Gaussian Distribution (4B)

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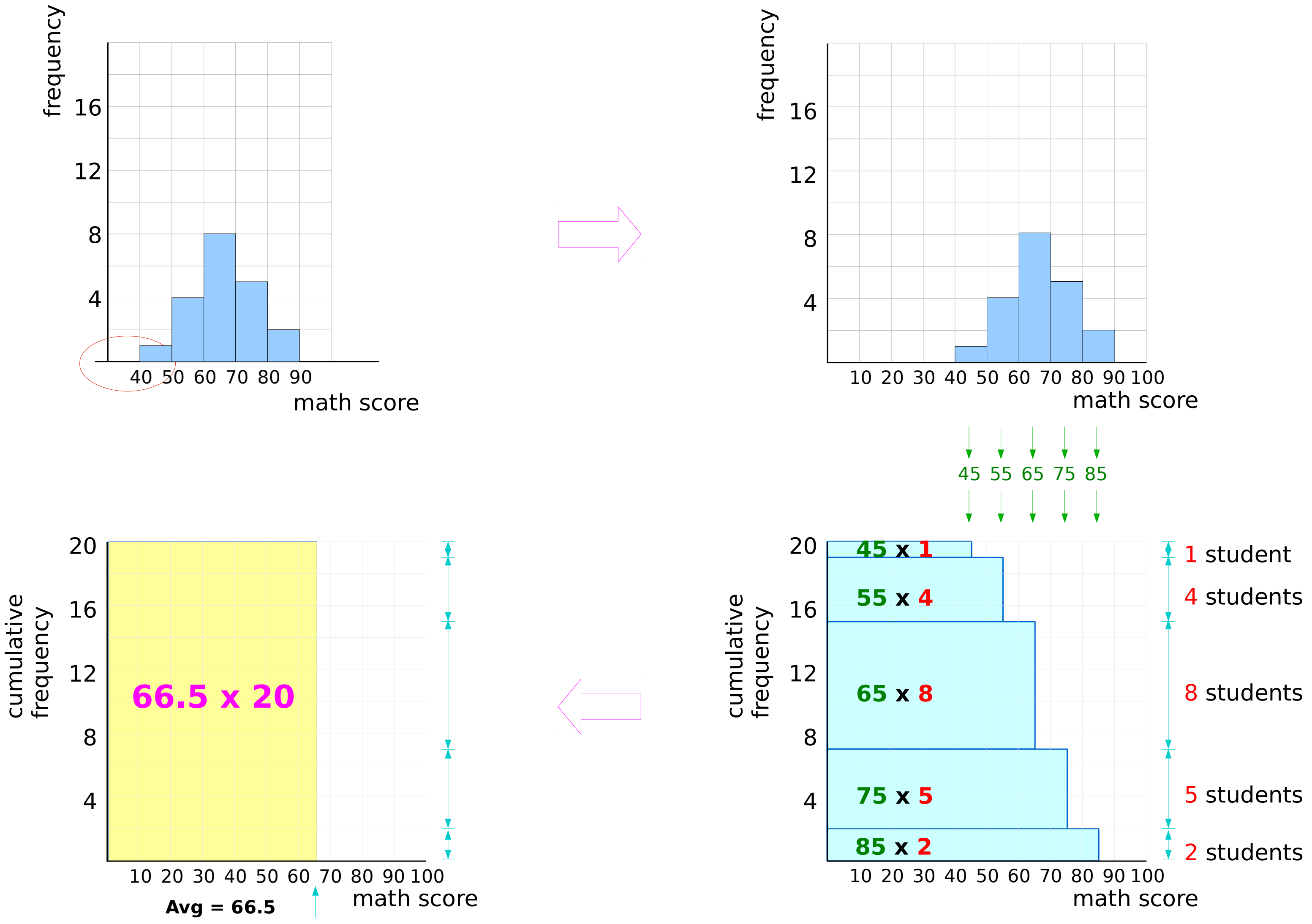
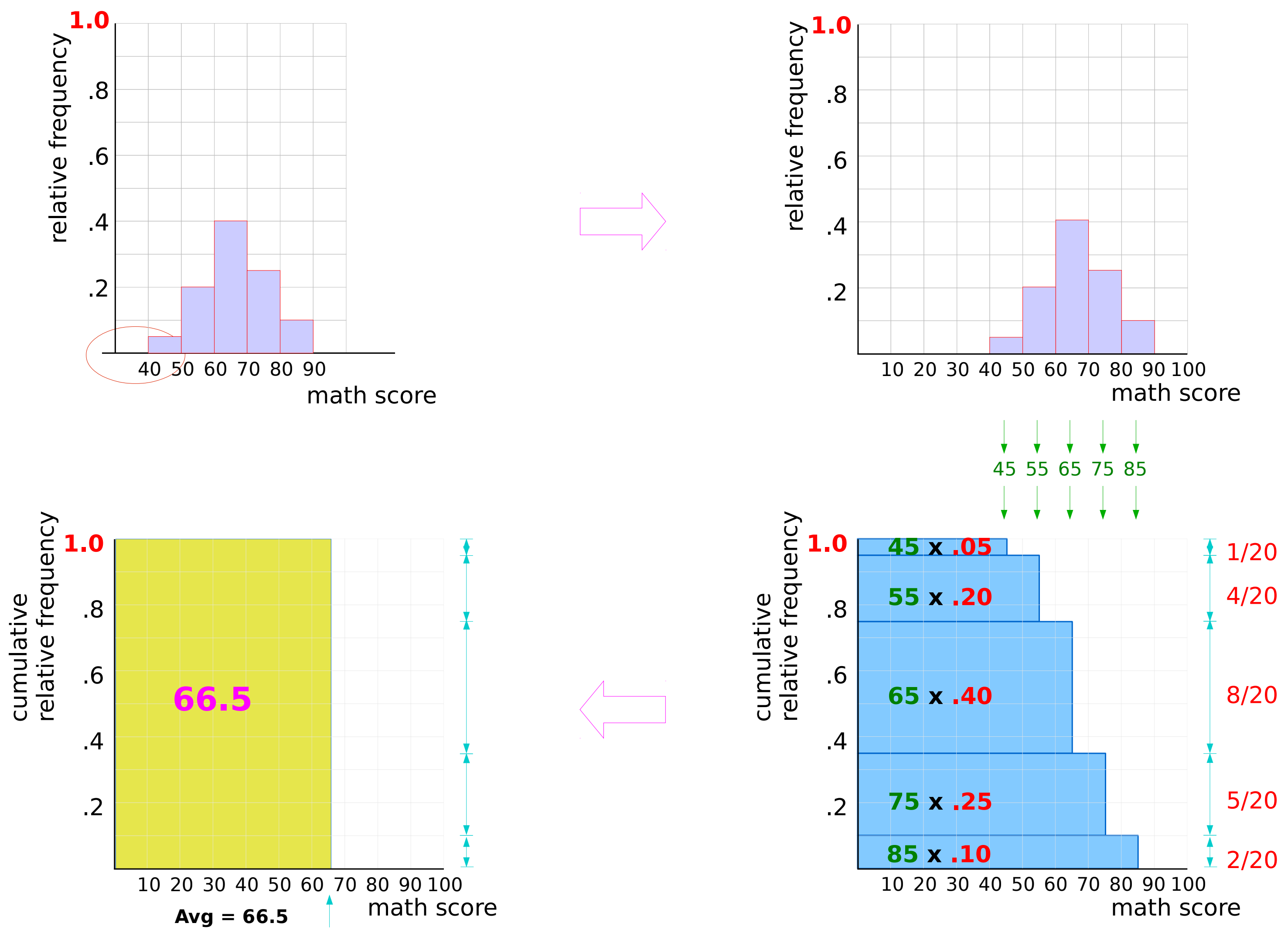
Absolute Frequency & Average

Absolute Frequency

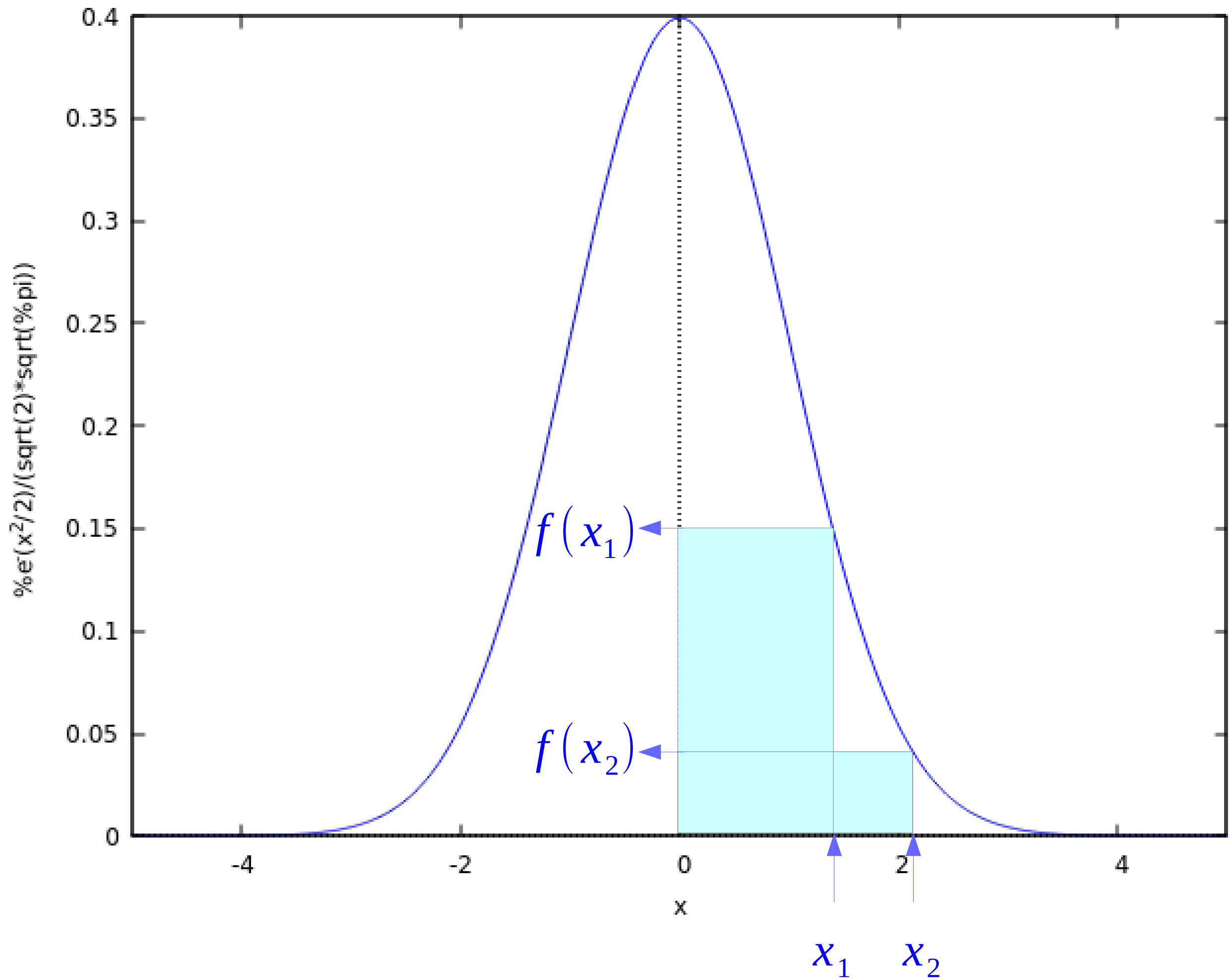


Relative Frequency & Average

Relative Frequency   **Group A**



Gaussian Distribution



$$\int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\sum_{k=-\infty}^{\infty} x_k f(x_k)\, \Delta_k$$

The Probability Density Function (pdf)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{\Delta_k \to 0} \sum_{k=-\infty}^{\infty} f(x_k)\Delta_k = 1$$

$$\int_{-\infty}^{\infty} x \cdot f(x)\,dx = \lim_{\Delta_k \to 0} \sum_{k=-\infty}^{\infty} x_k \cdot f(x_k)\Delta_k = \mu$$

$$\int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx = \lim_{\Delta_k \to 0} \sum_{k=-\infty}^{\infty} x_k^2 \cdot f(x_k)\Delta_k = \mu^2 + \sigma^2$$

Moment Functions

```
(%i6) f(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)));
```

(%o6) $f(x,\mu,\sigma):=\dfrac{\%e^{-\frac{(x-\mu)^2}{2\,\sigma^2}}}{\sqrt{2\,\pi\,\sigma^2}}$

```
(%i2) g(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)))*x;
```

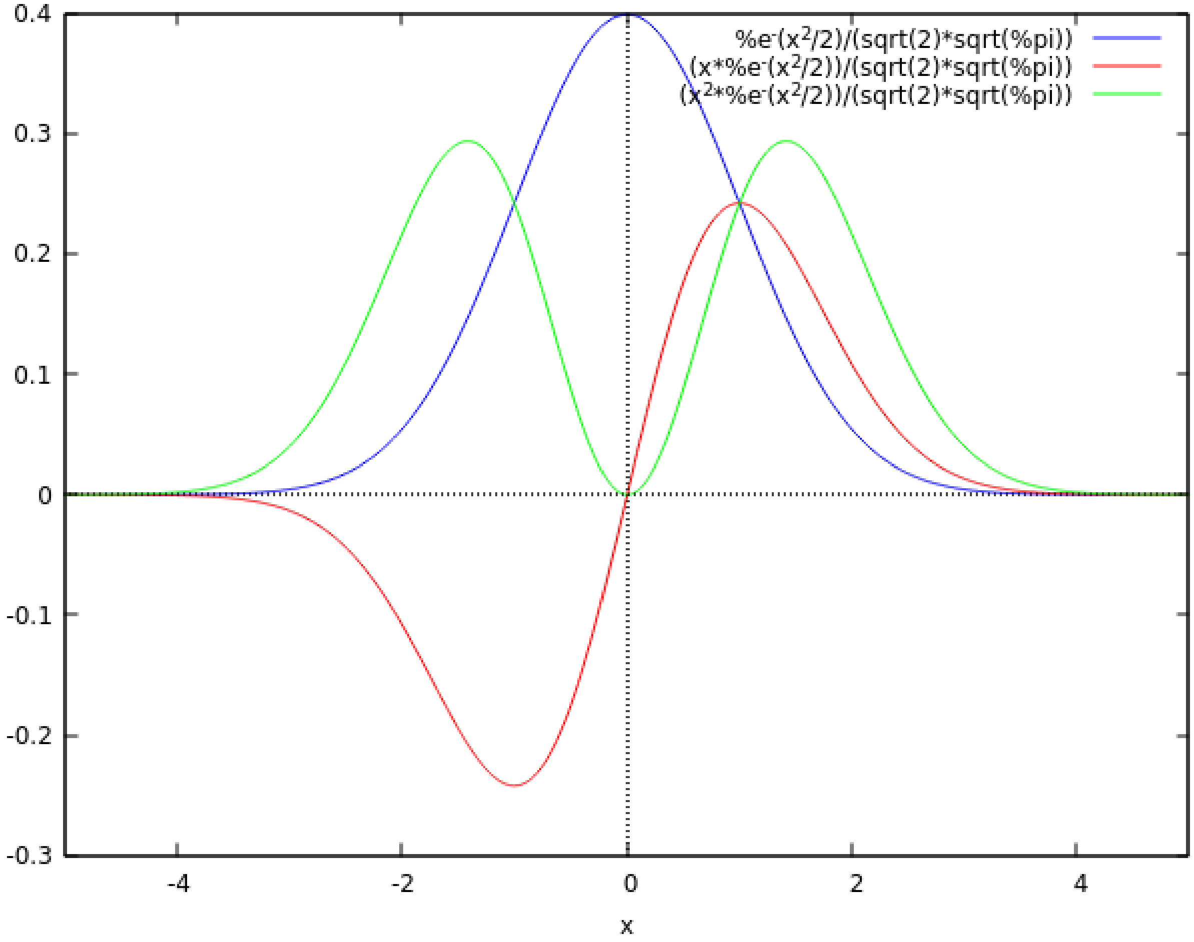
(%o2) $g(x,\mu,\sigma):=\dfrac{\%e^{-\frac{(x-\mu)^2}{2\,\sigma^2}}\,x}{\sqrt{2\,\pi\,\sigma^2}}$

```
(%i3) h(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)))*x^2;
```

(%o3) $h(x,\mu,\sigma):=\dfrac{\%e^{-\frac{(x-\mu)^2}{2\,\sigma^2}}\,x^2}{\sqrt{2\,\pi\,\sigma^2}}$

```
(%i7) plot2d( [f(x,0,1), g(x,0,1), h(x,0,1)], [x, -5, 5], [plot_format, gnuplot]);
(%o7) [/home/young/maxout.gnuplot]
```

Moment Function Plots



Numerical Integration Results

```
(%i16) float(integrate(f(x,0,1), x, -10, +10));
(%o16) 1.0

(%i15) float(integrate(g(x,0,1), x, -10, +10));
(%o15) 0.0

(%i27) float(integrate(h(x,0,1), x, -10, +10));
(%o27) 0.9999999999999998
```

The core function test

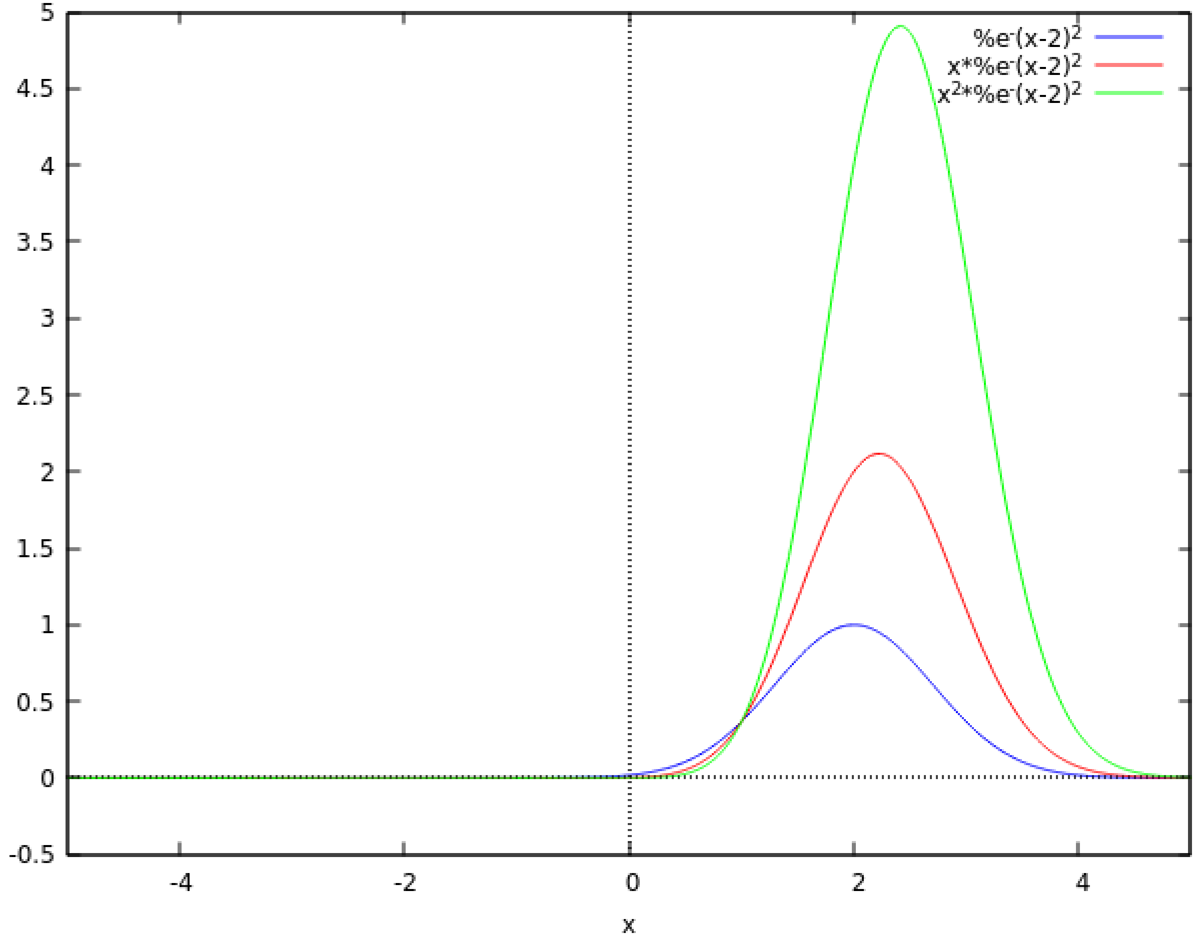
```
(%i28) m1(x) := %e^(-(x-2)^2);
(%o28) m1(x):=%e^(-(x-2)^2)

(%i29) m2(x) := m1(x) * x;
(%o29) m2(x):=m1(x) x

(%i30) m3(x) := m1(x) * x^2;
(%o30) m3(x):=m1(x) x^2

(%i31) plot2d([m1(x), m2(x), m3(x)], [x, -5, 5], [plot_format, gnuplot]);
(%o31) [/home/young/maxout.gnuplot]
```

The core function plots



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann