

Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

1 Linear Transformation of Gaussian Random Variables

Linear Transform

N Gaussian random variables

Definition

linear transform of Gaussian random variables X_1, X_2, \dots, X_N

$$Y_1 = a_{11}X_1 + a_{12}X_2 + \dots + a_{1N}X_N$$

$$Y_2 = a_{21}X_1 + a_{22}X_2 + \dots + a_{2N}X_N$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$Y_N = a_{N1}X_1 + a_{N2}X_2 + \dots + a_{NN}X_N$$

$$[Y] = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix}, [T] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix}, [X] = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}$$

Linear Transform

N Gaussian random variables

Corollary

linear transform of Gaussian random variables X_1, X_2, \dots, X_N

$$[Y] = [T][X], \quad [Y - \bar{Y}] = [T][X - \bar{X}]$$

$$[X] = [T]^{-1}[Y], \quad [X - \bar{X}] = [T]^{-1}[Y - \bar{Y}]$$

$$[Y] = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix}, \quad [\bar{Y}] = \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \\ \vdots \\ \bar{Y}_N \end{bmatrix}, \quad [X] = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}, \quad [\bar{X}] = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \vdots \\ \bar{X}_N \end{bmatrix}$$

