Laurent Series and z-Transform
- Geometric Series
Time Shift A

20180817 Fri

Causal Signal

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \cdots = \frac{1}{1-\left(\frac{z}{2}\right)}$$

$$X(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots = \frac{1}{1-\left(\frac{1}{2z}\right)}$$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\left(\frac{z}{2}\right)} \rightarrow \frac{2}{2-z} \quad (|z| < 2)$$

$$X(z) = \frac{1}{1-\left(\frac{1}{2z}\right)} \rightarrow \frac{z}{z-0.5} \quad (|z| > 0.5)$$

Anti-Causal Signal

$$a_n = \left(\frac{1}{2}\right)^n \qquad (n < 0)$$

$$f(z) = \left(\frac{1}{2}\right)^{-1} z^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-2} + \left(\frac{1}{2}\right)^{-3} z^{-3} + \cdots = \frac{\left(\frac{2}{z}\right)}{1 - \left(\frac{2}{z}\right)}$$

$$X(z) = \left(\frac{1}{2}\right)^{-1} z^{1} + \left(\frac{1}{2}\right)^{-2} z^{2} + \left(\frac{1}{2}\right)^{-3} z^{3} + \cdots = \frac{(2z)}{1 - (2z)}$$

$$a_n = \left(\frac{1}{2}\right)^n \qquad (n < 0)$$

$$f(z) = \frac{\left(\frac{2}{z}\right)}{1 - \left(\frac{2}{z}\right)} \rightarrow \frac{2}{z - 2} \qquad (|z| > 2)$$

$$X(z) = \frac{(2z)}{1 - (2z)} \rightarrow \frac{z}{0.5 - z} \qquad (|z| < 0.5)$$

$$a_n = -\left(\frac{1}{2}\right)^n \qquad (n < 0)$$

$$f(z) = \frac{-\left(\frac{2}{z}\right)}{1 - \left(\frac{2}{z}\right)} \rightarrow \frac{2}{2 - z} \qquad (|z| > 2)$$

$$X(z) = \frac{-(2z)}{1 - (2z)} \rightarrow \frac{z}{z - 0.5} \qquad (|z| < 0.5)$$

Inverse z

Causal

$$f(z) = \frac{2}{2-z} \quad (|z|<2)$$

$$X(z) = \frac{z}{z-0.5} \quad (|z|>0.5)$$

anti-Causal

$$f(z^{-1}) = \frac{2}{2-z^{-1}} \quad (|z^{-1}|<2) \longrightarrow f(z^{-1}) = X(z) = \frac{z}{z-0.5} \quad (|z|>0.5)$$

$$X(z^{-1}) = \frac{z^{-1}}{z^{-1}-0.5} \quad (|z^{-1}|>0.5) \longrightarrow X(z^{-1}) = f(z) = \frac{2}{2-z} \quad (|z|<2)$$

$f(z^{-1}) = X(z)$ Laurent Series (anti-causal signal)
with the same formula as causal X(z)

$X(z^{-1}) = f(z)$ z-Transform (anti-causal signal)
with the same formula as causal f(z)

Causal

$$f(z) = \frac{2}{2-z} \quad (|z|<2)$$

$$X(z) = \frac{z}{z-0.5} \quad (|z|>0.5)$$

anti-Causal

$$X(z^{-1}) = \frac{2}{2-z} \quad (|z|<2)$$

$$f(z^{-1}) = \frac{z}{z-0.5} \quad (|z|>0.5)$$

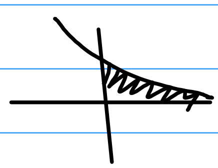
Causal $\qquad\qquad\qquad$ anti-Causal

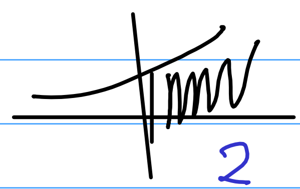
$$f(z) = \frac{2}{2-z} \quad (|z| < 2) \qquad\qquad f(z^{-1}) = \frac{z}{z-0.5} \quad (|z| > 0.5)$$

$$X(z) = \frac{z}{z-0.5} \quad (|z| > 0.5) \qquad\qquad X(z^{-1}) = \frac{2}{2-z} \quad (|z| < 2)$$

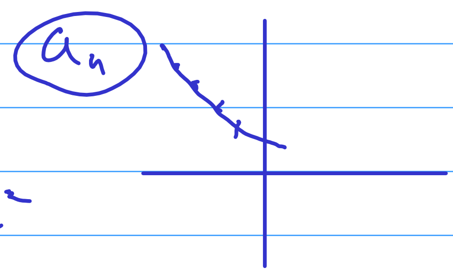
$$f(z) = \frac{2}{2-z} = \frac{1}{1-\left(\frac{z}{2}\right)} = \left(\tfrac{1}{2}\right)^0 z^0 + \left(\tfrac{1}{2}\right)^1 z^1 + \left(\tfrac{1}{2}\right)^2 z^2 + \cdots \qquad a_n = \left(\tfrac{1}{2}\right)^n$$

$$f(z^{-1}) = \frac{2}{2-z^{-1}} = \frac{1}{1-\left(\frac{1}{2z}\right)} = \left(\tfrac{1}{2}\right)^0 z^0 + \underbrace{\left(\tfrac{1}{2}\right)^{-1} z^{-1}}_{n=-1} + \underbrace{\left(\tfrac{1}{2}\right)^{-2} z^{-2}}_{n=-2} + \cdots \qquad a_{-n} = \left(\tfrac{1}{2}\right)^{-n}$$





$2 \qquad 2^2$



Inverse ROC

Causal | anti-Causal

$f(z) = \frac{2}{2-z} \quad (|z| < 2) \rightarrow -f(z) = -\frac{2}{2-z} \quad (|z| > 2)$

$X(z) = \frac{z}{z-0.5} \quad (|z| > 0.5) \rightarrow -X(z) = -\frac{z}{z-0.5} \quad (|z| < 0.5)$

	causal	anti-causal
p=2	$a_n = (\frac{1}{2})^n \quad (n \geq 0)$	$a_n = (\frac{1}{2})^n \quad (n < 0)$
	$f(z) = \frac{1}{1-(\frac{z}{2})} \quad (\lvert z \rvert < 2)$ $X(z) = \frac{1}{1-(\frac{1}{2z})} \quad (\lvert z \rvert > 0.5)$	$f(z) = \frac{(\frac{2}{z})}{1-(\frac{2}{z})} \quad (\lvert z \rvert > 2)$ $X(z) = \frac{(2z)}{1-(2z)} \quad (\lvert z \rvert < 0.5)$
	$f(z) = \frac{2}{2-z} \quad (\lvert z \rvert < 2)$ $X(z) = \frac{z}{z-0.5} \quad (\lvert z \rvert > 0.5)$	$f(z) = \frac{2}{z-2} \quad (\lvert z \rvert > 2)$ $X(z) = \frac{z}{0.5-z} \quad (\lvert z \rvert < 0.5)$

	causal	anti-causal
p=1/2	$a_n = (2)^n \quad (n \geq 0)$	$a_n = (2)^n \quad (n < 0)$
	$f(z) = \frac{1}{1-(2z)} \quad (\lvert z \rvert < 0.5)$ $X(z) = \frac{1}{1-(\frac{2}{z})} \quad (\lvert z \rvert > 2)$	$f(z) = \frac{(\frac{1}{2z})}{1-(\frac{1}{2z})} \quad (\lvert z \rvert > 0.5)$ $X(z) = \frac{(\frac{z}{2})}{1-(\frac{z}{2})} \quad (\lvert z \rvert < 2)$
	$f(z) = \frac{0.5}{0.5-z} \quad (\lvert z \rvert < 0.5)$ $X(z) = \frac{z}{z-2} \quad (\lvert z \rvert > 2)$	$f(z) = \frac{0.5}{z-0.5} \quad (\lvert z \rvert > 0.5)$ $X(z) = \frac{z}{2-z} \quad (\lvert z \rvert < 2)$

$f(z)$ $g(z^{-1})$	$f(z^{-1})$ $g(z)$

$X(z^{-1})$ $Y(z)$	$X(z)$ $Y(z^{-1})$

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Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

$n \geq 0$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n < 1$
causal a_n	$f(z)$	$\xleftrightarrow{z^{-1}}$	$f(z^{-1})$	anti-causal $-b_n$
	$\Vert$ $\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}$		$\cdot\frac{p}{p}\frac{z}{z}$ $\Vert$	
$-a_n$ anti-causal	$g(z^{-1})$	$\xleftrightarrow{z^{-1}}$	$g(z)$	b_n causal
$n < 0$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n \geq 1$

Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

$n < 1$	$\lvert z\rvert < p$		$\lvert z\rvert > p^{-1}$	$n \geq 0$
anti- causal $-y_n$	$X(z^{-1})$	$\xleftrightarrow{z^{-1}}$	$X(z)$	causal x_n
	$\Vert$ $\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}$		$\cdot\frac{p}{p}\frac{z}{z}$ $\Vert$	
causal y_n	$Y(z)$	$\xleftrightarrow{z^{-1}}$	$Y(z^{-1})$	anti- causal $-x_n$
$n \geq 1$	$\lvert z\rvert > p$		$\lvert z\rvert < p^{-1}$	$n < 0$

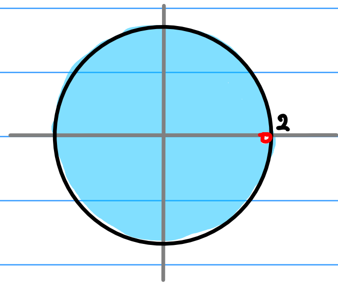
$n \in [0, \infty)$	$\lvert z \rvert < 2$		$\lvert z \rvert > 0.5$	$n \in (-\infty, 0]$
causal $a_n = 0.5^n$	$\boxed{\dfrac{z}{2-z}}$	$\xleftrightarrow{z^{-1}}$	$\dfrac{z}{z-0.5}$	anti-causal $-a_n = -0.5^n$
	$\cdot \dfrac{p}{p} \dfrac{z^{-1}}{z^{-1}}$		$\cdot \dfrac{p}{p} \dfrac{z}{z}$	
$-a_n = -0.5^n$ anti-causal	$\dfrac{z}{2-z}$	$\xleftrightarrow{z^{-1}}$	$\boxed{\dfrac{z}{z-0.5}}$	$a_n = 0.5^n$ causal
$n \in (-\infty, -1]$	$\lvert z \rvert > 2$		$\lvert z \rvert < 0.5$	$n \in [1, \infty)$

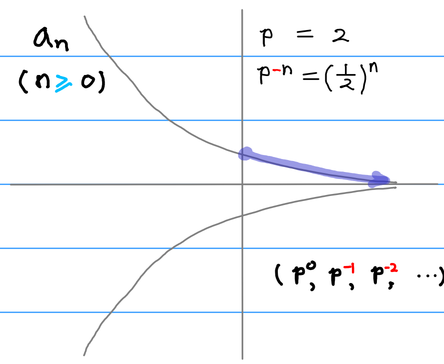
$n \in [0, \infty)$ $|z| < 2$

causal

$a_n = 0.5^n$

$\boxed{\dfrac{2}{2 - z}}$



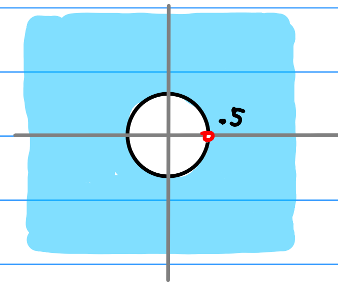


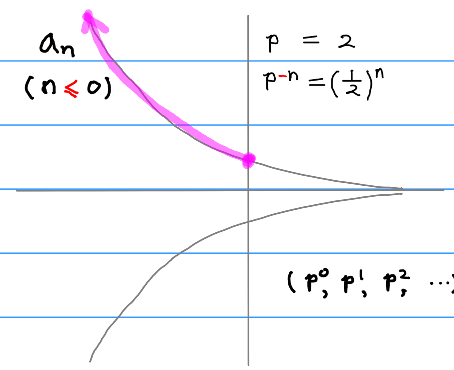
$|z| > 0.5$ $n \in (-\infty, 0]$

$\boxed{\dfrac{z}{z - 0.5}}$

anti-causal

$a_{-n} = 2^n$



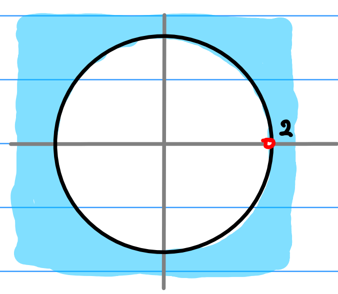


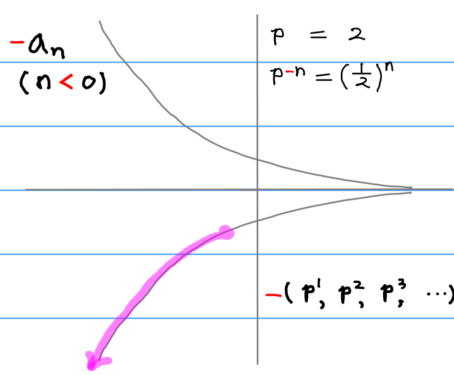
$-a_n = -0.5^n$

anti-causal

$\boxed{\dfrac{2}{2 - z}}$

$n \in (-\infty, -1]$ $|z| > 2$



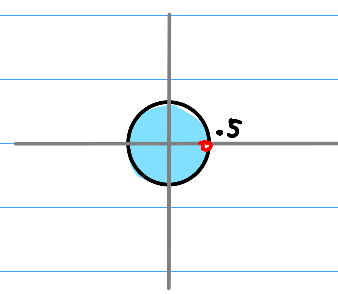


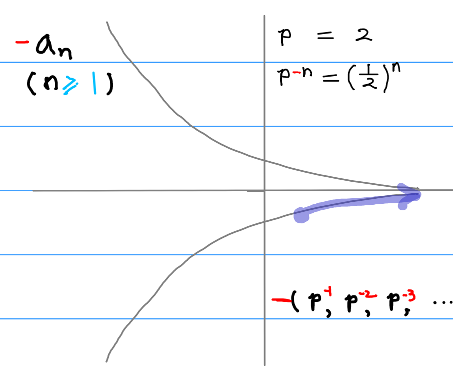
$\boxed{\dfrac{z}{z - 0.5}}$

$-a_{-n} = -2^n$

causal

$|z| < 0.5$ $n \in [1, \infty)$





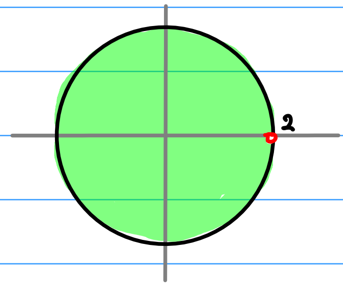
$n < 1$ $\quad |z| < 2$

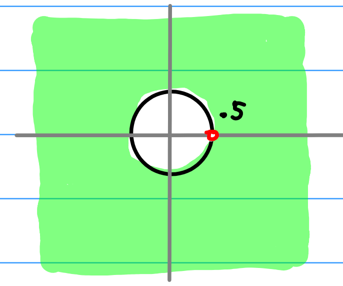
x_{-n} $\boxed{\text{anti-causal} \quad -(p^{-1}, p^{-2}, p^{-3}, \ldots)}$ $\boxed{-\dfrac{0.5}{1 - 0.5z}}$

$\xleftrightarrow{z^{-1}}$

$|z| > 0.5$ $\quad n \geq 0$

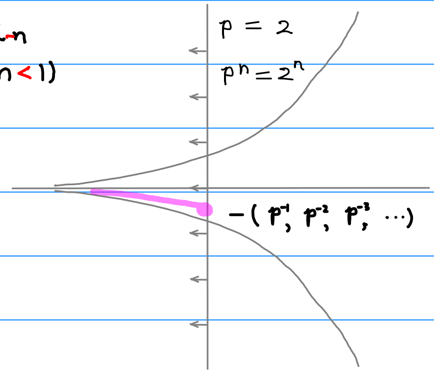
$\boxed{-\dfrac{0.5}{1 - 0.5z^{-1}}}$ $\boxed{\text{causal} \quad -(p^{-1}, p^{-2}, p^{-3}, \ldots)}$ x_n





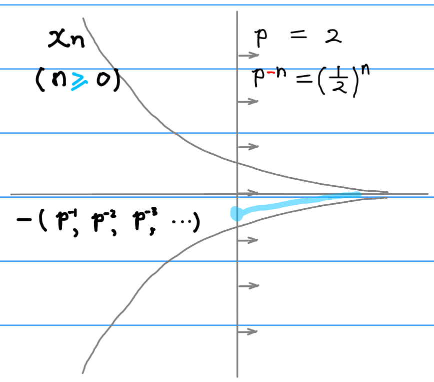
anti-causal $X(z^{-1})$

x_{-n}
$(n < 1)$



causal $X(z)$

x_n
$(n \geq 0)$



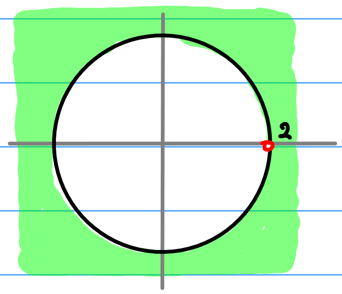
$n \geq 1$ $\quad |z| > 2$

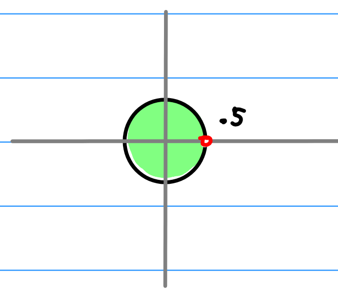
y_n $\boxed{(p^0, p^1, p^2, \ldots) \quad \text{causal}}$ $\boxed{\dfrac{z^{-1}}{1 - 2z^{-1}}}$

$\xleftrightarrow{z^{-1}}$

$|z| < 0.5$ $\quad n < 0$

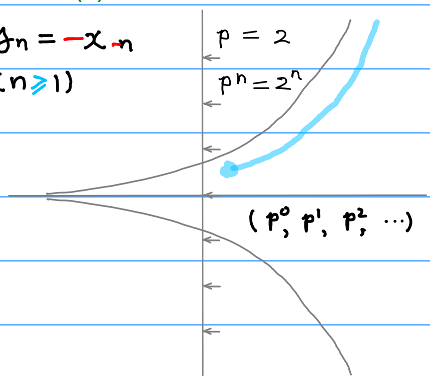
$\boxed{\dfrac{z}{1 - 2z}}$ $\boxed{(p^0, p^1, p^2, \ldots) \quad \text{anti-causal}}$ y_{-n}





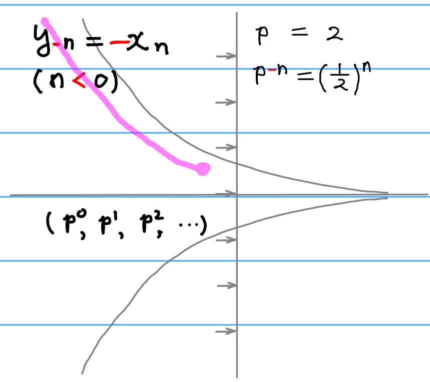
causal $Y(z)$

$y_n = -x_{-n}$
$(n \geq 1)$



anti-causal $Y(z^{-1})$

$y_{-n} = -x_n$
$(n < 0)$



2 formulas

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each

$$\boxed{\frac{1}{z-p}} \begin{cases} \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \triangleq \underset{\text{causal}}{f(z)} = \underset{\text{anti- causal}}{X(z^{-1})} \\ \qquad\qquad\qquad\quad \| \qquad\qquad \| \\ \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \triangleq \underset{\text{causal}}{Y(z)} = \underset{\text{anti- causal}}{g(z^{-1})} \end{cases}$$

$$\boxed{\frac{1}{z^{-1}-p}} \begin{cases} \boxed{\dfrac{z}{1-pz}} \triangleq \underset{\text{causal}}{g(z)} = \underset{\text{anti- causal}}{Y(z^{-1})} \\ \qquad\qquad\qquad\quad \| \qquad\qquad \| \\ \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq \underset{\text{causal}}{X(z)} = \underset{\text{anti- causal}}{f(z^{-1})} \end{cases}$$

Time Shift

$p = 2$

① $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = \frac{2}{2-z}$ $X(z) = \frac{z}{z-0.5}$

③ $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = -\frac{2}{2-z}$ $X(z) = -\frac{z}{z-0.5}$

⑤ $(n \geq 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = \frac{2z}{2-z}$ $X(z) = \frac{1}{z-0.5}$

⑦ $(n < 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = -\frac{2z}{2-z}$ $X(z) = -\frac{1}{z-0.5}$

⑨ $(n \geq -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $f(z) = \frac{2}{(2-z)z}$ $X(z) = \frac{z^2}{z-0.5}$

⑪ $(n < -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $f(z) = -\frac{2}{(2-z)z}$ $X(z) = -\frac{z^2}{z-0.5}$

Time Shift

$p = \frac{1}{2}$

② $(n \geq 0)$ $a_n = (2)^n$ $f(z) = \frac{0.5}{0.5 - z}$ $X(z) = \frac{z}{z-2}$

④ $(n < 0)$ $a_n = (2)^n$ $f(z) = -\frac{0.5}{0.5 - z}$ $X(z) = -\frac{z}{z-2}$

⑥ $(n \geq 1)$ $a_{n-1} = (2)^{n-1}$ $f(z) = \frac{0.5z}{0.5 - z}$ $X(z) = \frac{1}{z-2}$

⑧ $(n < 1)$ $a_{n-1} = (2)^{n-1}$ $f(z) = -\frac{0.5z}{0.5 - z}$ $X(z) = -\frac{1}{z-2}$

⑩ $(n \geq -1)$ $a_{n+1} = (2)^{n+1}$ $f(z) = \frac{0.5}{(0.5 - z)z}$ $X(z) = \frac{z^2}{z-2}$

⑫ $(n < -1)$ $a_{n+1} = (2)^{n+1}$ $f(z) = -\frac{0.5}{(0.5 - z)z}$ $X(z) = -\frac{z^2}{z-2}$

Time Shift

$2 \leftrightarrow \frac{1}{2}$

1	$(n \geq 0)$	$a_n = \left(\frac{1}{2}\right)^n$	$f(z) = \frac{2}{2-z}$	$X(z) = \frac{z}{z-0.5}$
2	$(n \geq 0)$	$a_n = (2)^n$	$f(z) = \frac{0.5}{0.5-z}$	$X(z) = \frac{z}{z-2}$
3	$(n < 0)$	$a_n = \left(\frac{1}{2}\right)^n$	$f(z) = -\frac{2}{2-z}$	$X(z) = -\frac{z}{z-0.5}$
4	$(n < 0)$	$a_n = (2)^n$	$f(z) = -\frac{0.5}{0.5-z}$	$X(z) = -\frac{z}{z-2}$
5	$(n \geq 1)$	$a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = \frac{2z}{2-z}$	$X(z) = \frac{1}{z-0.5}$
6	$(n \geq 1)$	$a_{n-1} = (2)^{n-1}$	$f(z) = \frac{0.5z}{0.5-z}$	$X(z) = \frac{1}{z-2}$
7	$(n < 1)$	$a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = -\frac{2z}{2-z}$	$X(z) = -\frac{1}{z-0.5}$
8	$(n < 1)$	$a_{n-1} = (2)^{n-1}$	$f(z) = -\frac{0.5z}{0.5-z}$	$X(z) = -\frac{1}{z-2}$
9	$(n \geq -1)$	$a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$	$f(z) = \frac{2}{(2-z)z}$	$X(z) = \frac{z^2}{z-0.5}$
10	$(n \geq -1)$	$a_{n+1} = (2)^{n+1}$	$f(z) = \frac{0.5}{(0.5-z)z}$	$X(z) = \frac{z^2}{z-2}$
11	$(n < -1)$	$a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$	$f(z) = -\frac{2}{(2-z)z}$	$X(z) = -\frac{z^2}{z-0.5}$
12	$(n < -1)$	$a_{n+1} = (2)^{n+1}$	$f(z) = -\frac{0.5}{(0.5-z)z}$	$X(z) = -\frac{z^2}{z-2}$

$2 \leftrightarrow \frac{1}{2}$

① $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = \frac{2}{2-z}$ $X(z) = \frac{z}{z-0.5}$

③ $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = -\frac{2}{2-z}$ $X(z) = -\frac{z}{z-0.5}$

⑤ $(n \geq 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = \frac{2z}{2-z}$ $X(z) = \frac{1}{z-0.5}$

⑦ $(n < 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = -\frac{2z}{2-z}$ $X(z) = -\frac{1}{z-0.5}$

⑨ $(n \geq -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $f(z) = \frac{2}{(2-z)z}$ $X(z) = \frac{z^2}{z-0.5}$

⑪ $(n < -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $f(z) = -\frac{2}{(2-z)z}$ $X(z) = -\frac{z^2}{z-0.5}$

$2 \longleftrightarrow \frac{1}{2}$

② $(n \geq 0)$ $a_n = (2)^n$

$f(z) = \frac{0.5}{0.5 - z}$ $X(z) = \frac{z}{z-2}$

$|z|<0.5$ $\frac{1}{1-2z}$ $|z|<0.5$ $\frac{1}{1-2z^{-1}}$

④ $(n < 0)$ $a_n = (2)^n$

$f(z) = -\frac{0.5}{0.5 - z}$ $X(z) = -\frac{z}{z-2}$

$|z|>0.5$ $\frac{-1}{1-2z}$ $|z|>0.5$ $\frac{-1}{1-2z^{-1}}$

⑥ $(n \geq 1)$ $a_{n-1} = (2)^{n-1}$

$f(z) = \frac{0.5z}{0.5 - z}$ $X(z) = \frac{1}{z-2}$

$|z|>0.5$ $\frac{z}{1-2z}$ $|z|>0.5$ $\frac{z^{-1}}{1-2z^{-1}}$

⑧ $(n < 1)$ $a_{n-1} = (2)^{n-1}$

$f(z) = -\frac{0.5z}{0.5 - z}$ $X(z) = -\frac{1}{z-2}$

$|z|>0.5$ $\frac{-z}{1-2z}$ $|z|>0.5$ $\frac{-z^{-1}}{1-2z^{-1}}$

⑩ $(n \geq -1)$ $a_{n+1} = (2)^{n+1}$

$f(z) = \frac{0.5}{(0.5-z)z}$ $X(z) = \frac{z^2}{z-2}$

$|z|<0.5$ $\frac{1}{(1-2z)z}$ $|z|<0.5$ $\frac{1}{(1-2z^{-1})z^{-1}}$

⑫ $(n < -1)$ $a_{n+1} = (2)^{n+1}$

$f(z) = -\frac{0.5}{(0.5-z)z}$ $X(z) = -\frac{z^2}{z-2}$

$|z|<0.5$ $\frac{-1}{(1-2z)z}$ $|z|<0.5$ $\frac{-1}{(1-2z^{-1})z^{-1}}$

Causality

$f(z)\ (|z|<p) \leftrightarrow a_n\ (n \geq 0)$ $\quad -(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$X(z^{-1})\ (|z|<p) \leftrightarrow x_{-n}\ (n < 1)$ $\quad -(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$f(z^{-1})\ (|z|>p^{-1}) \leftrightarrow a_{-n}\ (n < 1)$ $\quad -(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$X(z)\ (|z|>p^{-1}) \leftrightarrow x_n\ (n \geq 0)$ $\quad -(p^{-1}, p^{-2}, p^{-3}, \cdots)$

$f(z)\ (|z|>p) \leftrightarrow -a_n\ (n < 0)$ $\quad (p^0, p^1, p^2, \cdots)$

$X(z^{-1})\ (|z|>p) \leftrightarrow -x_{-n}\ (n \geq 1)$ $\quad (p^0, p^1, p^2, \cdots)$

$f(z^{-1})\ (|z|<p^{-1}) \leftrightarrow -a_{-n}\ (n \geq 1)$ $\quad (p^0, p^1, p^2, \cdots)$

$X(z)\ (|z|<p^{-1}) \leftrightarrow -x_n\ (n < 0)$ $\quad (p^0, p^1, p^2, \cdots)$

$$\boxed{\begin{array}{cc} f(z) & f(z^{-1}) \\ g(z^{-1}) & g(z) \end{array}} \qquad \boxed{\begin{array}{cc} X(z^{-1}) & X(z) \\ Y(z) & Y(z^{-1}) \end{array}} \qquad \boxed{\begin{array}{cc} a_n & a_{-n} \\ b_{-n} & b_n \end{array}} \qquad \boxed{\begin{array}{cc} x_{-n} & x_n \\ y_n & y_{-n} \end{array}}$$

$$\boxed{\begin{array}{cc} f(z) & f(z^{-1}) \\ f(z) & f(z^{-1}) \end{array}} \qquad \boxed{\begin{array}{cc} X(z^{-1}) & X(z) \\ X(z^{-1}) & X(z) \end{array}}$$

$$\boxed{\begin{array}{cc} -(p^{-1}, p^{-2}, p^{-3}, \dots) & -(p^{-1}, p^{-2}, p^{-3}, \dots) \\ (p^{0}, p^{1}, p^{2}, \dots) & (p^{0}, p^{1}, p^{2}, \dots) \end{array}} \qquad \boxed{\begin{array}{cc} -(p^{-1}, p^{-2}, p^{-3}, \dots) & -(p^{-1}, p^{-2}, p^{-3}, \dots) \\ (p^{0}, p^{1}, p^{2}, \dots) & (p^{0}, p^{1}, p^{2}, \dots) \end{array}}$$

$$\boxed{\begin{array}{cc} -\dfrac{p^{-1}}{1-p^{-1}z} & -\dfrac{p^{-1}}{1-p^{-1}z^{-1}} \\ \dfrac{z^{-1}}{1-pz^{-1}} & \dfrac{z}{1-pz} \end{array}} \qquad \boxed{\begin{array}{cc} -\dfrac{p^{-1}}{1-p^{-1}z} & -\dfrac{p^{-1}}{1-p^{-1}z^{-1}} \\ \dfrac{z^{-1}}{1-pz^{-1}} & \dfrac{z}{1-pz} \end{array}}$$

$f(z)$	$g(z)$
$f(z)$	$g(z)$

$Y(z)$	$X(z)$
$Y(z)$	$X(z)$

a_n	a_{-n}
$-a_n$	$-a_{-n}$

x_{-n}	x_n
$-x_{-n}$	$-x_n$

$\lvert z\rvert < p$	$\lvert z\rvert > p^{-1}$
$\lvert z\rvert > p$	$\lvert z\rvert < p^{-1}$

$\lvert z\rvert < p$	$\lvert z\rvert > p^{-1}$
$\lvert z\rvert > p$	$\lvert z\rvert < p^{-1}$

$[0, \infty)$	$(-\infty, 0]$
$(-\infty, -1]$	$[1, \infty)$

$(-\infty, 0]$	$[0, \infty)$
$[1, \infty)$	$(-\infty, -1]$

$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$
$$(p^{0}, p^{1}, p^{2}, \cdots)$$

$$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$$
$$(p^{0}, p^{1}, p^{2}, \cdots)$$

a_n	a_{-n}
$-a_n$	$-a_{-n}$

2^{-n}	2^n
-2^{-n}	-2^n

$a_n = -2^{-n}$

x_{-n}	x_n
$-x_{-n}$	$-x_n$

2^{-n}	2^n
-2^{-n}	-2^n

$x_n = -2^n$

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$
$(p^0, p^1, p^2, \cdots)$	$(p^0, p^1, p^2, \cdots)$

$-(2^{-1}, 2^{-2}, 2^{-3}, \cdots)$	$-(2^{-1}, 2^{-2}, 2^{-3}, \cdots)$
$(2^0, 2^1, 2^2, \cdots)$	$(2^0, 2^1, 2^2, \cdots)$

$-\dfrac{p^{-1}}{1-p^{-1}z}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$
$\dfrac{z^{-1}}{1-pz^{-1}}$	$\dfrac{z}{1-pz}$

$\dfrac{2^{-1}}{1-2^{-1}z}$	$\dfrac{2^{-1}}{1-2^{-1}z^{-1}}$
$-\dfrac{z^{-1}}{1-2z^{-1}}$	$-\dfrac{z}{1-2z}$

$\dfrac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$	$\dfrac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$
$-\dfrac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$	$-\dfrac{z}{1-2z}$

$\|z\| < p$	$\|z\| > p^{-1}$
$\|z\| > p$	$\|z\| < p^{-1}$

$\|z\| < 2$	$\|z\| > 2^{-1}$
$\|z\| > 2$	$\|z\| < 2^{-1}$

$[0, \infty)$	$(-\infty, 0]$
$(-\infty, -1]$	$[1, \infty)$

$[0, \infty)$	$(-\infty, 0]$
$(-\infty, -1]$	$[1, \infty)$

Shift to the right

shift to the right →	$\times z$	$\times z^{-1}$
delete a_0		
① $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$	$f(z) = \frac{2}{2-z}$	$X(z) = \frac{z}{z-0.5}$
⑤ $(n \geq 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = \frac{2z}{2-z}$	$X(z) = \frac{1}{z-0.5}$
② $(n \geq 0)$ $a_n = (2)^n$	$f(z) = \frac{0.5}{0.5-z}$	$X(z) = \frac{z}{z-2}$
⑥ $(n \geq 1)$ $a_{n-1} = (2)^{n-1}$	$f(z) = \frac{0.5z}{0.5-z}$	$X(z) = \frac{1}{z-2}$

shift to the right →	$\times z$	$\times z^{-1}$
insert a_0		
③ $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$	$f(z) = -\frac{2}{2-z}$	$X(z) = -\frac{z}{z-0.5}$
⑦ $(n < 1)$ $a_{n-1} = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = -\frac{2z}{2-z}$	$X(z) = -\frac{1}{z-0.5}$
④ $(n < 0)$ $a_n = (2)^n$	$f(z) = -\frac{0.5}{0.5-z}$	$X(z) = -\frac{z}{z-2}$
⑧ $(n < 1)$ $a_{n-1} = (2)^{n-1}$	$f(z) = -\frac{0.5z}{0.5-z}$	$X(z) = -\frac{1}{z-2}$

Shift to the left

Shift to the left $\leftarrow$ $\times z^{-1}$ $\times z$

delete a_0

① $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $\quad f(z) = \frac{2}{2-z}$ $\quad X(z) = \frac{z}{z-0.5}$

⑨ $(n \geq -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $\quad f(z) = \frac{2}{(2-z)z}$ $\quad X(z) = \frac{z}{z-0.5}$

② $(n \geq 0)$ $a_n = (2)^n$ $\quad f(z) = \frac{0.5}{0.5-z}$ $\quad X(z) = \frac{z}{z-2}$

⑩ $(n \geq -1)$ $a_{n+1} = (2)^{n+1}$ $\quad f(z) = \frac{0.5}{(0.5-z)z}$ $\quad X(z) = \frac{z}{z-2}$

Shift to the left $\leftarrow$ $\times z^{-1}$ $\times z$

insert a_0

③ $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $\quad f(z) = -\frac{2}{2-z}$ $\quad X(z) = -\frac{z}{z-0.5}$

⑪ $(n < -1)$ $a_{n+1} = \left(\frac{1}{2}\right)^{n+1}$ $\quad f(z) = -\frac{2}{(2-z)z}$ $\quad X(z) = -\frac{z}{z-0.5}$

④ $(n < 0)$ $a_n = (2)^n$ $\quad f(z) = -\frac{0.5}{0.5-z}$ $\quad X(z) = -\frac{z}{z-2}$

⑫ $(n < -1)$ $a_{n+1} = (2)^{n+1}$ $\quad f(z) = -\frac{0.5}{(0.5-z)z}$ $\quad X(z) = -\frac{z}{z-2}$

$n=-4$	$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n+1} $n=-2,-3,-4,\cdots$

b^{n+1} $n=-1,0,1,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n} $n=-1,-2,-3,\cdots$

b^{n} $n=0,1,2,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$		
	b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n-1} $n=0,-1,-2,\cdots$

b^{n-1} $n=1,2,3,\cdots$

$$1 \longleftrightarrow \frac{1}{1}$$

① $(n \geq 0) \quad a_n = (1)^n \qquad f(z) = \frac{1}{1-z} \qquad X(z) = \frac{z}{z-1}$

② $(n \geq 0) \quad a_n = (1^{-1})^n \qquad f(z) = \frac{1}{1-z} \qquad X(z) = \frac{z}{z-1}$

③ $(n < 0) \quad a_n = (1)^n \qquad f(z) = -\frac{1}{1-z} \qquad X(z) = -\frac{z}{z-1}$

④ $(n < 0) \quad a_n = (1^{-1})^n \qquad f(z) = -\frac{1}{1-z} \qquad X(z) = -\frac{z}{z-1}$

⑤ $(n \geq 1) \quad a_{n-1} = (1)^{n-1} \qquad f(z) = \frac{z}{1-z} \qquad X(z) = \frac{1}{z-1}$

⑥ $(n \geq 1) \quad a_{n-1} = (1^{-1})^{n-1} \qquad f(z) = \frac{z}{1-z} \qquad X(z) = \frac{1}{z-1}$

⑦ $(n < 1) \quad a_{n-1} = (1)^{n-1} \qquad f(z) = -\frac{z}{1-z} \qquad X(z) = -\frac{1}{z-1}$

⑧ $(n < 1) \quad a_{n-1} = (1^{-1})^{n-1} \qquad f(z) = -\frac{z}{1-z} \qquad X(z) = -\frac{1}{z-1}$

⑨ $(n \geq -1) \quad a_{n+1} = (1)^{n+1} \qquad f(z) = \frac{1}{(1-z)z} \qquad X(z) = \frac{z}{z-1}$

⑩ $(n \geq -1) \quad a_{n+1} = (1^{-1})^{n+1} \qquad f(z) = \frac{1}{(1-z)z} \qquad X(z) = \frac{z}{z-1}$

⑪ $(n < -1) \quad a_{n+1} = (1)^{n+1} \qquad f(z) = -\frac{1}{(1-z)z} \qquad X(z) = -\frac{z}{z-1}$

⑫ $(n < -1) \quad a_{n+1} = (1^{-1})^{n+1} \qquad f(z) = -\frac{1}{(1-z)z} \qquad X(z) = -\frac{z}{z-1}$

1 $(n \geq 0) \quad a_n = (1)^n \qquad f(z) = \dfrac{1}{1-z} \qquad X(z) = \dfrac{z}{z-1}$

2 $(n \geq 0) \quad a_n = (1^{-1})^n \qquad f(z) = \dfrac{1}{1-z} \qquad X(z) = \dfrac{z}{z-1}$

shift to the right → $\times z$ $\times z^{-1}$

delete a_0

5 $(n \geq 1) \quad a_{n-1} = (1)^{n-1} \qquad f(z) = \dfrac{z}{1-z} \qquad X(z) = \dfrac{1}{z-1}$

6 $(n \geq 1) \quad a_{n-1} = (1^{-1})^{n-1} \qquad f(z) = \dfrac{z}{1-z} \qquad X(z) = \dfrac{1}{z-1}$

3 $(n < 0) \quad a_n = (1)^n \qquad f(z) = -\dfrac{1}{1-z} \qquad X(z) = -\dfrac{z}{z-1}$

4 $(n < 0) \quad a_n = (1^{-1})^n \qquad f(z) = -\dfrac{1}{1-z} \qquad X(z) = -\dfrac{z}{z-1}$

shift to the right → $\times z$ $\times z^{-1}$

insert a_0

7 $(n < 1) \quad a_{n-1} = (1)^{n-1} \qquad f(z) = -\dfrac{z}{1-z} \qquad X(z) = -\dfrac{1}{z-1}$

8 $(n < 1) \quad a_{n-1} = (1^{-1})^{n-1} \qquad f(z) = -\dfrac{z}{1-z} \qquad X(z) = -\dfrac{1}{z-1}$

1 $(n \geq 0)$ $a_n = (1)^n$ $f(z) = \frac{1}{1-z}$ $X(z) = \frac{z}{z-1}$

2 $(n \geq 0)$ $a_n = (1^{-1})^n$ $f(z) = \frac{1}{1-z}$ $X(z) = \frac{z}{z-1}$

Shift to the left $\leftarrow$ $\times z^{-1}$ $\times z$

delete a_0

9 $(n \geq -1)$ $a_{n+1} = (1)^{n+1}$ $f(z) = \frac{1}{(1-z)z}$ $X(z) = \frac{z}{z-1}$

10 $(n \geq -1)$ $a_{n+1} = (1^{-1})^{n+1}$ $f(z) = \frac{1}{(1-z)z}$ $X(z) = \frac{z}{z-1}$

3 $(n < 0)$ $a_n = (1)^n$ $f(z) = -\frac{1}{1-z}$ $X(z) = -\frac{z}{z-1}$

4 $(n < 0)$ $a_n = (1^{-1})^n$ $f(z) = -\frac{1}{1-z}$ $X(z) = -\frac{z}{z-1}$

Shift to the left $\leftarrow$ $\times z^{-1}$ $\times z$

insert a_0

11 $(n < -1)$ $a_{n+1} = (1)^{n+1}$ $f(z) = -\frac{1}{(1-z)z}$ $X(z) = -\frac{z}{z-1}$

12 $(n < -1)$ $a_{n+1} = (1^{-1})^{n+1}$ $f(z) = -\frac{1}{(1-z)z}$ $X(z) = -\frac{z}{z-1}$

$n=-4$	$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n+1} $n=-2,-3,-4,\cdots$ b^{n+1} $n=-1,0,1,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

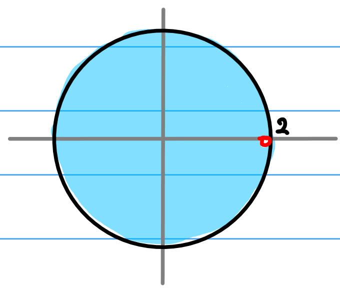
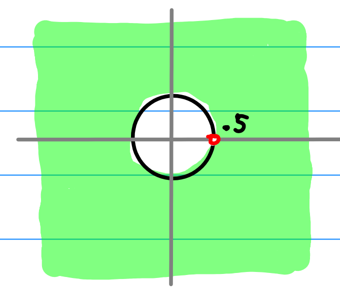
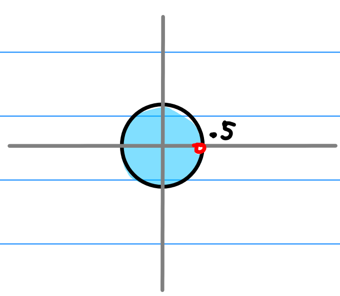
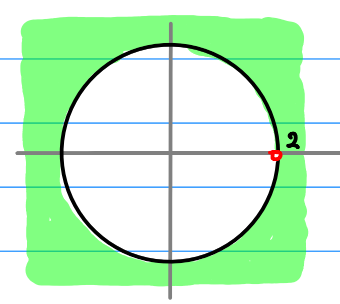
b^{n} $n=-1,-2,-3,\cdots$ b^{n} $n=0,1,2,\cdots$

$n=-3$

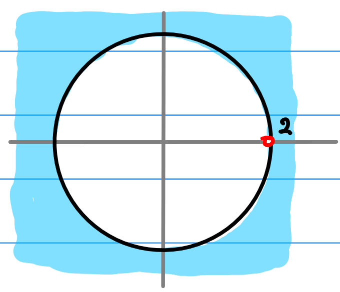
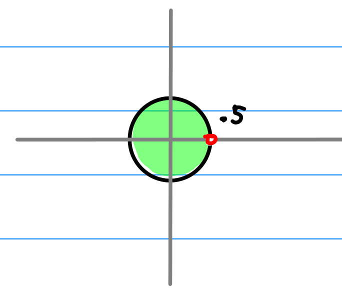
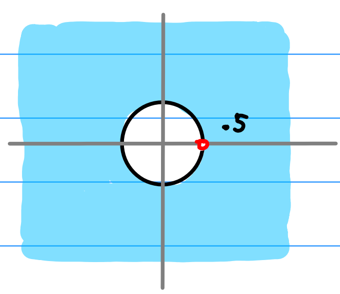
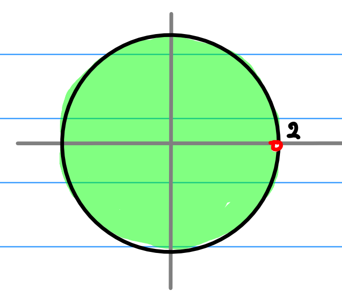
$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$		
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n-1} $n=0,-1,-2,\cdots$ b^{n-1} $n=1,2,3,\cdots$

Causal $(n \geq 0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

	$f(z)$		$X(z)$
$\left(\frac{1}{2}\right)^n$	$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$		$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$
	$f(z) = \frac{1}{1-\frac{z}{2}} \quad \lvert z\rvert < 2$	$\xleftrightarrow{z^{-1}}$	$X(z) = \frac{1}{1-\frac{1}{2z}} \quad \lvert z\rvert > 0.5$
	$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$		$= \frac{z}{z-0.5}$
			
	$\updownarrow p^{-1}$		$\updownarrow p^{-1}$
$(2)^n$	$a_n = (2)^n \quad (n \geq 0)$		$a_n = (2)^n \quad (n \geq 0)$
	$f(z) = \frac{1}{1-2z} \quad \lvert z\rvert < 0.5$	$\xleftrightarrow{z^{-1}}$	$X(z) = \frac{1}{1-\frac{2}{z}} \quad \lvert z\rvert > 2$
	$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$		$= \frac{z}{z-2}$
			

Anti-causal $(n<0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

	$f(z)$		$X(z)$
$\left(\frac{1}{2}\right)^n$	$a_n=\left(\frac{1}{2}\right)^n \quad (n<0)$ $f(z)=\dfrac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert>2$ $=-\dfrac{2}{2-z}$  	$\xleftrightarrow{z^{-1}}$	$a_n=\left(\frac{1}{2}\right)^n \quad (n<0)$ $X(z)=\dfrac{2z}{1-2z} \quad \lvert z\rvert<0.5$ $=-\dfrac{z}{z-0.5}$ 
	$\updownarrow p^{-1}$		$\updownarrow p^{-1}$
$(2)^n$	$a_n=(2)^n \quad (n<0)$ $f(z)=\dfrac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad \lvert z\rvert>0.5$ $=-\dfrac{0.5}{0.5-z}$  	$\xleftrightarrow{z^{-1}}$	$a_n=(2)^n \quad (n<0)$ $X(z)=\dfrac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert<2$ $=-\dfrac{z}{z-2}$ 

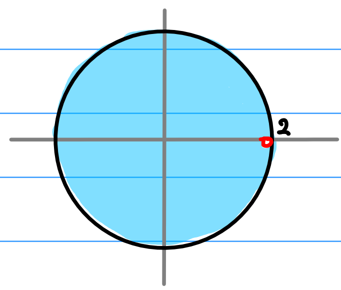
$f(z)$	Causal $(n \geq 0)$	Anti-causal $(n < 0)$

$(\frac{1}{2})^n$

$a_n = (\frac{1}{2})^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$

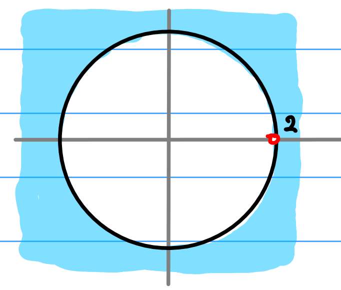


$\xleftrightarrow{-1}$

$a_n = (\frac{1}{2})^n \quad (n < 0)$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad |z| > 2$$

$$= -\frac{2}{2-z}$$



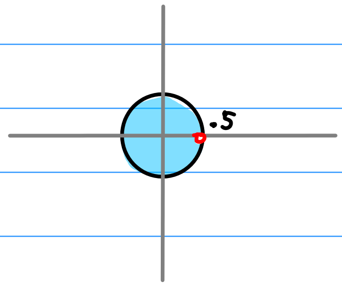
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$a_n = (2)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

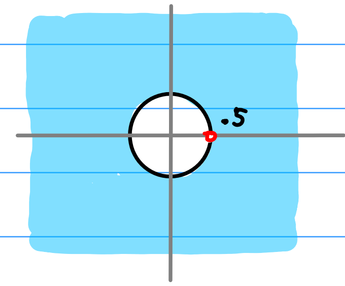


$\xleftrightarrow{-1}$

$a_n = (2)^n \quad (n < 0)$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= -\frac{0.5}{0.5-z}$$



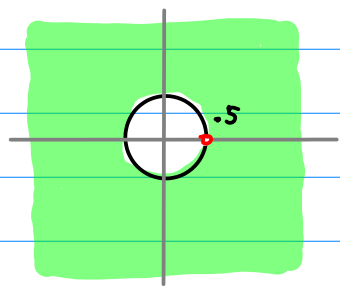
$X(z)$	Causal $(n \geq 0)$	Anti-causal $(n < 0)$

$\left(\frac{1}{2}\right)^n$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - \frac{1}{2z}} \quad |z| > 0.5$$

$$= \frac{z}{z - 0.5}$$

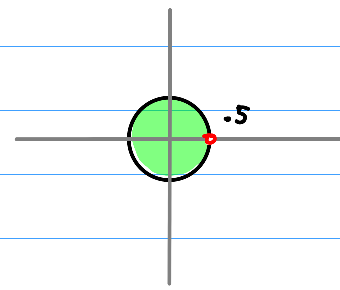


$\overset{-1}{\longleftrightarrow}$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$$

$$X(z) = \frac{2z}{1 - 2z} \quad |z| < 0.5$$

$$= -\frac{z}{z - 0.5}$$



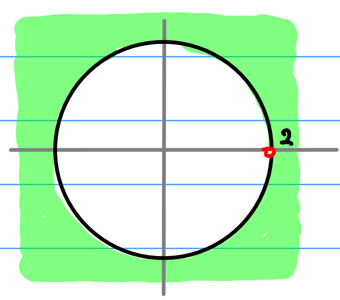
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$$a_n = (2)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - \frac{2}{z}} \quad |z| > 2$$

$$= \frac{z}{z - 2}$$

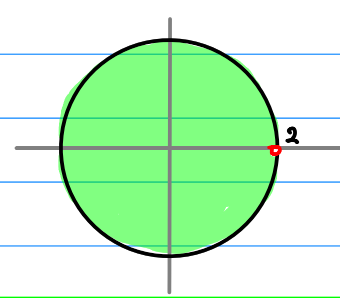


$\overset{-1}{\longleftrightarrow}$

$$a_n = (2)^n \quad (n < 0)$$

$$X(z) = \frac{\frac{2}{z}}{1 - \frac{2}{z}} \quad |z| < 2$$

$$= -\frac{z}{z - 2}$$



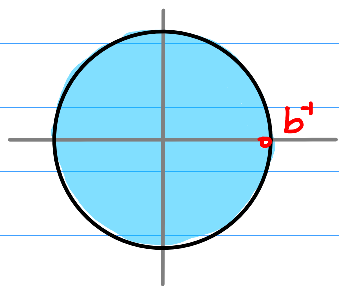
Causal b^n Anti-causal b^n

$f(z)$

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$f(z) = \frac{1}{1 - bz} \qquad |z| < b^{-1}$$

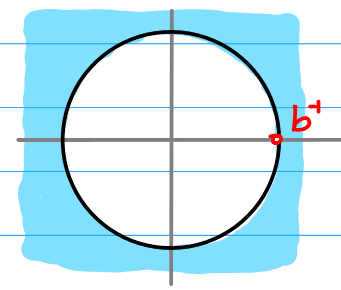
$$= \frac{b^{-1}}{b^{-1} - z}$$



$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1} z^{-1}}{1 - b^{-1} z^{-1}} \qquad |z| > b^{-1}$$

$$= -\frac{b^{-1}}{b^{-1} - z}$$

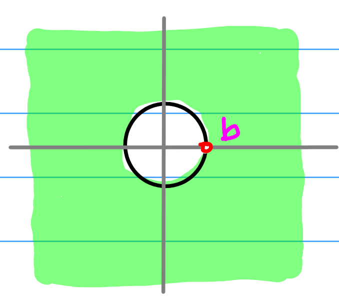


$X(z)$

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$X(z) = \frac{1}{1 - bz^{-1}} \qquad |z| > b$$

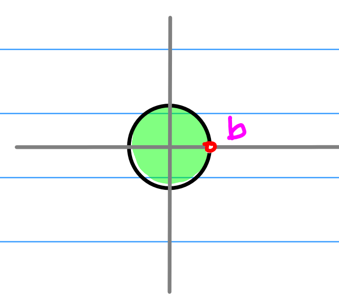
$$= \frac{z}{z - b}$$



$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1} z}{1 - b^{-1} z} \qquad |z| < b$$

$$= -\frac{z}{z - b}$$

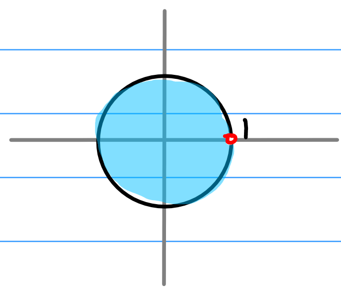


Causal $(n \geq 0)$ $\quad (1)^n, \ (1^{-1})^n$

	$f(z)$		$X(z)$
$(1)^n$	$a_n = (1)^n \quad (n \geq 0)$		$a_n = (1)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-z} \qquad |z| < 1$$

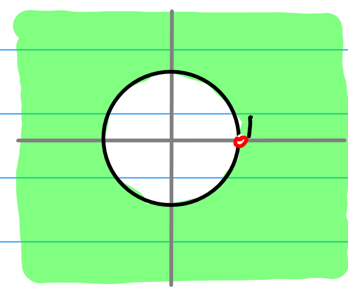
$$= \frac{z^{-1}}{z^{-1}-1} = \frac{1}{1-z}$$



$$\overset{z^{-1}}{\longleftrightarrow}$$

$$X(z) = \frac{1}{1-\frac{1}{z}} \qquad |z| > 1$$

$$= \frac{z}{z-1}$$



$\updownarrow p^{-1}$ $\qquad\qquad$ $\updownarrow p^{-1}$

$(1^{-1})^n$

Anti-causal $(n<0)$ $\quad (1)^n$, $(1^{-1})^n$

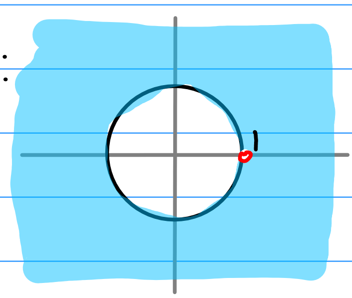
$f(z)$ $\qquad\qquad$ $X(z)$

$(1)^n$

$a_n = (1)^n \quad (n<0)$

$$f(z) = \frac{\frac{1}{z}}{1-\frac{1}{z}} \quad |z| > 1$$

$$= -\frac{1}{1-z}$$

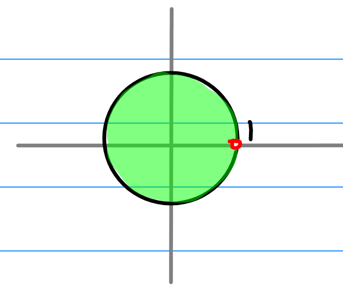


$\xleftrightarrow{z^{-1}}$

$a_n = (1)^n \quad (n<0)$

$$X(z) = \frac{z}{1-z} \quad |z| < 1$$

$$= -\frac{z}{z-1}$$



$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(1^{-1})^n$

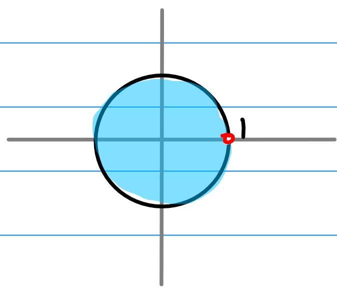
$f(z)$ Causal $(n \geq 0)$ Anti-causal $(n < 0)$

$(1)^n$

$a_n = (1)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-z} \qquad |z| < 1$$

$$= \frac{z^{-1}}{z^{-1} - 1} = \frac{1}{1-z}$$

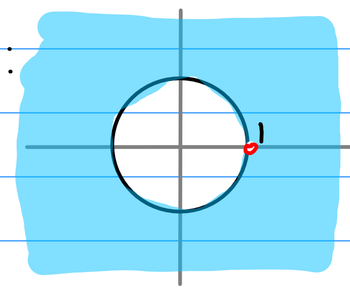


$\xleftrightarrow{-1}$

$a_n = (1)^n \quad (n < 0)$

$$f(z) = \frac{\frac{1}{z}}{1 - \frac{1}{z}} \qquad |z| > 1$$

$$= -\frac{1}{1-z}$$



$\updownarrow p^{-1}$ $\updownarrow p^{-1}$

$(1^{-1})^n$

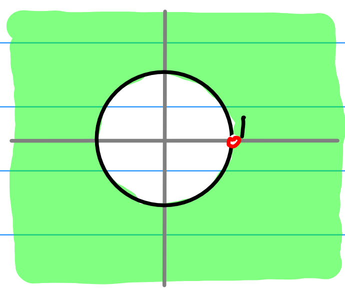
$X(z)$ Causal $(n \geq 0)$ Anti-causal $(n < 0)$

$(1)^n$

$a_n = (1)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{z}} \qquad |z| > 1$$

$$= \frac{z}{z-1}$$

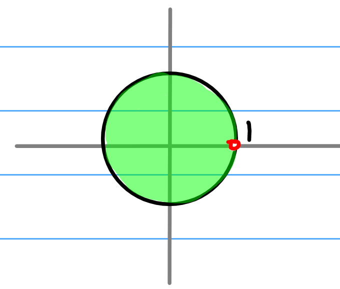


$\xleftrightarrow{-1}$

$a_n = (1)^n \quad (n < 0)$

$$X(z) = \frac{z}{1-z} \qquad |z| < 1$$

$$= -\frac{z}{z-1}$$



$\updownarrow p^{-1}$ $\updownarrow p^{-1}$

$(1^{-1})^n$

Causal b^n | Anti-causal b^n

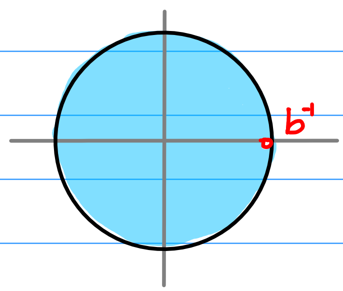
$f(z)$

Causal

$$a_n = (b)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1 - bz} \qquad |z| < b^{-1}$$

$$= \frac{b^{-1}}{b^{-1} - z}$$

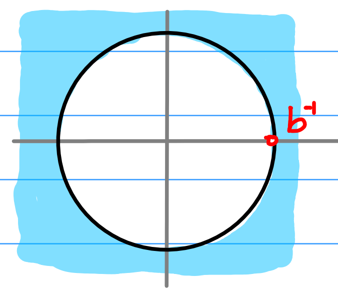


Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1} z^{-1}}{1 - b^{-1} z^{-1}} \qquad |z| > b^{-1}$$

$$= -\frac{b^{-1}}{b^{-1} - z}$$



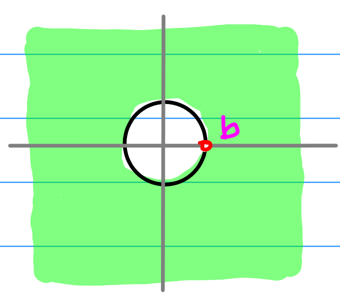
$X(z)$

Causal

$$a_n = (b)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - bz^{-1}} \qquad |z| > b$$

$$= \frac{z}{z - b}$$



Anti-causal

$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1} z}{1 - b^{-1} z} \qquad |z| < b$$

$$= -\frac{z}{z - b}$$