

FM Communication Systems

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

FM Waveform (1)

N Gaussian random variables

Definition

$$Z_{FM}(t) = A \cos \left[\omega_0 t + \Theta_0 + k_{FM} \int X(t) dt \right]$$

$$K_{cr}^2 = \frac{|X(t)|_{max}^2}{E[X^2(t)]} = \frac{|X(t)|_{max}^2}{X^2(t)}$$

$$\begin{aligned} W_{FM} &= 2\Delta\omega = 2k_{FM} |X(t)|_{max} \\ &= 2k_{FM} K |X(t)|_{cr} \sqrt{X^2(t)} \end{aligned}$$

FM Waveform (2)

N Gaussian random variables

Definition

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$$W_{FM} = 2\Delta\omega = 2k_{FM} |X(t)|_{max}$$

$$= 2k_{FM}K |X(t)|_{cr} \sqrt{X^2(t)}$$

$$\Delta\omega = k_{FM} |X(t)|_{max}$$

$$P_{FM} = E [Z_{FM}^2(t)] = \frac{A^2}{2}$$

FM Waveform (3)

N Gaussian random variables

Definition

$$Z_d(t) = K_D k_{FM} x(t)$$

$$Z_i = G_{ch}^2 \frac{A^2}{2}$$

$$Z_o = E [Z_d^2(t)] = K_D^2 k_{FM}^2 \overline{X^2(t)}$$

FM Waveform (4)

N Gaussian random variables

Definition

$$N_i = \frac{1}{2\pi} \int_{\omega_0 + \Delta\omega}^{\omega_0 + \Delta\omega} \frac{N_0}{2} d\omega = \frac{N_0 \Delta\omega}{\pi}$$

$$\left(\frac{S_i}{N_i} \right)_{FM} = \frac{\pi G_{ch}^2 A^2}{2N_0 \Delta\omega}$$

FM System (1)

N Gaussian random variables

Definition

$$G_{ch}A \cos(\omega_0 t + \Theta_0) + N_c(t) \cos(\omega_0 t + \Theta_0) - N_s(t) \sin(\omega_0 t + \Theta_0)$$

$$= A(t) \cos[\omega_0 t + \Theta_0 + \Psi(t)]$$

$$A(t) = \left\{ [G_{ch}A + N_c(t)]^2 + N_s^2(t) \right\}^{1/2}$$

$$\Psi(t) = \tan^{-1} \left\{ \frac{N_s(t)}{G_{ch}A + N_c(t)} \right\}$$

FM System (2)

N Gaussian random variables

Definition

$$\Psi(t) = \tan^{-1} \left\{ \frac{N_s(t)}{G_{ch}A + N_c(t)} \right\}$$

$$\Psi(t) \simeq \tan^{-1} \left\{ \frac{N_s(t)}{G_{ch}A} \right\} \simeq \frac{N_s(t)}{G_{ch}A}$$

$$A(t) \cos[\omega_0 t + \Theta_0 + \Psi(t)]$$

$$\simeq A(t) \cos \left[\omega_0 t + \Theta_0 + \frac{N_s(t)}{G_{ch}A} \right]$$

FM System (3)

N Gaussian random variables

Definition

$$\left(\frac{K_D}{G_{ch}A} \right) \frac{dN_s(t)}{dt}$$

$$\left(\frac{K_D}{G_{ch}A} \right)^2 \omega^2 S_{N_s N_s}(\omega)$$

$$\left(\frac{K_D}{G_{ch}A} \right)^2 \omega^2 [S_{N_s N_s}(\omega - \omega_0) + S_{N_s N_s}(\omega + \omega_0)]$$

$$|\omega| < \Delta\omega$$

FM System (4)

N Gaussian random variables

Definition

$$\begin{aligned} N_0 &= E [N_d^2(t)] \\ &= \frac{1}{2\pi} \int_{-W_x}^{W_x} \left(\frac{K_D}{G_{ch}A} \right)^2 \omega^2 [S_{N_s N_s}(\omega - \omega_0) + S_{N_s N_s}(\omega + \omega_0)] d\omega \\ &= \frac{K_D^2}{2\pi G_{ch}^2 A^2} \int_{-W_x}^{W_x} \omega^2 \left[\frac{N_0}{2} + \frac{N_0}{2} \right] d\omega \\ &= \frac{K_D^2 N_0 W_x^2}{3\pi G_{ch}^2 A^2} \end{aligned}$$

FM System (5)

N Gaussian random variables

Definition

$$\left(\frac{S_0}{N_0}\right)_{FM} = \frac{3\pi G_{ch}^2 A^2 k_{FM}^2 \overline{X^2(t)}}{N_0 W_X^3}$$

$$\left(\frac{S_0}{N_0}\right)_{FM} = \frac{6}{K_{cr}^2} \left(\frac{\Delta\omega}{W_X}\right) \left(\frac{S_i}{N_i}\right)_{FM}$$

