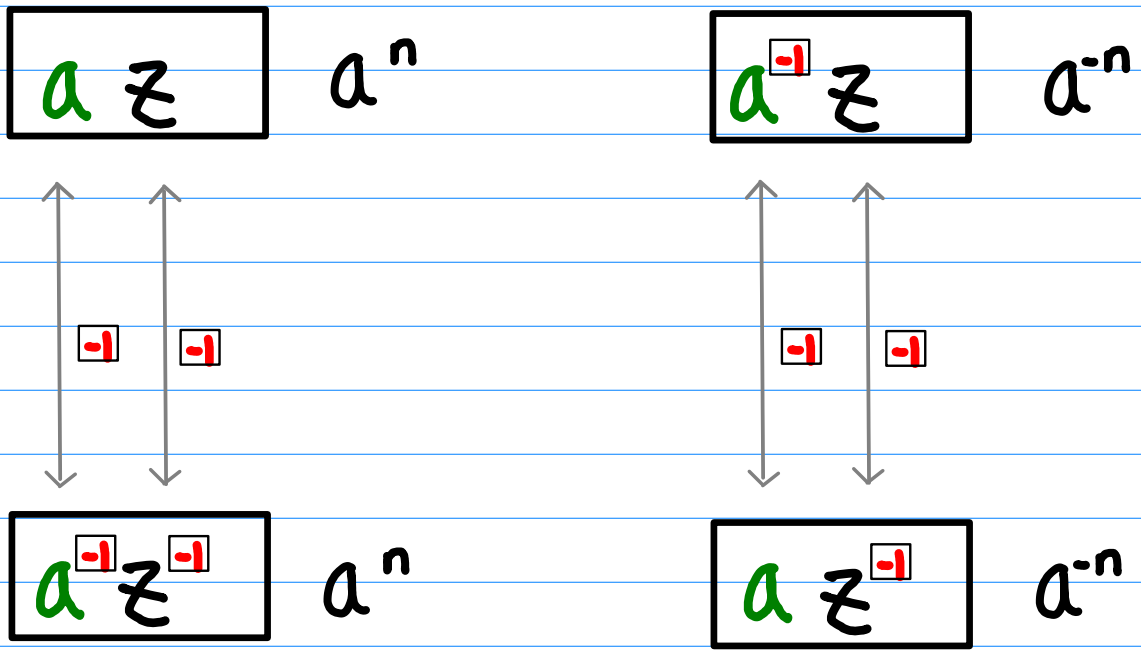
HW Butterfly FFT
z-Transform Properties

20200406 Mon

https://en.wikiversity.org/wiki/Complex_Analysis_in_plain_view
Geometric Series Examples
Applications (A.pdf, B.pdf)

Combinations of a and z
-- common ratio in a geometric series

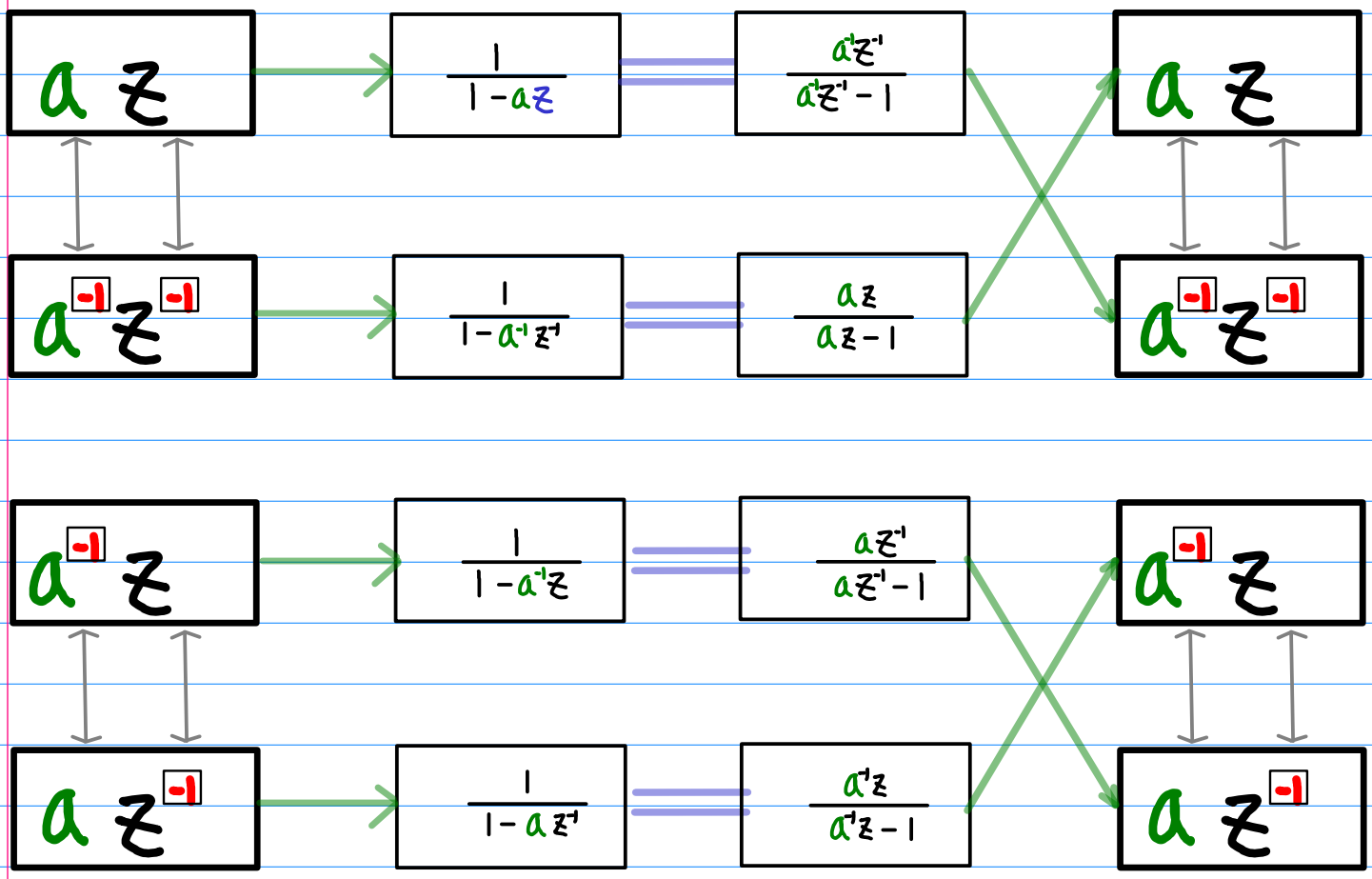


Geometric Series

the same formula,
different representations

common ratio

common ratio



two equivalent representations
of geometric series

the same formula with different ROCs

different Geometric Series

common ratio | **complementary ranges** | **inversed common ratio**

common ratio	causal u(n)	anti-causal u(-n-1)	inversed common ratio
$a\,z$	$\frac{1}{1-az}$ $\quad \lvert z\rvert < a^{-1}$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\quad \lvert z\rvert > a^{-1}$	$a^{-1}\,z^{-1}$
	$a^0z^0 + a^1z^1 + a^2z^2 + \cdots$	$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$	
	anti-causal u(-n)	**causal u(n-1)**	
$a^{-1}\,z^{-1}$	$\frac{1}{1-a^{-1}z^{-1}}$ $\quad \lvert z\rvert > a^{-1}$	$-\frac{az}{1-az}$ $\quad \lvert z\rvert < a^{-1}$	$a\,z$
	$a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots$	$-(a^1z^1 + a^2z^2 + a^3z^3 + \cdots)$	
	causal u(n)	**anti-causal u(-n-1)**	
$a^{-1}\,z$	$\frac{1}{1-a^{-1}z}$ $\quad \lvert z\rvert < a$	$-\frac{az^{-1}}{1-az^{-1}}$ $\quad \lvert z\rvert > a$	$a\,z^{-1}$
	$a^0z^0 + a^1z^1 + a^2z^2 + \cdots$	$-(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \cdots)$	
	anti-causal u(-n)	**causal u(n-1)**	
$a\,z^{-1}$	$\frac{1}{1-az^{-1}}$ $\quad \lvert z\rvert > a$	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\quad \lvert z\rvert < a$	$a^{-1}\,z$
	$a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots$	$-(a^1z^1 + a^2z^2 + a^3z^3 + \cdots)$	

geometric series starting with a unit term

non-shifted range
u(n), u(-n)

geometric series starting with a non-unit term

shifted range
u(n-1), u(-n-1)

the same formula with different ROCs

different Geometric Series

common ratio

the same common ratio

causal u(n)

$\frac{1}{1-az}$ $|z|<a^{-1}$

$a^0z^0 + a^1z^1 + a^2z^2 + \cdots$

left shifted u(-n-1)

$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$-(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \cdots)$

$a^{-1}z^{-1}$

anti-causal u(-n)

$\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots$

right shifted u(n-1)

$-\frac{az}{1-az}$ $|z|<a^{-1}$

$-(a^1z^1 + a^2z^2 + a^3z^3 + \cdots)$

az

causal u(n)

$\frac{1}{1-a^{-1}z}$ $|z|<a$

$a^0z^0 + a^1z^1 + a^2z^2 + \cdots$

left shifted u(-n-1)

$-\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$-(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \cdots)$

$a^{-1}z$

anti-causal u(-n)

$\frac{1}{1-az^{-1}}$ $|z|>a$

$a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots$

right shifted u(n-1)

$-\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$-(a^1z^1 + a^2z^2 + a^3z^3 + \cdots)$

$a^{-1}z$

geometric series starting with a unit term

non-shifted range
u(n), u(-n)

geometric series starting with a non-unit term

shifted range
u(n-1), u(-n-1)

Geometric Power Series Property (1)

Each representation has it own ROC
(Region of Convergence)

common ratio $a z \rightarrow |z| < a^{-1}$ ROC

common ratio $a^{-1} z^{-1} \rightarrow |z| > a^{-1}$ ROC

common ratio $a^{-1} z \rightarrow |z| < a$ ROC

common ratio $a z^{-1} \rightarrow |z| > a$ ROC

Geometric Power Series Property (2)

Starting terms

geometric series starting with

a unit term

$$\frac{1}{1-az}$$

$$\frac{1}{1-a^{-1}z^{-1}}$$

$$\frac{1}{1-a^{-1}z}$$

$$\frac{1}{1-az^{-1}}$$

geometric series starting with

a non-unit term (common ratio)

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$$

$$-\frac{az}{1-az}$$

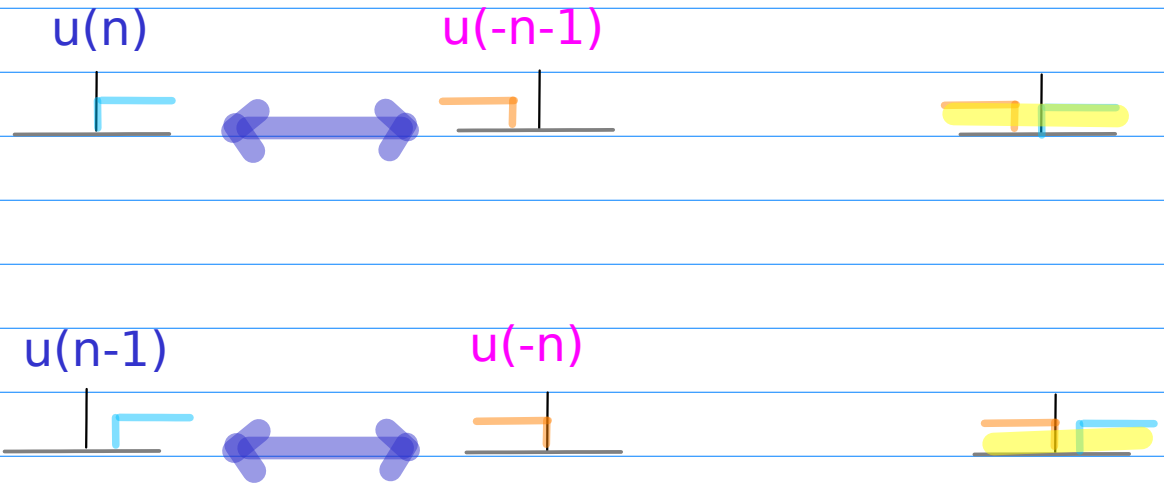
$$-\frac{az^{-1}}{1-az^{-1}}$$

$$-\frac{a^{-1}z}{1-a^{-1}z}$$

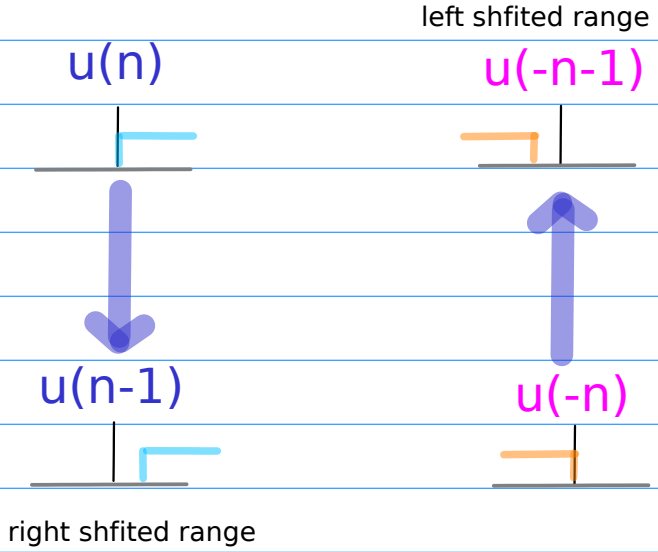
related to shifting

Geometric Power Series Property (3)

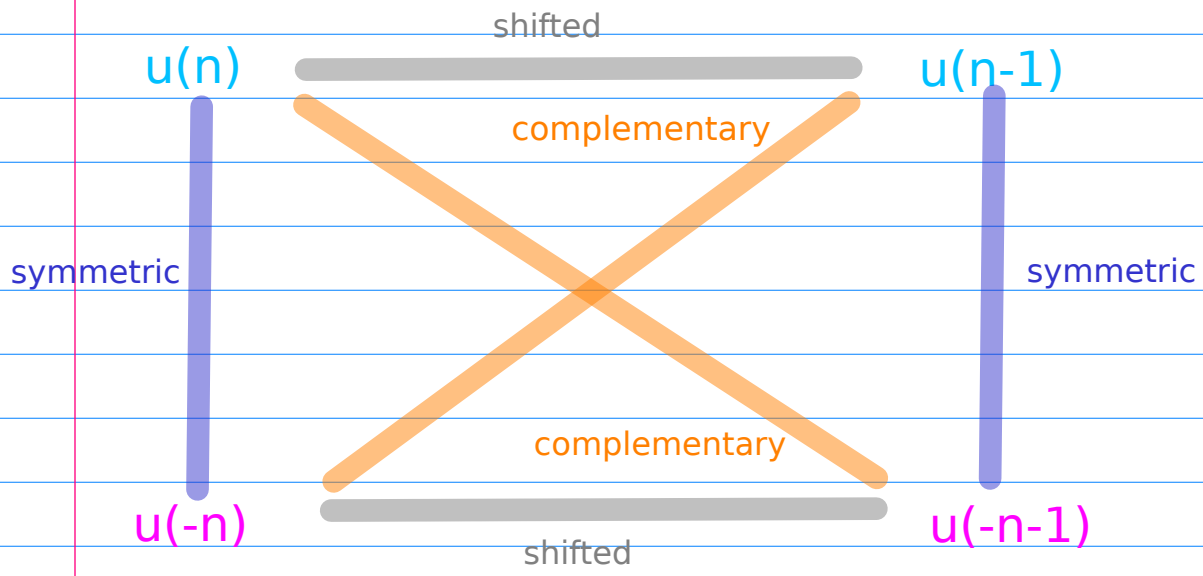
Complementary Ranges



Shifted Ranges

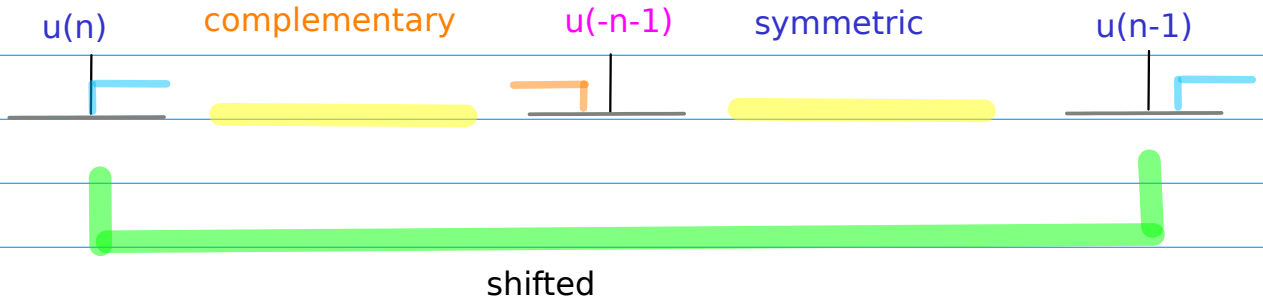


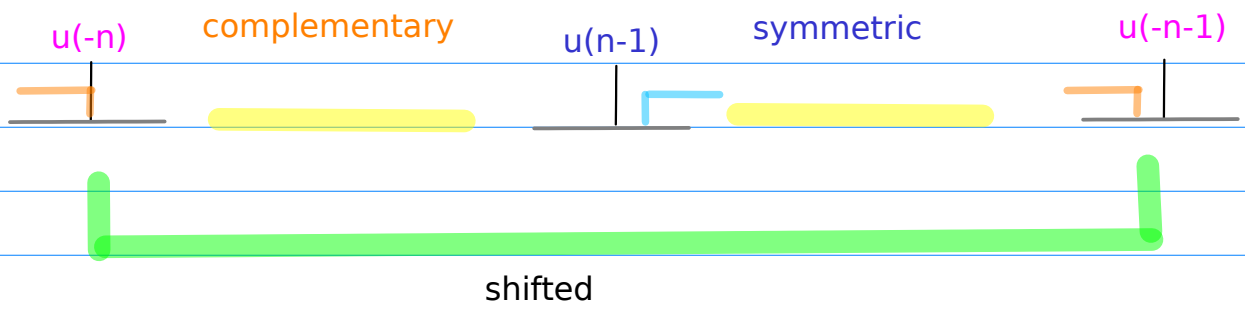
Geometric Power Series Property (4)



u(n) complementary u(-n-1) symmetric u(n-1)

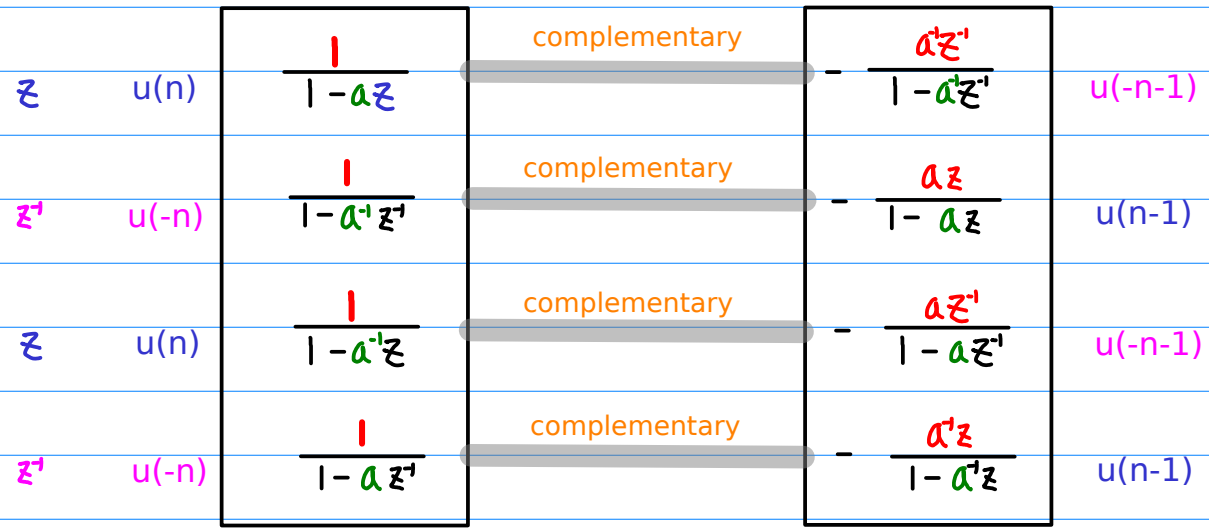
u(-n) complementary u(n-1) symmetric u(-n-1)

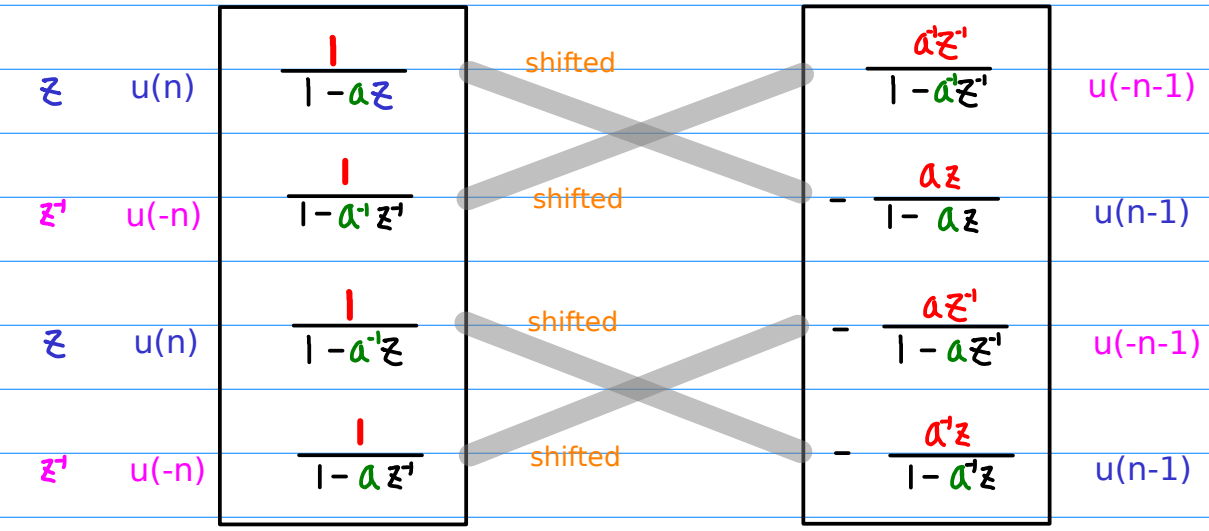




Geometric Power Series Property (5)

non-shifted range u(n), u(-n)	shifted range u(n-1), u(-n-1)
geometric series starting with	geometric series starting with
a unit term	a non-unit term (common ratio)

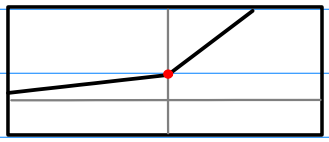




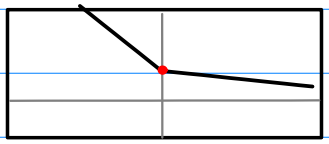
Geometric Power Series Property (6)

(*z)	**Right Shifted**	$u(n) \rightarrow u(n-1)$ $u(-n-1) \rightarrow u(-n)$
(/z)	**Left Shifted**	$u(n-1) \rightarrow u(n)$ $u(-n) \rightarrow u(-n-1)$
[*a]	**Right Shifted**	$a^{n} \rightarrow a^{n+1}$ $a^{-n} \rightarrow a^{-n+1}$
[/a]	**Left Shifted**	$a^{n} \rightarrow a^{n-1}$ $a^{-n} \rightarrow a^{-n-1}$

Geometric Power Series Property (7)

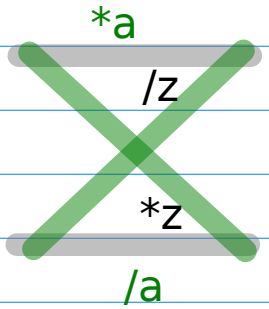


a^n



a^{-n}

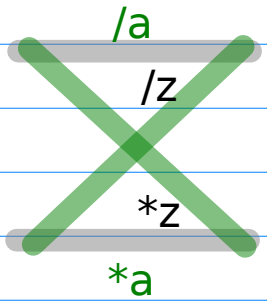
Causal Sequences



Left Shifted

Right Shifted

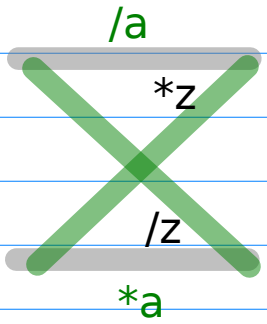
Causal Sequences



Left Shifted

Right Shifted

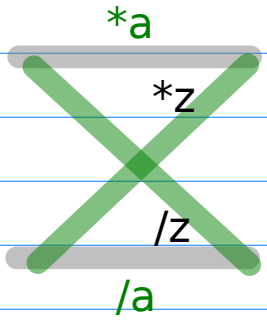
Anti-Causal Sequences



Right Shifted

Left Shifted

Anti-Causal Sequences



Right Shifted

Left Shifted