Temporal Characteristics of Random Processes

Young W Lim

March 14, 2021

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Random Variable Definition

A random variable

a real **function** over a **sample space** $S = \{s_1, s_2, s_3, ..., s_n\}$

$$s \to X(s)$$

$$x = X(s)$$

a random variable : a capital letter X
a particular value : a lowercase letter x

a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$
an element of S : s

Random Variable Example

Example

$$X(s_1) = x_1 \qquad\qquad s_1 \longrightarrow x_1$$
$$X(s_2) = x_2 \qquad\qquad s_2 \longrightarrow x_2$$
$$... \qquad\qquad\qquad ...$$
$$X(s_n) = x_n \qquad\qquad s_n \longrightarrow x_n$$

$$S = \{s_1, s_2, s_3, ..., s_n\} \qquad\qquad \text{a sample space}$$
$$X = \{x_1, x_2, x_3, ..., x_n\} \qquad\qquad \text{a random variable}$$

Random Process

A random process

a function of both **outcome** *s* and **time** *t*

$$X(t, s)$$

assigning a time function to every **outcome** s_i

$$s_i \rightarrow x(t, s_i)$$

the family of such time functions is called a random process

$$x(t, s_i) = X(t, s_i)$$

$$x(t, s) = X(t, s)$$

Ensemble of time functions

Time functions

A random process $X(t, s)$ represents
a <u>family</u> or <u>ensemble</u> of time functions

$X(t, s)$ represents
- a single time function $x(t, s)$
- when t is a variable and s is <u>fixed</u> at an outcome

$x(t, s)$ represents
- a sample function,
- an ensemble member,
- a realization of the process

Short-form notation for time functions
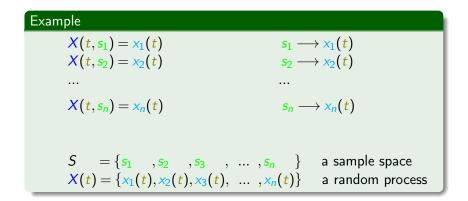

The short-form notation $x(t)$

to represent a specific waveform of a random process $X(t)$
for a given **outcome** s_i

$$x(t) = x(t, s)$$

$$X(t) = X(t, s)$$

Random Process Example

Example

$X(t, s_1) = x_1(t)$ $s_1 \longrightarrow x_1(t)$

$X(t, s_2) = x_2(t)$ $s_2 \longrightarrow x_2(t)$

...   ...

$X(t, s_n) = x_n(t)$ $s_n \longrightarrow x_n(t)$

$S \quad = \{s_1 \quad, s_2 \quad, s_3 \quad, \ ... \ , s_n \quad\}$ a sample space

$X(t) = \{x_1(t), x_2(t), x_3(t), \ ... \ , x_n(t)\}$ a random process

Random variables with time

a **random process** $X(t,s)$ represents a single time function
when t is a variable and s is <u>fixed</u> at an outcome

a random process $X(t,s)$ represents a single **random variable**
when both t and s are <u>fixed</u> at a time and an outcome, respectively

$$X_i = X(t_i, s) = X(t_i) \qquad \textbf{\textit{random variable}}$$

$$X(t,s) = X(t) \qquad \textbf{\textit{random process}}$$

An alphabet

the **alphabet** of $X(t)$

the set of its possible values

- the values of **time** t for which a random process is defined
- the **alphabet** of the random variable $X = X(t)$ at time t

Classification of Random Processes
(1) Types of time and alphabet

- the values of **time** t for which a random process is defined
  - continuous time
  - discrete time
- the **alphabet** of the random variable $X = X(t)$ at time t
  - continuous alphabet
  - discrete alphabet

Classification of Random Processes
(2) types of the random variable $X(t)$ and the time t

- a continuous **alphabet** continuous **time** random process
  - $X(t)$ has continuous values and t has continuous values
- a discrete **alphabet** continuous **time** random process
  - $X(t)$ has discrete values and t has continuous values
- a continuous **alphabet** discrete **time** random process
  - $X(t)$ has continuous values and t has discrete values
- a discrete **alphabet** discrete **time** random process
  - $X(t)$ has discrete values and t has discrete values

Deterministic and Non-deterministic Processes

- A process is **non-deterministic**
  if future values of any sample function
  cannot be predicted exactly from observed past values

- A process is **deterministic**
  if future values of any sample function
  can be predicted from observed past values

Deterministic Random Process Example

$$X(t) = A\cos(\omega_0 + \Theta)$$

A, Θ, or ω_0 (or all) can be random variables.

Any one <u>sample function</u> corresponds to the above equation
with particular values of these random variables.

Therefore the knowledge of the <u>sample function</u>
prior to any time instance fully allows
the prediction of the <u>sample function</u>'s future values
because all the necessary information is known