pe 24 displacement current

1 Capacitor: Radius = $\underbrace{4.2\,m}_{R}$ gap = $\underbrace{8E\text{-}3\,m}_{d}$ $Q = 45E\text{-}6$ Coul charge

$$C = \frac{\varepsilon_0 A}{d}$$

$\varepsilon_0 = 8.85E\text{-}12$ SI units

$$\oint E \cdot dA = \frac{1}{\varepsilon_0} Q \Rightarrow E = \frac{Q}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 \pi R^2}$$

electric field

$$= \frac{45E\text{-}6}{(8.85E\text{-}12)(\pi)(4.2)^2}$$

$$\boxed{E = 9.18\ N/C \text{ or } V/m}$$

2 $$c^{-2} \oint \vec{E} \cdot d\vec{A} = c^{-2} E A \quad \text{but } E = \frac{Q}{\varepsilon_0 A}$$

$$'' = \frac{1}{c^2} \frac{Q}{\varepsilon_0 A} A = \frac{Q}{c^2}$$

Here $Q = 49\mu C$

$$c^{-2} \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0 c^2} = \frac{49E\text{-}6}{(2.998E8)^2} \left(\frac{1}{8.85E\text{-}12}\right)$$

$$= \cancel{5.45E\text{-}22 \left(\frac{C s^2}{m^2}\right)} \rightarrow \text{~~units~~}$$

$$= 6.16E\text{-}11 \ \underbrace{\text{SI units}}$$

$$\left(\frac{sec}{meter}\right)^2 \frac{N/C}{1} (meter)^2$$

$$\frac{\text{Newton-Seconds}}{\text{Coul}}$$ – who cares?

3 Area $= \pi (4.9\text{m})^2$. $\tau_{decay} = RC$

where $C = \varepsilon_0 \frac{A}{d}$

$\varepsilon_0 = 8.85E-12$ SI
$d = 17E-3$ meters
$R = 9E3$ Ohms

$$\tau = RC = \frac{(9E3)(8.85E-12)(\pi)(4.9^2)}{17E-3}$$

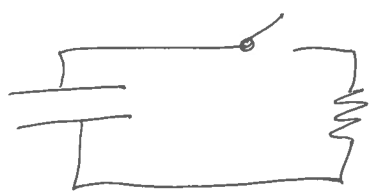
$$\boxed{\tau = 3.53\,E-4 \text{ sec}}$$

4 Two ways to do this:
FIRST:
Use Ampheres Law for the current through the wire

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$\mu_0 = 1.257 \times 10^{-6}$ SI
$\mu_0 = 4\pi \times 10^{-7}$ SI
newton-amp.

Get I from the current. $\varepsilon_0 = 8.85E-12$ SI
F/m



Close circuit at $t = 0$
$V = IR$ where
V is the cappie's voltage.

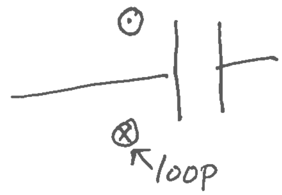
Define variables

r = radius of cappie = 3.3 meters
d = gap = 12 E-3 meters
Q = 93E-6 coul
R = 9E3 ohms.

$Q = CV$ where $C = \varepsilon_0 \frac{\pi r^2}{d}$

$V = QC^{-1} = \frac{Qd}{\varepsilon_0 \pi r^2}$ but $I = \frac{V}{R}$

$$I = \frac{Qd}{\varepsilon_0 \pi r^2} \frac{1}{R}$$



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I = 2\pi r B$$

$$B = \frac{\mu_0}{2\pi r} I = \frac{\mu_0}{2\pi r} \frac{Qd}{\varepsilon_0 \pi r^2} \frac{1}{R} = \boxed{\frac{\mu_0 Q d}{2\pi^2 \varepsilon_0 r^3 R}}$$

$$= \frac{4\pi E{-}7 \quad 93E{-}6}{(2\pi^2)(8.85E{-}12)} \frac{12E{-}3}{(3.3)^3 \quad 9E3} = \boxed{2.48E{-}8 \text{ Tesla}}$$

$$\boxed{\cancel{B = 2.4E{-}10 \text{ Tesla}}}$$

SECOND METHOD

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

$$2\pi r B = \mu_0 \varepsilon_0 \pi r^2 \frac{\partial E}{\partial t}$$

but $Ed = V$ and $Q = CV$

$$E = \frac{V}{d} = \frac{Q}{Cd} \rightarrow \frac{\partial E}{\partial t} = \frac{I}{Cd}$$

$$I = \frac{V}{R} \rightarrow \frac{\partial E}{\partial t} = \frac{V}{RCd} = \frac{1}{RC} \frac{Q}{Cd}$$

$$B = \frac{1}{2\pi r} \mu_0 \varepsilon_0 \pi r^2 \frac{Q}{RC^2 d} \qquad \frac{1}{C^2} = \frac{d^2}{\varepsilon_0^2 \pi^2 r^4}$$

$$B = \frac{1}{2\pi r} \mu_0 \varepsilon_0 \pi r^2 \frac{Q}{Rd} \frac{d^2}{\varepsilon_0^2 \pi^2 r^4}$$

$$\boxed{B = \frac{1}{2\pi^2} \frac{\mu_0}{\varepsilon_0} \frac{1}{r^3} \frac{Qd}{R}}$$

(Whew!

I had no idea this paoblem was so tedius.

Suggested Method:

Don't find capacitance. Instead use Gauss Law: $\oint E \cdot dA = \frac{1}{\varepsilon_0} Q$

It helps to break it by calculating $I = .409$ amps

$$EA = \frac{1}{\varepsilon_0} Q$$

$$V = Ed = \frac{Q}{\varepsilon_0 A} \cdot d = IR$$

$$I = \frac{Qd}{\varepsilon_0 A R} = \frac{(93E-6)(12E-3)}{(8.85E-12)(\pi)(3.3)^2(9E3)} = .409$$

Also $2\pi r B = \mu_0 I = \oint \vec{B} \cdot d\ell$

$$B = \frac{\mu_0}{2\pi r} I = \frac{(1.257E-6)}{2\pi(3.3)}(.409) = 2.48E-8$$