Instructors with only modest wiktext skills should copy/paste this to another Wikversity page. It is easy to add material to this because it uses transclusions. Note that all wikilinks are permalinks and therefore immune to vandalism and capricious editing. Material customized for WSU Lake in green

Introduction

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1. The circumference of a circle is \( C_\odot = 2\pi r \) and the circle's area is \( A_\odot = \pi r^2 \) is its area.

2. The surface area of a sphere is \( A_\odot = 4\pi r^2 \) and sphere's volume is \( V_\odot = \frac{4}{3}\pi r^3 \)

3. 1 kilometer = .621 miles and 1 MPH = 1 mi/hr = .447 m/s

4. Typical air density is 1.2kg/m\(^3\), with pressure 10\(^5\)Pa. The density of water is 1000kg/m\(^3\).

5. Earth's mean radius ≈ 6371km, mass ≈ 6 × 10\(^{24}\) kg

6. Universal gravitational constant = \( G = 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \)

7. Speed of sound = 340m/s and the speed of light = \( c \approx 3\times10^8 \text{m/s} \)

8. One light-year ≈ 9.5×10\(^{15}\)m ≈ 63240AU (Astronomical unit) ... <These 8 equations were added for WSU-L exams>

Units_and_Measurement

The base SI units are mass: kg (kilogram); length: m (meter); time: s (second). Percent error (http://wiki.ubc.ca/index.php?title=Uncertainty_and_Error&oldid=81540) is \( (\delta A/A) \times 100\% \)
Vectors

Vector \( \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \) involves components \((A_x,A_y,A_z)\) and three orthonormal unit vectors.

- If \( \vec{A} + \vec{B} = \vec{C} \), then \( A_x + B_x = C_x \), etc, and vector subtraction is defined by \( \vec{B} = \vec{C} - \vec{A} \).
- The two-dimensional displacement from the origin is \( \vec{r} = x \hat{i} + y \hat{j} \). The magnitude is \( A = |A| = \sqrt{A_x^2 + A_y^2} \). The angle (phase) is \( \theta = \tan^{-1} (y/x) \).
- Scalar multiplication \( \alpha \vec{A} = \alpha A_x \hat{i} + \alpha A_y \hat{j} + \ldots \)
- Any vector divided by its magnitude is a unit vector and has unit magnitude: \( |\vec{V}| = 1 \)
- Dot product \( \vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + \ldots \) and \( \vec{A} \cdot \vec{A} = A^2 \)
- Cross product \( \vec{A} = \vec{B} \times \vec{C} \Rightarrow A_\alpha = B_\beta C_\gamma - C_\beta B_\gamma \) where \((\alpha, \beta, \gamma)\) is any cyclic permutation of \((x, y, z)\), i.e., \((\alpha, \beta, \gamma)\) represents either \((x, y, z)\) or \((y, z, x)\) or \((z, x, y)\).
- Cross-product magnitudes obey \( A = BC \sin \theta \) where \( \theta \) is the angle between \( \vec{B} \) and \( \vec{C} \), and \( \vec{A} \perp \{ \vec{B}, \vec{C} \} \) by the right hand rule.
- Vector identities \( c(\vec{A} + \vec{B}) = c\vec{A} + c\vec{B} \)
- \( \vec{A} + \vec{B} = \vec{B} + \vec{A} \)
- \( \vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C} \)
- \( \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \)
- \( \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \)
- \( (\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C} \)
- \( (\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + \vec{B} \times \vec{C} \)
- \( \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = (\vec{A} \times \vec{B}) \cdot \vec{C} \)
- \( \vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C} \)
- \( (\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{D}) - (\vec{B} \cdot \vec{C}) (\vec{A} \cdot \vec{D}) \)

Motion_Along_a_Straight_Line

Delta as difference \( \Delta x = x_f - x_i \to dx \to 0 \) in limit of differential calculus.
- Average velocity \( \bar{v} = \Delta x / \Delta t \to v = dx / dt \) (instantaneous velocity)
- Acceleration \( \bar{a} = \Delta v / \Delta t \to a = dv / dt \).
WLOG set $\Delta t = t$ and $\Delta x = x - x_0$ if $t_i = 0$. Then $\Delta v = v - v_0$, and $v(t) = \int_0^t a(t') dt' + v_0$.

$$x(t) = \int_0^t v(t') dt' + x_0 = x_0 + \bar{v}t,$$
where $\bar{v} = \frac{1}{t} \int_0^t v(t')dt'$ is the average velocity.

At constant acceleration: $\bar{v} = \frac{v_0 + v}{2}$, $v = v_0 + at$, $x = x_0 + v_0t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\Delta x$.

For free fall, replace $x \rightarrow y$ (positive up) and $a \rightarrow -g$, where $g = 9.81 \, \text{m/s}^2$ at Earth’s surface.

Motion in Two and Three Dimensions

Instantaneous velocity: $\bar{v}(t) = v_x(t) \hat{i} + v_y(t) \hat{j} + v_z(t) \hat{k} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$

$$\bar{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t},$$
where $\vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} + z(t) \hat{k}$

Acceleration $\bar{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$, where $a_x(t) = \frac{dv_x}{dt} = a_t^2 x / dt^2$.

Average values: $\bar{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$, and $\bar{a}_{ave} = \frac{\Delta \bar{v}}{\Delta t} = \frac{\bar{v}(t_2) - \bar{v}(t_1)}{t_2 - t_1}$

Free fall time of flight $T_{of} = \frac{2(v_0 \sin \theta_0)}{g}$,

Trajectory $y = (\tan \theta_0) x - \left[ \frac{g}{2(v_0 \cos \theta_0)^2} \right] x^2$,

Range $R = \frac{v_0^2 \sin 2\theta_0}{g}$

Uniform circular motion: position $\vec{r}(t)$, velocity $\bar{v}(t) = d\vec{r}(t)/dt$, and acceleration $\bar{a}(t) = d\bar{v}(t)/dt$:

$$\vec{r} = A \cos \omega t \hat{i} + A \sin \omega t \hat{j}, \quad \bar{v} = -A \omega \sin \omega t \hat{i} + A \omega \cos \omega t \hat{j}, \quad \bar{a} = -A \omega^2 \cos \omega t \hat{i} - A \omega^2 \sin \omega t \hat{j}.$$ Note that if $A = r$ then $|\bar{a}| = a_C = \omega^2 r = v^2 / r$ where $v \equiv ||\bar{v}|| = \omega r$.

Tangential and centripetal acceleration $\bar{a} = \bar{a}_c + \bar{a}_T$ where $a_T = d|\bar{v}|/dt$.

Relative motion: $\bar{r}_{PS} = \bar{r}_{PS'} + \hat{s}' \hat{s}$, $\bar{v}_{PS} = \bar{v}_{PS'} + \bar{v}_{s'} \hat{s}$, $\bar{a}_{PC} = \bar{v}_{PA} + \bar{v}_{AB} + \bar{v}_{BC}$, $\bar{a}_{PS} = \bar{a}_{PS'} + \bar{a}_{s' s}$

This is a transclusion, added two days before Test 1

$$x = x_0 + v_{0x} \Delta t + \frac{1}{2} a_x \Delta t^2 \quad v_x = v_{0x} + a_x \Delta t \quad v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$y = y_0 + v_{0y} \Delta t + \frac{1}{2} a_y \Delta t^2 \quad v_y = v_{0y} + a_y \Delta t \quad v_y^2 = v_{y0}^2 + 2a_y \Delta y$$

$v^2 = v_{0}^2 + 2a_x \Delta x + 2a_y \Delta y$ ... in advanced notation this becomes $\Delta(v^2) = 2\bar{a} \cdot \Delta \bar{z}$.

In free fall we often set, $a_x = 0$ and $a_y = -g$. If angle is measured with respect to the x axis:

$$v_x = v \cos \theta \quad v_y = v \sin \theta \quad v_{x0} = v_0 \cos \theta_0 \quad v_{y0} = v_0 \sin \theta_0$$
Newton's Laws of Motion

Newton's 2nd Law \( \mathbf{m} \ddot{\mathbf{a}} = \sum \mathbf{F}_j \), where \( \mathbf{p} = \mathbf{m} \dot{\mathbf{v}} \) is momentum, \( \mathbf{m} \) is mass, and \( \sum \mathbf{F}_j \) is the sum of all forces. This sum needs only include external forces because all internal forces cancel by the 3rd law \( \mathbf{F}_{AB} = -\mathbf{F}_{BA} \). The 1st law is that velocity is constant if the net force is zero.

- Weight \( \mathbf{w} = \mathbf{m} \mathbf{g} \).
- Normal force is a component of the contact force by the surface. If the only forces are contact and weight, \( |\mathbf{N}| = \mathbf{N} = mg \cos \theta \) where \( \theta \) is the angle of incline.
- Hooke's law \( \mathbf{F} = -k \mathbf{x} \) where \( k \) is the spring constant.

Applications of Newton's Laws

\( f_s \leq \mu_s N \) and \( f_k = \mu_k N \): \( f \) = friction, \( \mu_{s,k} \) = coefficient of (static, kinetic) friction, \( N \) = normal force.
- Centripetal force \( \mathbf{F}_c = \mathbf{mv}^2/r = \mathbf{mr}\omega^2 \) for uniform circular motion. Angular velocity \( \omega \) is measured in radians per second.
- Ideal angle of banked curve: \( \tan \theta = v^2/(rg) \) for curve of radius \( r \) banked at angle \( \theta \).
- Drag equation \( \mathbf{F}_D = \frac{1}{2} \rho C v^2 A \) where \( C \) = Drag coefficient, \( \rho \) = mass density, \( A \) = area, \( v \) = speed. Holds approximately for large Reynold's number \( \mathbf{Re} = \rho v L / \eta \), where \( \eta \) = dynamic viscosity; \( L \) = characteristic length.
- Stokes's law models a sphere of radius \( r \) at small Reynold's number: \( \mathbf{F}_s = 6\pi r \eta v \).

The x and y components of the three forces of tension on the small grey circle where the three "massless" ropes meet are:

\[
\begin{align*}
T_{1x} &= -T_1 \cos \theta_1, \\
T_{1y} &= T_1 \sin \theta_1, \\
T_{2x} &= 0, \\
T_{2y} &= -mg, \\
T_{3x} &= T_3 \cos \theta_3, \\
T_{3y} &= T_3 \sin \theta_3.
\end{align*}
\]

Also transcluded from Physics equations:

- \( 2\pi \text{ rad} = 360 \text{ deg} = 1 \text{ rev} \) relates the radian, degree, and revolution.
- \( f = \frac{\# \text{ revs}}{\# \text{ secs}} \) is the number of revolutions per second, called frequency.
- \( T = \frac{\# \text{ secs}}{\# \text{ revs}} \) is the number of seconds per revolution, called period. Obviously \( fT = 1 \).
- \( \omega = \frac{\Delta \theta}{\Delta t} \) is called angular frequency (\( \omega \) is called omega, and \( \theta \) is measured in radians). Obviously \( \omega T = 2\pi \).
- \( a = \frac{v^2}{r} = \omega v = \omega^2 r \) is the acceleration of uniform circular motion, where \( v \) is speed, and \( r \) is radius.
- \( v = \omega r = 2\pi r/T \), where \( T \) is period.