

Vector Calculus (H.1)

Multiple Integrals

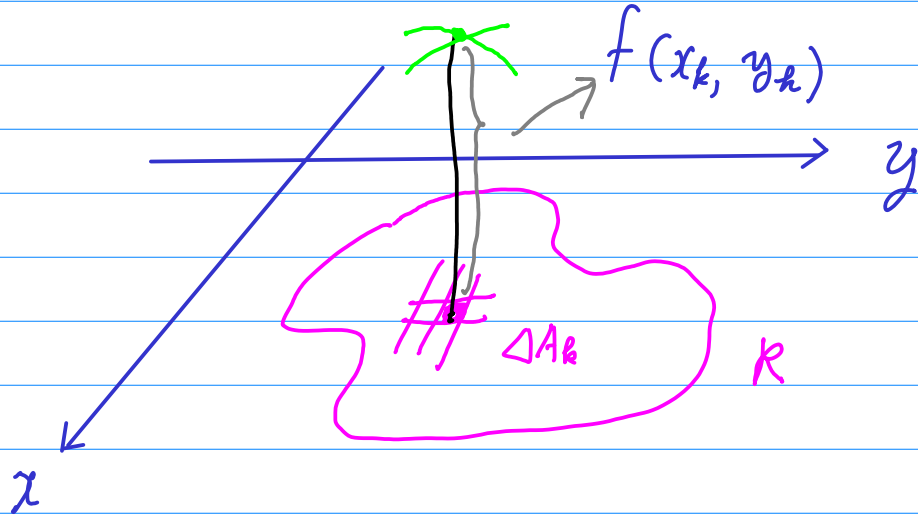
20160107

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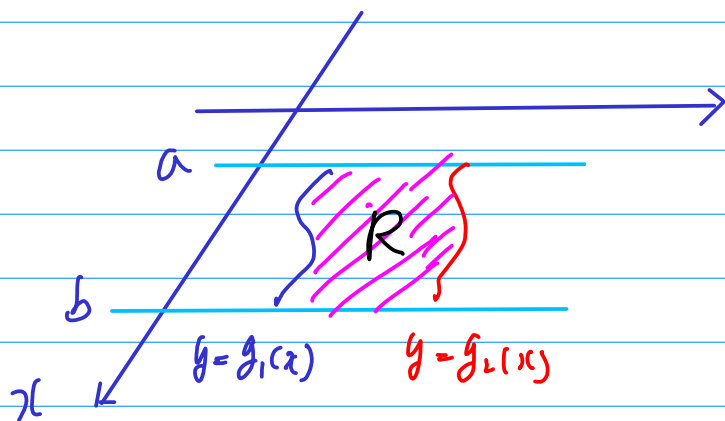
9.10 Double Integral

$$\iint_R f(x, y) dA = \lim_{(P) \rightarrow 0} \sum_{k=1}^n f(x_k, y_k) \Delta A_k$$



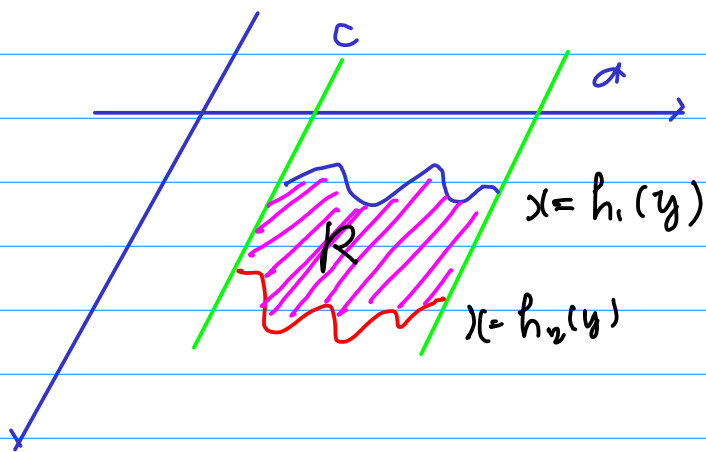
$$\iint_R 1 \cdot dA = \text{area} \quad A = \iint_R dA$$

Fubini's Theorem



$$\iint_R f(x, y) dA$$

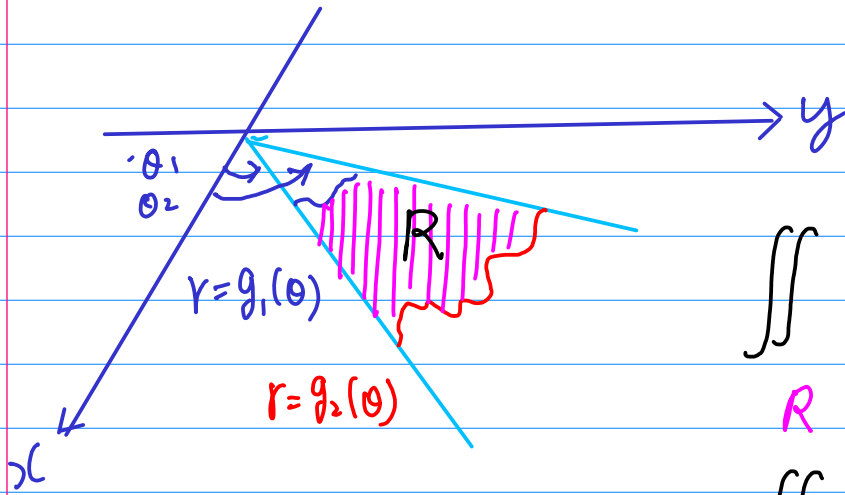
$$= \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x, y) dy \right] dx$$



$$\iint_R f(x, y) dA$$

$$= \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x, y) dx \right] dy$$

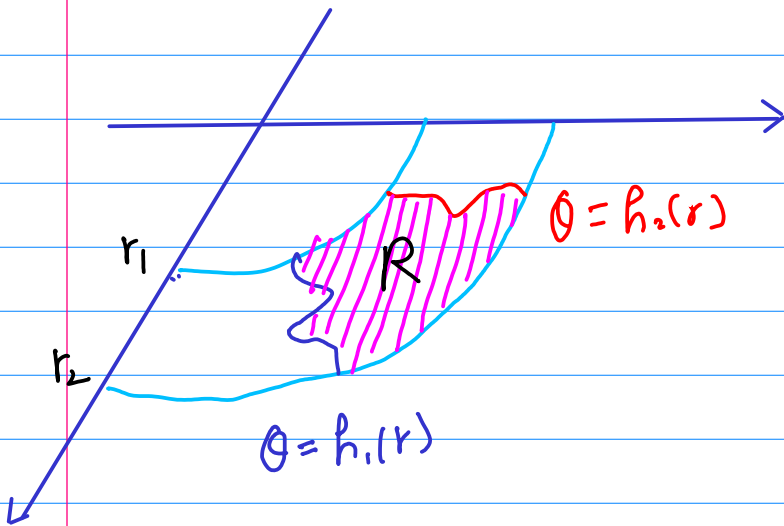
9.11 Double Integral in Polar Coord.



$$\iint_R f(x, y) dA$$

$$\iint_R f(r, \theta) dA$$

$$= \int_{\theta_1}^{\theta_2} \left[\int_{g_1(\theta)}^{g_2(\theta)} f(r, \theta) r dr \right] d\theta$$



$$= \int_{r_1}^{r_2} \left[\int_{h_1(r)}^{h_2(r)} f(r, \theta) r d\theta \right] dr$$

Change of variables

$$\iint_R f(x, y) dA = \int_{\alpha}^{\beta} \int_{\theta_1(\alpha)}^{\theta_2(\alpha)} f(r \cos \theta, r \sin \theta) r dr d\theta$$

x, y $x \leftarrow r \cos \theta$
 $y \leftarrow r \sin \theta$ r, θ

Jacobian

$$\iint_R f(x,y) dA$$

$$\int_C f(x,y) ds$$

