

# Gaussian Distribution (4B)

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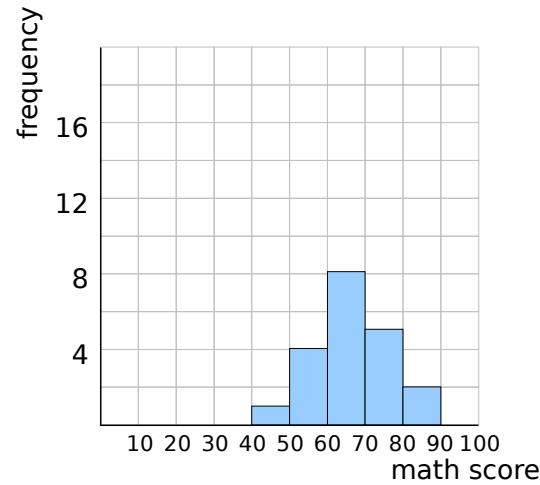
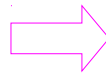
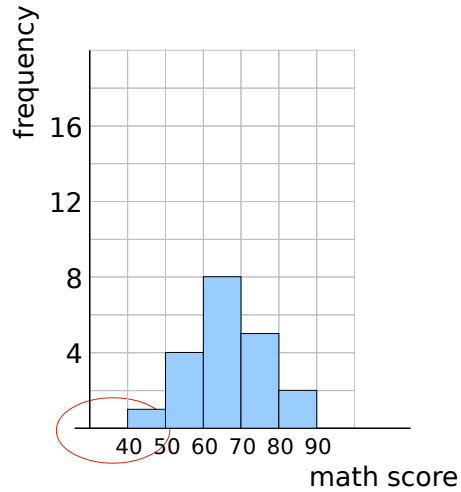
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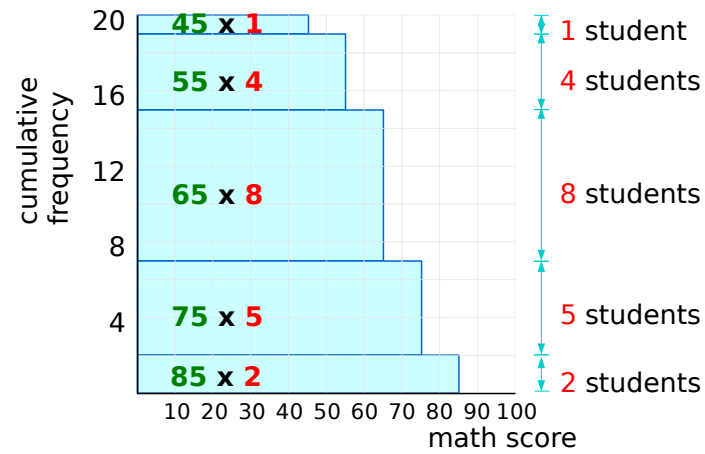
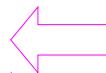
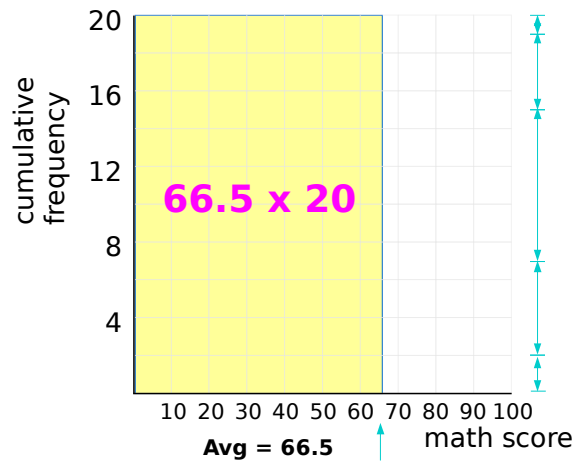
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# Absolute Frequency & Average

## Absolute Frequency

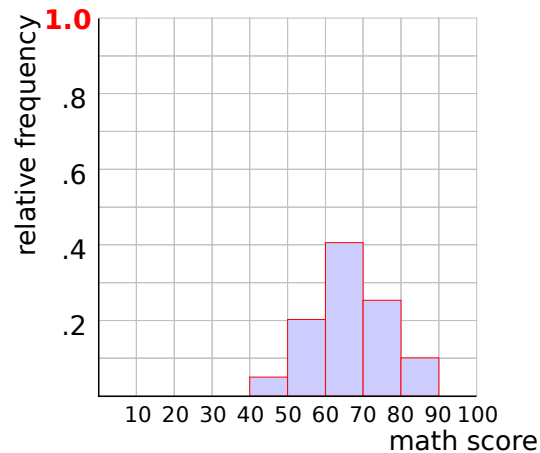
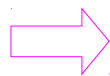
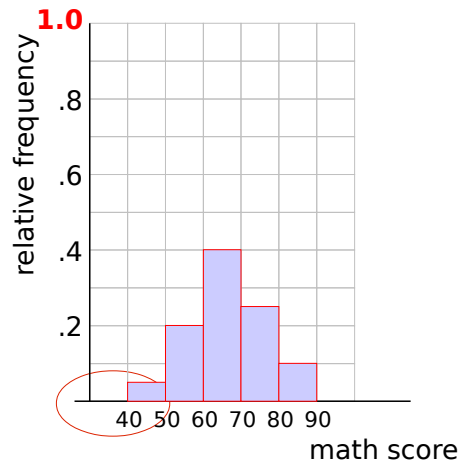


45 55 65 75 85

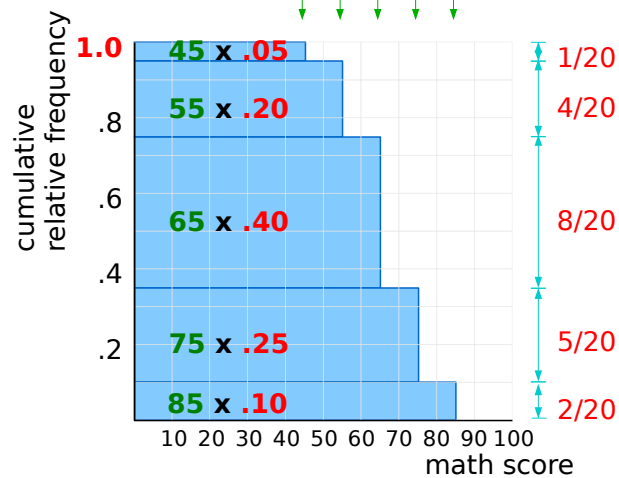
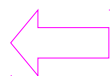
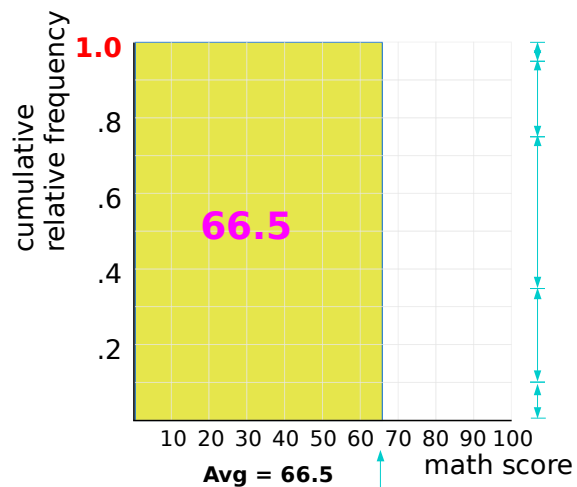


# Relative Frequency & Average

## Relative Frequency Group A

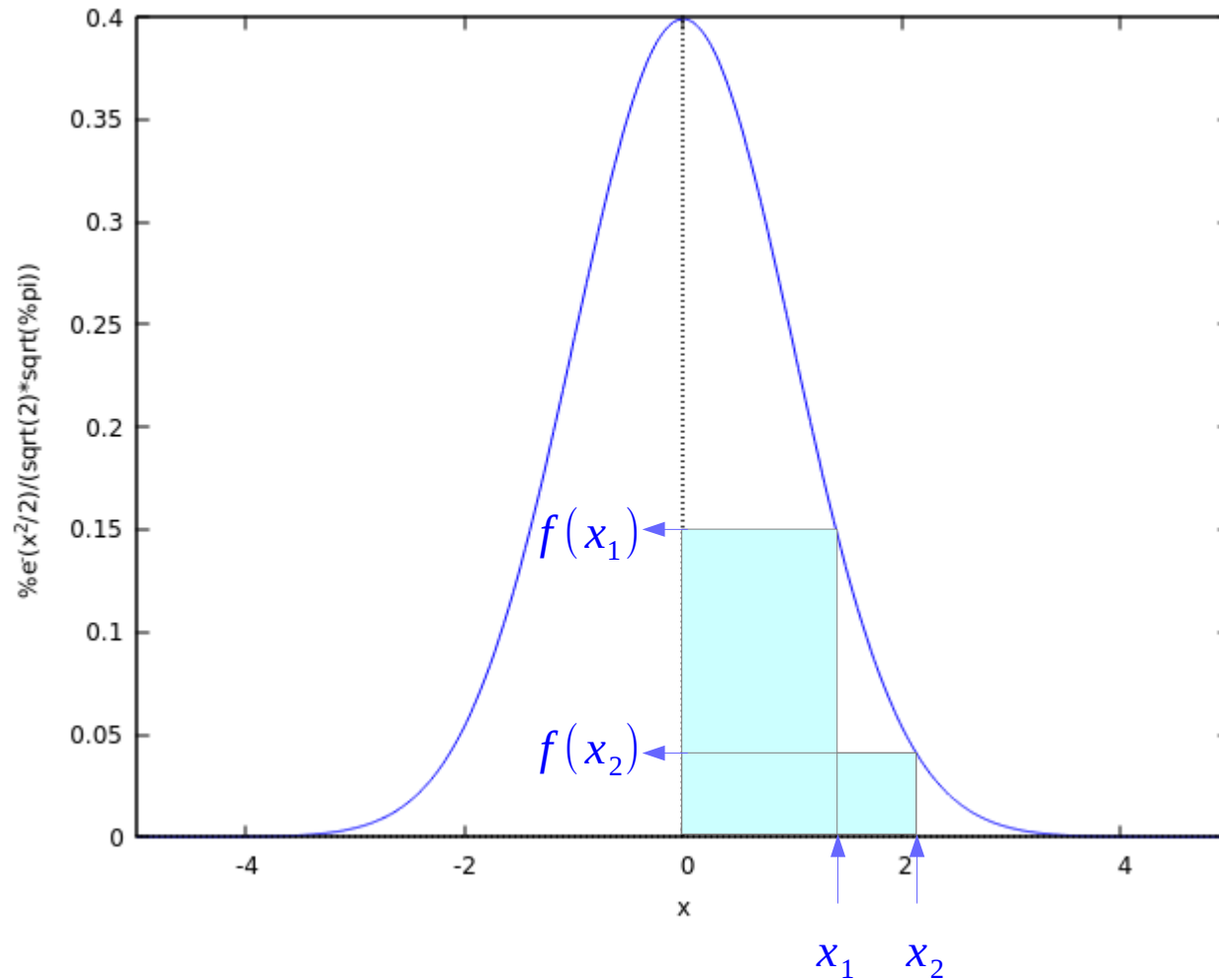


45 55 65 75 85



Expected Value (2A)

# Gaussian Distribution



$$\int_{-\infty}^{\infty} x f(x) dx$$

$$\sum_{k=-\infty}^{\infty} x_k f(x_k) \Delta_k$$

# The Probability Density Function (pdf)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} f(x) dx = \lim_{\Delta_k \rightarrow 0} \sum_{k=-\infty}^{\infty} f(x_k) \Delta_k = 1$$

$$\int_{-\infty}^{\infty} x \cdot f(x) dx = \lim_{\Delta_k \rightarrow 0} \sum_{k=-\infty}^{\infty} x_k \cdot f(x_k) \Delta_k = \mu$$

$$\int_{-\infty}^{\infty} x^2 \cdot f(x) dx = \lim_{\Delta_k \rightarrow 0} \sum_{k=-\infty}^{\infty} x_k^2 \cdot f(x_k) \Delta_k = \mu^2 + \sigma^2$$

# Moment Functions

```
(%i6) f(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)));
```

$$(\%o6) f(x, \mu, \sigma) := \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

```
(%i2) g(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)))*x;
```

$$(\%o2) g(x, \mu, \sigma) := \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}} x}{\sqrt{2\pi\sigma^2}}$$

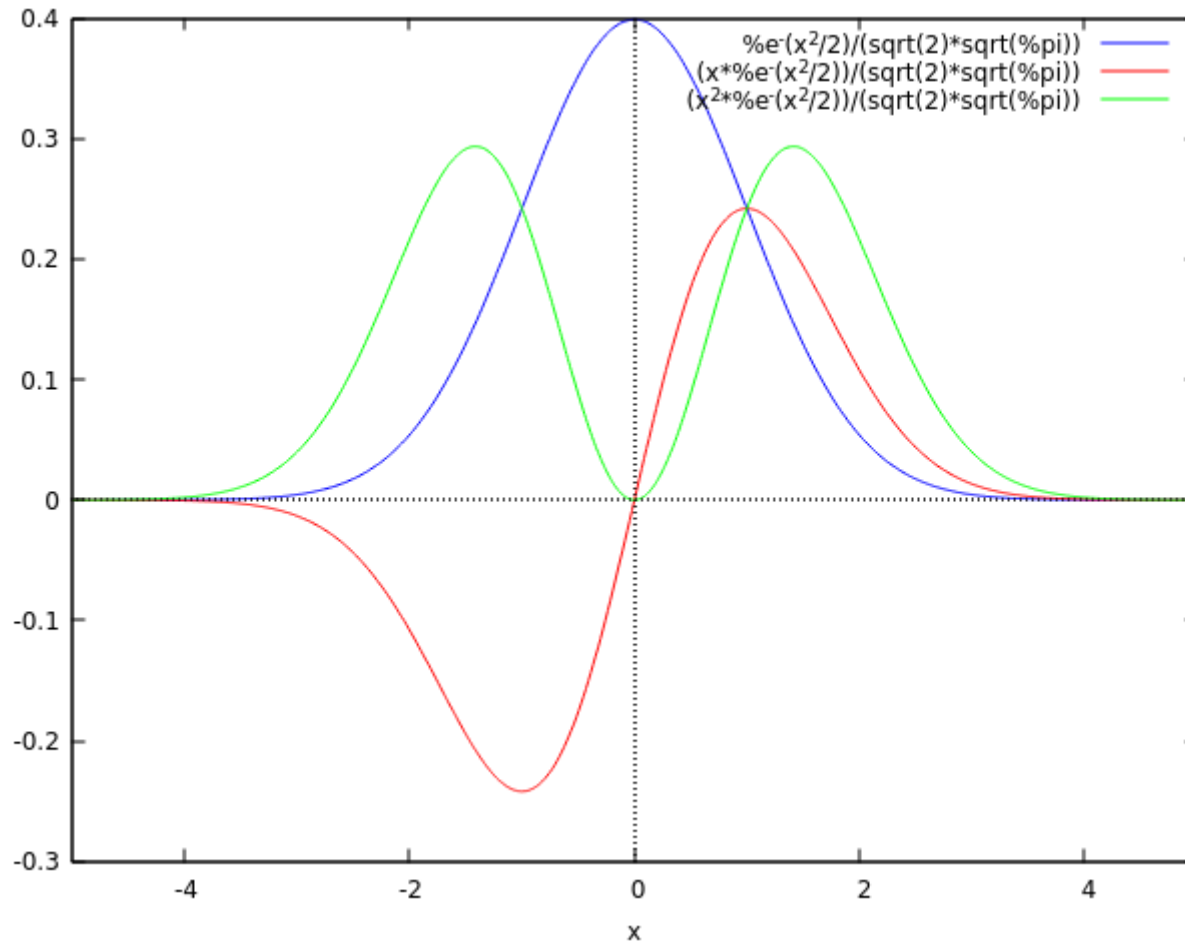
```
(%i3) h(x, %mu, %sigma) := (1/sqrt(2*%pi*%sigma^2)) * %e^(-((x-%mu)^2 / (2 * %sigma^2)))*x^2;
```

$$(\%o3) h(x, \mu, \sigma) := \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}} x^2}{\sqrt{2\pi\sigma^2}}$$

```
(%i7) plot2d( [f(x,0,1), g(x,0,1), h(x,0,1)], [x, -5, 5], [plot_format, gnuplot]);
```

```
(%o7) [/home/young/maxout.gnuplot]
```

# Moment Function Plots





# Numerical Integration Results

```
(%i16) float(integrate(f(x,0,1), x, -10, +10));  
(%o16) 1.0
```

```
(%i15) float(integrate(g(x,0,1), x, -10, +10));  
(%o15) 0.0
```

```
(%i27) float(integrate(h(x,0,1), x, -10, +10));  
(%o27) 0.9999999999999998
```

# The core function test

```
(%i28) m1(x) := %e^(-(x-2)^2);
```

```
(%o28) m1(x) := %e-(x-2)2
```

```
(%i29) m2(x) := m1(x) * x;
```

```
(%o29) m2(x) := m1(x) x
```

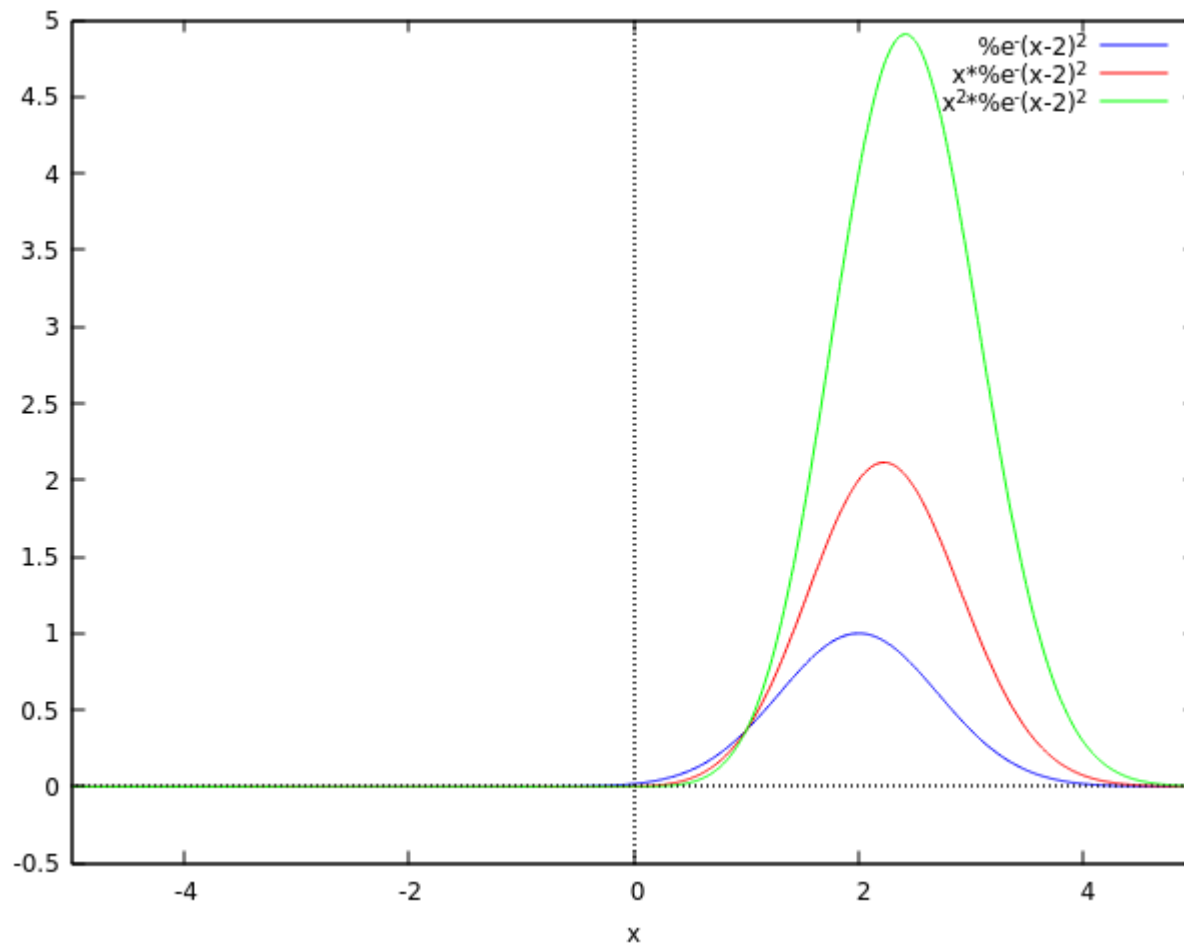
```
(%i30) m3(x) := m1(x) * x^2;
```

```
(%o30) m3(x) := m1(x) x2
```

```
(%i31) plot2d([m1(x), m2(x), m3(x)], [x, -5, 5], [plot_format, gnuplot]);
```

```
(%o31) [/home/young/maxout.gnuplot]
```

# The core function plots



## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings
  
- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann