IVP

$$\begin{cases} \frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2y(t) = x(t) \\ y(0) = k_0 , \quad y'(0) = k_1 \end{cases}$$

$$\begin{cases} (y''(t)) + 3 (y'(t)) + 2y(t) = x(t) \\ y(0) = k_0 , \quad y'(0) = k_1 \end{cases}$$

$$(s^2 Y(s) - s y(0) - y'(0)) + 3 (s Y(s) - y(0)) + 2 Y(s) = X(s)$$

$$(s^2 + 3s + 2) Y(s) = s y(0) + y'(0) + 3 y(0) + X(s)$$

$$(s^2 + 3s + 2) Y(s) = k_0 s + k_1 + 3k_0 + X(s)$$

$$Y(s) = \underbrace{\frac{\overset{\text{state}}{k_0 s + k_1 + 3k_0}}{(s^2+3s+2)}}_{\text{Zero Input Rsp}} + \underbrace{\frac{\overset{\text{input}}{X(s)}}{(s^2+3s+2)}}_{\text{zero state Rsp}}$$

Transfer function

Zero.. state

$$H(s) = \frac{Y(s)}{X(s)} \quad \text{.... " zero state "}$$

$$= \frac{Y(s)}{X(s)} = \frac{1}{(s^2+3s+2)}$$

Signal Flow Graph (H.1)

$$Y(s) = \cancel{\frac{k_0 s + k_1 + 3k_0}{(s^2+3s+2)}} + \frac{X(s)}{(s^2+3s+2)}$$

Zero state $\left(y(0) = y'(0) = 0 \right)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s^2+3s+2)}$$

Impulse Response

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

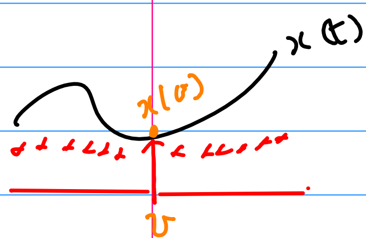
$$\delta(t) \qquad\qquad h(t)$$

$\delta(t-\tau)$ $\qquad$ $h(t-\tau)$

$a \cdot \delta(t-\tau)$ $\qquad$ $a \cdot h(t-\tau)$

$x(\tau)\delta(t-\tau)$ $\qquad$ $x(\tau)h(t-\tau)$

$x(\tau)\,\delta(t-\tau)$ $\qquad$ $x(\tau)\,h(t-\tau)$



$$\int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,dt \qquad \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = y(t)$$

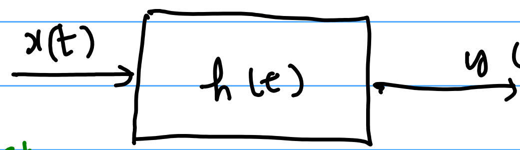
$$x(t) * h(t)$$

$x(\tau) = 0 \qquad \underline{\tau < 0}$

$h(t-\tau) = 0 \qquad t-\tau < 0$

$\underline{t < \tau}$

$\underline{0 < \tau < t}$



$$t - \nu \geq 0$$

$$y(t) = \int_0^{\infty} h(\nu)\, x(t-\nu)\, d\nu$$

e^{st}

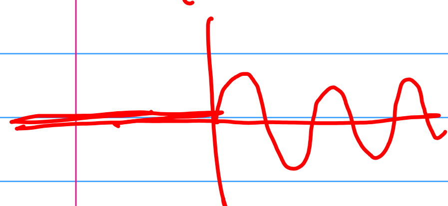
$$y(t) = \int_0^{\infty} h(\nu)\, e^{s(t-\nu)}\, d\nu$$

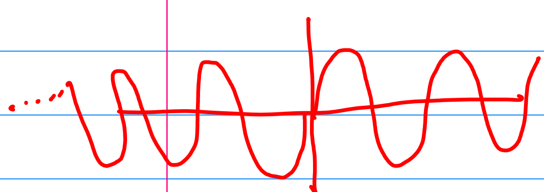
$$= e^{st} \int_0^{\infty} h(\nu)\, e^{-s\nu}\, d\nu$$

$s = \sigma + j\omega$

e^{st}

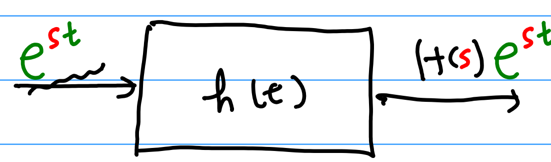
$$e^{st} \cdot \underbrace{\int_0^{\infty} h(\nu)\, e^{-s\nu}\, d\nu}_{= H(s)}$$





$$F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} f(\nu)\, e^{-s\nu}\, d\nu$$



$h(t) \Longleftrightarrow H(s)$

Laplace Transform

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * y(t)$$

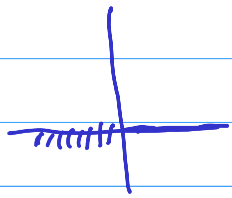
$$\Updownarrow$$

$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = X(s) \cdot H(s)$$

$$y^{(3)}(t) + a\,y^{(2)}(t) + b\,y^{(1)}(t) + c\,y(t) = m\,x^{(1)}(t) + n\,x(t)$$

Zero-state assumed $\qquad y(0) = y'(0) = y''(0) = 0$

Transfer Fn 구할 때 $\qquad x(0) = 0 \qquad x(t)$



$$s^3 Y(s) + a s^2 Y(s) + b s Y(s) + c Y(s) = m s X(s) + n X(s)$$

$$(s^3 + a s^2 + b s + c)\, Y(s) = (m s + n)\, X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{m s + n}{s^3 + a s^2 + b s + c}$$

$$y'' + 3y' + 2y = x$$

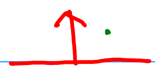
$\boxed{\text{Zero state}}$

$$s^2 Y(s) + 3sY(s) + 2Y(s) = X(s)$$

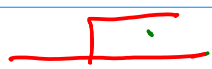
$$(s^2 + 3s + 2)\,Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)}$$

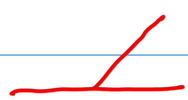
$$x(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

impulse 함수  impulse 응답 $g(t)$

$\Downarrow$ 적분 $\Downarrow$ 적분

step 함수  step 응답 $\int g(t)\,dt$

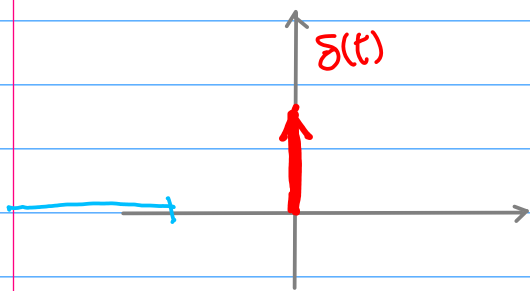
$\Downarrow$ $\Downarrow$ 적분

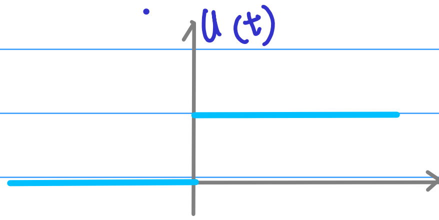
ramp 함수  ramp 응답 $\int_0^t \int_0^t g(\tau)\,d\tau\,dt$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

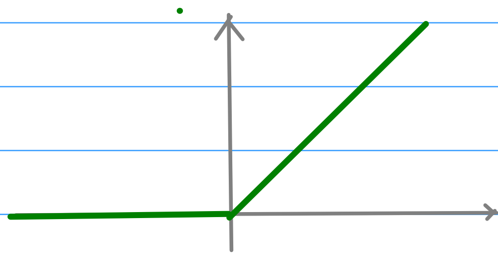
$$x'(t) \longrightarrow \boxed{h(t)} \longrightarrow y'(t)$$

$$x''(t) \longrightarrow \boxed{h(t)} \longrightarrow y''(t)$$

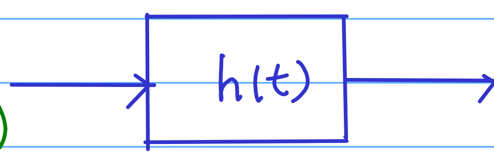




$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$



$$r(t) = \int_{-\infty}^{t} u(t)\, dt$$

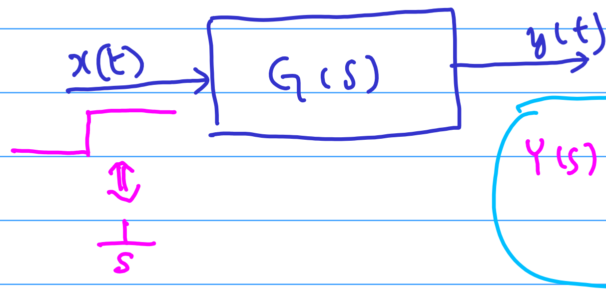


$\delta(t)$	$h(t)$
$u(t)$	$\int_{-\infty}^{t} h(t)\, dt$
$r(t)$	$\int_{-\infty}^{t}\int_{-\infty}^{t} h(t)\, dt\, dt$

$$s^2 Y(s) + 3s Y(s) + 2 Y(s) = X(s)$$

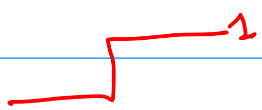
$$(s^2 + 3s + 2)\, Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)}$$

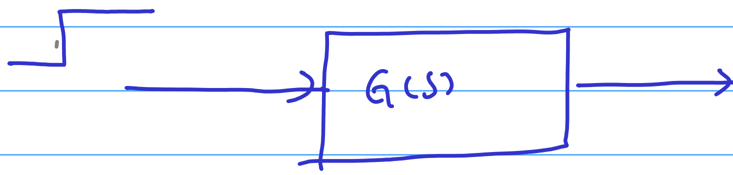


$$Y(s) = G(s) \cdot X(s) = \frac{1}{s\,(s+1)(s+2)}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)}\right)$$



$$= \underbrace{A \cdot 1}_{y_p} + \underbrace{B e^{-1\cdot t} + C e^{-2\cdot t}}_{y_h}$$



$$\frac{k_0}{s} + \frac{k_1}{s+\sigma_1} + \frac{k_2}{s+\sigma_2}$$

$$\Updownarrow$$

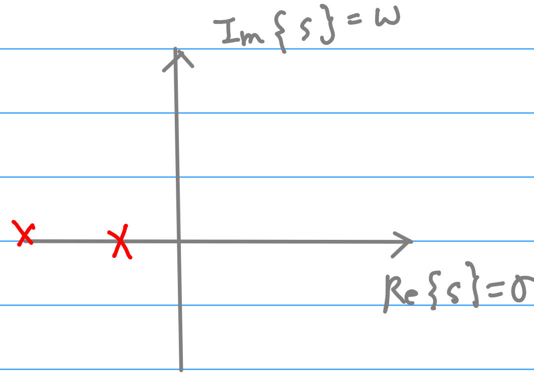
$$k_0 + k_1 e^{-\sigma_1 t} + k_2 e^{-\sigma_2 t}$$

$$G(s) = \frac{1}{s^2+3s+2}$$

$\sigma_1, \sigma_2 \Leftarrow$ 분모 $= 0$ $\quad (s^2+3s+2)=0$

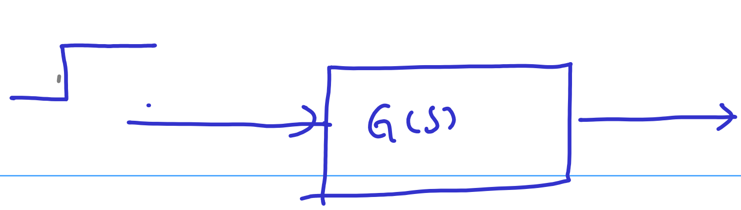
분모가 0이 되는 s들은 pole

$$s = \sigma + j\omega$$



σ_1, σ_2 는 서로 다른 두 실근

$\sigma_1 = \sigma_2$ 중근

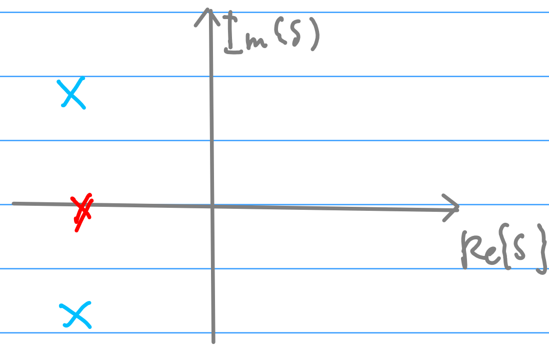


$$\frac{1}{s(s-\sigma_1)^2} = \frac{k_0}{s} + \frac{k_1}{(s-\sigma_1)} + \frac{k_2}{(s-\sigma_1)^2}$$

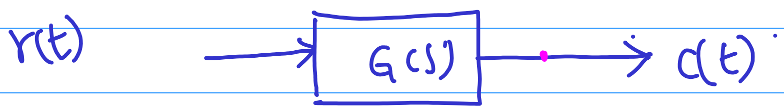
$$\Updownarrow$$

$$k_0 + k_1 e^{-\sigma_1 t} + k_2 t e^{-\sigma_1 t}$$

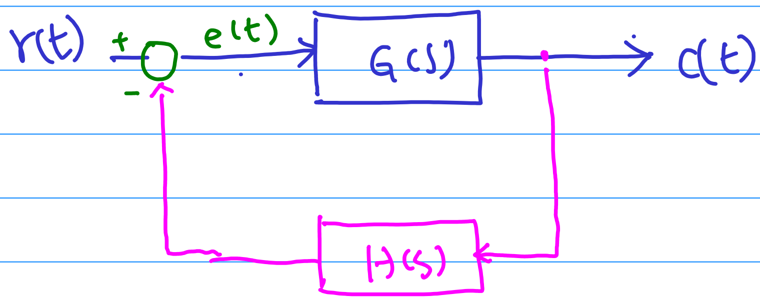
$$G(s) = \frac{1}{s^2+3s+2}$$



Feedback

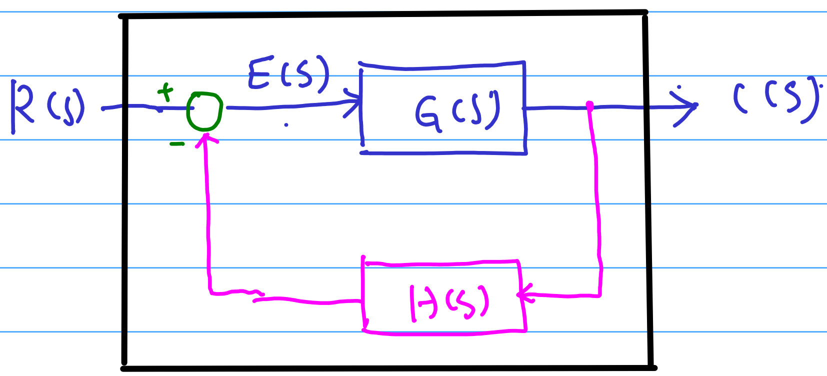


$$C(s) = G(s) \cdot R(s)$$



$$C(s) = G(s) \cdot E(s)$$

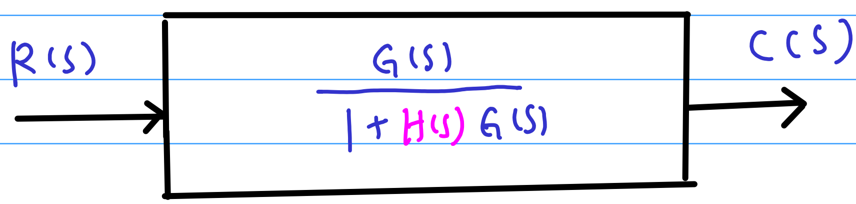
$$E(s) = R(s) - H(s)\,C(s)$$

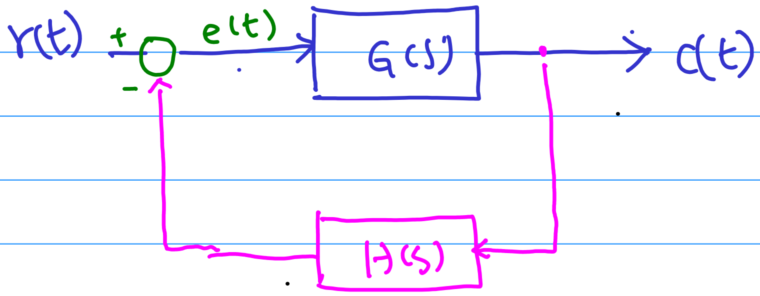


$$\frac{C(s)}{R(s)} = \frac{G(s) \cdot E(s)}{E(s) + H(s)\,C(s)}$$

$$= \frac{G(s) \cdot E(s)}{E(s) + H(s)\,G(s) \cdot E(s)}$$

$$= \frac{G(s)}{1 + H(s)\,G(s)}$$





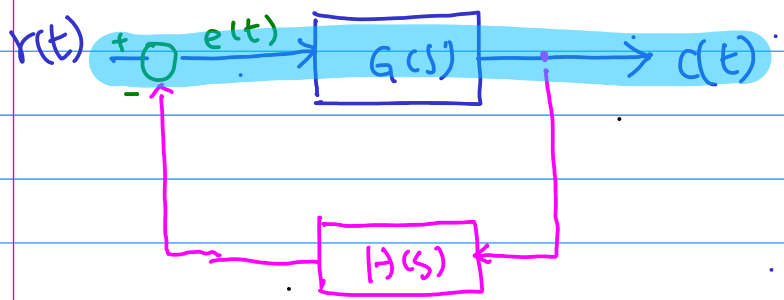
$$C(s) = G(s)E(s)$$
$$E(s) = R(s) - H(s)C(s)$$
$$E(s) + H(s)C(s) = R(s)$$
$$\frac{E(s)}{R(s)} = \frac{E(s)}{E(s) + H(s)C(s)}$$
$$= \frac{E(s)}{E(s) + H(s)G(s)E(s)}$$
$$= \frac{1}{1 + H(s)G(s)}$$

$$C(s) = G(s)E(s)$$
$$E(s) = R(s) - H(s)C(s)$$

$$\frac{E(s)}{R(s)} = \frac{R(s) - H(s)C(s)}{R(s)}$$
$$= 1 - \frac{H(s)G(s)E(s)}{R(s)}$$

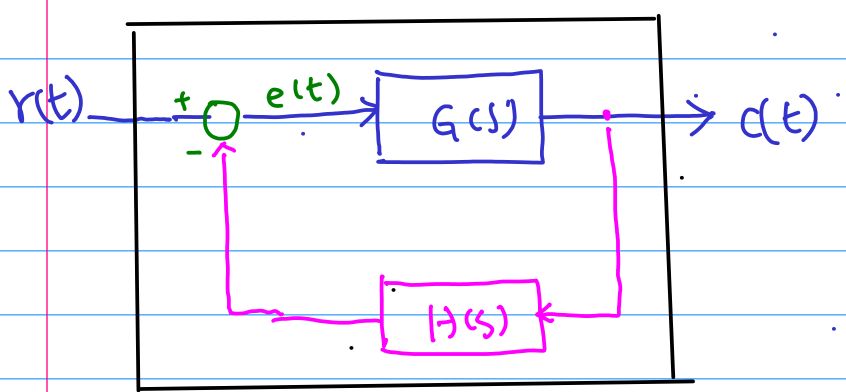
$$\frac{E(s)}{R(s)} + H(s)G(s)\frac{E(s)}{R(s)} = 1$$
$$(1 + H(s)G(s))\frac{E(s)}{R(s)} = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + H(s)G(s)}$$

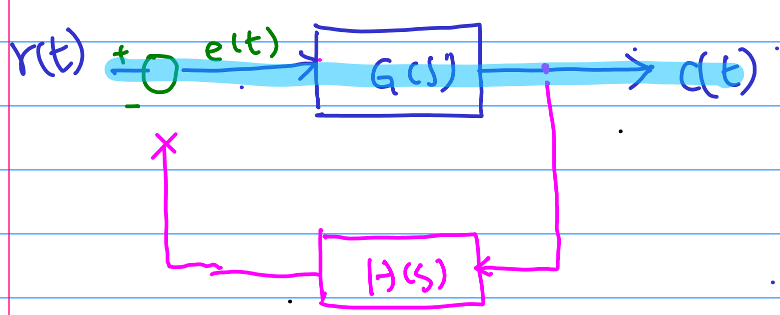


forward transfer fn : G(s)

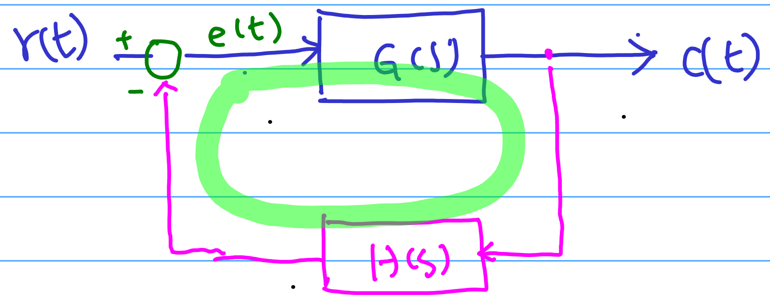
feedback transfer fn : H(s)



closed loop transfer fn : $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$



Open loop transfer fn : G(s)



loop transfer fn : $G(s)H(s)$

Unit feed back $H(s)=1$

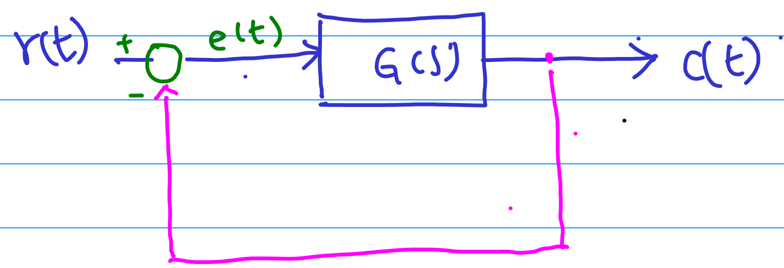
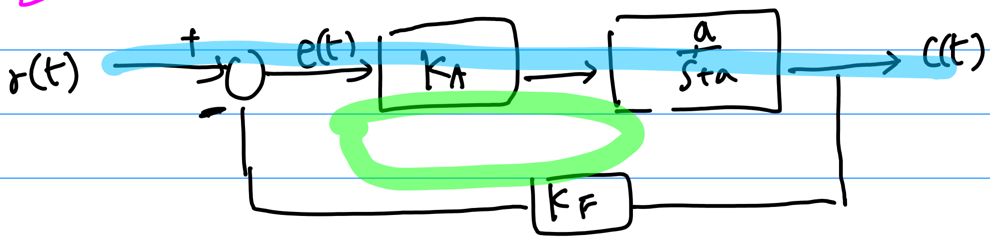
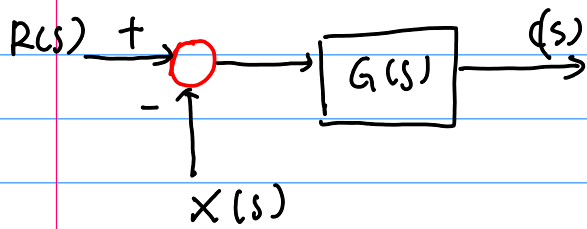


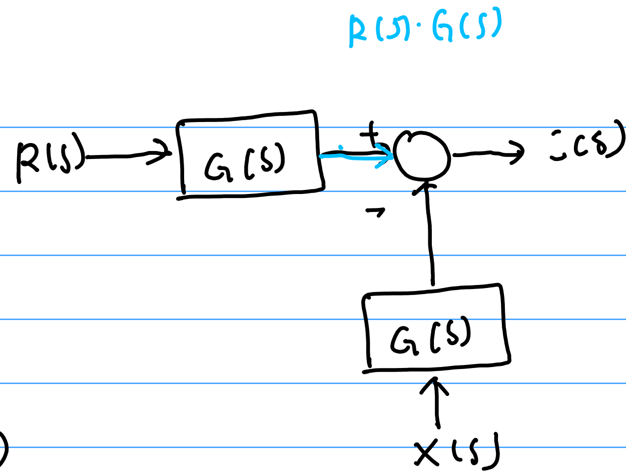
그림 2-10



$$\frac{K_A a}{(s+a)} \qquad\qquad - \frac{K_A K_F a}{(s+a)}$$

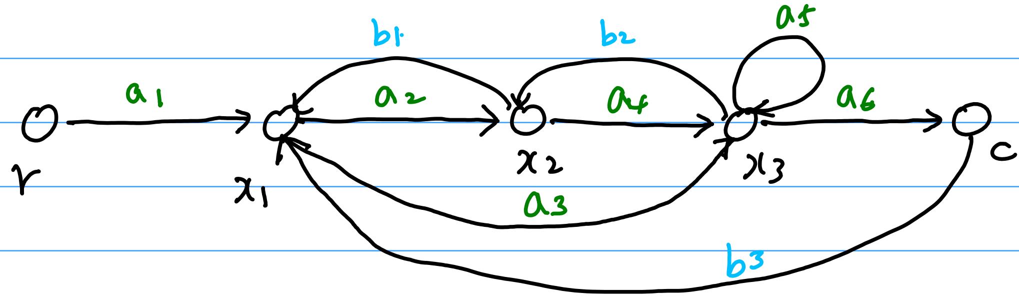
$$G_{CL}(s) = \frac{\quad}{1 - \quad} = \frac{\frac{K_A a}{(s+a)}}{1 - \left(- \frac{K_A K_F a}{(s+a)}\right)} = \frac{K_A a}{s+a+K_A K_F a}$$





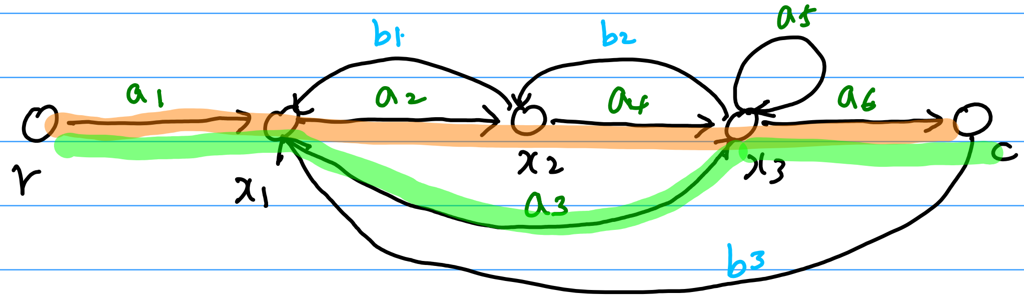
$$C(s) = \left(R(s) - X(s)\right) \cdot G(s)$$

$$= R(s) \cdot G(s) - X(s) \cdot G(s)$$



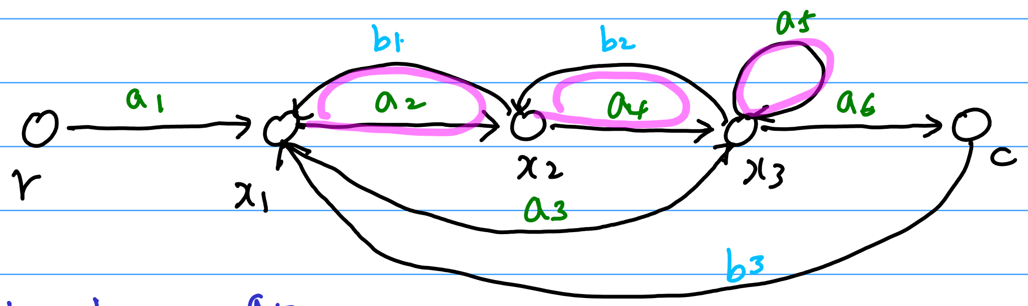
a1
a2
b1
b2
a4
a5
a6
a3
b3
r
x1
x2
x3
c

Forward path

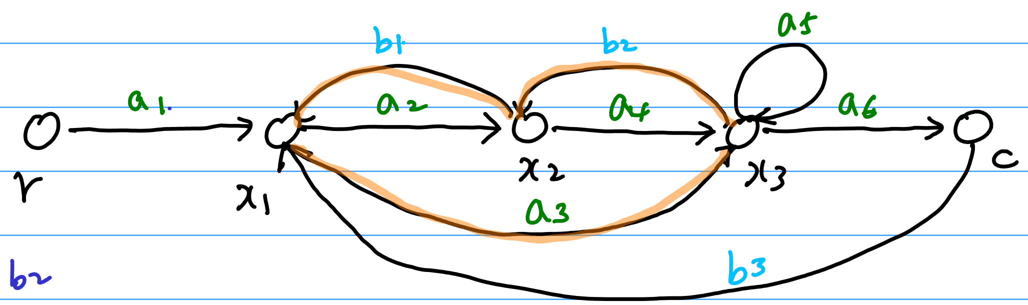


Forward Gain $\begin{cases} a_1 a_2 a_4 a_6 \\ a_1 a_3 a_6 \end{cases}$

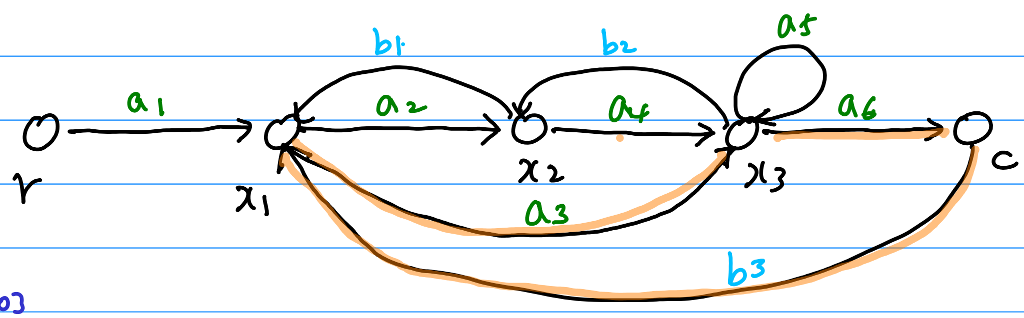
Loop



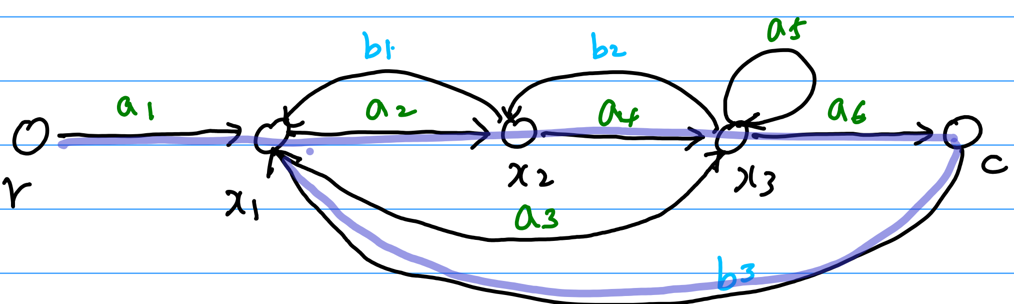
loop gain b_1, b_2, a_5



$a_3 b_1 b_2$

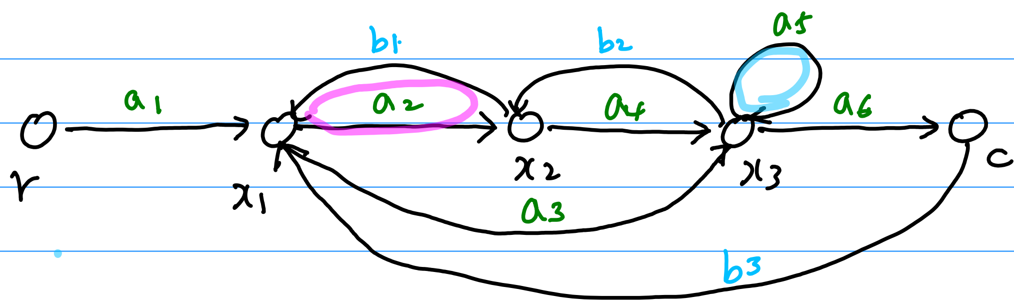


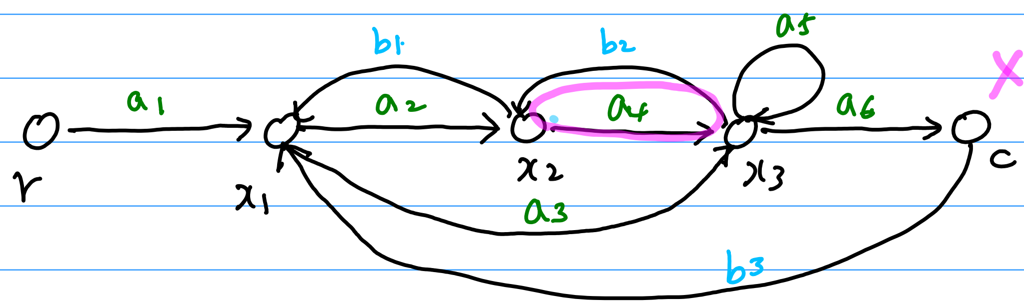
$a_3 a_4 b_3$

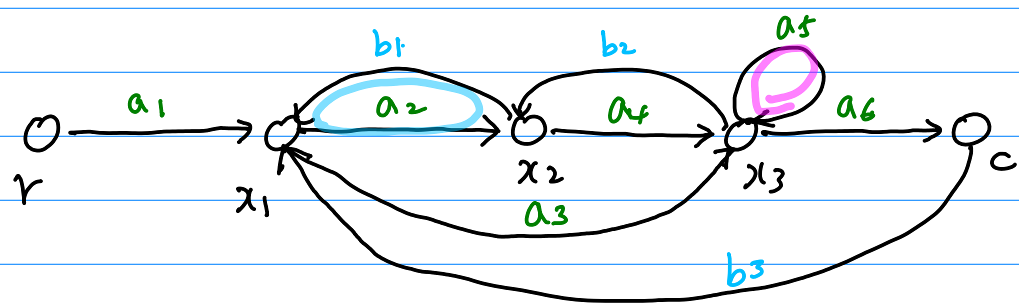


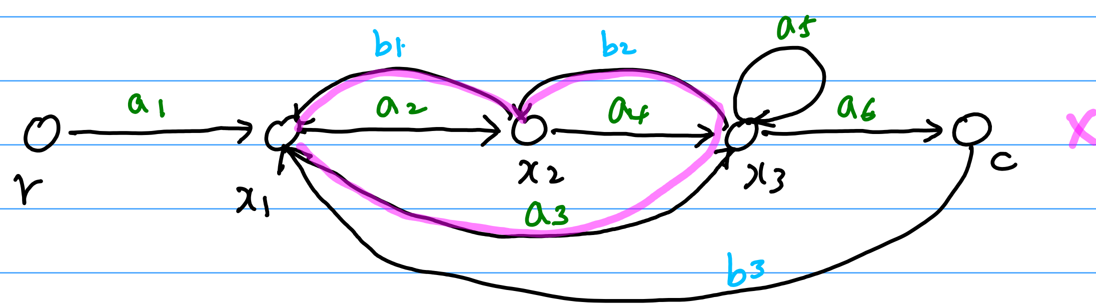
$a_1 a_2 a_4 a_6 b_3$

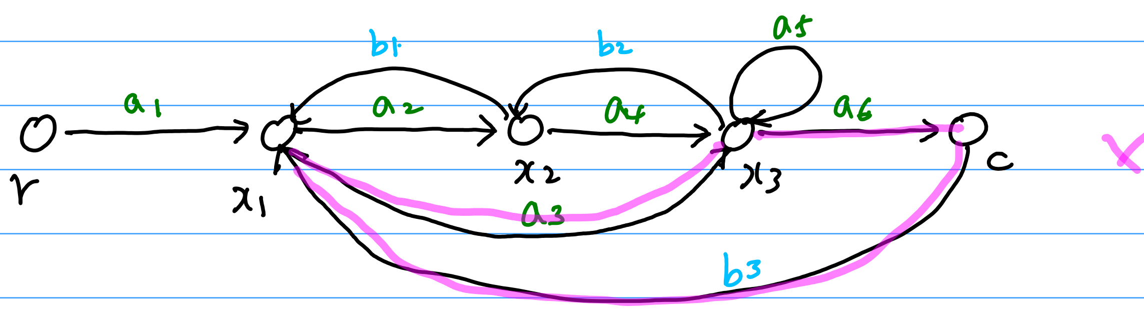
Non touching loop

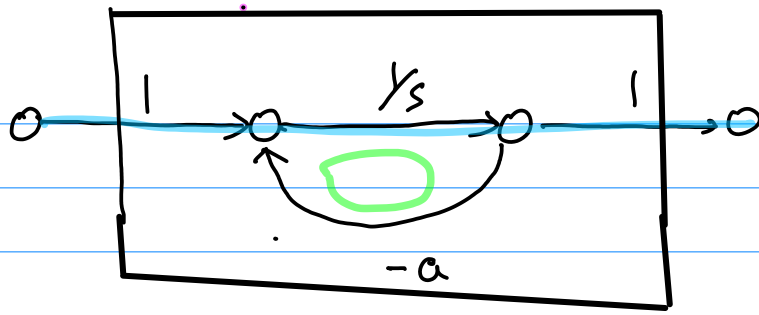








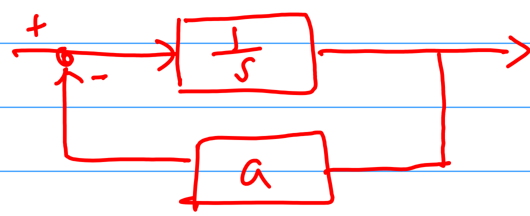




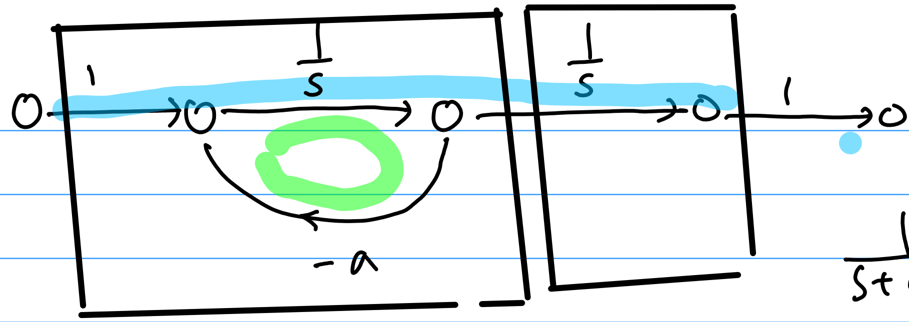
$\frac{1}{s}$

$-\frac{a}{s}$

$$\frac{\frac{1}{s}}{1-\left(-\frac{a}{s}\right)} = \frac{\frac{1}{s}}{1+\frac{a}{s}} = \frac{1}{s+a}$$



$\frac{1}{s^2}$

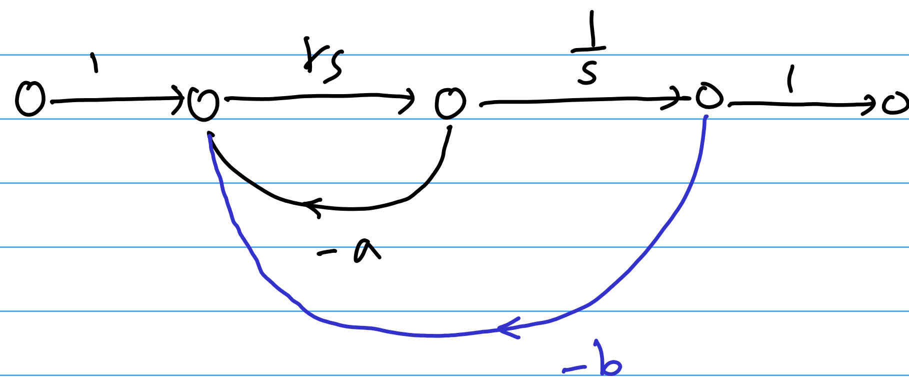


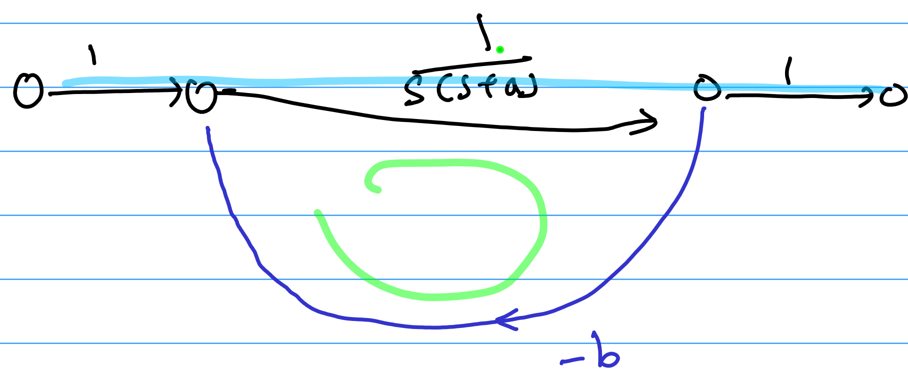
$$\frac{1}{s+a} \times \frac{1}{s} = \frac{1}{s(s+a)}$$

$\frac{1}{s^2}$

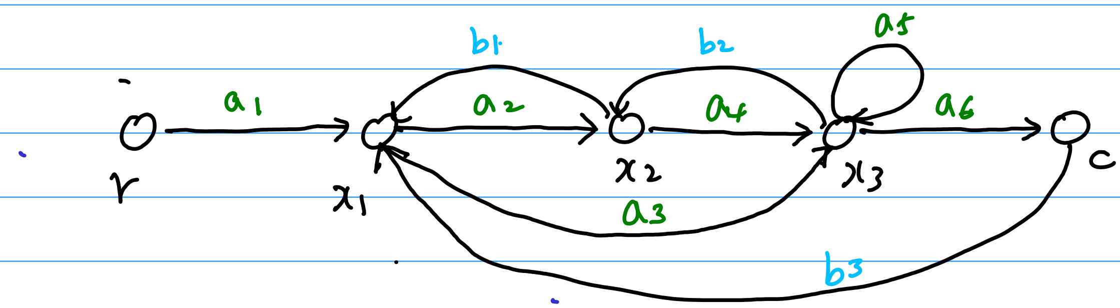
$-\frac{a}{s}$

$$\frac{\frac{1}{s^2}}{1-\left(-\frac{a}{s}\right)} = \frac{1}{s^2+as} = \frac{1}{s(s+a)}$$

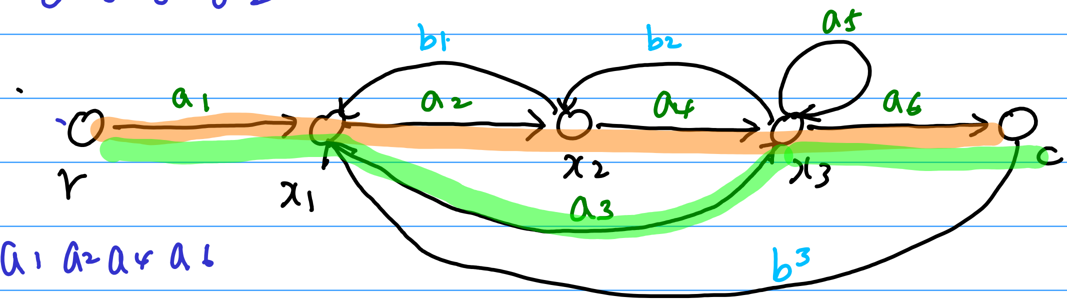




$$\frac{\frac{1}{s(s+a)}}{1+\frac{b}{s(s+a)}} = \frac{1}{s(s+a)+b} = \frac{1}{s^2+as+b}$$



N=2개의 전 방향 경로

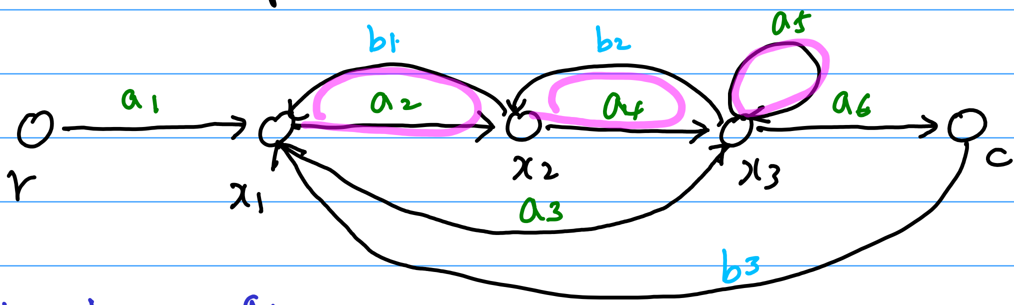


$a_1\ a_2\ a_4\ a_6$

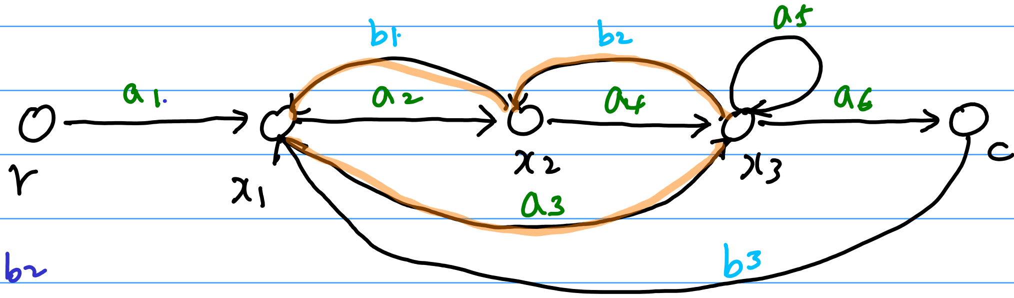
$a_1\ a_3\ a_6$

K=6개의 loop

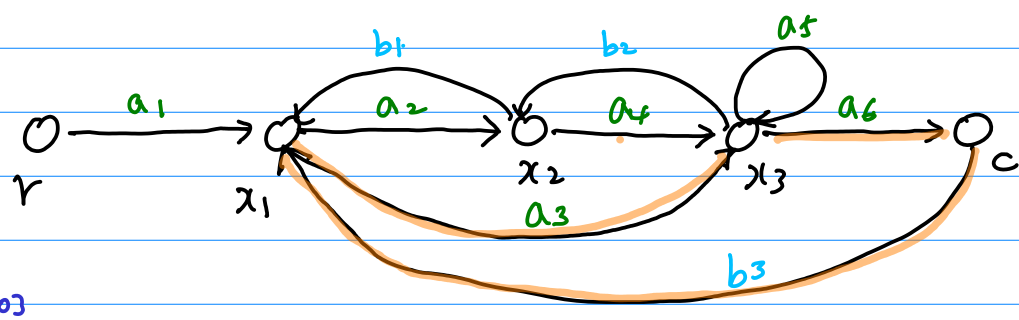
K=6개의 loop



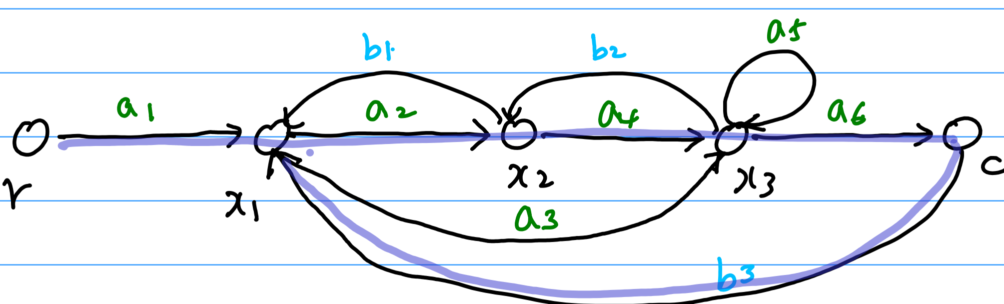
loop gain b_1, b_2, a_5



$a_3 b_1 b_2$



$a_3 a_6 b_3$



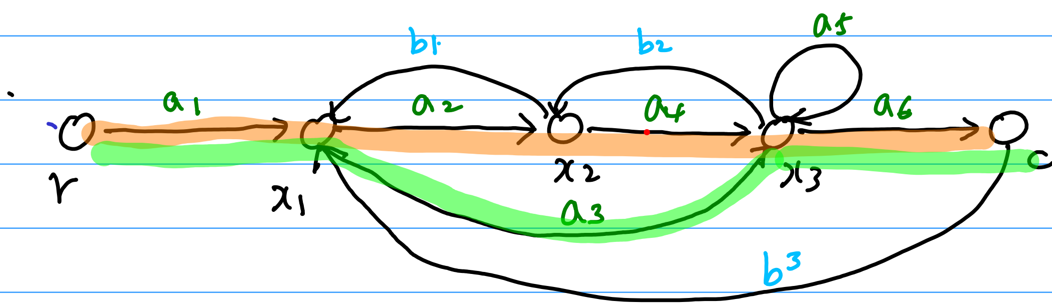
$a_1 a_2 a_4 a_6 b_3$

$N=2$의 forward path

$K=6$의 loop ↔ k는 forward loop의 index

$k=1, 2.$

$$G = \frac{1}{\Delta} \sum_{k=1}^{N} G_k \Delta_k$$



$k=1$ $G_1 = \underline{a_1 a_2 a_4 a_6}$

$k=2$ $G_2 = \underline{a_1 a_3 a_6}$

$$\Delta = 1 - \sum_{l} L_{l1} + \sum_{m} L_{m2} - \sum_{n} L_{n3} + \sum L$$

L_{l1} loop 1개씩 combination

loop 1 loop 2 loop 3 loop 4

L_{l2} non-touching loop 2개씩 combi

loop1 loop2

loop1 loop 3

loop1 loop4

loop2 loop3

loop 2 loop4

loop loop

L_{l3} non-touching loop 3개씩 Combi

loop1 loop2 loop3

loop1 loop2 loop 4

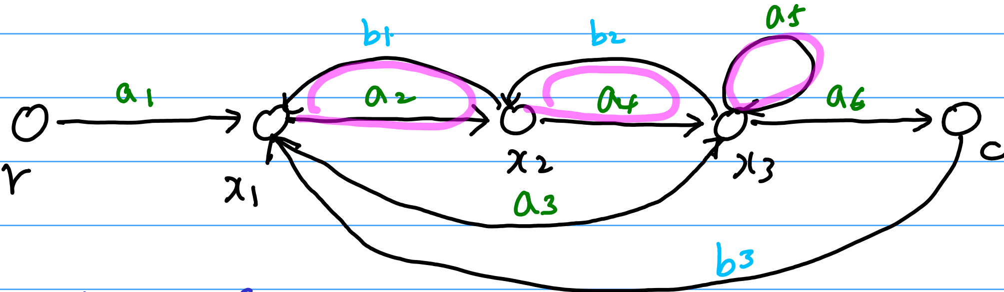
loop1 loop3 loop 4

loop 2 loop 3 loop 4

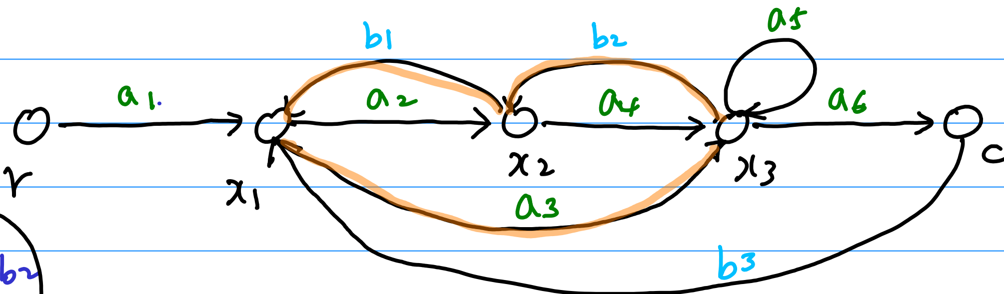
L_{l1} loop 1개씩 combination

1

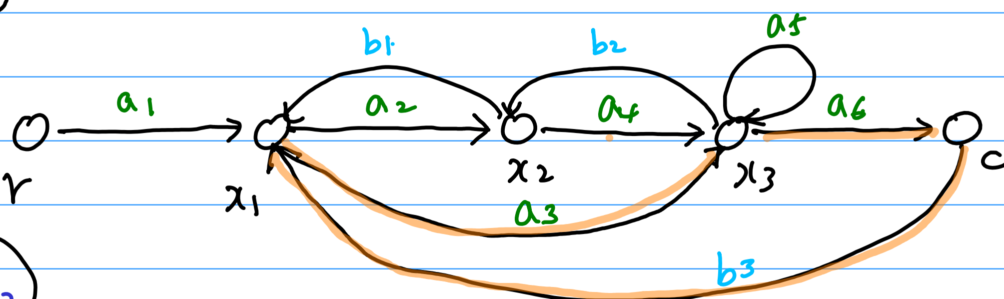
$L_{21} = a_2 b_1$ $L_{31} = a_4 b_2$ $L_{11} = a_5$



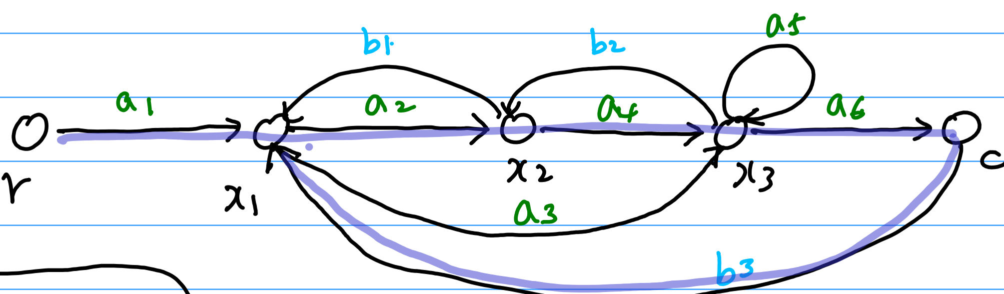
loop gain b_1, b_2, a_5



$L_{41} = a_3 b_1 b_2$



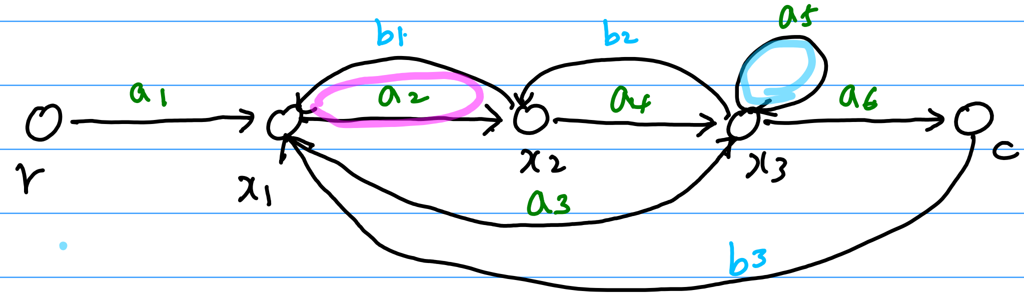
$L_{51} = a_3 a_6 b_3$



$L_{61} = a_1 a_2 a_4 a_6 b_3$

L_{12} non-touching loop 2개씩 combination

$L_{21} = a_2 b_1$ $L_{11} = a_5$



$L_{12} = L_{11} L_{21} = a_2 a_5 b_1$

$$\Delta = 1 \ominus \sum_{l} L_{l1} \oplus \sum_{m} L_{m2} \ominus \sum_{n} L_{n3} \oplus \sum$$

$$= 1 - (L_{11} + L_{21} + L_{31} + L_{41} + L_{51}$$

$$+ (L_{12})$$

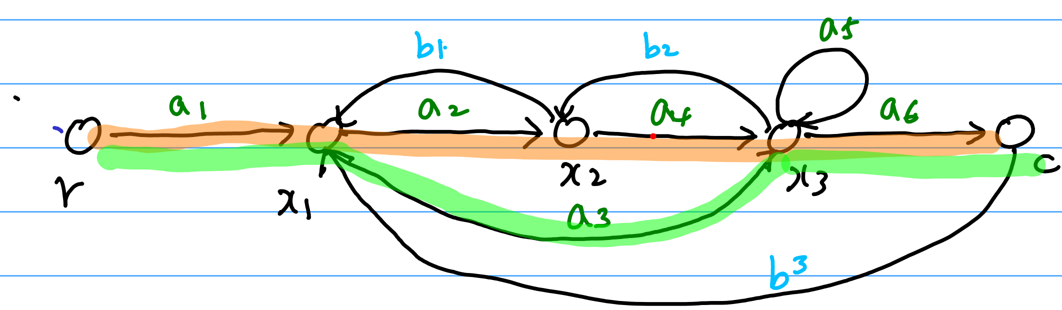
$$\sum_{l} L_{l1} \qquad \sum_{m} L_{m2}$$

$$\Delta = 1 - \sum_{l} L_{l1} + \sum_{m} L_{m2} - \sum_{n} L_{n3} + \sum L$$

$$\Delta = 1 - (a_5 + a_2 b_1 + a_4 b_1 + a_3 b_1 b_2 + a_3 a_6 b_3 + a_2 a_4 a_6 a_3) + (a_2 a_5 b_1)$$

Δ_1 = terms in Δ that are non-touching forward path 1

Δ_2 = terms in Δ that are non-touching forward path 2



$k=1$ $G_1 = a_1 a_2 a_4 a_6$

$k=2$ $G_2 = a_1 a_3 a_6$

$$G = \frac{1}{\Delta} \sum_{k=1}^{N} G_k \Delta_k$$

$$= \frac{1}{\Delta} \left(G_1 \Delta_1 + G_2 \Delta_2 \right)$$

$$= \frac{1}{\Delta} \left(a_1 a_2 a_4 a_6 + a_1 a_3 a_6 \right)$$

$$=$$

$$f(t) \Longleftrightarrow F(s)$$

$$f'(t) \Longleftrightarrow s\,F(s)$$

$$\int_{-\infty}^{t} f(t)\,dt \Longleftrightarrow \frac{F(s)}{s}$$

$\boxed{\text{differentiator}}$ 미분기

$$r(t) \xrightarrow{\;s\;} c(t) = \frac{d}{dt} r(t)$$

$$R(s) \qquad\qquad C(s) = s\,R(s)$$

Transfer function $= \dfrac{Y(s)}{X(s)} = \dfrac{C(s)}{R(s)} = s$

$s = \sigma + j\omega$

0

$G(s) = s$

Frequency Response $\quad s \leftarrow j\omega$

$$|G(j\omega)| = |j\omega| = \omega$$

$\boxed{\text{integrator}}$ 적분기

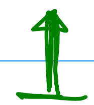
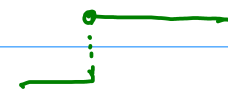
$$r(t) \xrightarrow{\;\frac{1}{s}\;} c(t) = \int_{-\infty}^{t} r(t)\,dt$$

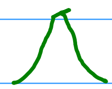
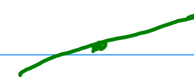
$$R(s) \qquad\qquad C(s) = \frac{R(s)}{s}$$

$\boxed{\text{integrator}}$

$$r(t) \xrightarrow{\frac{1}{s}} c(t) = \int_{-\infty}^{t} r(t)\,dt$$

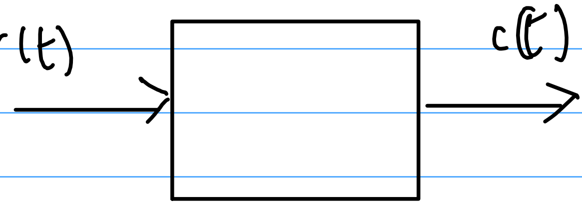
$$R(s) \qquad C(s) = \frac{R(s)}{s}$$

ideal  

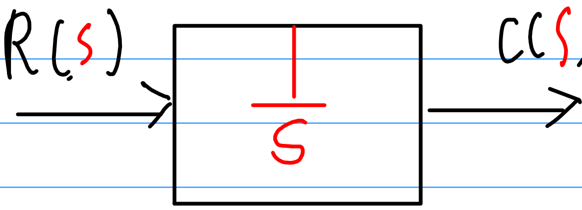
pract  

$$c(t) = \int_{-\infty}^{t} r(t)\,dt$$

$$\frac{d}{dt} c(t) = r(t)$$



$$r(t) \qquad c(t) = \int_{-\infty}^{t} r(t)\,dt$$



$$R(s) \qquad C(s) = r(s) \cdot \frac{1}{s}$$

State diagram

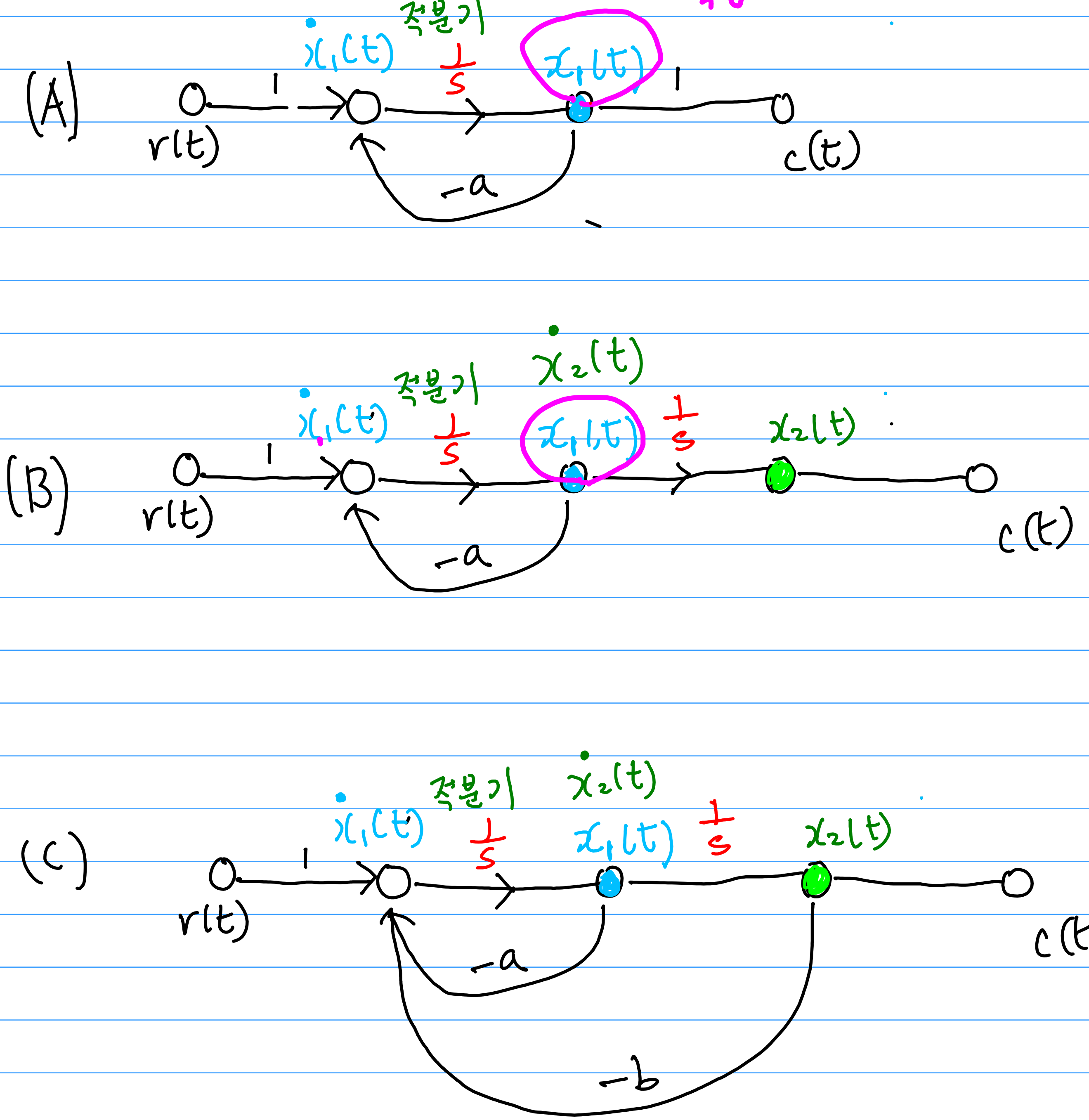
integrator + 비례기

상수

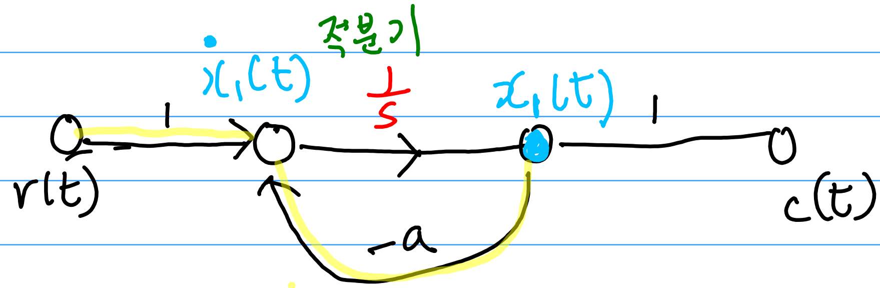
$\frac{1}{s}$

2권 3-17

적분된 신호 → State



(A)



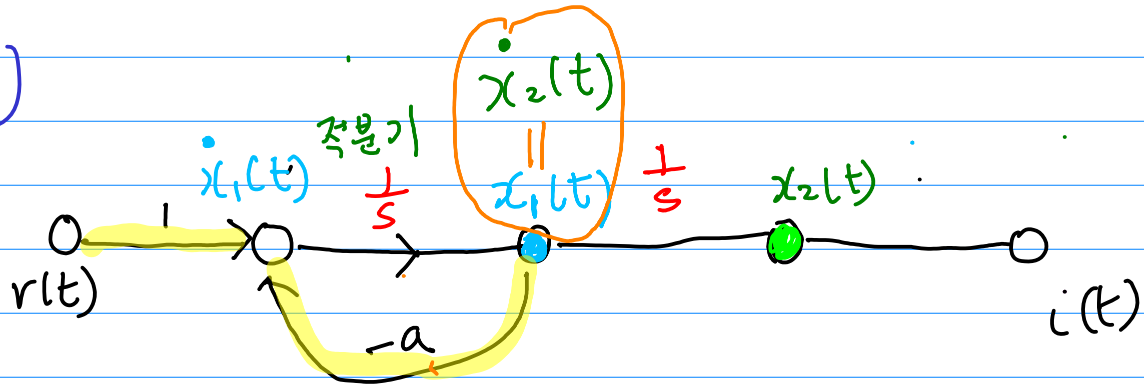
$$\underbrace{\dot{x}_1(t)}_{\text{state를 미분한것}} = -a\,\underbrace{x_1(t)}_{\text{state}} + r(t)$$

state eq

$$c(t) = x_1(t)$$

output

(13)



$$\dot{x}_1(t) = -a\, x_1(t) + r(t)$$

$$\dot{x}_2(t) = x_1(t) \ *$$

state eq

• 있는 상태변수

• 이 없는 상태변수

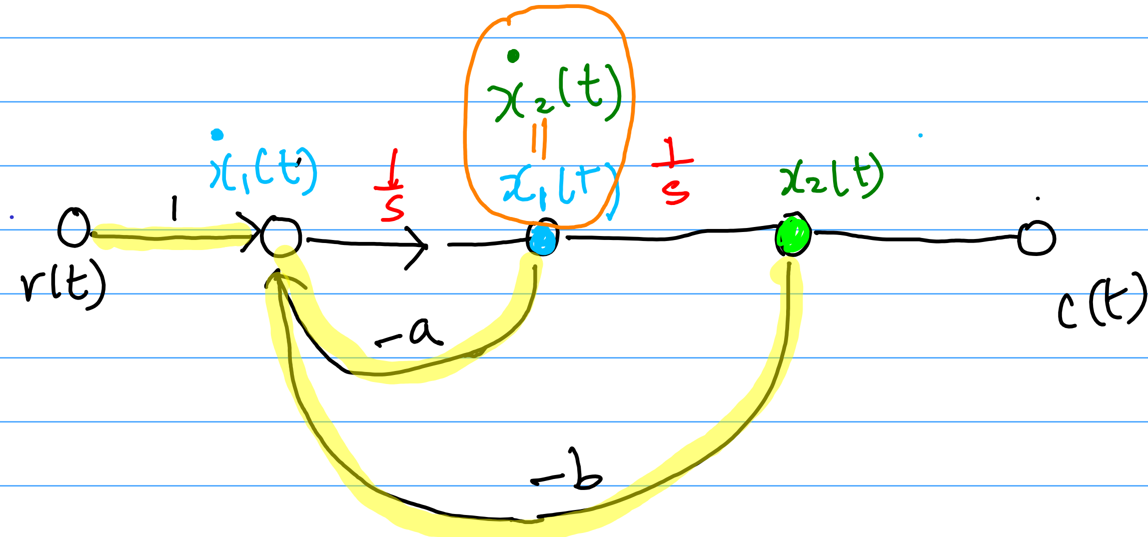
① $\dot{x}_2(t) = x_1(t)$ (ok).

② $\dot{x}_2(t) = \frac{1}{s} \cdot x_1(t)$ ~~(ok)~~

$$c(t) = x_2(t)$$

output equation

(c)



$$\begin{cases} \dot{x}_1(t) = -a\,x_1(t) - b\,x_2(t) + r(t) \\ \dot{x}_2(t) = x_1(t) \end{cases}$$

$\boxed{\text{State e}}$

$$c(t) = x_2(t)$$

$\boxed{\text{output eq}}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r($$

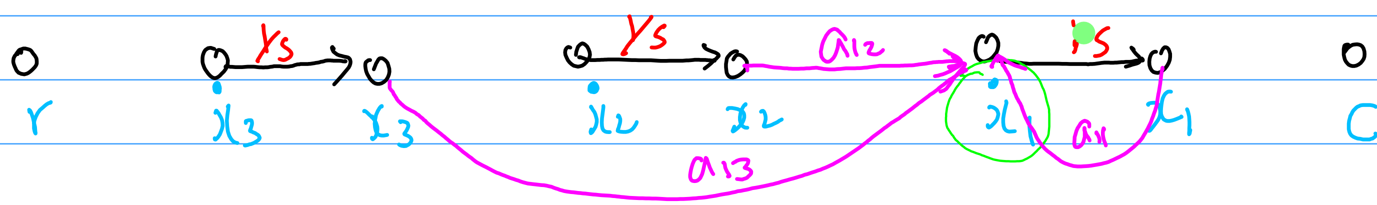
P54)

상수행렬

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} r($$

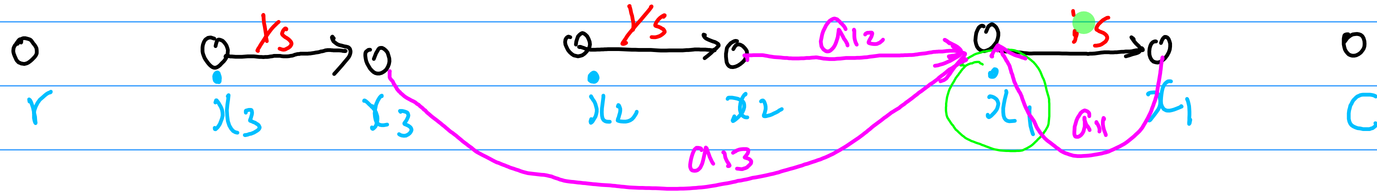
... $\boxed{\text{state eq}}$

$$c(t) = \begin{bmatrix} \underline{c_1 \quad c_2 \quad c_3} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$



$$
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} r(t)
$$

$$
\dot{x}_1(t) = a_{11}\, x_1(t) + a_{12}\, x_2(t) + a_{13}\, x_3(t) + b
$$



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} r(t)$$

$$\dot{x}_1(t) = a_{11} x_1(t) + a_{12} x_2(t) + a_{13} x_3(t) + b$$

Transfer function $\longrightarrow$ State Space

$$\text{Transfer function} = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{b_0}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(}{R}$$

$$(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)\,C(s) = b_0\,R(s)$$

$$(s^n C(s) + a_{n-1}s^{n-1}C(s) + \cdots + a_1 s C(s) + a_0 C(s)) = b_0$$

$$\frac{d^n c}{dt^n} + a_{n-1}\underset{\displaystyle x_n}{\underset{\|}{\frac{d^{n-1}c}{dt^{n-1}}}} + \cdots + a_1 \underset{\displaystyle x_2}{\underset{\|}{\frac{dc}{dt}}} + a_0 \underset{\displaystyle x_1}{\underset{\|}{c(t)}} = b_0$$

ODE

$$\frac{d^n c}{dt^n} + a_{n-1}\underbrace{\frac{d^{n-1} c}{dt^{n-1}}}_{x_n} + \cdots + a_1 \underbrace{\frac{dc}{dt}}_{x_2} + a_0 \underbrace{c(t)}_{x_1} = b_0\, r(t)$$

state

$$\begin{aligned}
x_1(t) &= c(t) \\
x_2(t) &= \frac{d}{dt} c(t) \\
&\vdots \\
x_n(t) &= \frac{d^{n-1}}{dt^{n-1}} c(t)
\end{aligned}$$

$$\begin{aligned}
\dot{x}_1(t) &= \frac{d}{dt} c(t) = x_2(t) \\
\dot{x}_2(t) &= \frac{d^2}{dt^2} c(t) = x_3(t) \\
&\vdots \\
\dot{x}_n(t) &= \frac{d^n}{dt^n} c(t) = \,?
\end{aligned}$$

$$-a_{n-1} x_n - \cdots - a x_2 - a_0 x_1 + b_0 r(t)$$

$$= -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1} x_n + b_0 r(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_{n-1}(t) \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix}$$

$$c(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}$$

Transfer function $\rightarrow$ State Space

$$\text{Transfer function} = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{b_0}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(s)}{R(s)}$$

$$\boxed{G(s) = \frac{(b_{n-1}s^{n-1} + \cdots + b_1 s + b_0)}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} = \frac{C(s)}{R(s)}$$

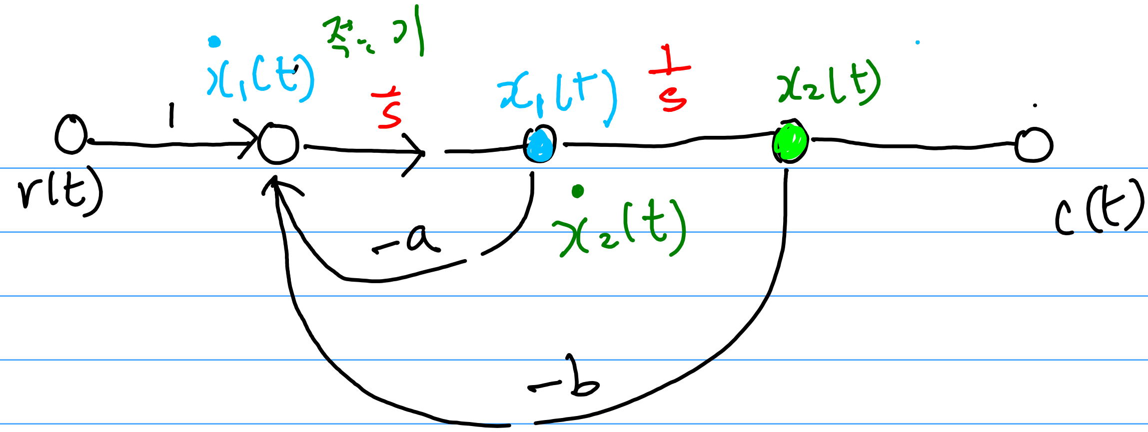
$$\rightarrow \boxed{\frac{1}{(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}} \rightarrow \boxed{(b_{n-1}s^{n-1} + \cdots + b_1 s + b_0)}$$

Transfer fn $\longrightarrow$ State Space

State Space $\longrightarrow$ Transfer fn.

$$\dot{\mathbb{x}}(t) = \mathbb{A}\,\mathbb{x}(t) + \mathbb{B}\,\mathbb{u}(t)$$

$$\mathbb{y}(t) = \mathbb{C}\,\mathbb{x}(t) + \mathbb{D}\,\mathbb{u}(t)$$



$$\begin{cases} \dot{x}_1(t) = -a\,x_1(t) - b\,x_2(t) + r(t) \\ \dot{x}_2(t) = x_1(t) \end{cases}$$ $\boxed{\text{state eq}}$

$$c(t) = x_2(t)$$ $\boxed{\text{output eq}}$

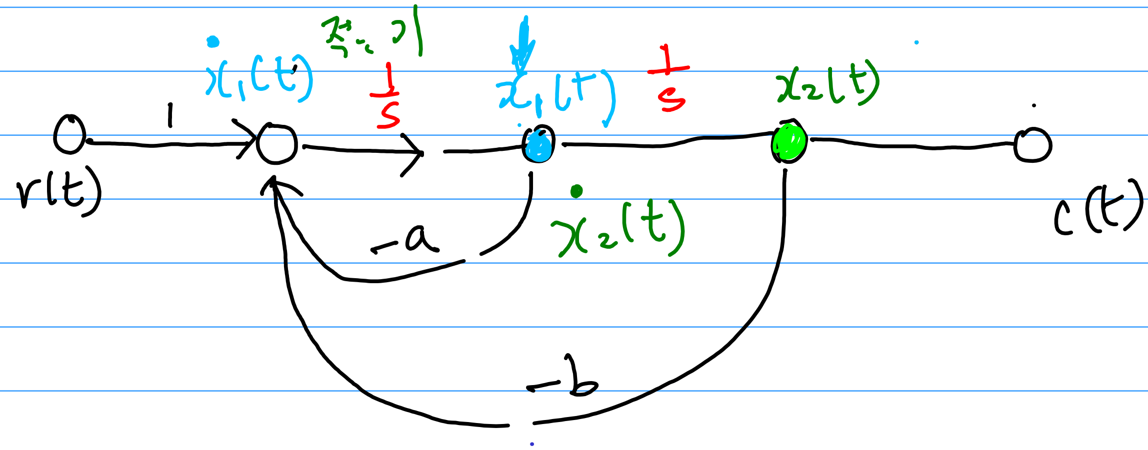
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t)$$

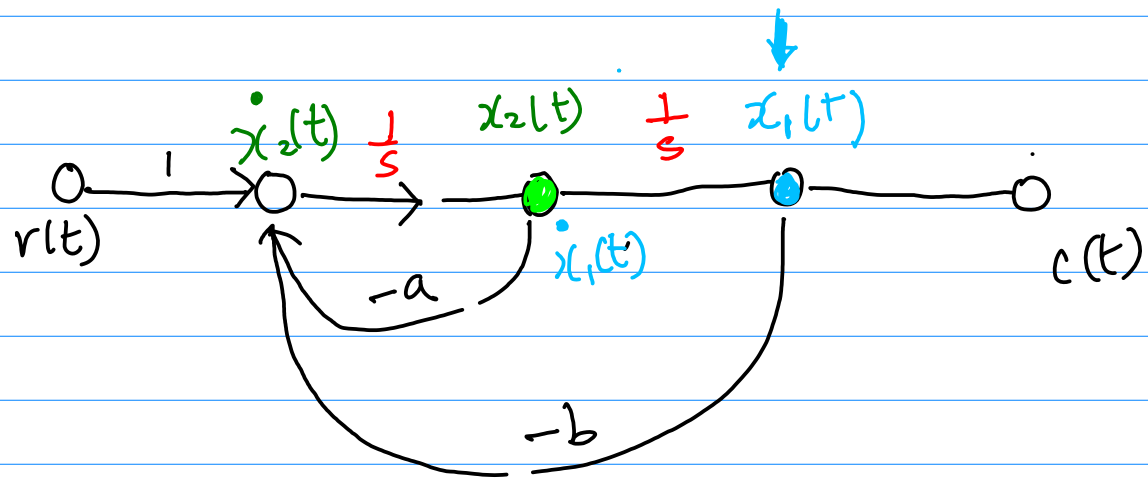
$$c(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 \cdot r(t)$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{r}(t)$$

$$\mathbf{c}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{r}(t)$$

(C)

① 

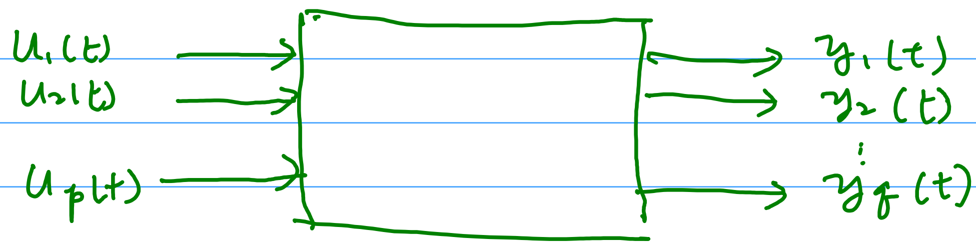
② 

$$\dot{x}_2(t) = \underline{-b\,x_1(t) - a\,x_2(t)} + r(t)$$
$$\dot{x}_1(t) = x_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{d^2 c(t)}{dt^2} + a\frac{dc(t)}{dt} + b\,c(t) = r(t)$$

MIMO



$$\mathbf{u}(t) = p\left\{\overbrace{\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{bmatrix}}^{1}\right. \qquad \mathbf{y}(t) = q\left\{\overbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{bmatrix}}^{1}\right.$$

$$\mathbf{U}(s) = p\left\{\overbrace{\begin{bmatrix} U_1(s) \\ U_2(s) \\ \vdots \\ U_p(s) \end{bmatrix}}^{1}\right. \qquad \mathbf{Y}(s) = q\left\{\overbrace{\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ \vdots \\ Y_q(s) \end{bmatrix}}^{1}\right.$$

$p=1 \dashrightarrow u(t) \qquad q=1 \cdots\cdots y(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0 \cdot u(t)$$

$$\begin{bmatrix} sX_1(s) \\ sX_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} -a & -b \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} U(s)$$

$$Y(s) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \underbrace{0}_{D}$$

$$s\,\mathbf{X}(s) = \mathbf{A}\,\mathbf{X}(s) + \mathbf{B}\,U(s)$$

$$Y(s) = \mathbf{C}\,\mathbf{X}(s) + \mathbf{D}\,U(s)$$

$$s\,X(s) = A\,X(s) + B\,u(s)$$

$$Y(s) = C\,X(s) + D\,u(s)$$

$$s\,X(s) - A\,X(s) = B\,u(s)$$

$$s\,I\,X(s) - A\,X(s) = B\,u(s)$$

$$(sI - A)\,X(s) = B\,u(s)$$

$$X(s) = (sI - A)^{-1}\,B\,u(s)$$

$$sX(s) = AX(s) + Bu(s)$$
$$Y(s) = CX(s) + Du(s)$$

$$X(s) = (sI - A)^{-1} Bu(s)$$
$$Y(s) = CX(s) + Du(s)$$

$$= C(sI - A)^{-1} Bu(s) + Du(s)$$
$$Y(s) = \{C(sI - A)^{-1}B + D\} u(s)$$

$$\frac{Y(s)}{u(s)} = \left\{ \underbrace{\underbrace{C}_{1\times 2}\underbrace{(sI - A)^{-1}}_{2\times 2}\underbrace{B}_{2\times 1}}_{1\times 1} + D \right\}$$

$$
n\left\{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}\right. = n\left\{\overbrace{\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}}^{n}\right. \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + n\left\{\overbrace{\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix}}^{p}\right. \begin{bmatrix} r_1(t) \\ \vdots \\ r_p(t) \end{bmatrix}
$$

$$
n \times 1 \quad = \quad n \times n \quad n \times 1 \quad n \times p \times p \times 1
$$

$$
q\left\{\begin{bmatrix} c_1(t) \\ \vdots \\ c_q(t) \end{bmatrix}\right. = q\left\{\overbrace{\begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix}}^{n}\right. \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + q\left\{\overbrace{\begin{bmatrix} & & \\ & D & \\ & & \end{bmatrix}}^{p}\right. \begin{bmatrix} r_1(t) \\ \vdots \\ r_p(t) \end{bmatrix}
$$

q개 출력   p개 입력

$q \times p$

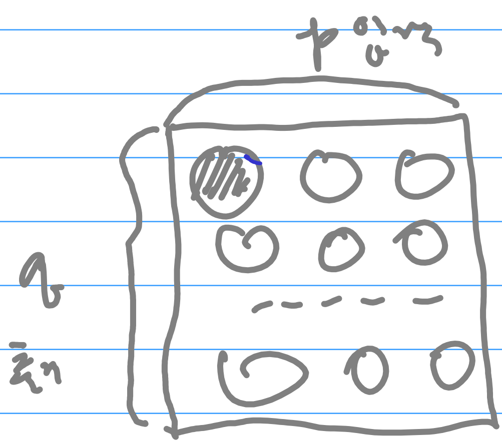
$$
\frac{Y(s)}{U(s)} = \underbrace{\left\{ \overset{q \times n}{C} \overset{n \times n}{(sI - A)^{-1}} \overset{n \times p}{B} + \overset{q \times p}{D} \right\}}_{q \times p}
$$

q개의 출력 $\begin{cases} c_1(t) \\ c_2(t) \\ \vdots \\ c_q(t) \end{cases}$ p개 입력 $\begin{cases} r_1(t) \\ r_2(t) \\ \vdots \\ r_p(t) \end{cases}$

q개 출력 p개 입력

$$\underset{q \times p}{\frac{Y(s)}{U(s)}} = \underbrace{\left\{ \underset{q \times n}{C} \, \underset{n \times n}{(sI - A)^{-1}} \, \underset{n \times p}{B} + \underset{q \times p}{D} \right\}}_{q \times p}$$

q개의 출력 $\begin{cases} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{cases}$ p개 입력 $\begin{cases} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{cases}$



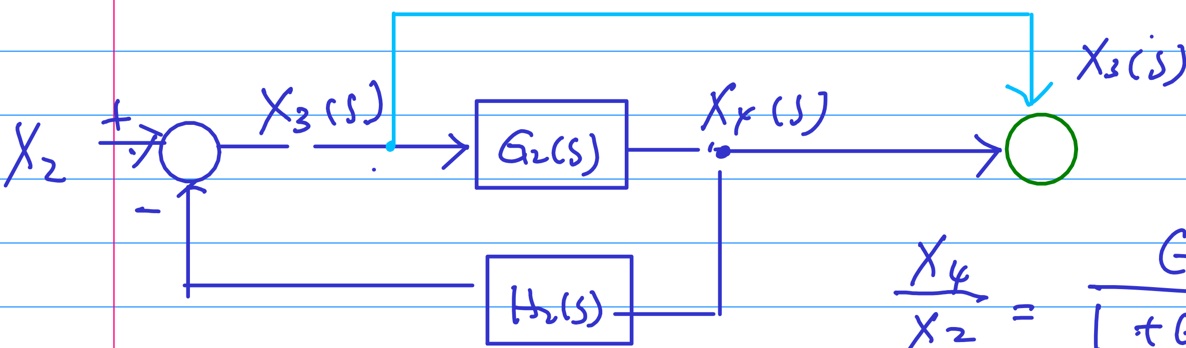
$\frac{Y_1(s)}{U_1(s)}$ $\frac{Y_1(s)}{U_2(s)}$ ...

$\frac{Y_2(s)}{U_1(s)}$

p개 입력

$$\left\{\begin{array}{l} u_1(t) \\ u_2(t) \\ \vdots \\ u_p(t) \end{array}\right. \qquad \left.\begin{array}{l} y_1(t) \\ y_2(t) \\ \vdots \\ y_q(t) \end{array}\right\}$$

q개의 출력



$$\frac{X_4}{X_2} = \frac{G_2}{1 + G_2(s)H_2}$$

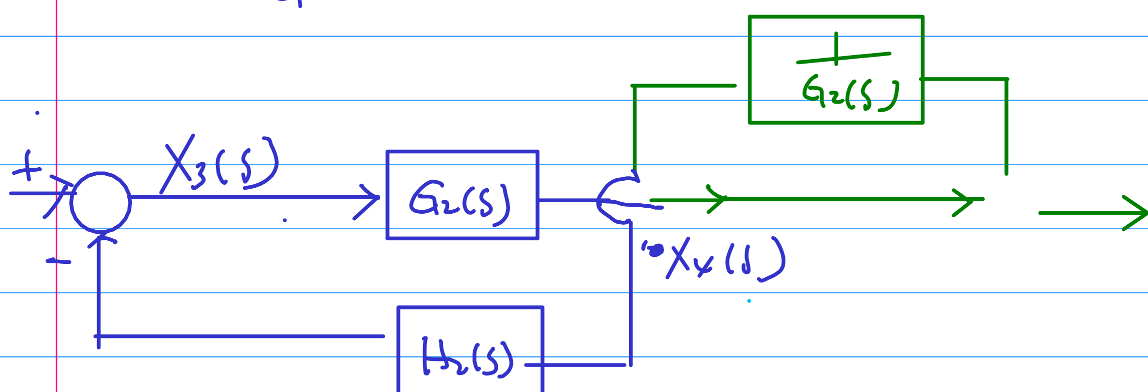
$$\frac{X_2}{X_4} = \frac{1 + G_2 H_2}{G_2}$$

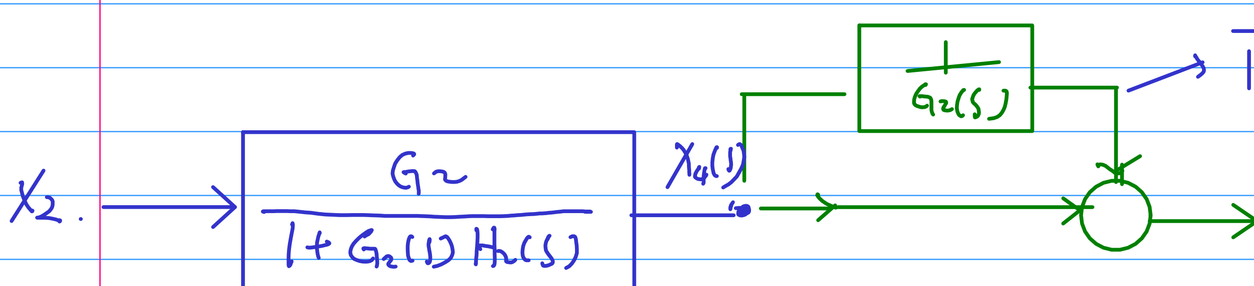
$$X_2 = (\qquad) X_4$$

$$X_3 = X_2 - X_4 =$$

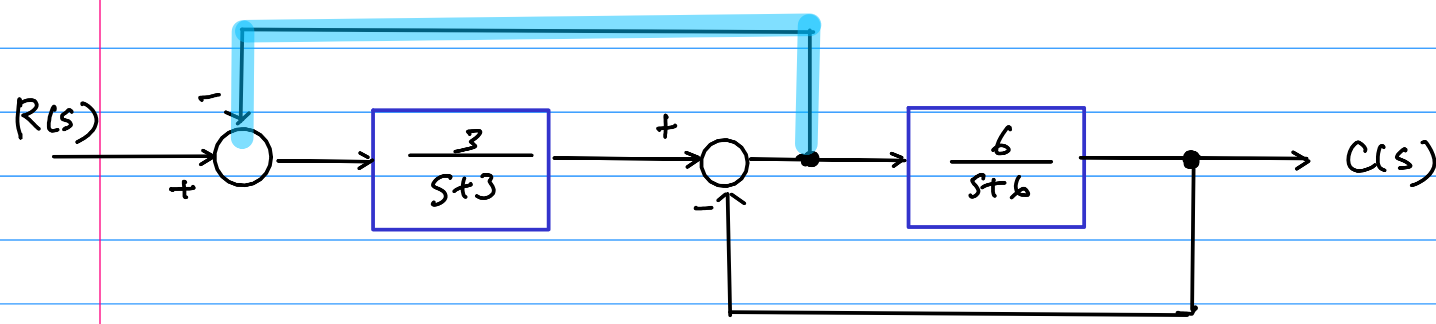
$$= (\qquad) X_4 - X_4$$

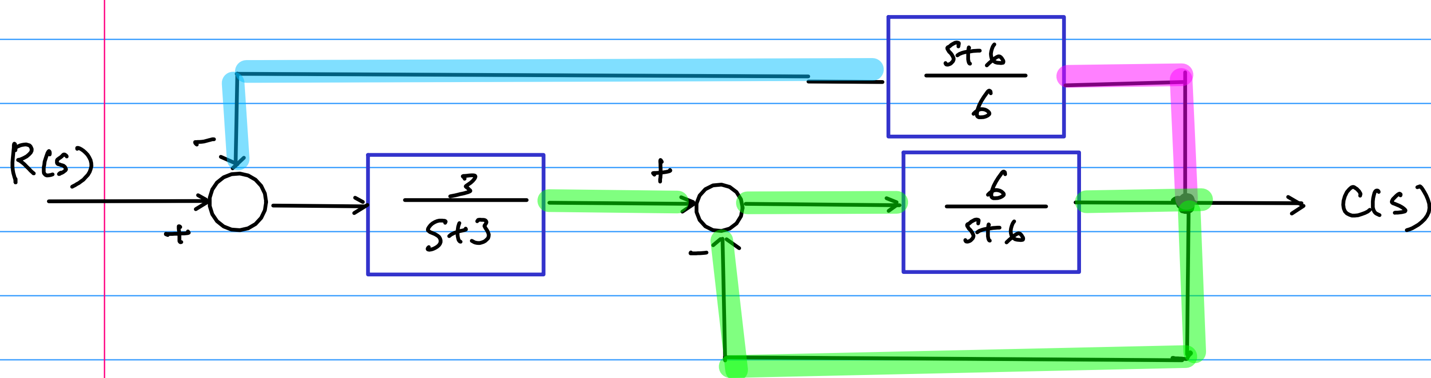
$$= \left(\frac{1 + G_2 H_2}{G_2} - 1\right) X_4$$

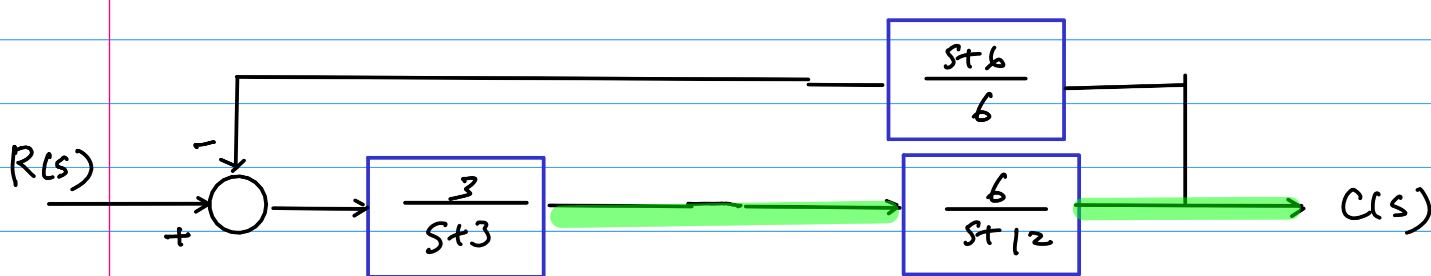




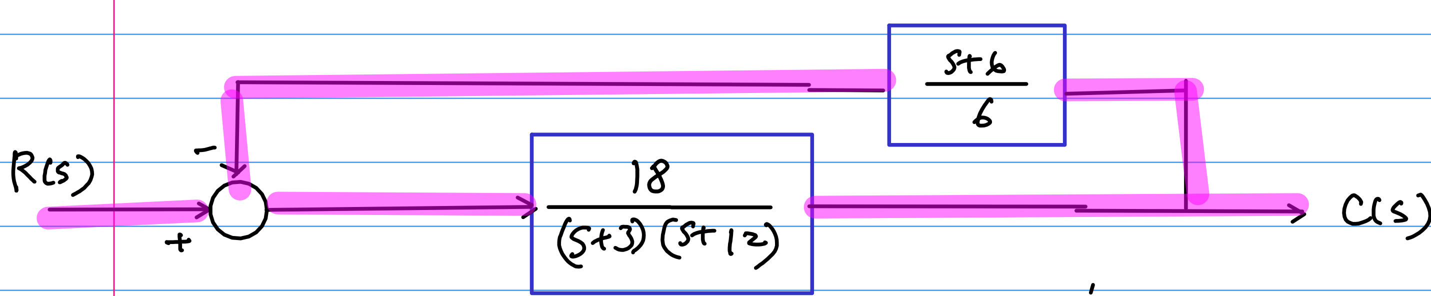
$$\frac{1}{1 + G_2(s)H_2(s)}$$

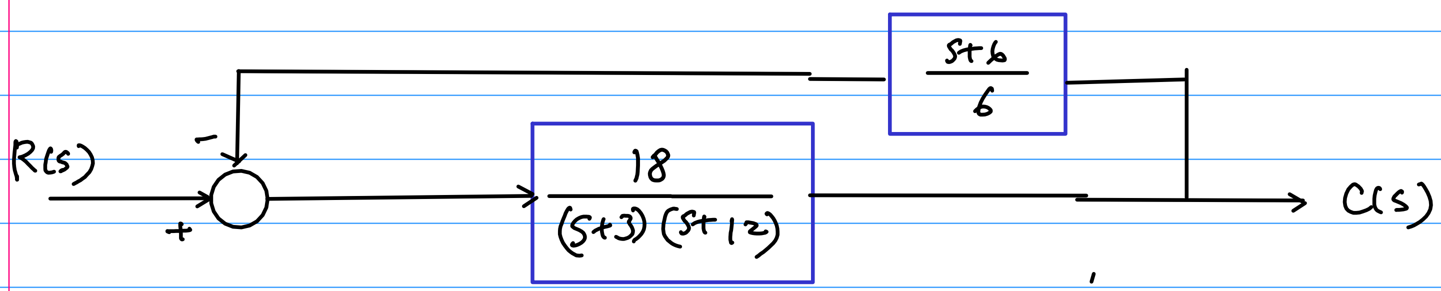






$$\frac{\frac{6}{s+6}}{1+\frac{6}{s+6}} = \frac{6}{s+12}$$



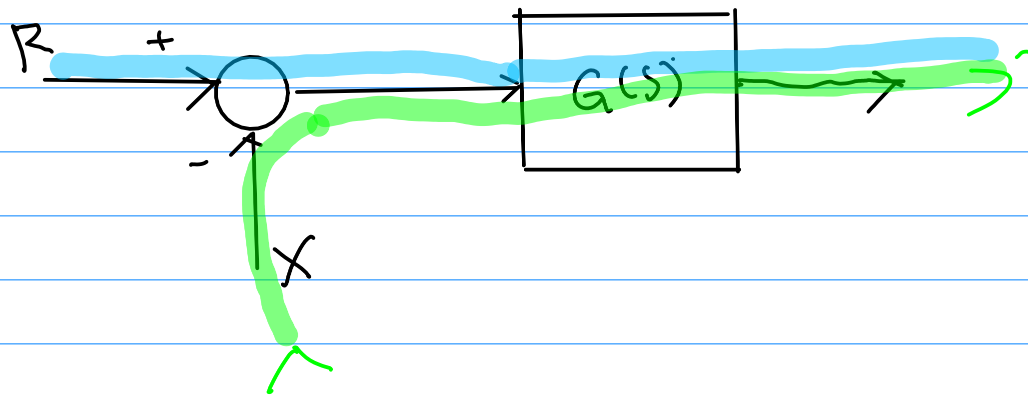


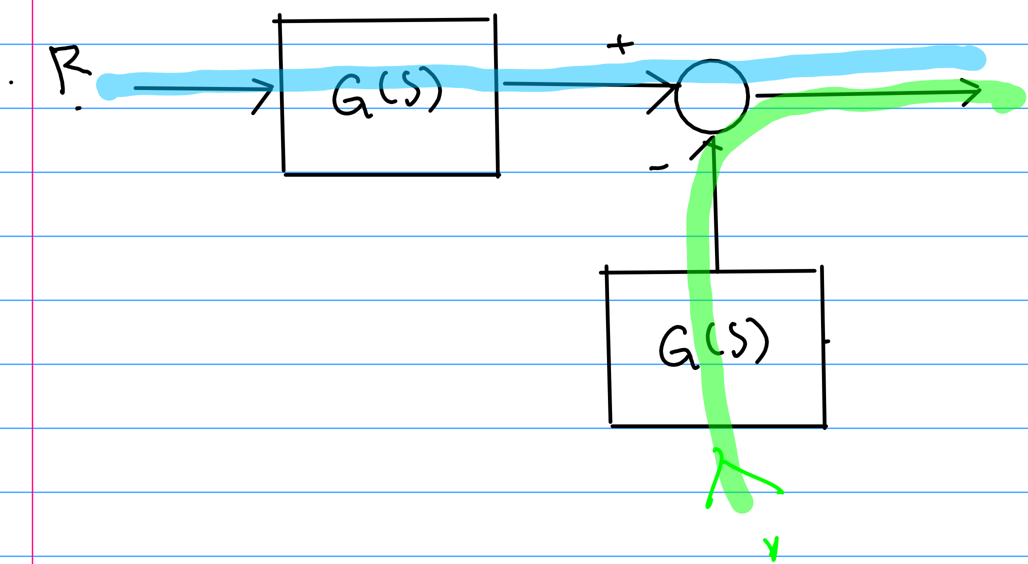
$$\frac{\dfrac{\cancel{18}}{(s+3)(s+12)}}{1+\dfrac{s+6}{6}\,\dfrac{\cancel{18}}{(s+3)(s+12)}} = \frac{6}{6(s+3)(s+12)+(s+6)}$$

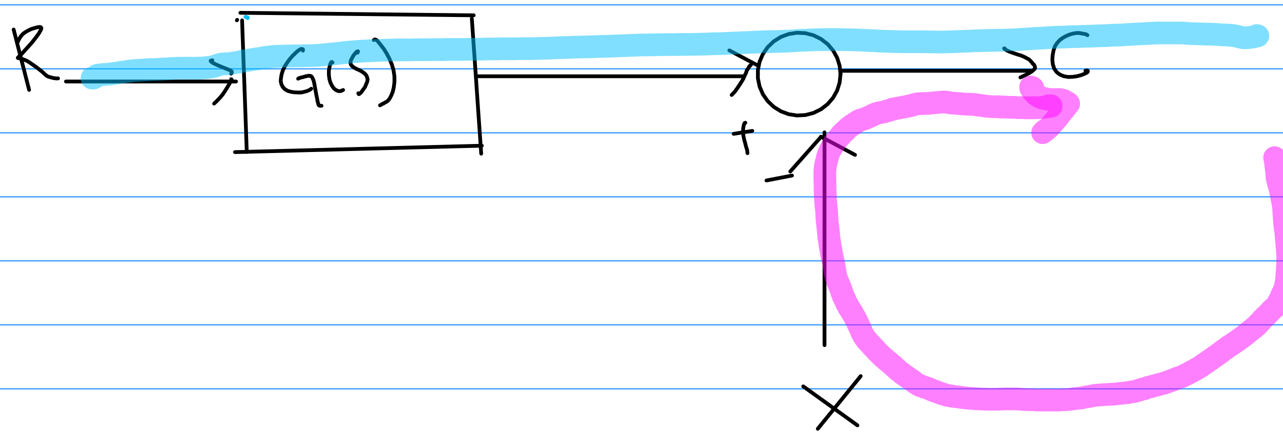
$$6(s^2+15s+36)+(s+6)$$

$$\frac{6}{6s^2+91s+222}$$

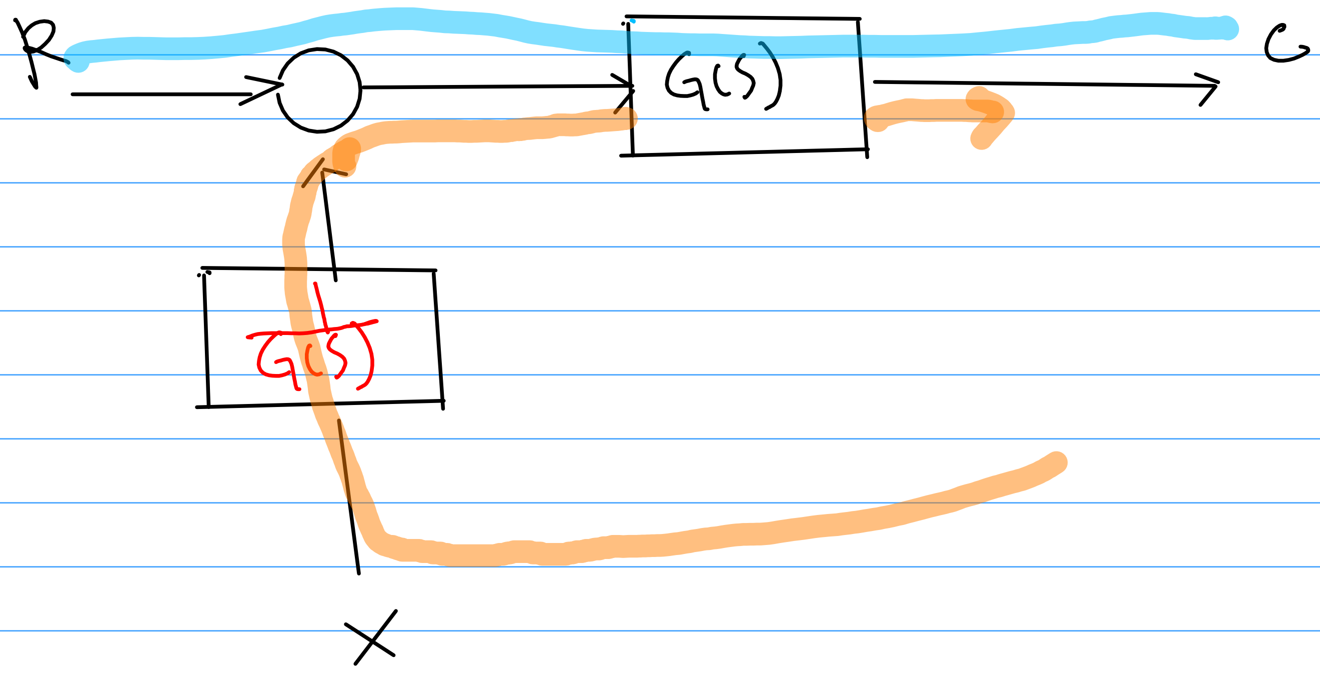
$$\begin{array}{r} 36 \\ \times\; 6 \\ \hline 216 \\ +\; 6 \\ \hline 222 \end{array}$$


R
+
-
G(s)
X

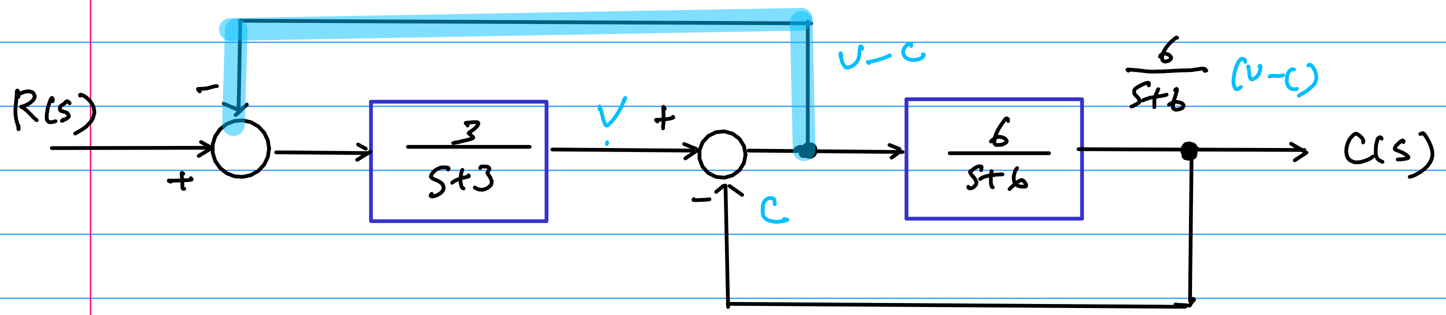

R
G(s)
+
-
G(s)
Y



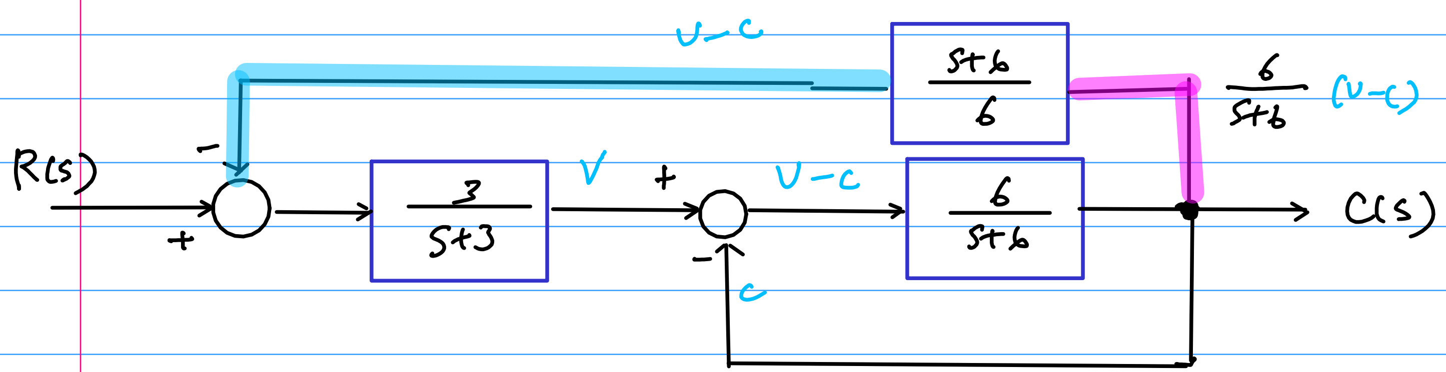
R
G(s)
C
+
−
X



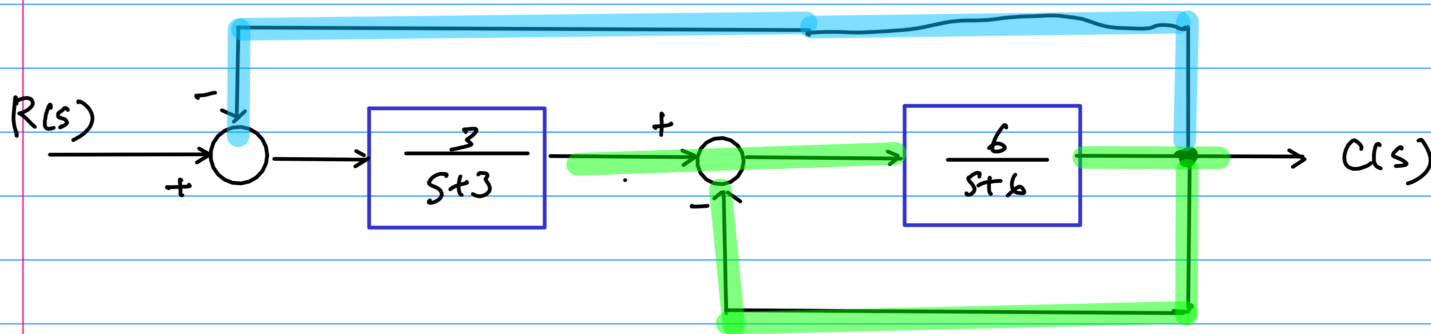
R
G(s)
C
1/G(s)
X



R(s)
C(s)
$\frac{3}{s+3}$
$\frac{6}{s+6}$
V
v-c
$\frac{6}{s+6}(v-c)$
c
+
-



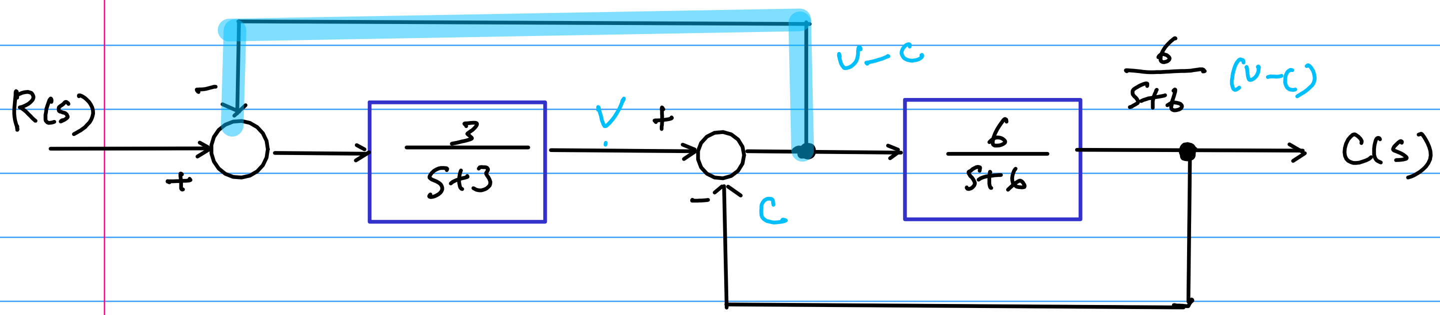
R(s)
C(s)
$\frac{3}{s+3}$
$\frac{6}{s+6}$
$\frac{s+6}{6}$
V
v-c
$\frac{6}{s+6}(v-c)$
c
+
-

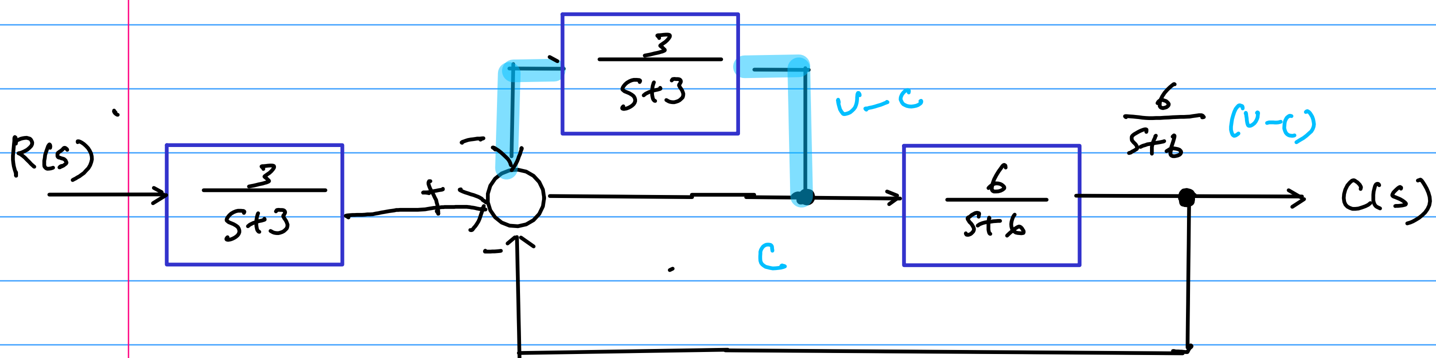


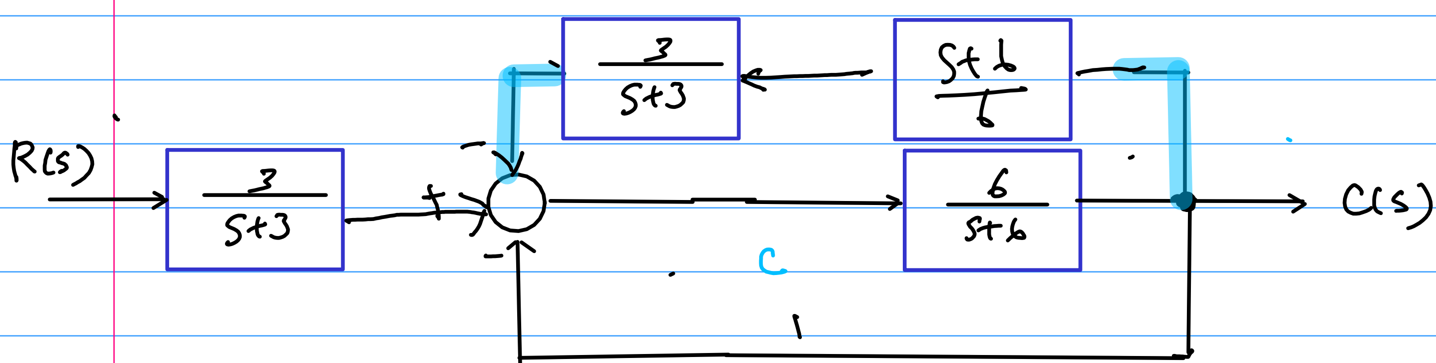
$$\frac{\frac{6}{s+6}}{1+\frac{1}{s+6}} = \frac{6}{s+12}$$

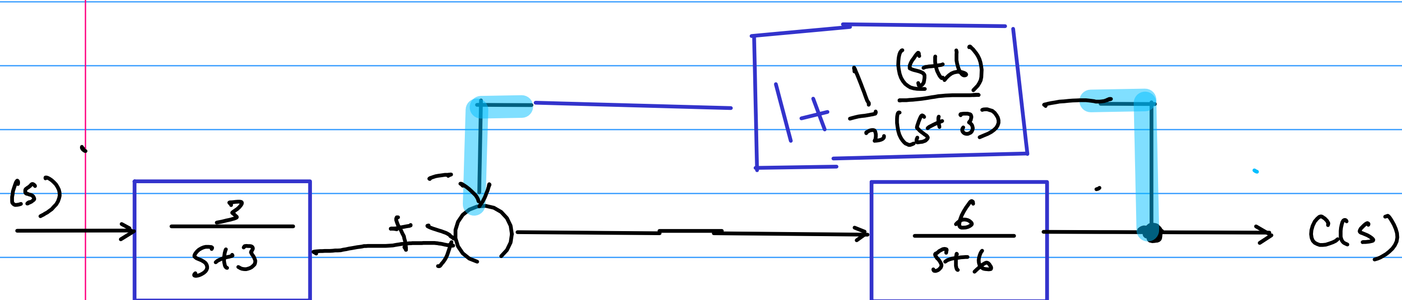
$$\frac{\frac{3\cdot 6}{(s+3)(s+12)}}{1 \quad \frac{3}{s+3}\cdot\frac{6}{s+12}}$$

$+$

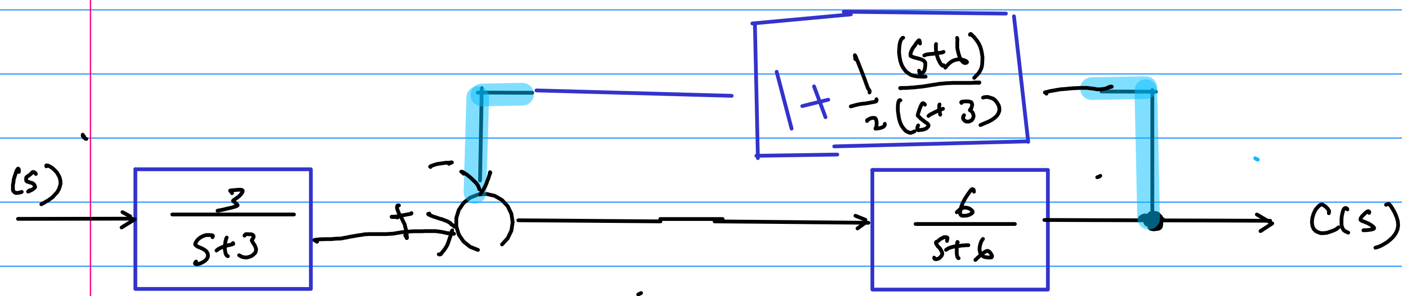








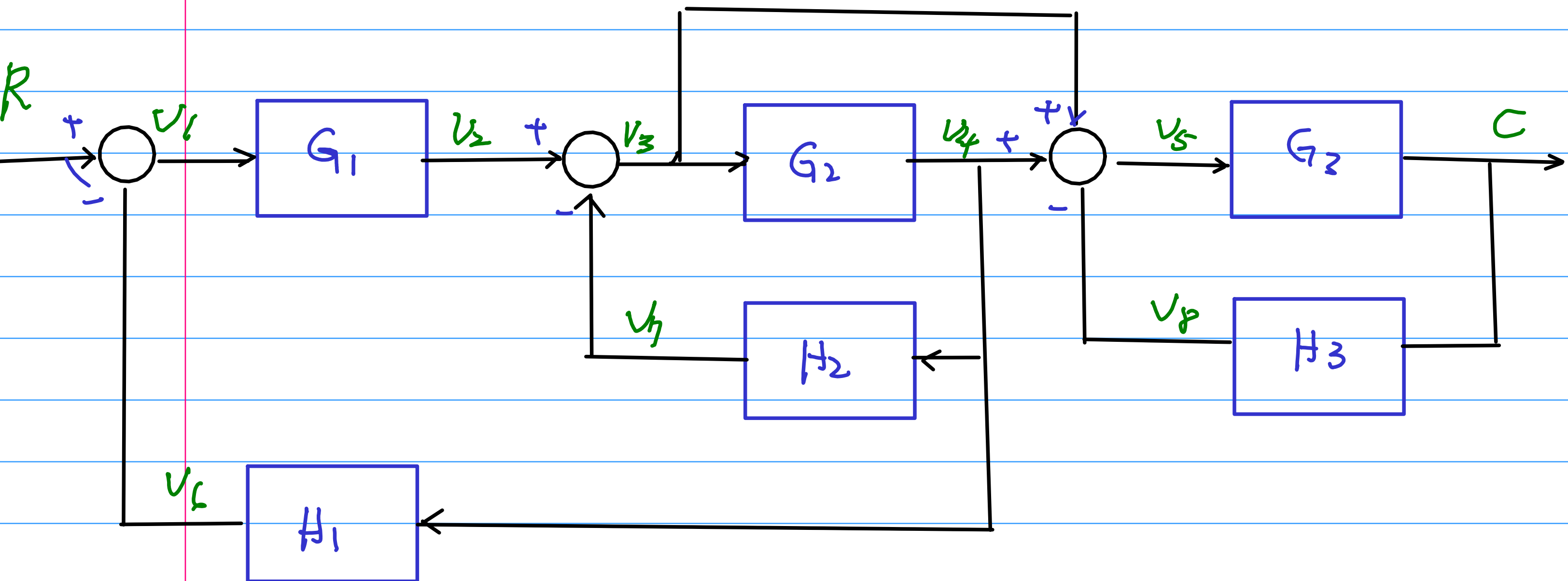
$$\frac{\frac{6}{S+6}}{1+\frac{6}{\cdots}\left(1+\frac{1}{\cdots}\frac{(S+6)}{(S+3)}\right)}$$

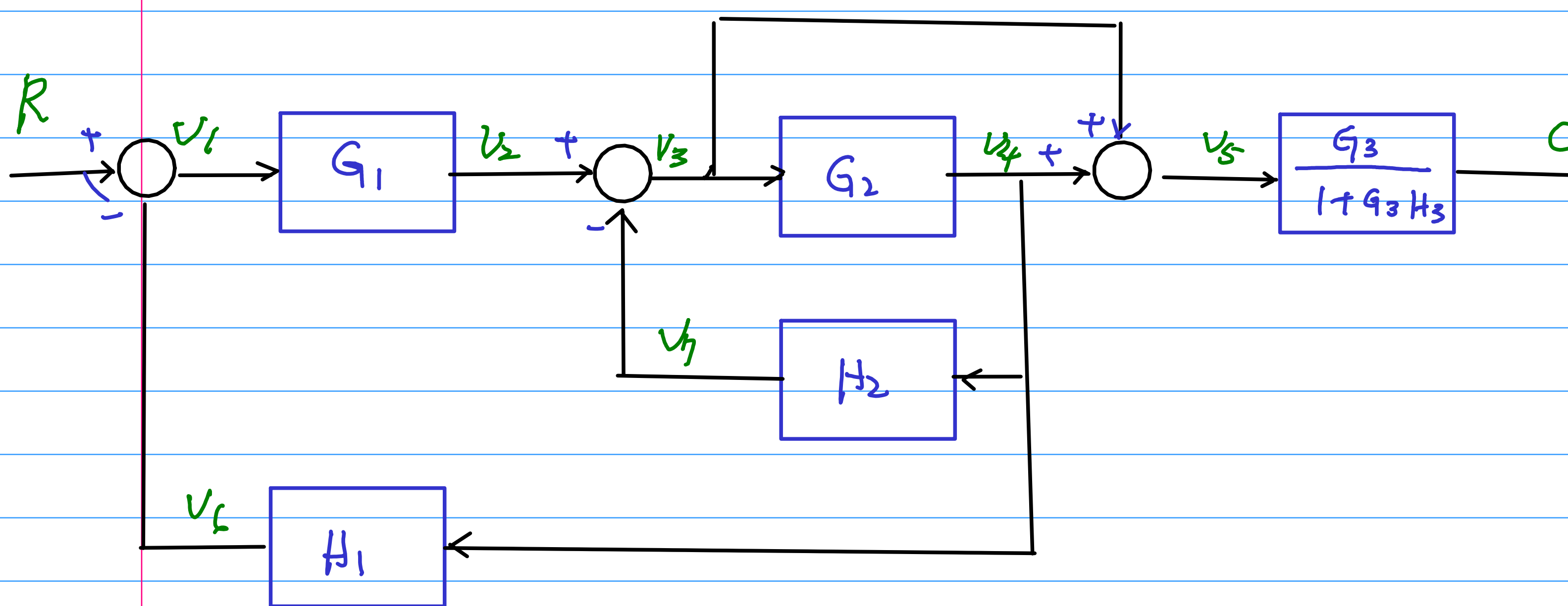


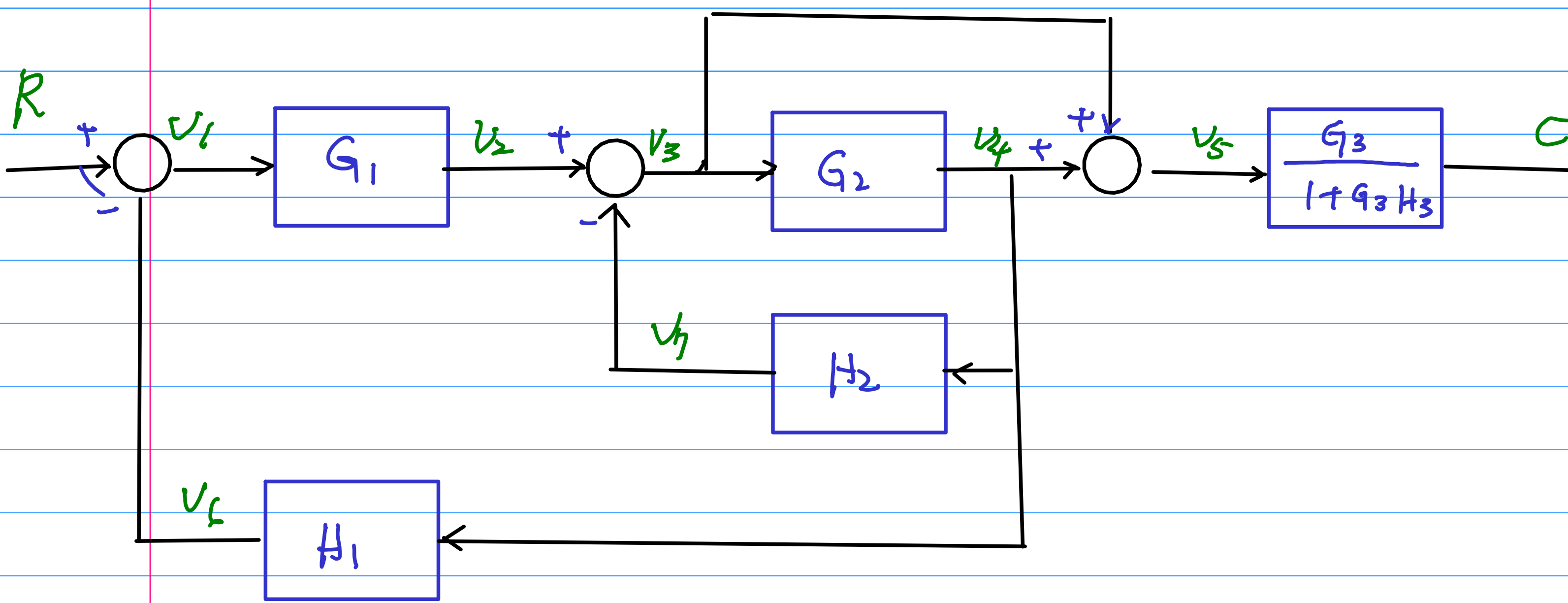
$$= \frac{\cancel{\frac{6}{s+6}}}{\frac{s+6}{6} + \cancel{\frac{6}{s+6}}\left(1 + \frac{1}{2}\frac{(s+1)}{(s+3)}\right)}$$

$$\frac{1}{\frac{s+6}{6} + 1 + \frac{1}{2}(s+6)}$$

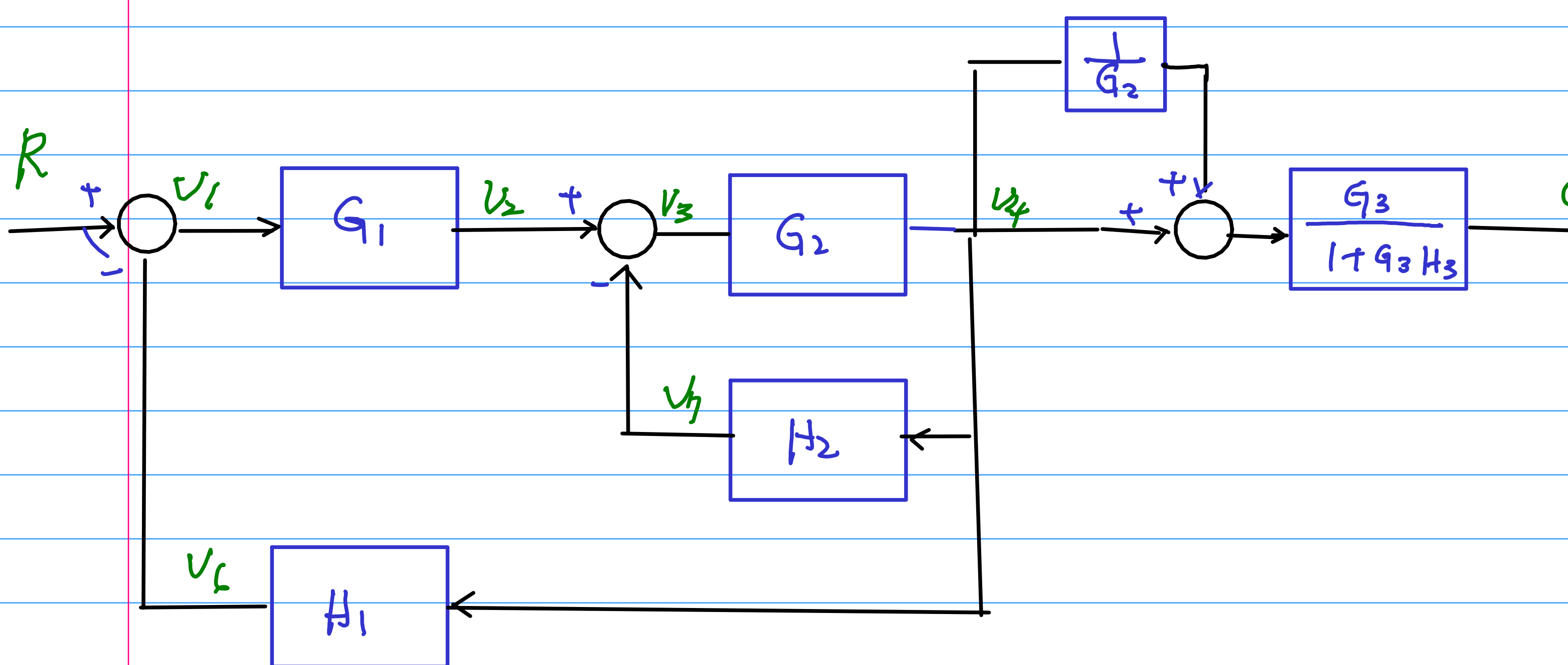
1


R
V1
G1
V2
V3
G2
V4
V5
G3
C
V7
H2
V8
H3
V6
H1

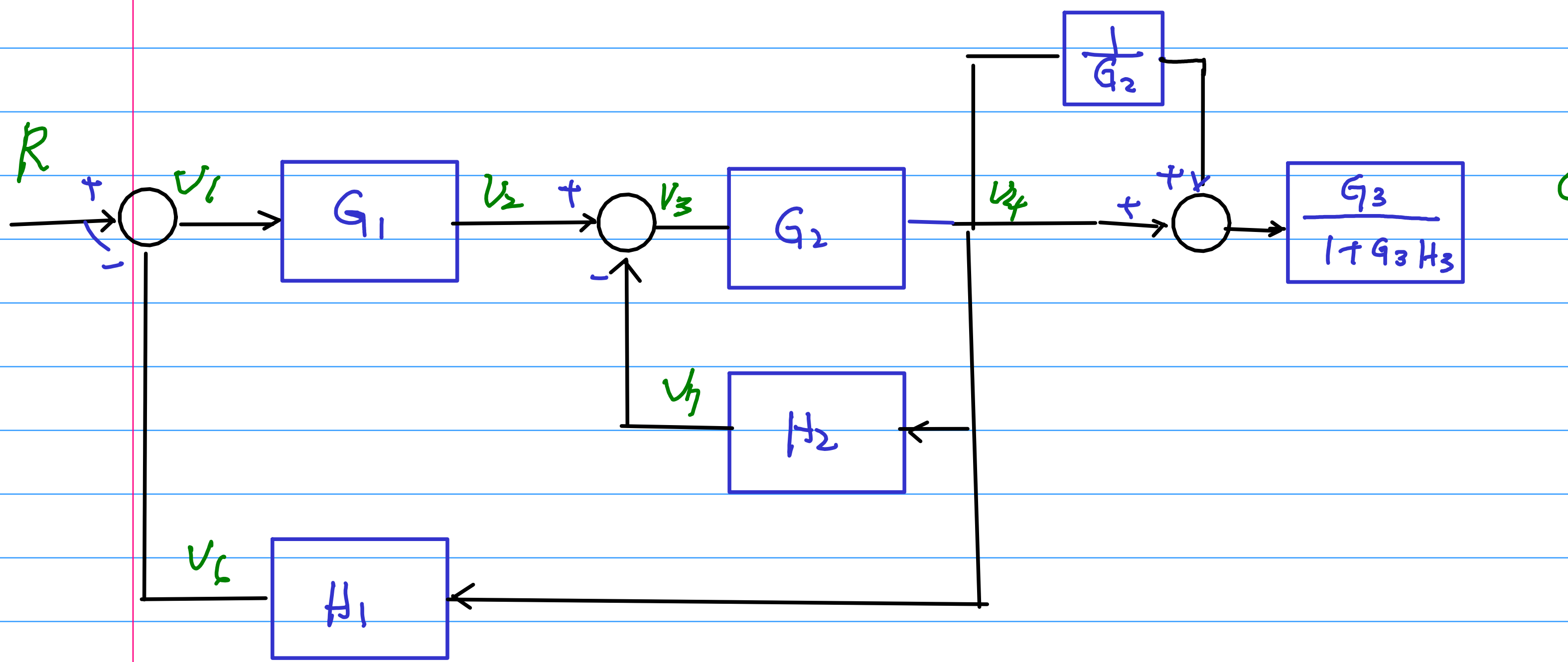

R
V1
G1
V2
V3
G2
V4
V5
G3 / 1+G3H3
V7
H2
V6
H1

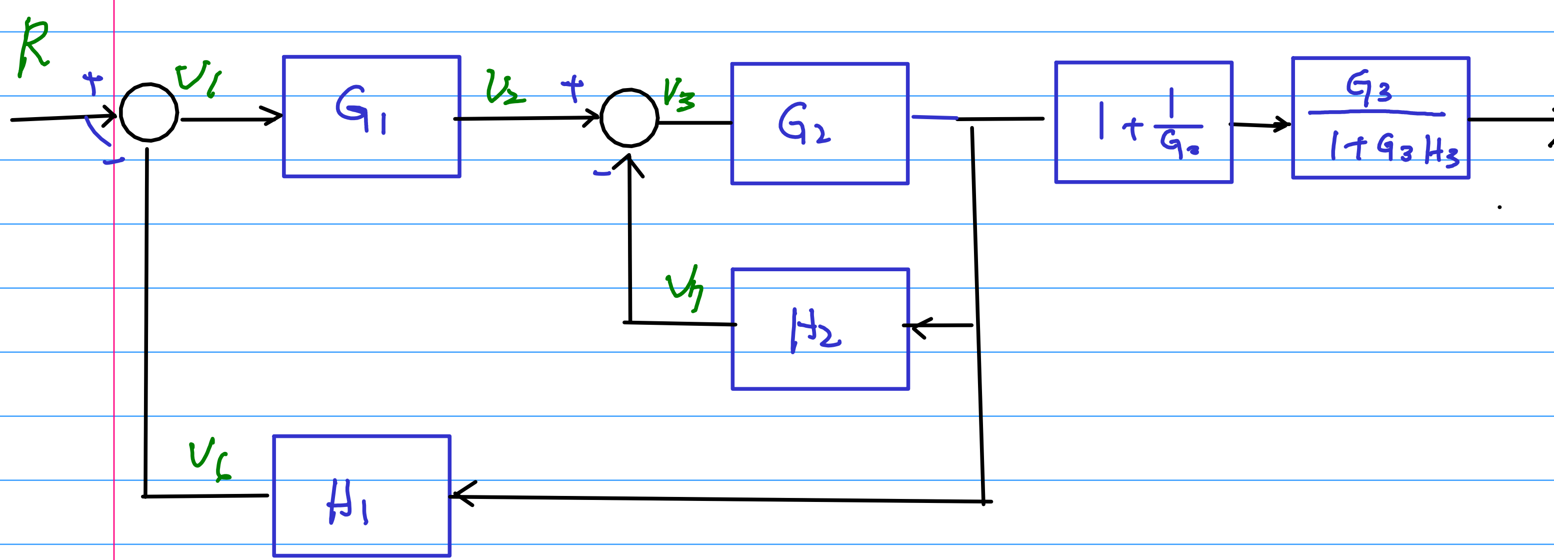


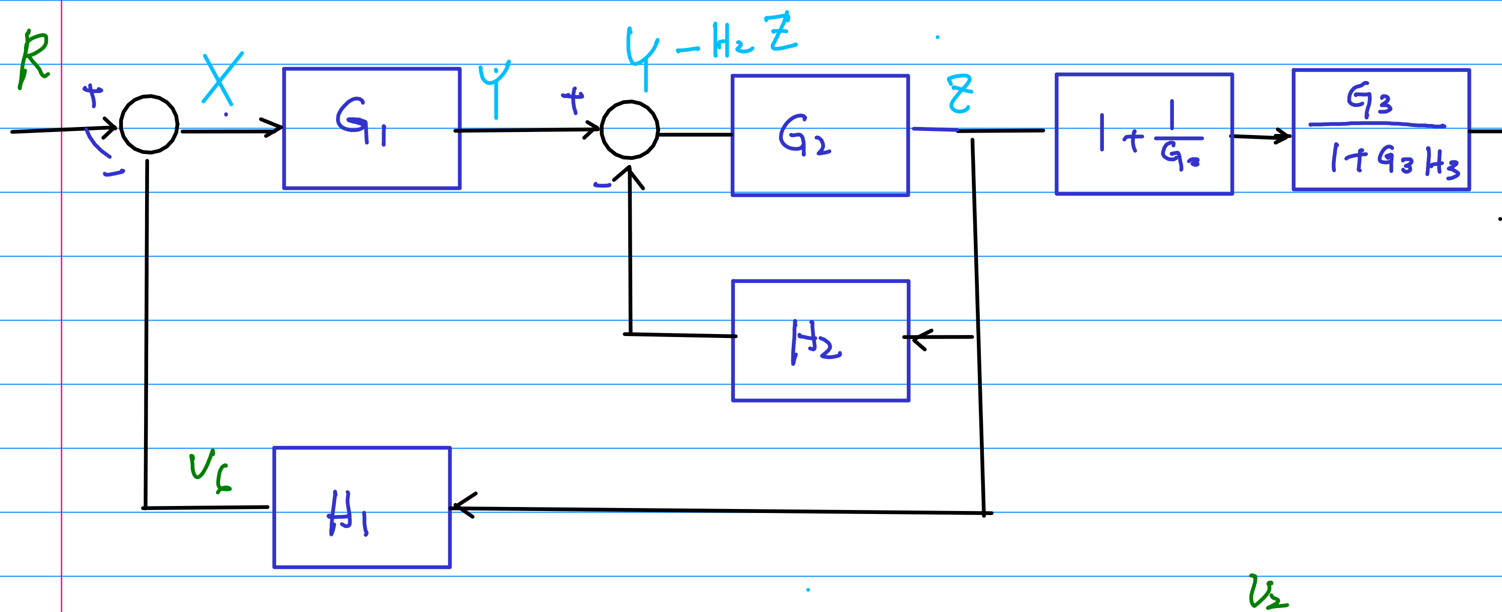
R
+
-
V_1
G_1
V_2
+
V_3
-
G_2
V_4
+
+
V_5
$\frac{G_3}{1+G_3H_3}$
C
V_7
H_2
V_6
H_1

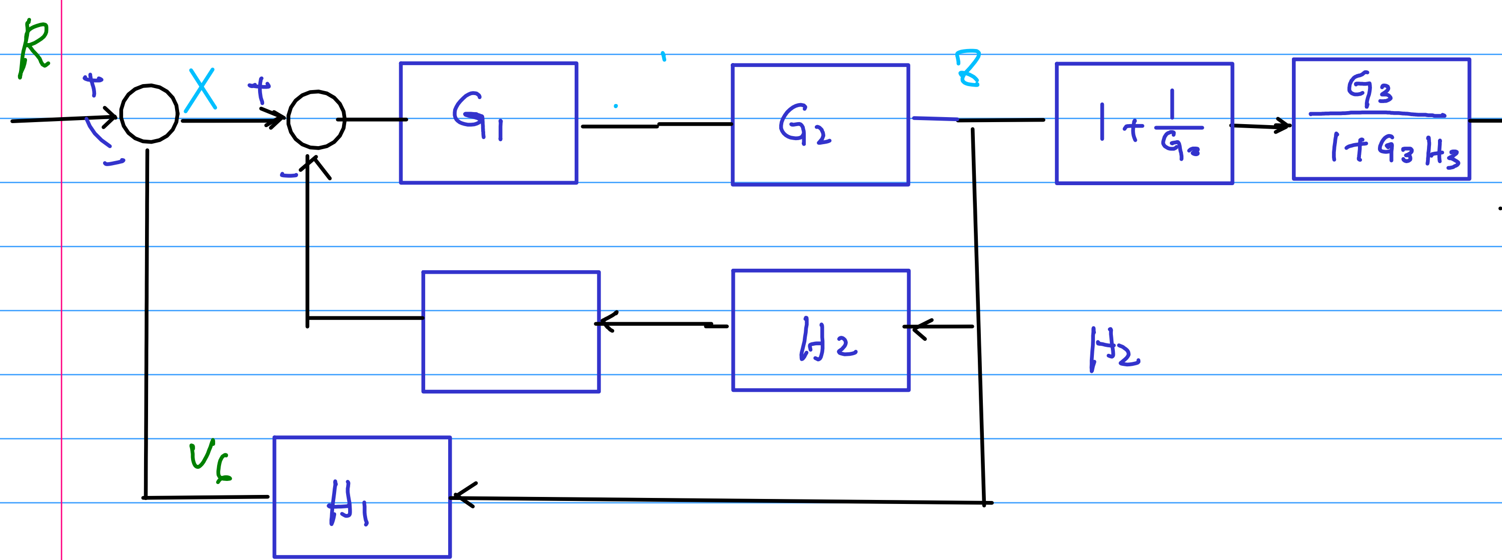


R
+
-
V_1
G_1
V_2
+
V_3
-
G_2
$\frac{1}{G_2}$
V_4
+
+
$\frac{G_3}{1+G_3H_3}$
V_7
H_2
V_6
H_1


R
+
-
v1
G1
v2
+
v3
G2
v4
1 / G2
+
+
G3 / 1+G3H3
H2
H1

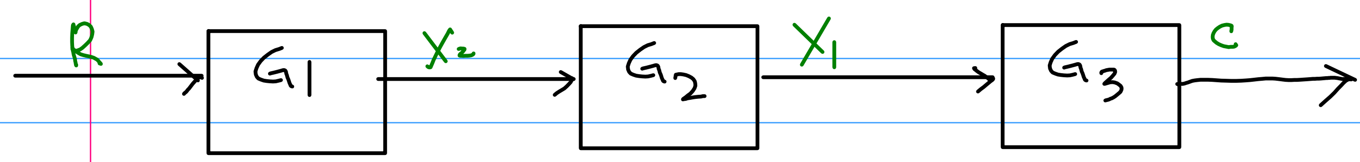

R
+
-
v1
G1
v2
+
v3
G2
1 + 1/G2
G3 / 1+G3H3
H2
H1


R
+
−
X
G1
Y
+
Y − H2 Z
G2
Z
1 + 1/G0
G3 / (1 + G3 H3)
−
H2
Vf
H1
Vs

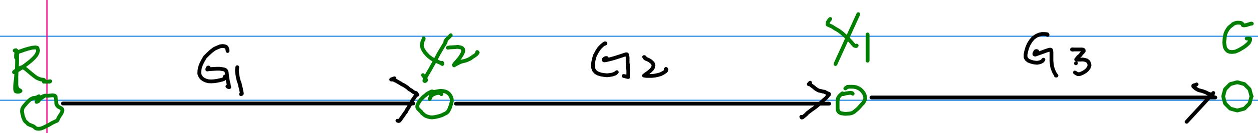

R
+
−
X
+
G1
G2
Z
1 + 1/G0
G3 / (1 + G3 H3)
−
H2
H2
Vf
H1

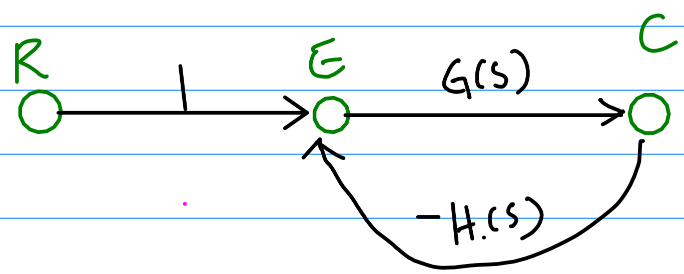
Block Diagram & Signal Flow Grap

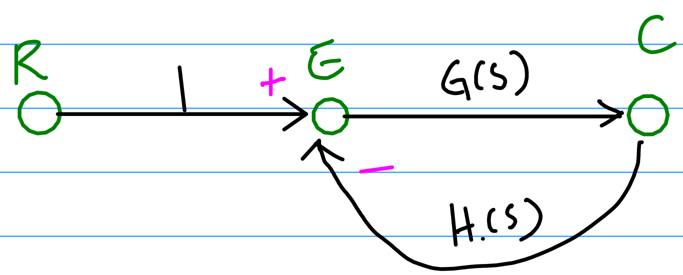
Block Diagram

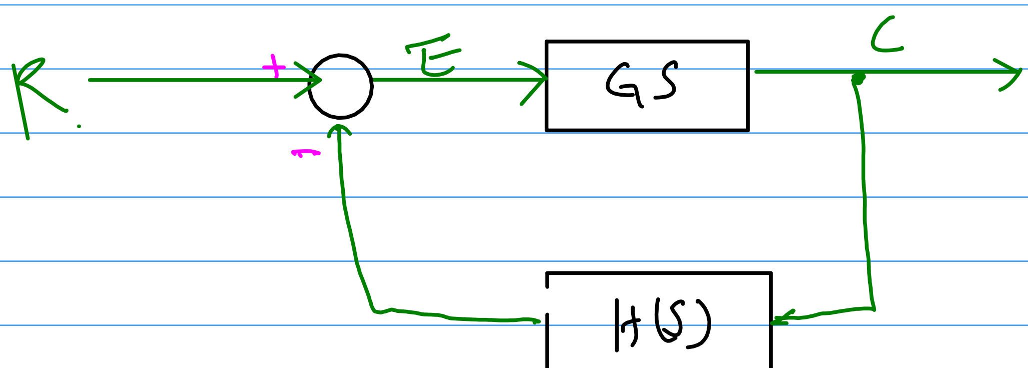


Signal Flow Graph.

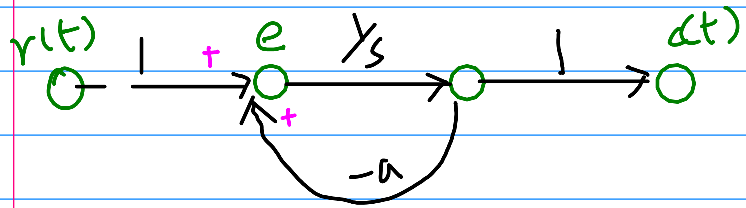




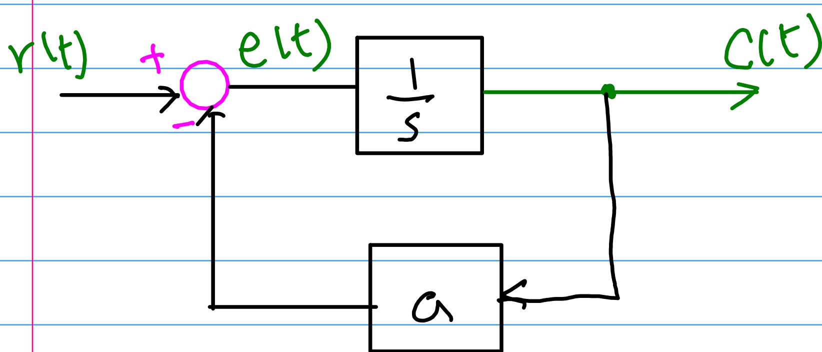




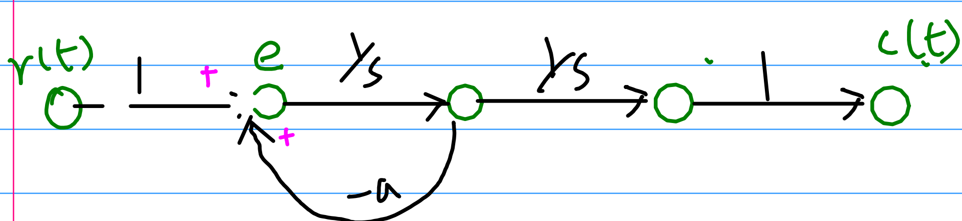
P48. 22) 3.16



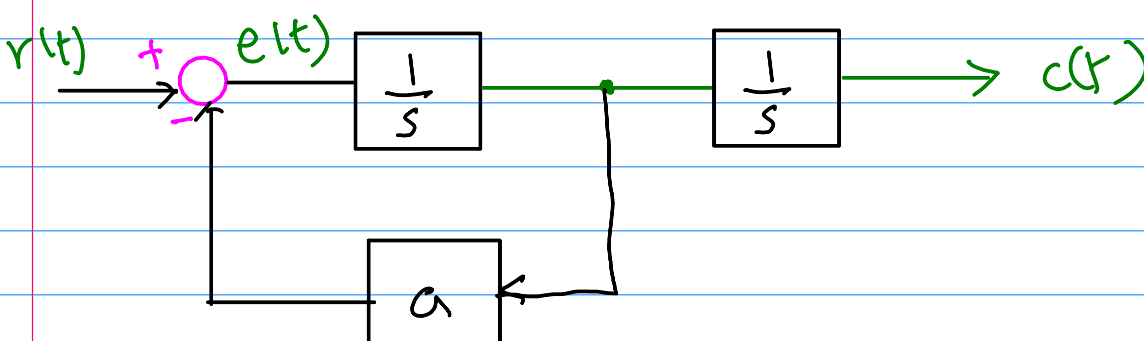
$$\frac{\frac{1}{s}}{1+\frac{a}{s}} = \frac{1}{s+a}$$

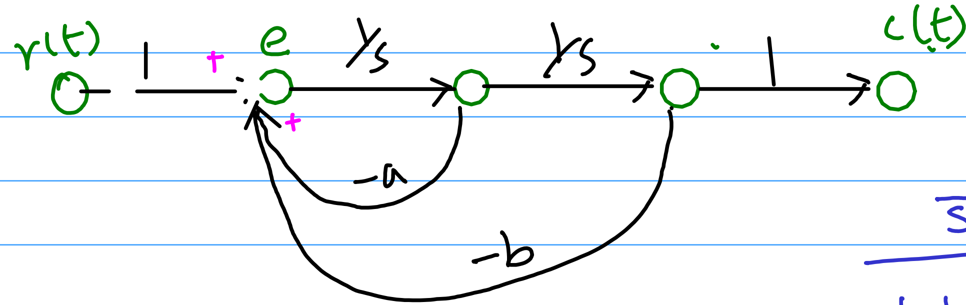


$$\frac{1}{s+a} \times \frac{1}{s}$$

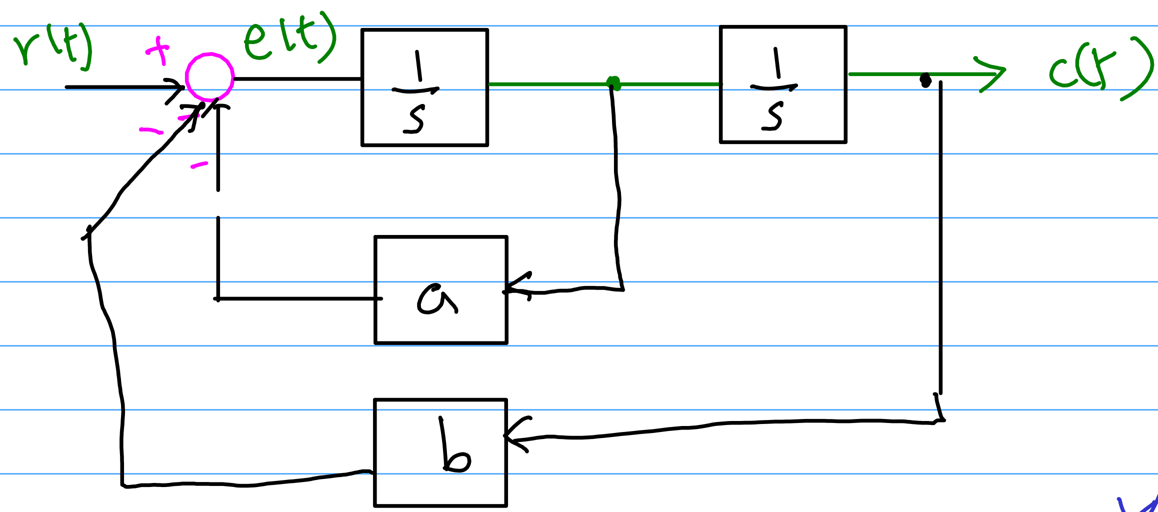


$$= \frac{1}{s(s+a)}$$





$$\frac{\frac{1}{s(s+a)}}{1+\frac{b}{s(s+a)}}$$



$$\frac{1}{s^2+as+b}$$

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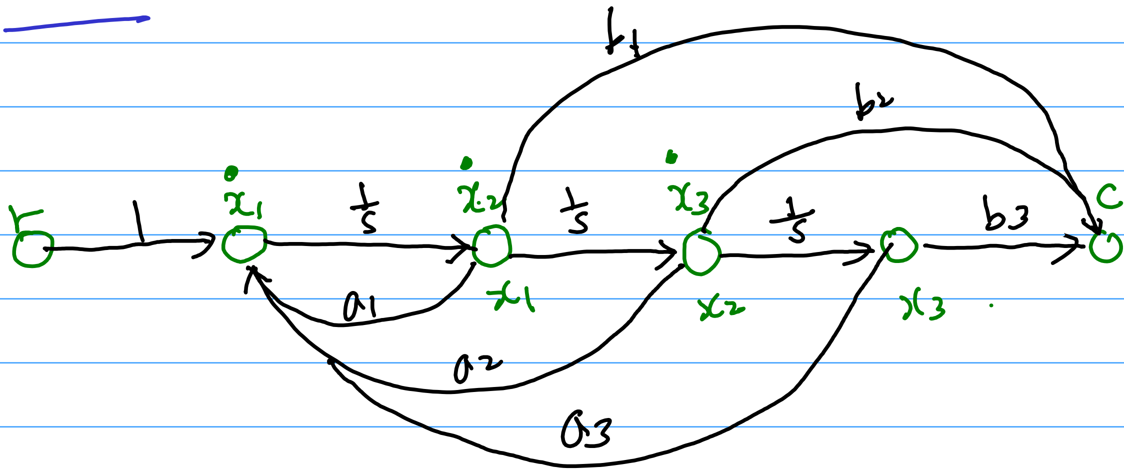

$$\dot{x}_1 = r + a_1x_1 + a_2x_2 + a_3x_3$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

$$c = b_1x_1 + b_2x_2 + b_3x_3$$

$$c = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots \\ \dot{x}_n\end{bmatrix} = \begin{bmatrix} & 1 & & 0 \\ & & 1 & \\ 0 & & & 1\\ -a_0 & -a_1 & \cdots & -a_{n-1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix} + \begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}u$$

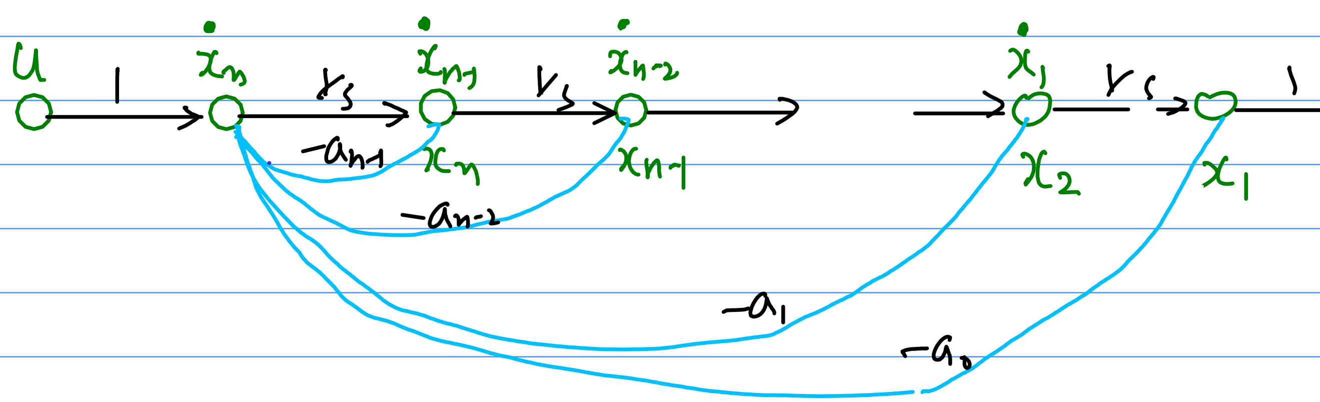
$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\vdots$$

$$\dot{x}_n = -(a_0 x_1 + a_1 x_2 + \cdots + a_{n-1} x^n) + u$$

$$y^{(n)} + a_{n-1} \overset{x_n}{y^{(n-1)}} + \cdots + a_1 \overset{x_2}{y^{(1)}} + a_0 \overset{x_1}{y} = u$$



$$\overset{x_n}{} \qquad \overset{x_2}{} \qquad \overset{x_1}{}$$

$$y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y^{(1)} + a_0 y = u$$

$$s^n Y(s) + a_{n-1} s^{n-1} Y(s) \qquad + a_1 s Y(s) + a_0 Y(s) = U(s)$$

$$\left(s^n + a_{n-1} s^{n-1} \qquad + a_1 s + a_0\right) Y(s) = U(s)$$

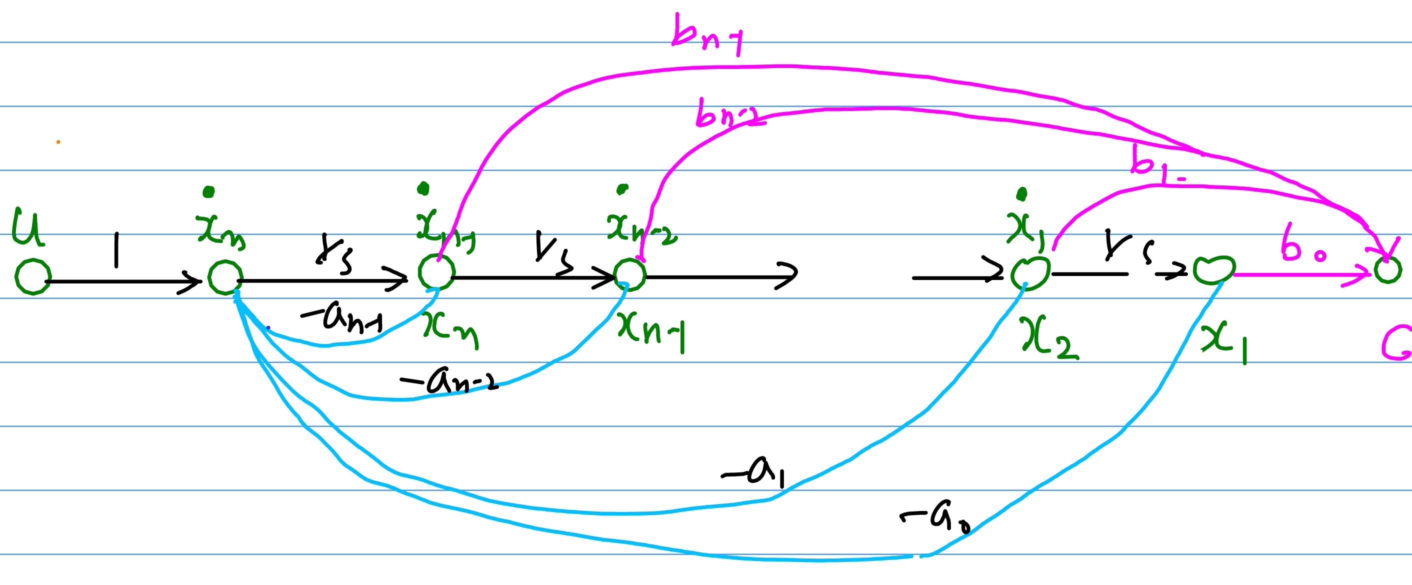
$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

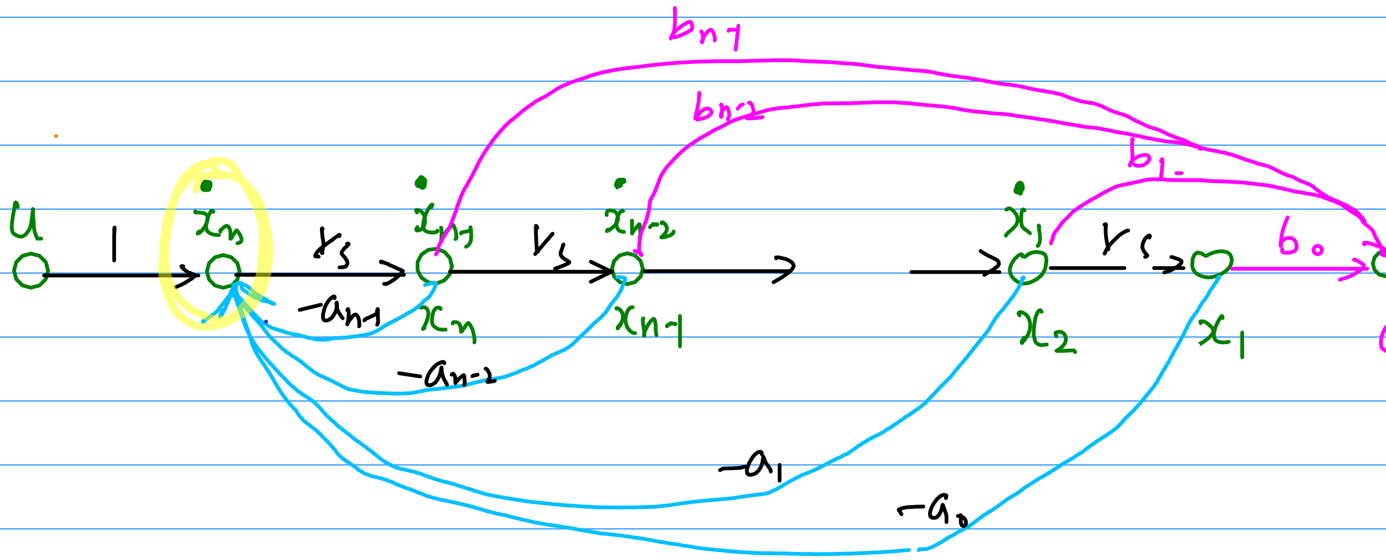
$$\frac{C(s)}{U(s)} = \frac{b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \quad b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= \frac{Y(s)}{U(s)} \left(b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \quad b_1 s + b_0\right)$$

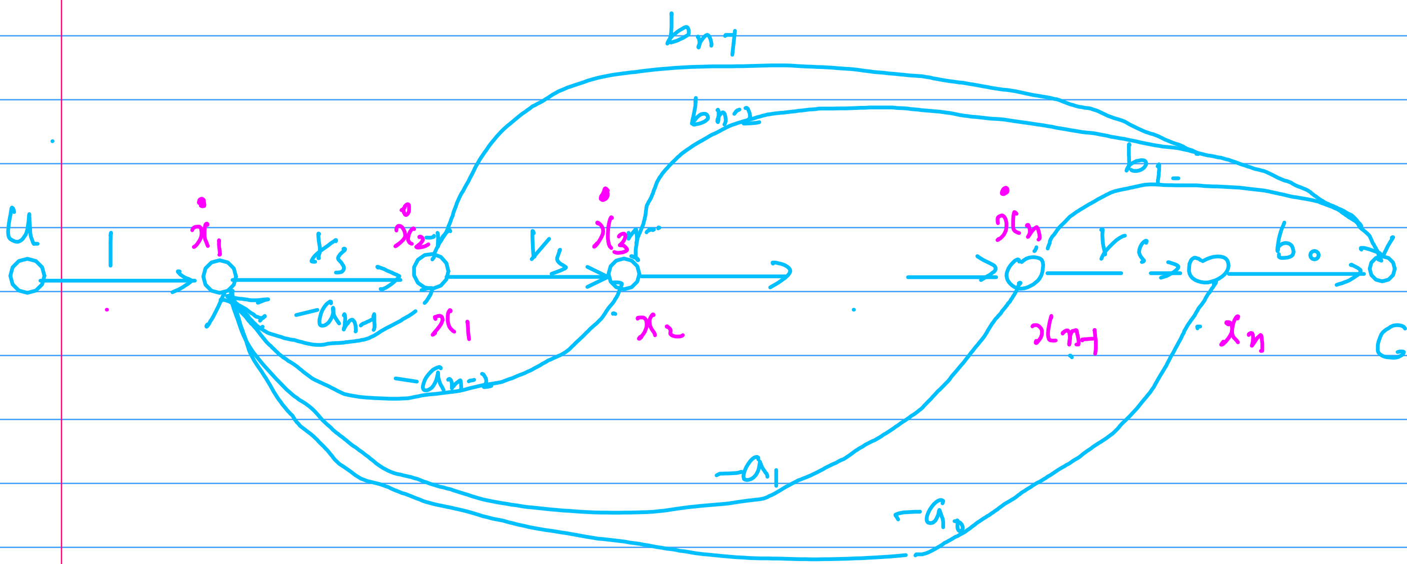
$$c(t) = [\,b_0 \quad b_1 \qquad\qquad b_{n-1}\,] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= b_0 x_1 + b_1 x_2 + \dots \quad b_{n-1} x_n$$





$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \\ \\ \dot x_n\end{bmatrix} = \begin{bmatrix} & 1 & & \\ & & 1 & \\ 0 & & & 1\\ -a_0 & -a_1 & \cdots & -a_{n-1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ \\ x_n\end{bmatrix} + \begin{bmatrix}0\\ \vdots\\ \vdots\\ 0\\ 1\end{bmatrix}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & & & & \\ & \ddots & 0 & & \\ & 0 & \ddots & & \\ & & & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

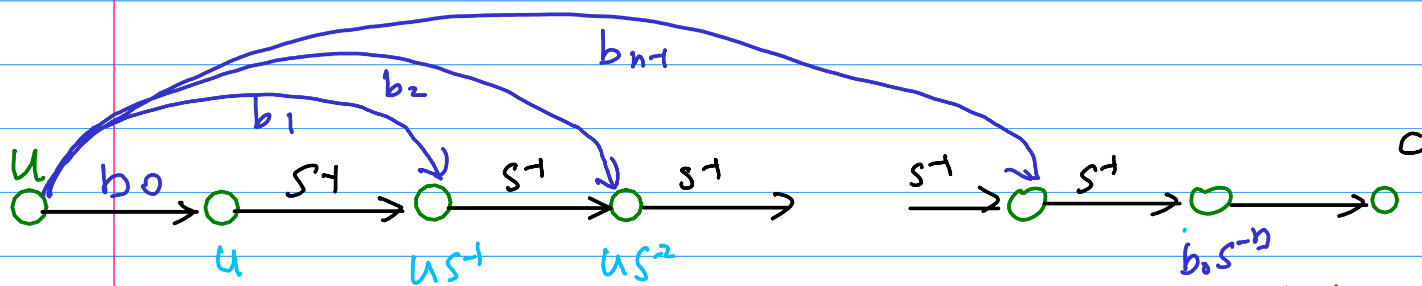
$$\frac{C(s)}{U(s)} = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \quad b_1 s + b_0}{s^{n} + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

$$\frac{C(s)}{U(s)} = \frac{b_{n-1}s^{-1} + b_{n-2}s^{-2} + \quad b_1 s^{-n+1} + b_0 s^{-n}}{1 \quad + a_{n-1}s^{-1} + \dots + a_1 s^{-n+1} + a_0 s^{-n}}$$

$$\left(1 + a_{n-1}s^{-1} + \dots + a_1 s^{-n+1} + a_0 s^{-n}\right) C(s) =$$
$$\left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \quad b_1 s^{-n+1} + b_0 s^{-n}\right) U(s)$$

$$C(s) = -\left(+a_{n-1}s^{-1} + a_{n-2}s^{-2} + \dots + a_1 s^{-n+1} + a_0 s^{-n}\right) C(s)$$
$$+ \left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \dots + b_1 s^{-n+1} + b_0 s^{-n}\right) U(s)$$

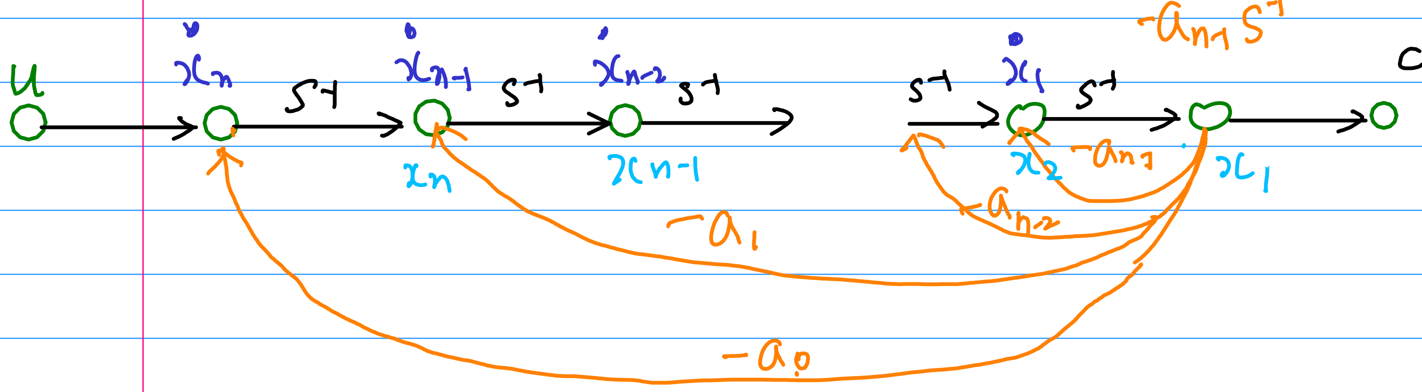
$$C(s) = -\left(a_{n-1}s^{-1} + a_{n-2}s^{-2} + \cdots + a_1 s^{-n+1} + a_0 s^{-n}\right) C(s)$$
$$+ \left(b_{n-1}s^{-1} + b_{n-2}s^{-2} + \cdots + b_1 s^{-n+1} + b_0 s^{-n}\right) U(s)$$

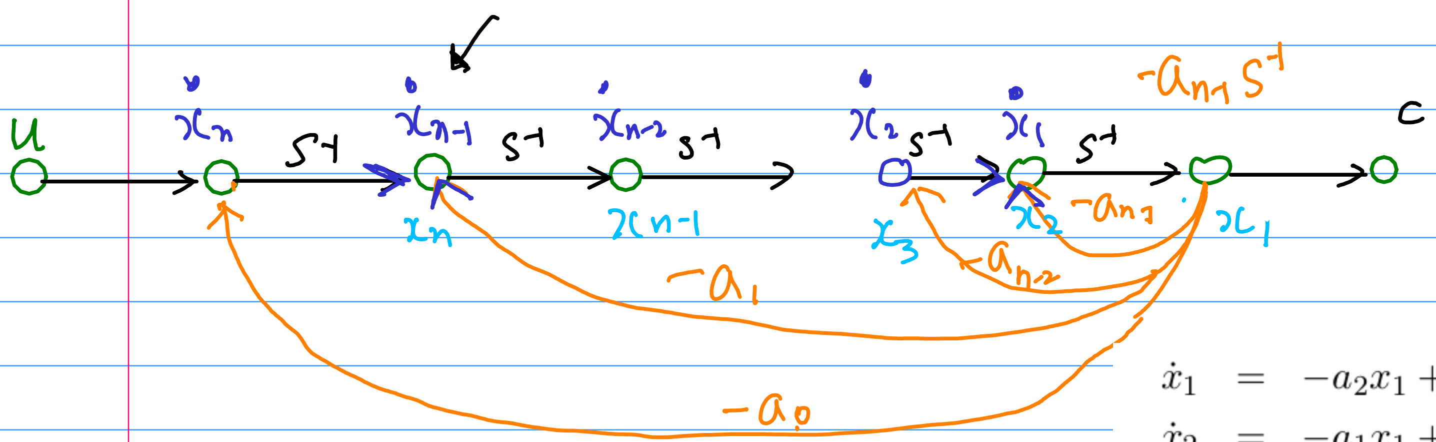


$b_0 s^{-n}$
$+ b_1 s^{-n+1}$
$+ b_2 s^{-n+2}$
$\vdots$
$+ b_{n-1} \cdot s^{-1}$

$-a_0 s^{-n}$
$-a_1 s^{-n+1}$

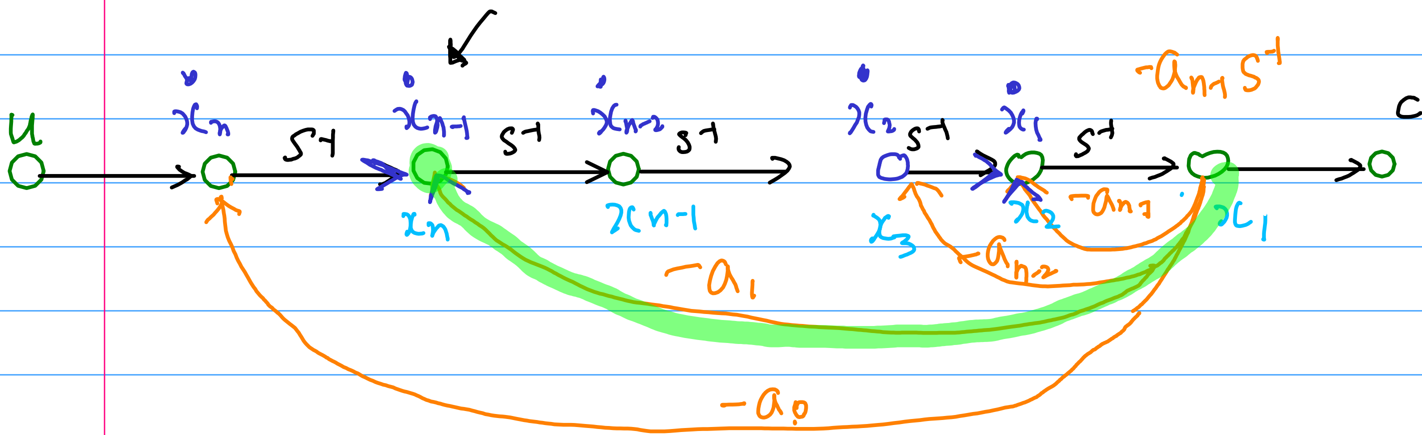
$-a_{n-2} s^{-2}$
$-a_{n-1} s^{-1}$





$$
\begin{aligned}
\dot{x}_1 &= -a_2x_1 + x_2 + b_2 \\
\dot{x}_2 &= -a_1x_1 + x_3 + b_1 \\
\dot{x}_3 &= -a_0x_1 + b_0u \\
y &= x_1
\end{aligned}
$$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_{n-1} & 1 & & & \\ -a_{n-2} & 0 & 1 & & \\ \vdots & & & \ddots & \\ -a_1 & & & & 1 \\ -a_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} u.
$$



$$\dot{x}_n = -a_0 x_1$$

$$\dot{x}_{n-1} = -a_1 x_1 + s^{-1}(\dot{x}_n)$$

$$-a_1 x_1 + x_n$$

$$\dot{x}_2 = -a_{n-1} \cdot x_1 + s^{-1}(\dot{x}_3)$$

$$= -a_{n-1} \cdot x_1 + x_3$$