FM Communication Systems

Young W Lim

February 21, 2020

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$Z_{FM}(t) = A\cos\left[\omega_0 t + \Theta_0 + k_{FM}\int X(t)dt\right]$$

$$K_{cr}^2 = \frac{|X(t)|_{max}^2}{E\left[X^2(t)\right]} = \frac{|X(t)|_{max}^2}{\overline{X^2(t)}}$$

$$W_{FM} = 2\Delta\omega = 2k_{FM}\,|X(t)|_{max}$$

$$= 2k_{FM}K\,|X(t)|_{cr}\sqrt{\overline{X^2(t)}}$$

Definition

\

$$W_{FM} = 2\Delta\omega = 2k_{FM}\left|X(t)\right|_{max}$$

$$= 2k_{FM}K\left|X(t)\right|_{cr}\sqrt{\overline{X^2(t)}}$$

$$\Delta\omega = k_{FM}\left|X(t)\right|_{max}$$

$$P_{FM} = E\left[Z_{FM}^2(t)\right] = \frac{A^2}{2}$$

Definition

\

$$Z_d(t) = K_D k_{FM} x(t)$$

$$Z_i = G_{ch}^2 \frac{A^2}{2}$$

$$Z_o = E\left[Z_d^2(t)\right] = K_D^2 k_{FM}^2 \overline{X^2(t)}$$

Definition

$$N_i = \frac{1}{2\pi} 2 \int_{\omega_0+\Delta\omega}^{\omega_0+\Delta\omega} \frac{N_0}{2} d\omega = \frac{N_o \Delta\omega}{\pi}$$

$$\left(\frac{S_i}{N_i}\right)_{FM} = \frac{\pi G_{ch}^2 A^2}{2 N_0 \Delta\omega}$$

Definition

$$G_{ch}Acos\,(w_0t+\Theta_0)+N_c(t)cos\,(w_0t+\Theta_0)-N_s(t)\sin\,(w_0t+\Theta_0)$$

$$=A(t)\cos\,[\omega_0t+\Theta_0+\Psi(t)]$$

$$A(t)=\left\{[G_{ch}A+N_c(t)]^2+N_s^2(t)\right\}^{1/2}$$

$$\Psi(t)=\tan^{-1}\left\{\frac{N_s(t)}{G_{ch}A+N_c(t)}\right\}$$

Definition

$$\Psi(t) = \tan^{-1}\left\{\frac{N_s(t)}{G_{ch}A + N_c(t)}\right\}$$

$$\Psi(t) \simeq \tan^{-1}\left\{\frac{N_s(t)}{G_{ch}A}\right\} \simeq \frac{N_s(t)}{G_{ch}A}$$

$$A(t)\cos\left[\omega_0 t + \Theta_0 + \Psi(t)\right]$$

$$\simeq A(t)\cos\left[\omega_0 t + \Theta_0 + \frac{N_s(t)}{G_{ch}A}\right]$$

Definition

$$\left(\frac{K_D}{G_{ch}A}\right)\frac{dN_s(t)}{dt}$$

$$\left(\frac{K_D}{G_{ch}A}\right)^2 \omega^2 S_{N_sN_s}(\omega)$$

$$\left(\frac{K_D}{G_{ch}A}\right)^2 \omega^2 [S_{N_sN_s}(\omega-\omega_0)+S_{N_sN_s}(\omega-\omega_0)]$$

$$|\omega| < \Delta\omega$$

Definition

$$N_0 = E\left[N_d^2(t)\right]$$

$$= \frac{1}{2\pi}\int_{-W_x}^{W_x}\left(\frac{K_D}{G_{ch}A}\right)^2 \omega^2\left[S_{N_sN_s}(\omega-\omega_0)+S_{N_sN_s}(\omega-\omega_0)\right]d\omega$$

$$= \frac{K_D^2}{2\pi G_{ch}^2 A^2}\int_{-W_x}^{W_x}\omega^2\left[\frac{N_0}{2}+\frac{N_0}{2}\right]d\omega$$

$$= \frac{K_D^2 N_0 W_X^2}{3\pi G_{ch}^2 A^2}$$

Definition

$$\left(\frac{S_0}{N_0}\right)_{FM} = \frac{3\pi G_{ch}^2 A^2 k_{FM}^2 \overline{X^2(t)}}{N_0 W_X^3}$$

$$\left(\frac{S_0}{N_0}\right)_{FM} = \frac{6}{K_{cr}^2}\left(\frac{\Delta\omega}{W_X}\right)\left(\frac{S_i}{N_i}\right)_{FM}$$