Applicative Methods (3B)

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The definition of Applicative

```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```

f (a -> b) :: a **function** wrapped in f

f a :: a **value** wrapped in f

The class has a two methods :

pure brings arbitrary values into the functor

(<*>) takes a **function** wrapped in a functor f
and a **value** wrapped in a functor f
and returns the result of the application
which is also wrapped in a functor f

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Maybe instance of Applicative

```
instance Applicative Maybe where
    pure                     = Just
    (Just f) <*> (Just x)    = Just (f x)
    _        <*> _           = Nothing
```

pure wraps the value with **Just**;

(<*>) applies

the function wrapped in **Just**

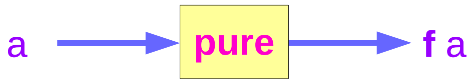
to the value wrapped in **Just** if both exist,

and results in **Nothing** otherwise.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

An Instance of the Applicative Typeclass

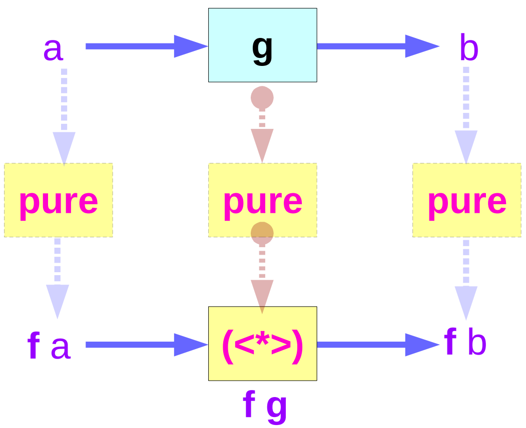
```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```



(**Functor f**) => **Applicative f**

f : Functor, Applicative

```
instance Applicative Maybe where
    pure = Just
    Nothing <*> _ = Nothing
    (Just f) <*> something = fmap f something
```



f : function in a context

(**Functor f**) => **Applicative f**

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

fmap and <*>

a bonus law about the relation between fmap and (<*>):

fmap f x = pure f <*> x   **-- fmap**

Applying a "**pure**" function with **(<*>)**

is equivalent to using **fmap**.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Left associative <*>, fmap, and <$>

pure g <*> x <*> y <*> z

fmap g x <*> y <*> z

g <$> x <*> y <*> z

g :: f (a -> b -> c -> d)
x :: f a
y :: f b
z :: f c

infix operator <$>

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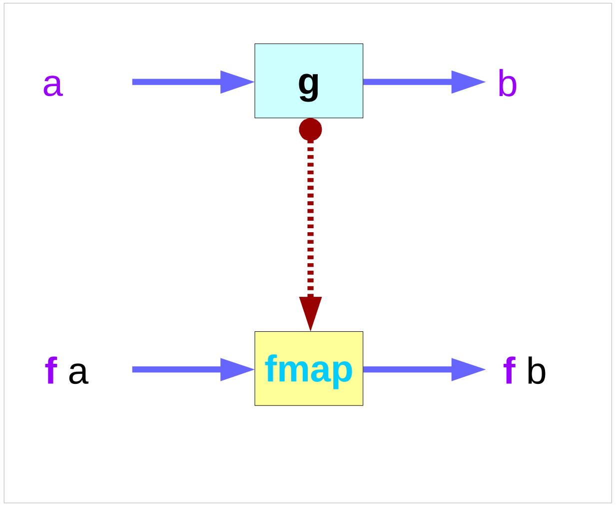
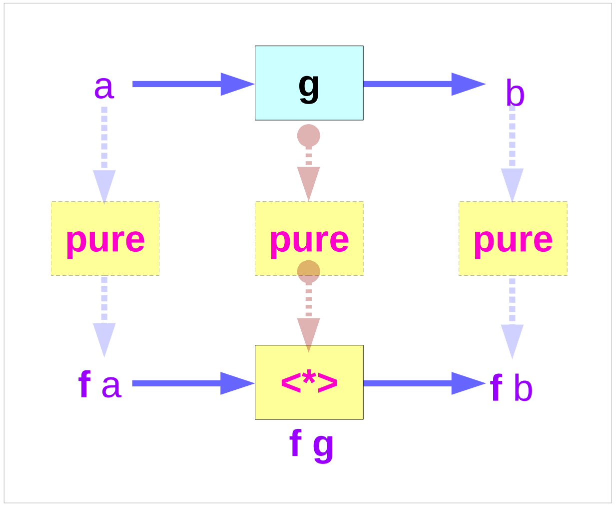
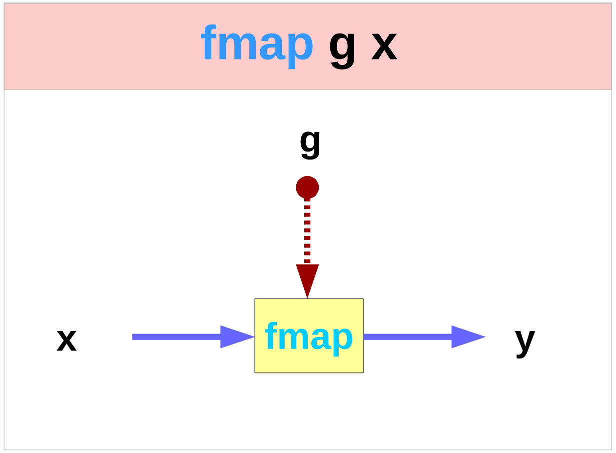
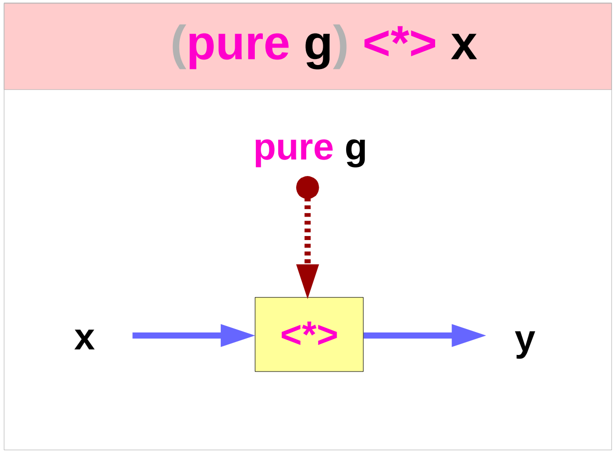




pure = f



x :: f a
y :: f b



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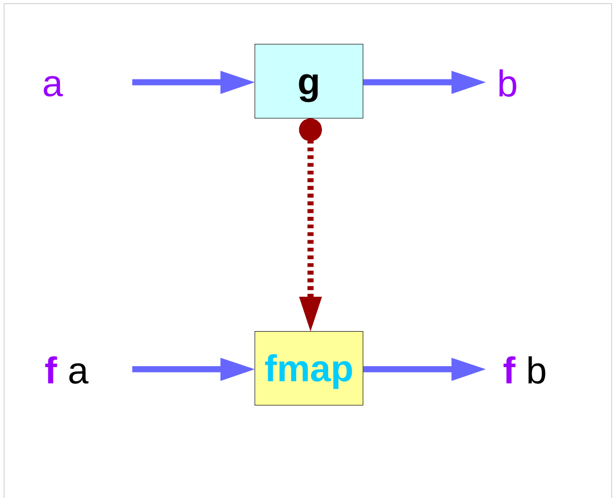
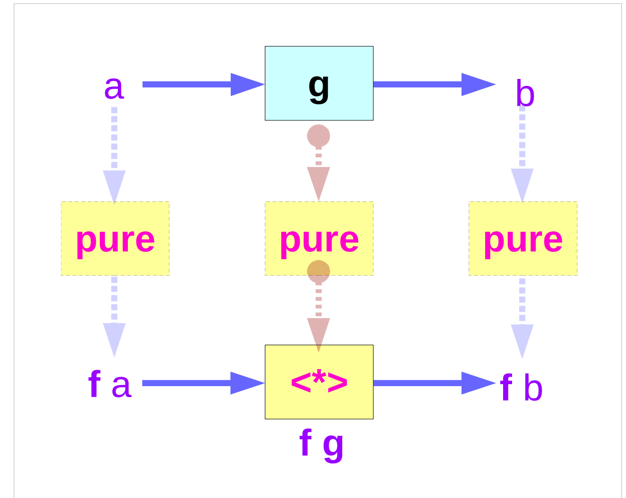
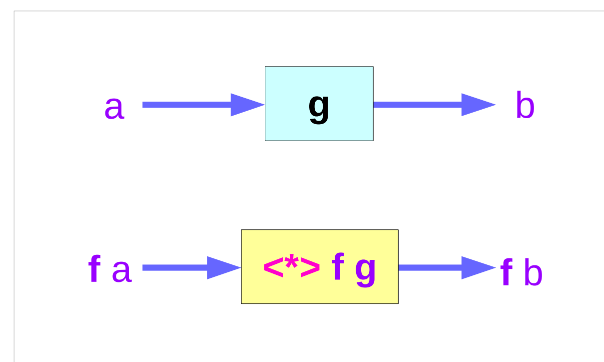
f a <*> f g





pure = f



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Left associative <*> examples

```
ghci> pure (+) <*> Just 3 <*> Just 5
Just 8
```

```
pure (+) <*> Just 3 <*> Just 5

Just (+3) <*> Just 5

Just 8
```

```
ghci> pure (+) <*> Just 3 <*> Nothing
Nothing
```

```
ghci> pure (+) <*> Nothing <*> Just 5
Nothing
```

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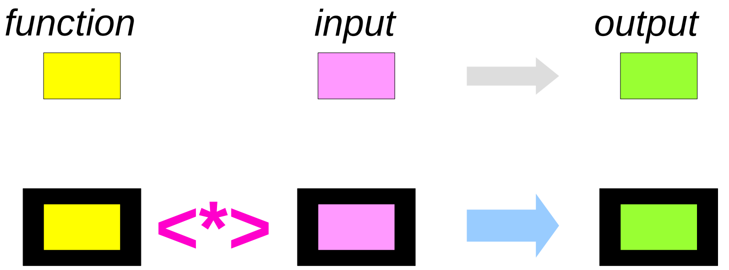
Infix Operators <*> vs <$> - overview

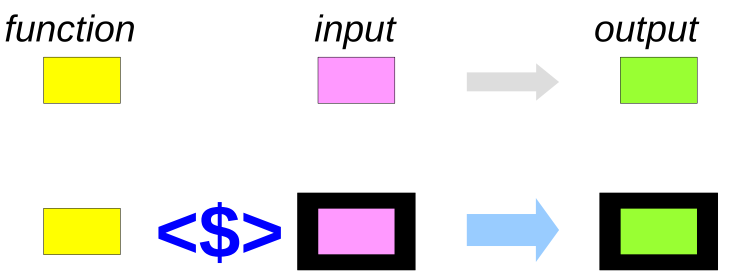
h <*> x <*> y

h :: **f** (a -> b -> c)
x :: **f** a
y :: **f** b



g <$> x <*> y

g :: (a -> b -> c)
x :: **f** a
y :: **f** b



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Infix Operators <*> vs <$> - a type view

```
h <*> x <*> y
```

```
h :: f (a -> b -> c)
x :: f a
y :: f b
```

```
h :: f (a -> b -> c)
x :: f a
h <*> x :: f (b -> c)
```

```
h :: f (a -> b -> c)
x :: f a
h <*> x :: f (b -> c)
y :: f b
h <*> x <*> y :: f c
```

```
g <$> x <*> y
```

```
g :: (a -> b -> c)
x :: f a
y :: f b
```

```
g :: (a -> b -> c)
x :: f a
g <$> x :: f (b -> c)
```

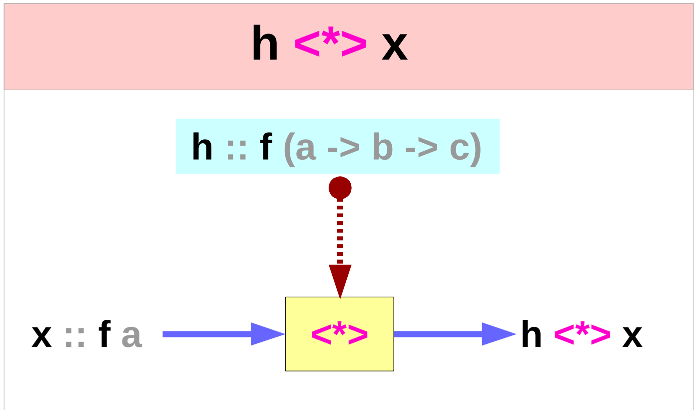
```
g :: (a -> b -> c)
x :: f a
g <$> x :: f (b -> c)
y :: f b
g <$> x <*> y :: f c
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Infix Operators <*> vs <$> - a curried function view

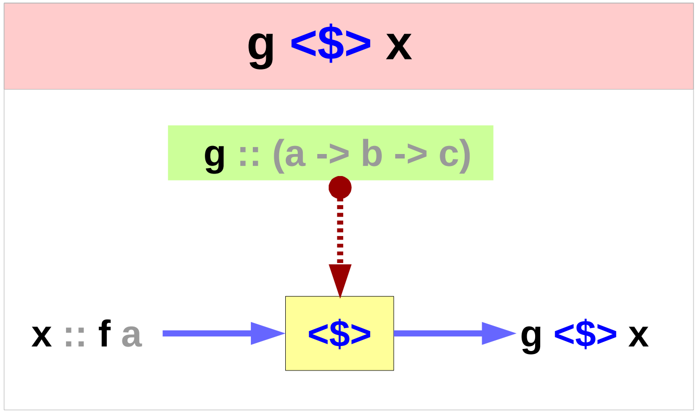
h <*> x <*> y

h :: **f** (a -> b -> c)
x :: **f** a
y :: **f** b



g <$> x <*> y

g :: (a -> b -> c)
x :: **f** a
y :: **f** b



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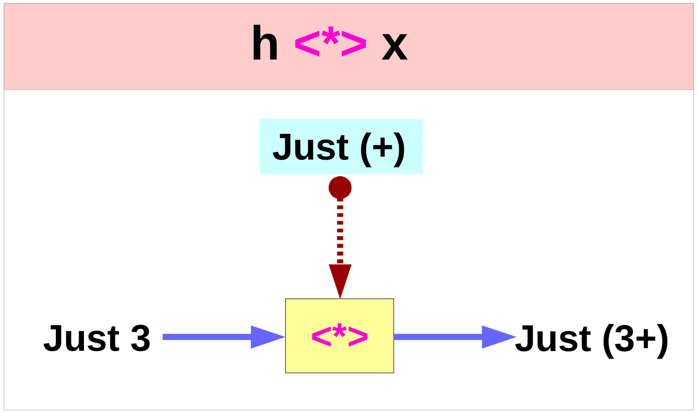
Infix Operators <*> vs <$> examples

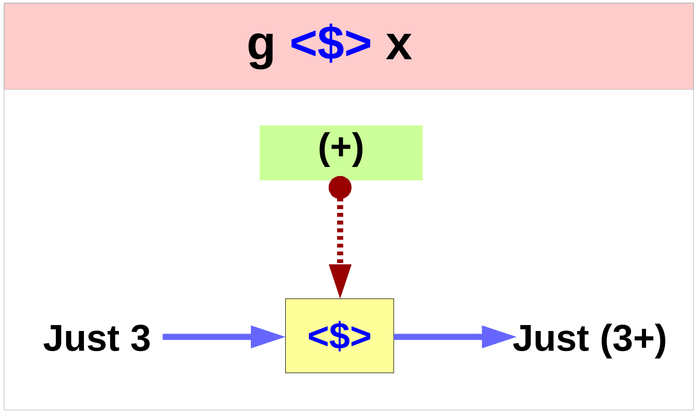
h <*> x <*> y

Just (+) <*> Just 3 <*> Just 2
Just (+3) <*> Just 2
Just 5



g <$> x <*> y

(+) <$> Just 3 <*> Just 2
Just (+3) <*> Just 2
Just 5



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

the minimal complete definition

```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```

the minimal complete definition

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

Not in the minimal complete definition

g :: a -> b,   x :: f a

```
instance Applicative Maybe where
    pure = Just
    Nothing <*> _ = Nothing
    (Just g) <*> something = fmap g something
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

The Applicative Typeclass

Applicative is a superclass of **Monad**.

every **Monad** is also a **Functor** and an **Applicative**

fmap, **pure**, **(<*>)** can all be used with **monad**s.

a **Monad** instance

requires **Functor** and **Applicative** instances.

defines the types and roles of **return** and **(>>)**

fmap :	defined in **Functors**
pure, (**<*>**) :	defined in **Applicatives**
return, (**>>**) :	defined in **Monads**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(<$>) vs ($)

(<$>) infix operator

`(<$>) :: (Functor f) => (a -> b) -> f a -> f b`

`g <$> x = fmap g x`

The $ operator is for avoiding parentheses

```
putStrLn (show (1 + 1))
putStrLn $ show (1 + 1)
putStrLn $ show $ 1 + 1          – right associative
```

(**$**) calls the <u>function</u> which is its left-hand argument of **$**

on the <u>value</u> which is its right-hand argument of **$**

The Applicative Laws

The identity law:	pure id <*> v = v	id :: a -> a v :: f a
Homomorphism:	pure g <*> pure x = pure (g x)	g :: a -> b x :: a
Interchange:	u <*> pure y = pure ($ y) <*> u	u :: f (a -> b) y :: a
Composition:	u <*> (v <*> w) = pure (.) <*> u <*> v <*> w	w :: f a v :: f (a -> b) u :: f (b -> c)
Left associative	u <*> v <*> w = (u <*> v) <*> w	u :: f (c -> b -> a) v :: f c u <*> v :: f (b -> a) w :: f b u <*> v <*> w = f a

The Identity Law

The identity law	**pure id <*> v = v**	**id :: a -> a**	**v :: f a**

pure to inject values into the functor
in a *default*, *featureless* way,
so that the result is as close as possible
to the plain value.

applying the **pure id** morphism does nothing,
exactly like with the plain **id** function.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Homomorphism Law

The homomorphism law **pure g <*> pure x = pure (g x)** **g :: a -> b** **x :: a**

applying a "**pure**" <u>function</u> to a "**pure**" <u>value</u> is the same as
applying the <u>function</u> to the <u>value</u> in the *ordinary way*
and then using **pure** on the result.
means **pure** <u>preserves</u> function application.

applying a <u>non-effectful</u> function **g**
to a <u>non-effectful</u> argument **x** in an <u>effectful</u> <u>context</u> **<u>pure</u>**
is the same as just **applying** the function **g** to the argument **x**
and then injecting the result **(f x)**
into the <u>effectual</u> <u>context</u> with **pure**.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Interchange Law

The interchange law **u <*> pure y = pure ($ y) <*> u** **u :: f (a -> b)** **y :: a**

($ **y**) is the *function* that supplies **y**
as argument to another function
– a higher order function

Function $ Argument
$ y
(**y**) as a single argument

applying a morphism **u** to a "**pure**" value **pure y**
is the same as applying **pure** ($ **y**) to the morphism **u**

Just (+3) <*> Just 2
Just ($ 2) <*> Just (+3)

when evaluating the application of
an effectful function (**u)** to a pure argument (**pure y**),
the order doesn't matter – commutative.

u :: f (a -> b) **y :: a**

u <*> pure y :: f b **pure y :: f a**

pure ($ y) <*> u :: f b **pure ($ y) :: f (a)**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law

The composition law	**pure (.) <*> u <*> v <*> w = u <*> (v <*> w)**	**w :: f a v :: f (a -> b) u :: f (b -> c)**

pure (.) <u>composes</u> morphisms similarly
to how **(.)** <u>composes</u> functions:

applying the <u>composed</u> mourphism
pure (.) <*> u <*> v *to* **w**
gives the same <u>result</u> (**u <*> (v <*> w))**
as applying **u** *to* the <u>result</u> **(v <*> w)**
of applying **v** to **w**

it is expressing a sort of <u>associativity</u> property of **(<*>)**.

w :: f a -- value
v :: f (a -> b) -- func1
u : f (b -> c) -- func2

v <*> w :: f b
u <*> (v <*> w) :: f c

pure (.) <*> u <*> v :: f (a -> c)
pure (.) <*> u <*> v <*> w :: f c

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law and Left Associativity

The composition law `pure (.) <*> u <*> v <*> w = u <*> (v <*> w)`   `w :: f a    v :: f (a -> b)    u :: f (b -> c)`

```
f (b -> c)          f (a -> b)
pure g              pure h
         pure (.)
         f (a -> c)
```

`pure (.) <*> pure g <*> pure h <*> pure x`   `(g . h)  x`

`((pure (.) <*> pure g) <*> pure h) <*> pure x`

`= pure g <*> (pure h <*> pure x)`   `g   (h  x)`

```
u = pure g :: f (b -> c)      g :: (b -> c)
v = pure h :: f (a -> b)      h :: (a -> b)
w = pure x :: f a             x :: a
```

Left associative `u <*> v <*> w = (u <*> v) <*> w`   `u :: f (c -> b -> a)    v :: f c    w :: f b`

```
u :: f (c -> b -> a)
v :: f c
u <*> v :: f (b -> a)
w :: f b
u <*> v <*> w = f a
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

liftA2

liftA2 :: **(a -> b -> c)** **->** **f a** **->** **f b** **->** **f c**

lift a binary function (**a->b->c**) to actions.

Some functors support an implementation of **liftA2**

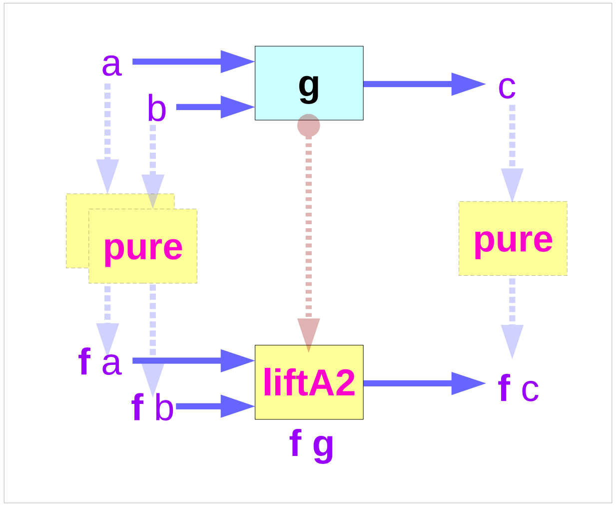
that is more efficient than the default one.

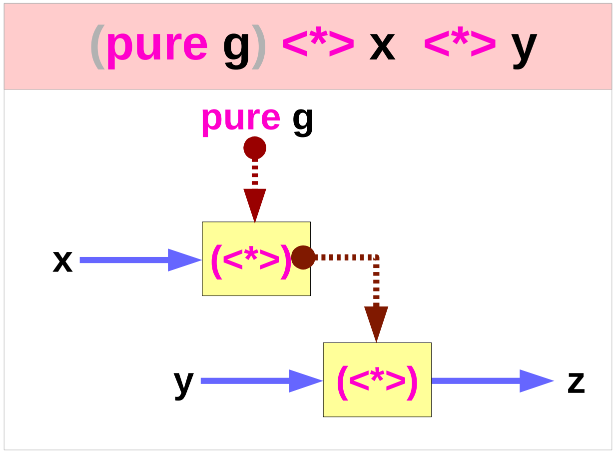
liftA2 may have an efficient implementation

whereas **fmap** is an expensive operation,

sometimes better to use **liftA2** than

to use **fmap** over the structure and then use **<*>**.

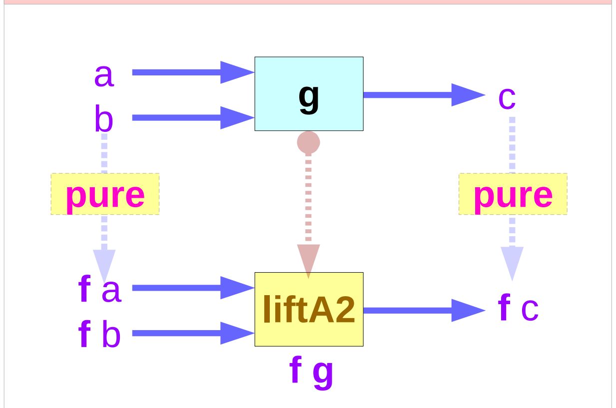




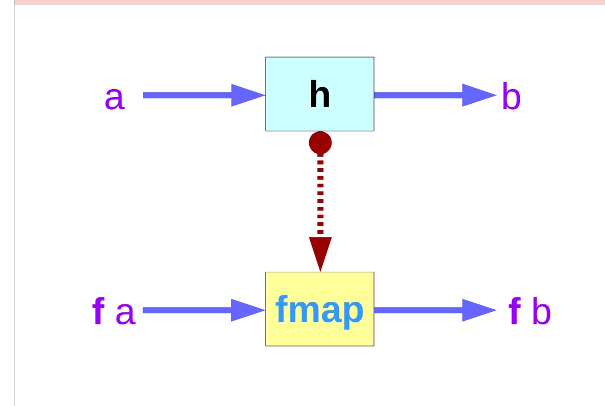
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

liftA2, <*>, fmap, <$>

liftA2 g x y



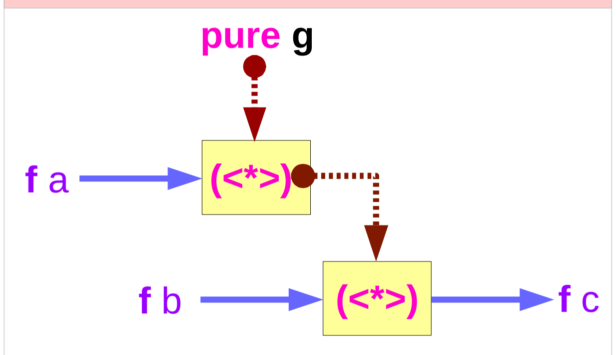
fmap h x



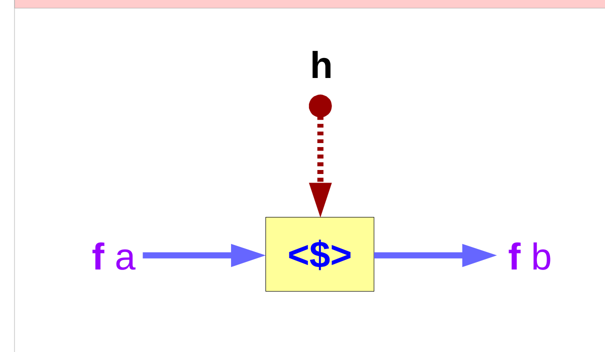
g :: a -> b -> c

h :: a -> b

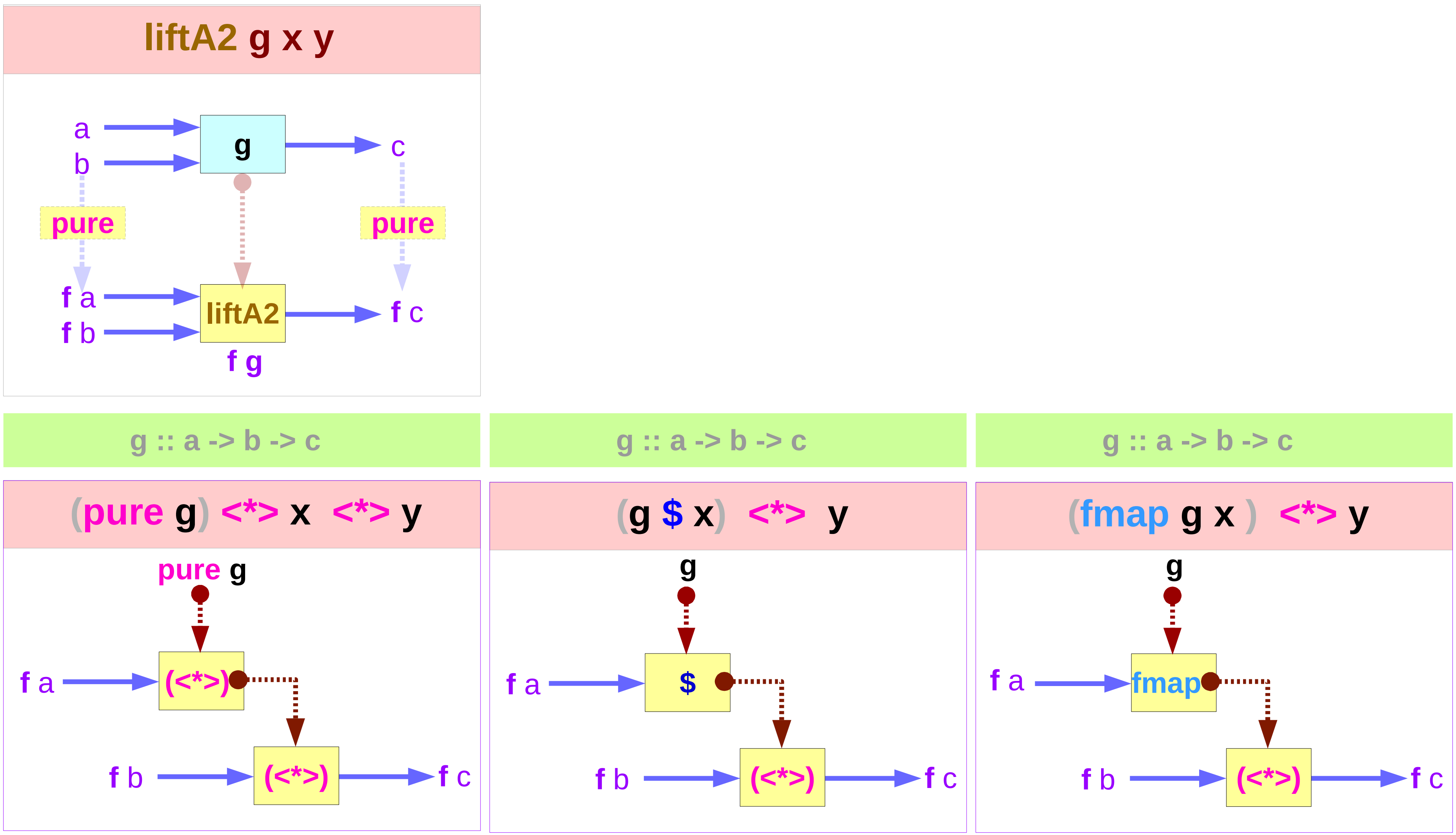
(pure g) <*> x <*> y



h <$> x



pure g <*> x <*> y equivalent



liftA2

liftA2 g x y

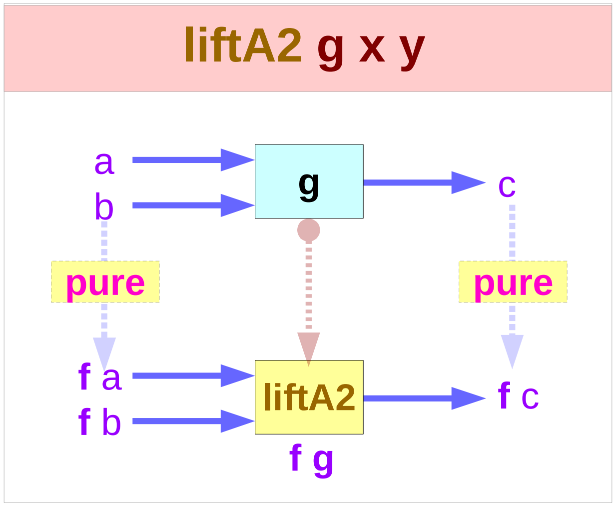
liftA2 :: (a -> b -> c) -> f a -> f b -> f c

g :: a -> b -> c

x :: f a

y :: f b

liftA2 g x y :: f c



g :: a -> b -> c

pure g <*> x <*> y

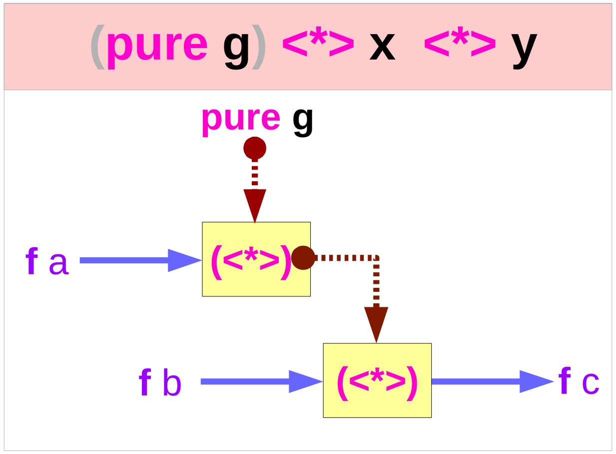
g :: a -> b -> c

x :: f a

y :: f b

z :: f c

pure g <*> x <*> y :: f c



https://wiki.haskell.org/Applicative_functor

Limitations of Functors

(a -> b -> c) -> (f a -> f b -> f c) – let fmap2 Functor as an extension of fmap

fmap :: (a -> b) -> (f a -> f b)
fmap2 :: Functor f => (a -> b -> c) -> (f a -> f b -> f c)

fmap2 h fa fb = undefined
h :: a -> b -> c
fa :: f a
fb :: f b

h :: a -> (b -> c)
fmap h :: f a -> f (b -> c)
fmap h fa :: f (b -> c) – now **f (b -> c)** must be applied to **f b**

fmap gives us a way to apply functions (**a -> b**) to values (**f a**) inside a Functor context,
but **fmap** cannot be used to apply a functions **f (b -> c)**
which are themselves in a Functor context to values **f b** in a Functor context.

http://www.openhaskell.com/lectures/applicative.html

pure, fmap, and liftA2

```
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
pure    :: a -> f a                             – fmap0 → pure
fmap    :: (a -> b) -> f a -> f b               – fmap1 → fmap
fmap2   :: (a -> b -> c) -> f a -> f b -> f c   – fmap2 → liftA2
```

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 h fa fb = (h `fmap` fa) <*> fb
liftA2 h fa fb = h <$> fa <*> fb
```

```
(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

```
liftA2 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc
```

http://www.openhaskell.com/lectures/applicative.html

liftA2 examples

liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

liftA2 (+) (Just 5) (Just 6) = Just 11

liftA2 h fa fb = (h `fmap` fa) <*> fb

liftA2 h fa fb = h <$> fa <*> fb

fmap (+) (Just 5) = Just (+5)

(+) <$> (Just 5) = Just (+5)

<*> :: Applicative f => f (a -> b) -> f a -> f b

(Just (+5)) <*> (Just 6) = Just 11

let v1 = IO (Just (+5))

let v2 = IO (Just 6)

liftA2 (<*>) v1 v2 = IO (Just 11)

https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html

<*> or liftA2 implementations

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

A minimal complete definition :

either one of the two

1) pure and **<*>**
2) pure and **liftA2**

1) pure g <*> x <*> y
2) liftA2 g x y

If it defines both, then they must behave the same

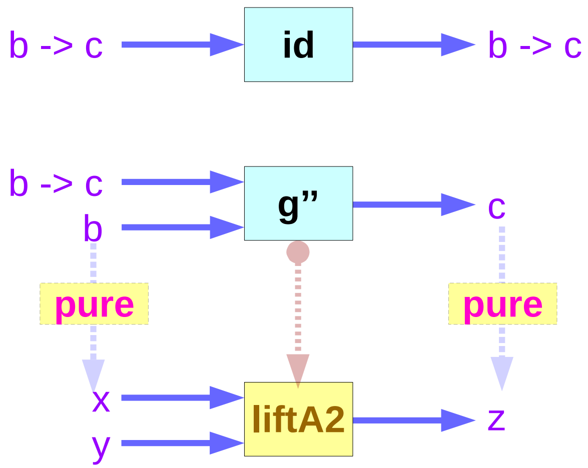
as their default definitions:

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

(<*>) = liftA2 id

liftA2 id x y = id <$> x <*> y = x <*> y

liftA2 id x y = x <*> y

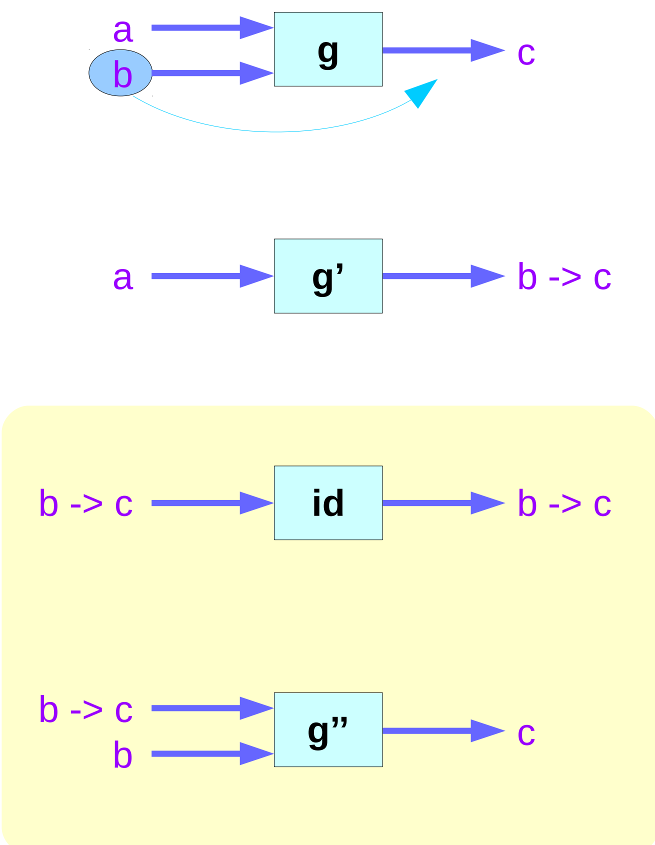


liftA2 g x y = g <$> x <*> y	g :: a -> b -> c	x :: f a	y :: f b
liftA2 g” x y = g” <$> x <*> y	g” :: (b -> c) -> b -> c	x :: f (b -> c)	y :: f b
liftA2 id x y = id <$> x <*> y = x <*> y	id :: (b -> c) -> (b -> c)	x :: f (b -> c)	y :: f b

(<*>) = liftA2 id

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

g’’ :: (b->c) -> b->c



g :: a -> b -> c

g’ :: a -> (b -> c)

id :: (b -> c) -> (b -> c)

g’’ :: (b -> c) -> (b -> c)

view the function
as having one input only

consider the case
when **a** is (**b -> c**)

Then **g’’** is the same as **id**

Results and effects in a scope

Actually, using the **liftA** commands

we can pull results of applicative functors

into a scope where we can talk

exclusively about functor results **c**

and not about effects. **f c**

Note that functor results can also be functions. **c**

This scope is simply a function,

which contains the code that we used in the non-functorial setting.

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

a **b** **c**

scope **(a -> b -> c)**

using only results
but not effects

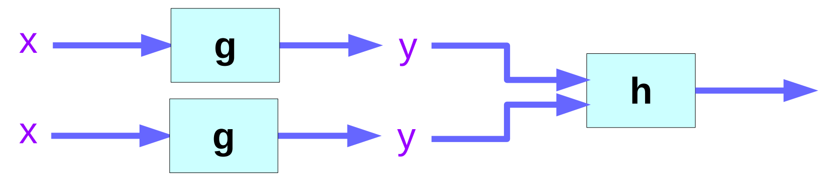
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

liftA3 – a non-functorial expression

Consider the non-functorial expression:

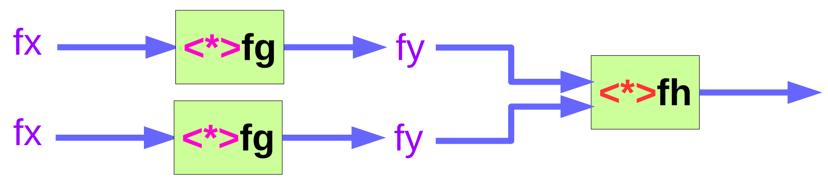
x :: x
g :: x -> y
h :: y -> y -> z

let y = g x
in h y y



generalization

fx :: f x
fg :: f (x -> y)
fh :: f (y -> y -> z)



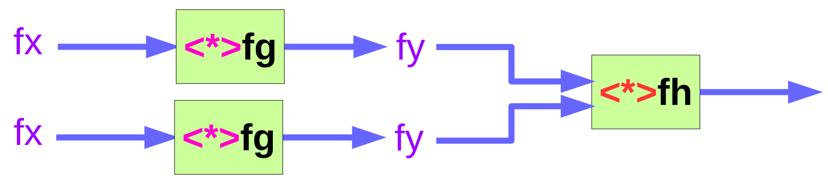
https://wiki.haskell.org/Applicative_functor

liftA3 – using <*> only

```
let fy = fg <*> fx
  in  fh <*> fy <*> fy
```

if **fy** writes something to the terminal

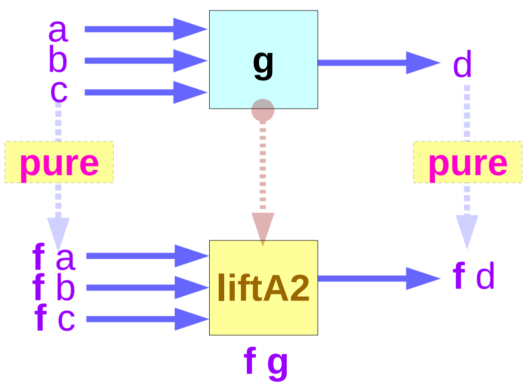
then **fh <*> fy <*> fy** writes twice.



this runs the effect of **fy** twice.

How the effect is run only once and the result is used twice?

→ utilize **liftA3**



https://wiki.haskell.org/Applicative_functor

liftA3 – using three input function

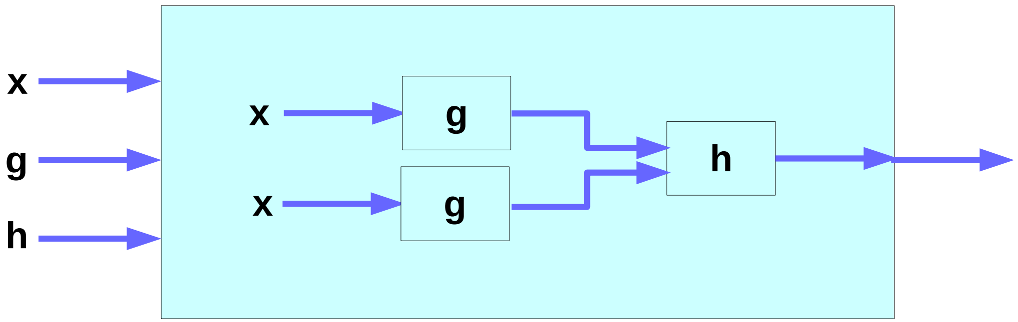
liftA3 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d

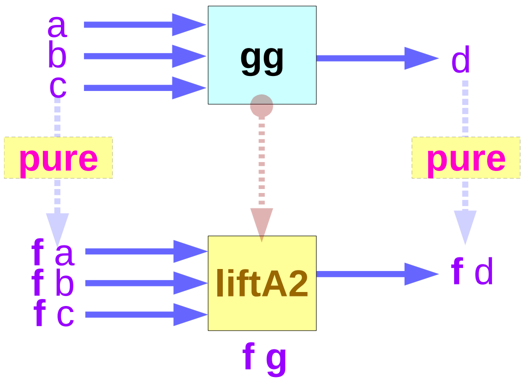
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc

liftA3

(\x g h -> let y = g x in h y y)

fx fg fh





http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

liftA3 – effects, results and scopes

Actually, using the **liftA** commands	
we can pull results of applicative functors	**y** from **fy**
into a scope where we can talk	**y -> y -> z**
exclusively about functor results	**y**
and not about effects.	**fy**
Note that functor results can also be functions.	**y**
This scope is simply a function,	**y -> y -> z**
which contains the code that we used in the non-functorial setting.	

```
liftA3
  (\x g h -> let y = g x in h y y)
  fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

https://wiki.haskell.org/Applicative_functor

liftA2 (<*>)

liftA2 (<*>) can be used to compose applicative functors.

It's easy to see how to use it from its type:

```
o :: (Applicative f, Applicative f1) =>    f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)
o = liftA2 (<*>)
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

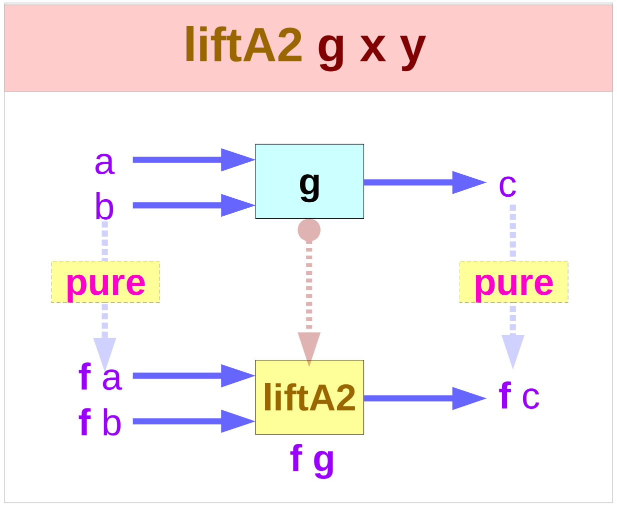
liftA2 (<*>) for composite applicative functors

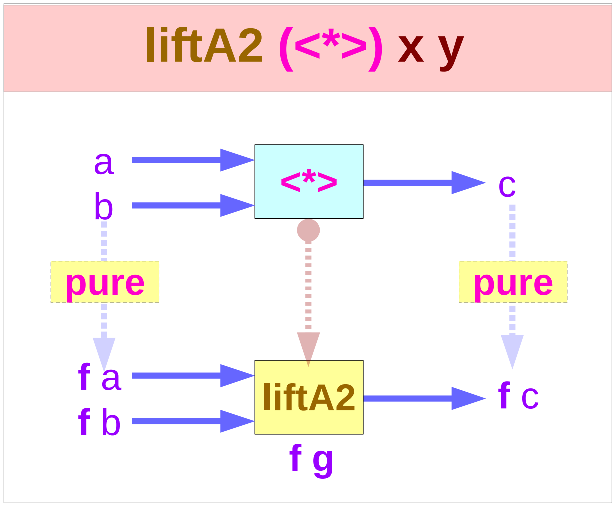
o :: (Applicative f, Applicative f1) =>
f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)

o = liftA2 (<*>)

f1 (a -> b) <*> f1 a <*> f1 b

liftA2 (<*>) can be used to <u>compose</u> <u>applicative</u> <u>functors</u>.





https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

liftA2 (<*>) Examples (1)

if **f** is **Maybe** and **f1** is **[ ]** we get:

[(+1), (+6)] **[1, 6]**

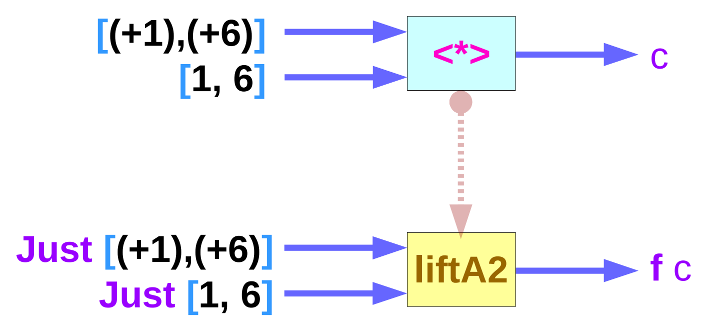
(+1) [1, 6]

(+6) [1, 6]

Just [(+1),(+6)] \`o\` Just [1, 6]
Just [2,7,7,12]

liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]
Just ([(+1),(+6)] <*> [1, 6])
Just [2, 7, 7, 12])

liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

liftA2 (<*>) Examples (2)

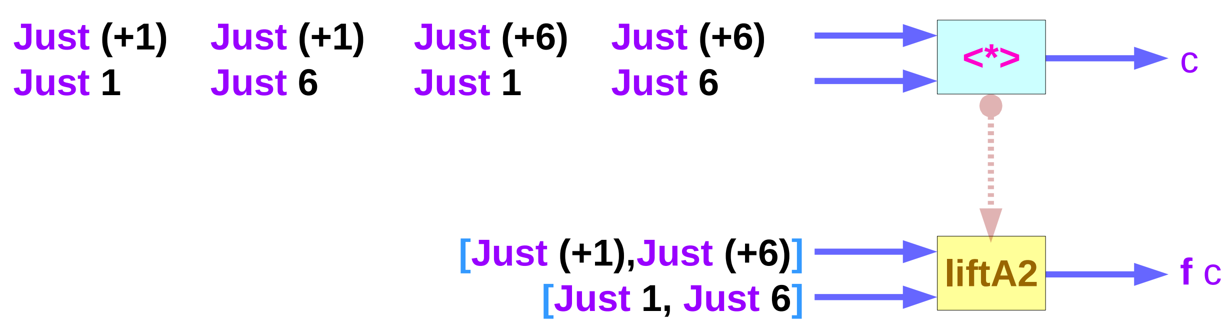
if f is Maybe and f1 is [ ] we get: [Just (+1),Just (+6)] [Just 1, Just 6]

[Just (+1),Just (+6)] \`o\` [Just 1, Just 6]
[Just 2, Just 7, Just 7, Just 12]

Just (+1) [Just 1, Just 6]
Just (+6) [Just 1, Just 6]

liftA2 (<*>) [Just (+1),Just (+6)] [Just 1, Just 6]
[Just (+1) <*> Just 1, Just (+1) <*> Just 6, Just (+6) <*> Just 1, Just (+6) <*> Just 6]
[Just 2, Just 7, Just 7, Just 12]

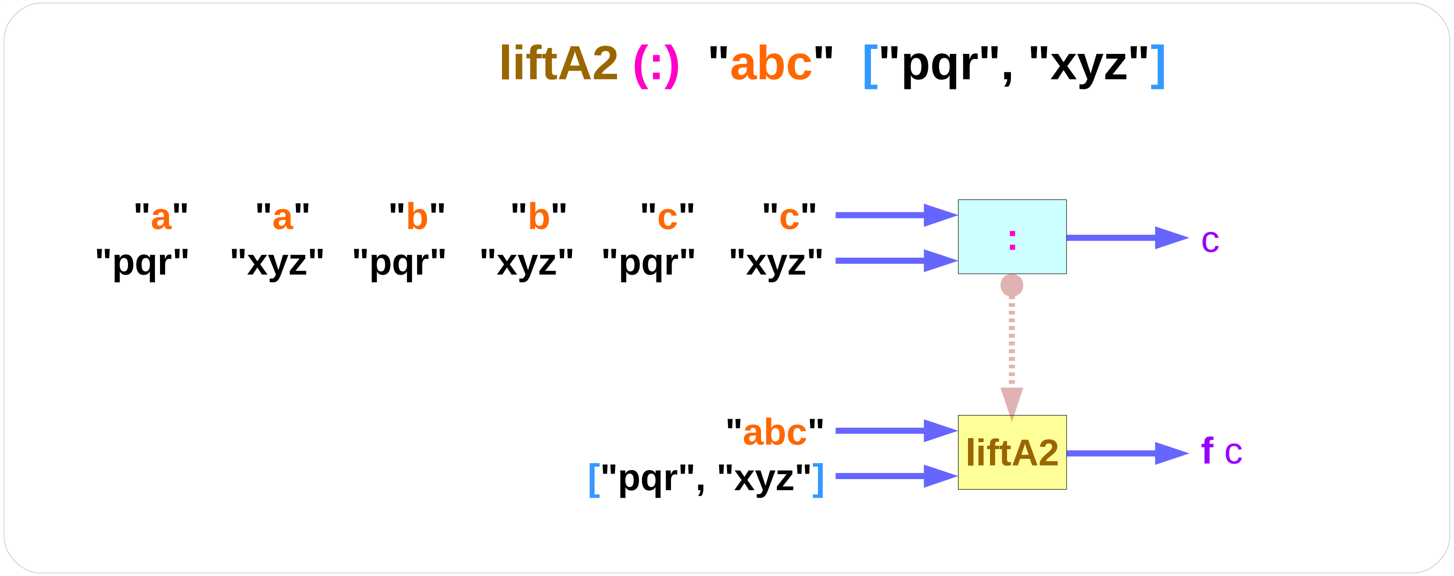
liftA2 (<*>) [Just (+1),Just (+6)] [Just 1, Just 6]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

liftA2 (:)

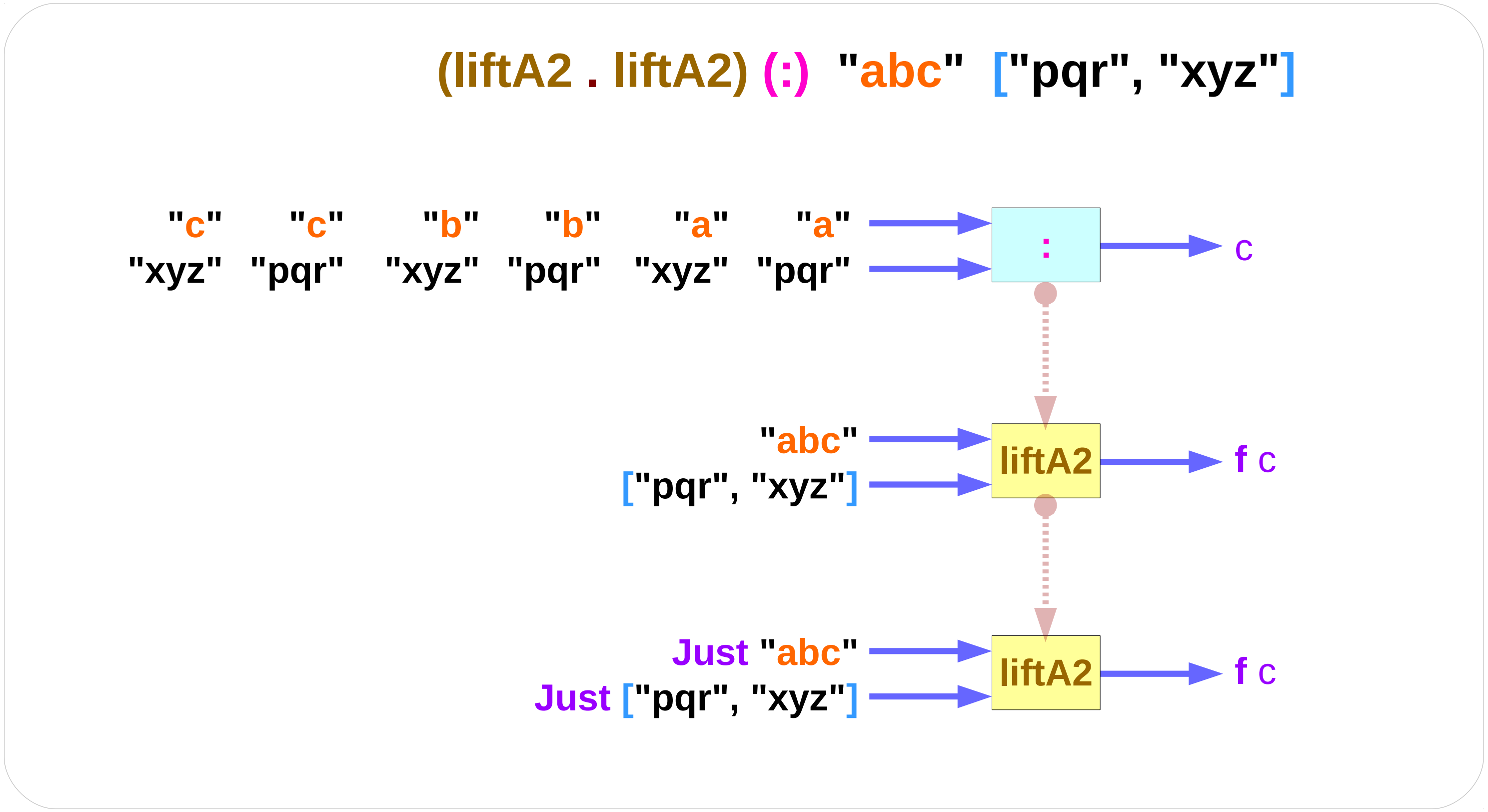
liftA2 (:) "abc" ["pqr", "xyz"]
["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

liftA2 (:)

(liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]



https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf