torsten Wiesel to have response properties that resemble Gabor filters, which are band-pass.

## References

## External links



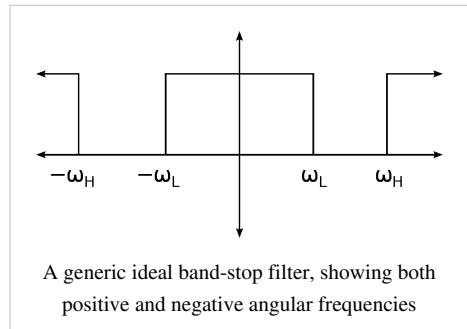
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## Band-stop filter

In signal processing, a **band-stop filter** or **band-rejection filter** is a filter that passes most frequencies unaltered, but attenuates those in a specific range to very low levels. It is the opposite of a band-pass filter. A **notch filter** is a band-stop filter with a narrow stopband (high Q factor).

Narrow notch filters (optical) are used in Raman spectroscopy, live sound reproduction (public address systems, or PA systems) and in instrument amplifiers (especially amplifiers or preamplifiers for acoustic instruments such as acoustic guitar, mandolin, bass instrument amplifier, etc.) to reduce or prevent audio feedback, while having little noticeable effect on the rest of the frequency spectrum (electronic or software filters). Other names include 'band limit filter', 'T-notch filter', 'band-elimination filter', and 'band-reject filter'.

Typically, the width of the stopband is 1 to 2 decades (that is, the highest frequency attenuated is 10 to 100 times the lowest frequency attenuated). However, in the audio band, a notch filter has high and low frequencies that may be only semitones apart.



## Examples

### In the audio domain

#### Anti-hum filter

For countries using 60 Hz power lines:

- Low Freq: 59 Hz

- High Freq: 61 Hz

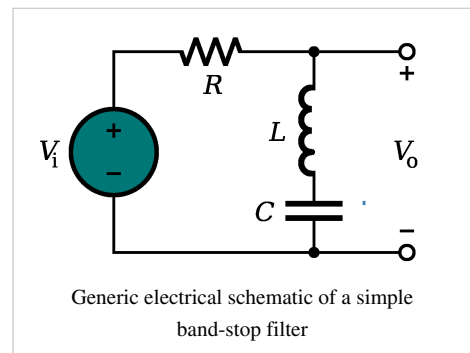
This means that the filter passes all frequencies, except for the range of 59–61 Hz. This would be used to filter out the mains hum from the 60 Hz power line, though its higher harmonics could still be present.

For countries where power transmission is at 50Hz, the filter would have a 49–51 Hz range.

#### Anti-presence filter

- Low Freq: 1 kHz
- High Freq: 4 kHz

For attenuating presence.



## **In the radio frequency (RF) domain**

### **Non-linearities of power amplifiers**

When measuring the non-linearities of power amplifiers, a very narrow notch filter can be very useful to avoid the carrier frequency. Use of the filter may ensure that the maximum input power of a spectrum analyser used to detect spurious content will not be exceeded.

### **Wave trap**

A notch filter, usually a simple LC circuit, is used to remove a specific interfering frequency. This is a technique used with radio receivers that are so close to a transmitter that it swamps all other signals. The wave trap is used to remove, or greatly reduce, the signal from the local transmitter.<sup>[1]</sup>

## **In the optical domain**

Optical notch filters rely on destructive interference.

## **Notes**

[1] Joseph J. Carr, *The technician's radio receiver handbook: wireless and telecommunication technology*, p.282, Newnes, 2001 ISBN 0-7506-7319-2.

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