Laurent Series and z-Transform
- Geometric Series
Applications B

20210405 Mon

$\cdots \quad a_{-3} \quad a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots$

$\cdots \quad z^{-3} \quad z^{-2} \quad z^{-1} \quad z^0 \quad z^1 \quad z^2 \quad z^3 \quad \cdots$

causal Laurent $f(z)$

$a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots$

anti-causal Laurent $g(z^{-1})$

$\cdots + a_{-3} z^{-3} + a_{-2} z^{-2} + a_{-1} z^{-1}$

$\cdots \quad a_{-3} \quad a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad \cdots$

$\cdots \quad z^{-3} \quad z^{-2} \quad z^{-1} \quad z^0 \quad z^{-1} \quad z^{-2} \quad z^{-3} \quad \cdots$

causal z-transform $Y(z)$

$a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots$

anti-causal z-transform $X(z^{-1})$

$\cdots + a_{-3} z^3 + a_{-2} z^2 + a_{-1} z^1$

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A unit starting Geometric Series unshifted

Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

(1) $\boxed{+\frac{1}{1-\boxed{az}}}$ $\boxed{|z|<a^{-1}}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

a^n u(n)	$(n \geq 0)$

(2) $\boxed{+\frac{1}{1-\boxed{a^{-1}z}}}$ $\boxed{|z|<a}$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + \cdots)$

$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(3) $\boxed{-\frac{1}{1-\boxed{a^{-1}z^{-1}}}}$ $\boxed{|z|>a^{-1}}$

$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$-a^n$ u(-n)	$(n<1)$

(4) $\boxed{-\frac{1}{1-\boxed{az^{-1}}}}$ $\boxed{|z|>a}$

$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + \cdots)$

$-(\frac{1}{a})^n$ u(-n)	$(n<1)$

Geometric Series - a unit start term

z-Transform (n → -n)

(1) $+\dfrac{1}{1-\boxed{az}}$ $\quad\boxed{|z|<a^{-1}}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

a^{-n} u((-n))	$(-n \geq 0)$
$(\frac{1}{a})^{n}$ u(-n)	$(n < 1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\quad\boxed{|z|<a}$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{1} z^1 + (\frac{1}{a})^{2} z^2 + \cdots)$

$(\frac{1}{a})^{-n}$ u((-n))	$(-n \geq 0)$
a^{n} u(-n)	$(n < 1)$

(3) $-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad\boxed{|z|>a^{-1}}$

$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{1} z^{-1} + (\frac{1}{a})^{2} z^{-2} + \cdots)$

$-a^{-n}$ u(-(-n))	$(-n < 1)$
$-(\frac{1}{a})^{n}$ u(n)	$(n \geq 0)$

(4) $-\dfrac{1}{1-\boxed{az^{-1}}}$ $\quad\boxed{|z|>a}$

$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$-(\frac{1}{a})^{-n}$ u(-(-n))	$(-n < 1)$
$-a^{n}$ u(n)	$(n \geq 0)$

Geometric Series

Laurent Series vs. z-Transform $(n \to -n)$

(1) $+\dfrac{1}{1-\boxed{az}}$ $\quad \boxed{|z|<a^{-1}}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^1 + (\frac{1}{a})^{-2}z^2 + \cdots)$

Laurent	$a^n \; u(n)$	$(n \geq 0)$
z-Trans	$(\frac{1}{a})^n \; u(-n)$	$(n<1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\quad \boxed{|z|<a}$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + \cdots)$

Laurent	$(\frac{1}{a})^n \; u(n)$	$(n \geq 0)$
z-Trans	$a^n \; u(-n)$	$(n<1)$

(3) $-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\quad \boxed{|z|>a^{-1}}$

$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + \cdots)$

Laurent	$-a^n \; u(-n)$	$(n<1)$
z-Trans	$-(\frac{1}{a})^n \; u(n)$	$(n \geq 0)$

(4) $-\dfrac{1}{1-\boxed{az^{-1}}}$ $\quad \boxed{|z|>a}$

$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + \cdots)$

Laurent	$-(\frac{1}{a})^n \; u(-n)$	$(n<1)$
z-Trans	$-a^n \; u(n)$	$(n \geq 0)$

Laurent Series

z-Transform

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

(5) $-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z| > a^{-1}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-a^{n}\ u(-n-1)$ $\quad (n < 0)$

(6) $-\frac{az^{-1}}{1-az^{-1}}$ $\quad |z| > a$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

$-(\frac{1}{a})^{n}\ u(-n-1)$ $\quad (n < 0)$

(7) $+\frac{az}{1-az}$ $\quad |z| < a^{-1}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$a^{n}\ u(n-1)$ $\quad (n \geq 1)$

(8) $+\frac{a^{-1}z}{1-a^{-1}z}$ $\quad |z| < a$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

$(\frac{1}{a})^{n}\ u(n-1)$ $\quad (n \geq 1)$

Geometric Series - a non-unit start term

z-Transform (n → -n)

(5) $\boxed{-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}}$ $\boxed{|z|>a^{-1}}$

$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^{1}z^{-1} + \left(\frac{1}{a}\right)^{2}z^{-2} + \left(\frac{1}{a}\right)^{3}z^{-3} + \cdots\right)$

$-a^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-\left(\frac{1}{a}\right)^{n}$ u(n-1)	$(n \geq 1)$

(6) $\boxed{-\dfrac{az^{-1}}{1-az^{-1}}}$ $\boxed{|z|>a}$

$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^{-1}z^{-1} + \left(\frac{1}{a}\right)^{-2}z^{-2} + \left(\frac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$

$-\left(\frac{1}{a}\right)^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-a^{n}$ u(n-1)	$(n \geq 1)$

(7) $\boxed{+\dfrac{az}{1-az}}$ $\boxed{|z|<a^{-1}}$

$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^{-1}z^{1} + \left(\frac{1}{a}\right)^{-2}z^{2} + \left(\frac{1}{a}\right)^{-3}z^{3} + \cdots\right)$

a^{-n} u((-n)-1)	$(-n \geq 1)$
$\left(\frac{1}{a}\right)^{n}$ u(-n-1)	$(n < 0)$

(8) $\boxed{+\dfrac{a^{-1}z}{1-a^{-1}z}}$ $\boxed{|z|<a}$

$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^{1}z^{1} + \left(\frac{1}{a}\right)^{2}z^{2} + \left(\frac{1}{a}\right)^{3}z^{3} + \cdots\right)$

$\left(\frac{1}{a}\right)^{-n}$ u((-n)-1)	$(-n \geq 1)$
a^{n} u(-n-1)	$(n < 0)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform (n → -n)

(5) $-\dfrac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}$ $\quad |z| > a^{-1}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-((\tfrac{1}{a})^{1}z^{-1} + (\tfrac{1}{a})^{2}z^{-2} + (\tfrac{1}{a})^{3}z^{-3} + \cdots)$

Laurent	$-a^{n}$ u(-n-1)	$(n < 0)$
z-Trans	$-(\frac{1}{a})^{n}$ u(n-1)	$(n \geq 1)$

(6) $-\dfrac{\boxed{a z^{-1}}}{1-\boxed{a z^{-1}}}$ $\quad |z| > a$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\tfrac{1}{a})^{-1}z^{-1} + (\tfrac{1}{a})^{-2}z^{-2} + (\tfrac{1}{a})^{-3}z^{-3} + \cdots)$

Laurent	$-(\frac{1}{a})^{n}$ u(-n-1)	$(n < 0)$
z-Trans	$-a^{n}$ u(n-1)	$(n \geq 1)$

(7) $+\dfrac{\boxed{a z}}{1-\boxed{a z}}$ $\quad |z| < a^{-1}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$((\tfrac{1}{a})^{-1}z^{1} + (\tfrac{1}{a})^{-2}z^{2} + (\tfrac{1}{a})^{-3}z^{3} + \cdots)$

Laurent	a^{n} u(n-1)	$(n \geq 1)$
z-Trans	$(\frac{1}{a})^{n}$ u(-n-1)	$(n < 0)$

(8) $+\dfrac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}$ $\quad |z| < a$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\tfrac{1}{a})^{1}z^{1} + (\tfrac{1}{a})^{2}z^{2} + (\tfrac{1}{a})^{3}z^{3} + \cdots)$

Laurent	$(\frac{1}{a})^{n}$ u(n-1)	$(n \geq 1)$
z-Trans	a^{n} u(-n-1)	$(n < 0)$

4 cases of geometric series **Simple Pole Form**

- 2 representations for each case

	using z				using $1/z$			
	simple pole p		simple pole $1/p$		simple pole $1/p$		simple pole p
(A)	$\dfrac{1}{z - p}$	(B)	$\dfrac{1}{z - p^{-1}}$	(C)	$\dfrac{1}{z^{-1} - p}$	(D)	$\dfrac{1}{z^{-1} - p^{-1}}$
/p	$-\dfrac{p^{-1}}{1 - p^{-1}z}$	*p	$-\dfrac{p}{1 - pz}$	/p	$-\dfrac{p^{-1}}{1 - p^{-1}z^{-1}}$	*p	$-\dfrac{p}{1 - pz^{-1}}$
/z	$\dfrac{z^{-1}}{1 - pz^{-1}}$	/z	$\dfrac{z^{-1}}{1 - p^{-1}z^{-1}}$	*z	$\dfrac{z}{1 - pz}$	*z	$\dfrac{z}{1 - p^{-1}z}$

p^{-1} (A, B) $\qquad$ p^{-1} (C, D)

z^{-1}

/p	$-p^{-n-1}$ u(n)	*p	$-p^{n+1}$ u(n)	/p	$-p^{n-1}$ u(-n)	*p	$-p^{-n+1}$ u(-n)
/z	p^{-n-1} u(-n-1)	/z	p^{n+1} u(-n-1)	*z	$-p^{n-1}$ u(n-1)	*z	$-p^{-n+1}$ u(n-1)

4 cases of geometric series **Simple Pole Form**

- 2 representations for each case

	using p				using $1/p$		
	simple pole p		simple pole $1/p$		simple pole $1/p$		simple pole p
(A)	$\dfrac{1}{z - p}$	(C)	$\dfrac{1}{z^{-1} - p}$	(B)	$\dfrac{1}{z - p^{-1}}$	(D)	$\dfrac{1}{z^{-1} - p^{-1}}$
/p	$-\dfrac{p^{-1}}{1 - p^{-1}z}$	/p	$-\dfrac{p^{-1}}{1 - p^{-1}z^{-1}}$	*p	$-\dfrac{p}{1 - pz}$	*p	$-\dfrac{p}{1 - pz^{-1}}$
/z	$\dfrac{z^{-1}}{1 - pz^{-1}}$	*z	$\dfrac{z}{1 - pz}$	/z	$\dfrac{z^{-1}}{1 - p^{-1}z^{-1}}$	*z	$\dfrac{z}{1 - p^{-1}z}$

z^{-1} (A)–(C) z^{-1} (B)–(D)

p^{-1} (C)–(B)

/p	$-p^{-n-1}$ u(n)	/p	$-p^{n-1}$ u(-n)	*p	$-p^{n+1}$ u(n)	*p	$-p^{-n+1}$ u(-n)
/z	p^{-n-1} u(-n-1)	*z	$-p^{n-1}$ u(n-1)	/z	p^{n+1} u(-n-1)	*z	$-p^{-n+1}$ u(n-1)

(A) $\boxed{\dfrac{1}{z - p}}$ $\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$ $\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$

p^{2} p^{1} p^{0} p^{-1} p^{-2} p^{-3} p^{-4}

z^{-3} z^{-2} z^{-1} z^{0} z^{1} z^{2} z^{3}

$$\cdots + a_{-3} z^{3} + a_{-2} z^{2} + a_{-1} z^{1} \quad p^{-1} z^{0} + p^{-2} z^{1} + p^{-3} z^{2} + p^{-4} z^{3} + \cdots$$

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$ $\boxed{-\dfrac{p}{1 - p z}}$ $\boxed{\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}}$

(C) $\boxed{\dfrac{1}{z^{-1} - p}}$ $\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$ $\boxed{\dfrac{z}{1 - p z}}$

(D) $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$ $\boxed{-\dfrac{p}{1 - p z^{-1}}}$ $\boxed{\dfrac{z}{1 - p^{-1} z}}$

when the pole is expressed as p

2 formulas **Simple Pole Form**

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each **Shifted Geometric Series Form**

(A) $\boxed{\dfrac{1}{z-p}}$

causal Laurent / anti- causal z-transform

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1})$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} \triangleq g(z^{-1}) = Y(z)$$

anti- causal Laurent / causal z-transform

(C) $\boxed{\dfrac{1}{z^{-1}-p}}$

anti- causal Laurent / causal z-transform

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq f(z^{-1}) = X(z)$$

$$\boxed{\frac{z}{1-pz}} \triangleq g(z) = Y(z^{-1})$$

causal Laurent / anti- causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

when the pole is expressed as 1/p

2 formulas — **Simple Pole Form**

$$\boxed{\frac{1}{z - p^{-1}}} \qquad \boxed{\frac{1}{z^{-1} - p^{-1}}}$$

2 representations each — **Shifted Geometric Series Form**

(B) $\boxed{\dfrac{1}{z - p^{-1}}}$

causal Laurent — anti-causal z-transform

$$\boxed{-\frac{p}{1 - pz}} \triangleq f(z) = X(z^{-1})$$

$$\| \qquad\qquad \|$$

$$\boxed{\frac{z^{-1}}{1 - p^{-1}z^{-1}}} \triangleq g(z^{-1}) = Y(z)$$

anti-causal Laurent — causal z-transform

(D) $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$

anti-causal Laurent — causal z-transform

$$\boxed{-\frac{p}{1 - pz^{-1}}} \triangleq f(z^{-1}) = X(z)$$

$$\| \qquad\qquad \|$$

$$\boxed{\frac{z}{1 - p^{-1}z}} \triangleq g(z) = Y(z^{-1})$$

causal Laurent — anti-causal z-transform

Simple Pole Form — **Shifted Geometric Series Form**

(A) $\dfrac{1}{z - p}$

$-\dfrac{p^{-1}}{1 - p^{-1} z}$

$-(p^{-1}z^{0} + p^{-2}z^{1} + p^{-3}z^{2} + \cdots)$ $f(z)$ causal Laurent

$-((\tfrac{1}{p})^{1}z^{0} + (\tfrac{1}{p})^{2}z^{1} + (\tfrac{1}{p})^{3}z^{2} + \cdots)$ $X(z^{-1})$ anti-causal z-transform

$\dfrac{z^{-1}}{1 - p z^{-1}}$

$(p^{0}z^{-1} + p^{1}z^{-2} + p^{2}z^{-3} + \cdots)$ $g(z^{-1})$ anti-causal Laurent

$((\tfrac{1}{p})^{0}z^{-1} + (\tfrac{1}{p})^{-1}z^{-2} + (\tfrac{1}{p})^{-2}z^{-3} + \cdots)$ $Y(z)$ causal z-transform

(C) $\dfrac{1}{z^{-1} - p}$

$-\dfrac{p}{1 - p z^{-1}}$

$-(p^{-1}z^{0} + p^{-2}z^{-1} + p^{-3}z^{-2} + \cdots)$ $f(z^{-1})$ anti-causal Laurent

$-((\tfrac{1}{p})^{1}z^{0} + (\tfrac{1}{p})^{2}z^{-1} + (\tfrac{1}{p})^{3}z^{-2} + \cdots)$ $X(z)$ causal z-transform

$\dfrac{z}{1 - p z}$

$(p^{0}z^{1} + p^{1}z^{2} + p^{2}z^{3} + \cdots)$ $g(z)$ causal Laurent

$((\tfrac{1}{p})^{0}z^{1} + (\tfrac{1}{p})^{-1}z^{2} + (\tfrac{1}{p})^{-2}z^{3} + \cdots)$ $Y(z^{-1})$ anti-causal z-transform

(B) $\dfrac{1}{z - p^{-1}}$

$-\dfrac{p}{1 - p z}$

$-(p^{1}z^{0} + p^{2}z^{1} + p^{3}z^{2} + \cdots)$ $f(z)$ causal Laurent

$-((\tfrac{1}{p})^{-1}z^{0} + (\tfrac{1}{p})^{-2}z^{1} + (\tfrac{1}{p})^{-3}z^{2} + \cdots)$ $X(z^{-1})$ anti-causal z-transform

$\dfrac{z^{-1}}{1 - p^{-1} z^{-1}}$

$(p^{0}z^{-1} + p^{-1}z^{-2} + p^{-2}z^{-3} + \cdots)$ $g(z^{-1})$ anti-causal Laurent

$((\tfrac{1}{p})^{0}z^{-1} + (\tfrac{1}{p})^{1}z^{-2} + (\tfrac{1}{p})^{2}z^{-3} + \cdots)$ $Y(z)$ causal z-transform

(D) $\dfrac{1}{z^{-1} - p^{-1}}$

$-\dfrac{p}{1 - p z^{-1}}$

$-(p^{1}z^{0} + p^{2}z^{-1} + p^{3}z^{-2} + \cdots)$ $f(z^{-1})$ anti-causal Laurent

$-((\tfrac{1}{p})^{-1}z^{0} + (\tfrac{1}{p})^{-2}z^{-1} + (\tfrac{1}{p})^{-3}z^{-2} + \cdots)$ $X(z)$ causal z-transform

$\dfrac{z}{1 - p^{-1} z}$

$(p^{0}z^{1} + p^{-1}z^{2} + p^{-2}z^{3} + \cdots)$ $g(z)$ causal Laurent

$((\tfrac{1}{p})^{0}z^{1} + (\tfrac{1}{p})^{1}z^{2} + (\tfrac{1}{p})^{2}z^{3} + \cdots)$ $Y(z^{-1})$ anti-causal z-transform

Shifted Geometric Series (1) p

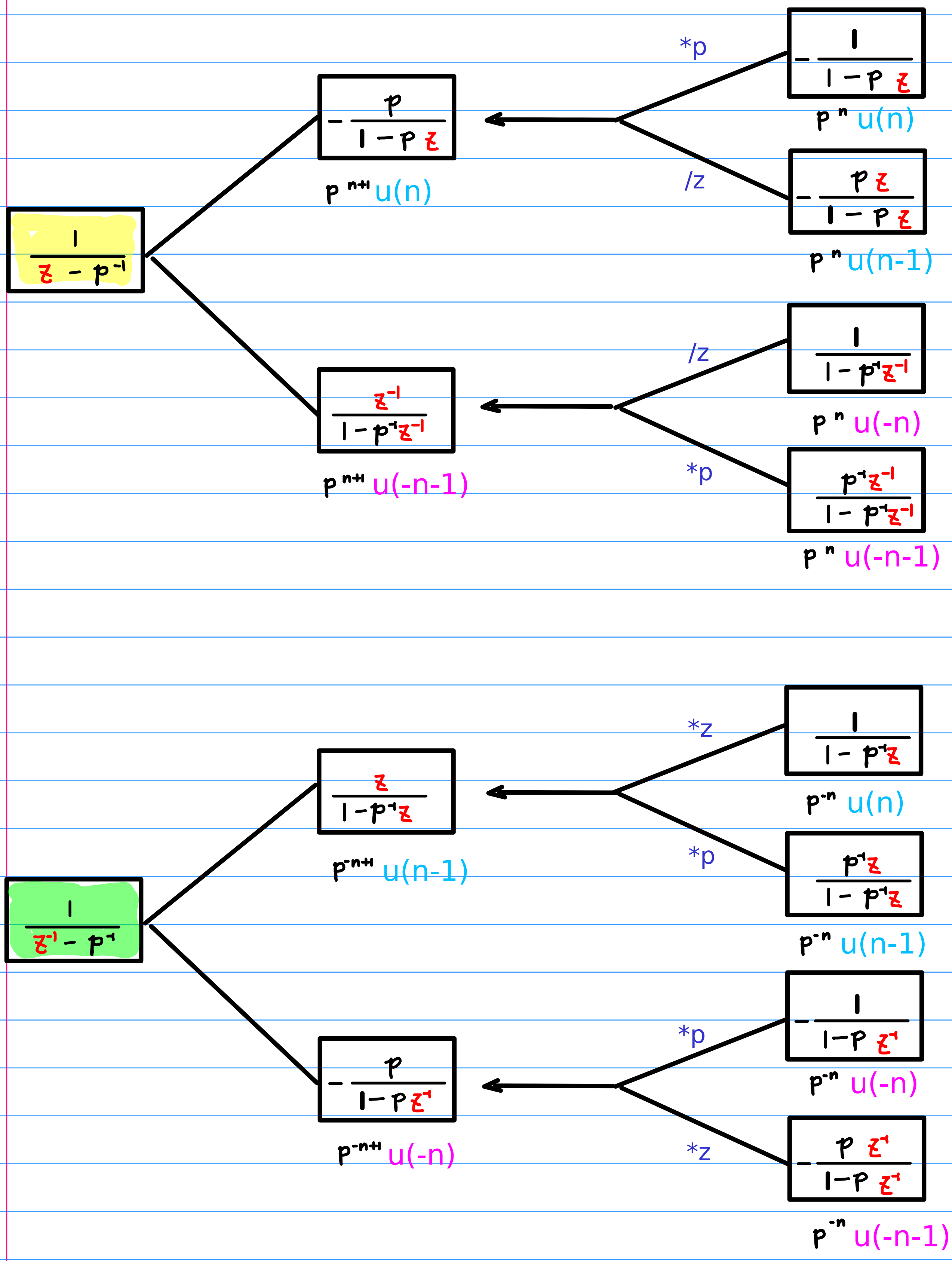
$$\boxed{\frac{1}{z-p}}$$

- $-\dfrac{p^{-1}}{1-p^{-1}z}$ $\quad p^{-n-1}u(n)$
  - /p: $-\dfrac{1}{1-p^{-1}z}$ **unshifted** $\quad p^{-n}u(n)$
  - /z: $-\dfrac{p^{-1}z}{1-p^{-1}z}$ **complementary** $\quad p^{-n}u(n-1)$
- $\dfrac{z^{-1}}{1-pz^{-1}}$ $\quad p^{-n-1}u(-n-1)$
  - /z: $\dfrac{1}{1-pz^{-1}}$ **unshifted** $\quad p^{-n}u(-n)$
  - /p: $\dfrac{pz^{-1}}{1-pz^{-1}}$ **complementary** $\quad p^{-n}u(-n-1)$

$$\boxed{\frac{1}{z^{-1}-p}}$$

- $-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$ $\quad p^{n-1}u(-n)$
  - /p: $-\dfrac{1}{1-p^{-1}z^{-1}}$ **unshifted** $\quad p^{n}u(-n)$
  - *z: $-\dfrac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}$ **complementary** $\quad p^{n}u(-n-1)$
- $\dfrac{z}{1-pz}$ $\quad p^{n-1}u(n-1)$
  - *z: $\dfrac{1}{1-pz}$ **unshifted** $\quad p^{n}u(n)$
  - /p: $\dfrac{pz}{1-pz}$ **complementary** $\quad p^{n}u(n-1)$

Shifted Geometric Series (2) 1/p



2 formulas   Simple Pole Form

$\boxed{\dfrac{1}{z-p}}$ $\boxed{\dfrac{1}{z^{-1}-p}}$

2 representations each   Geometric Series Form

causal   anti- causal

$$\boxed{\dfrac{1}{z-p}} \to \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{\dfrac{1}{z-p}} \to \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \triangleq Y(z) = g(z^{-1})$$

causal   anti- causal

causal   anti- causal

$$\boxed{\dfrac{1}{z^{-1}-p}} \to \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq X(z) = f(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{\dfrac{1}{z^{-1}-p}} \to \boxed{\dfrac{z}{1-pz}} \triangleq g(z) = Y(z^{-1})$$

causal   anti- causal

Simple Pole Form | Geometric Series Form

Simple Pole Form

1

$-\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$-a^{n}$	$(n \geq 0)$

2

$-\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-\left(\frac{1}{a}\right)^{n}$	$(n < 1)$

3

$\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^{n}	$(n < 1)$

4

$\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

5

$-\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$-\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

6

$-\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-a^{n}$	$(n < 1)$

7

$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^{n}$	$(n < 1)$

8

$\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^{n}	$(n \geq 0)$

Geometric Series : $f(z)$, $g(z)$, $\bar{f}(z)$, $\bar{g}(z)$

$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n+1}$	$(n \geq 0)$

$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n-1}$	$(n \geq 1)$

Geometric Series :

$f(z) = g(z^{-1})$ the same algebraic formula but the complement ROC's

$g(z) = f(z^{-1})$ the same algebraic formula but the complement ROC's

$f(z) = f_a(z)$ two variable function

$g(z) = g_a(z)$ two variable function

$\bar{f}(z) = f_{a^{-1}}(z)$ inverse a

$\bar{g}(z) = g_{a^{-1}}(z)$ inverse a

associated simple pole forms

$a^{-1} f(z),\ z^{-1} g(z),\ a\,\bar{f}(z),\ z^{-1}\bar{g}(z)$

1

$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^n$	$(n \geq 0)$

2

$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$

3

$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

4

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

5

$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

6

$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

7

$z\,\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

8

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$
$\bar{f}(z)$	$\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$	$\bar{g}(z)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

neg(n) Sym(n)

inv(z) inv(z)

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

(1)	(2)	(1)	(2)
(3)	(4)	(3)	(4)
(5)	(6)	(5)	(6)
(7)	(8)	(7)	(8)

$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$
$\bar{f}(z)$	$\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$	$\bar{g}(z)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{a}{1-az}$	$\lvert z \rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\frac{a}{1-az^{-1}}$	$\lvert z \rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$\bar{f}(z) = -\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z \rvert < a$
$a_n = -(\frac{1}{a})^{n+1}$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$\bar{f}(z^{-1}) = -\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

③

$a_n = a^{n+1}$	$(n < 0)$
$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z \rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$
$g(z) = \frac{z}{1-a^{-1}z}$	$\lvert z \rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$\bar{g}(z^{-1}) = \frac{z^{-1}}{1-az^{-1}}$	$\lvert z \rvert > a$
$a_n = (\frac{1}{a})^{n+1}$	$(n < 0)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$\bar{g}(z) = \frac{z}{1-az}$	$\lvert z \rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$ $\quad a^{-1}f(z^{-1})$

$zg(z^{-1})$ $\quad z^{-1}g(z)$

$a\bar{f}(z)$ $\quad a\bar{f}(z^{-1})$

$z\bar{g}(z^{-1})$ $\quad z^{-1}\bar{g}(z)$

a unit nominator

①

$a_n = -a^n$	$(n \geq 0)$
$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$
$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

③

$zg(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

④

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑤

$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$
$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

neg(n) sym(n)

inv(z) inv(z)

⑥

$a_n = -a^n$	$(n < 1)$
$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

⑦

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$ $\quad a^{-1}f(z^{-1})$

$zg(z^{-1})$ $\quad z^{-1}g(z)$

$a\bar{f}(z)$ $\quad a\bar{f}(z^{-1})$

$z\bar{g}(z^{-1})$ $\quad z^{-1}\bar{g}(z)$

a unit nominator

①

$a_n = -a^n$	$(n \geq 0)$
$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$
$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

③

$a_n = a^n$	$(n < 1)$
$zg(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$
$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

Simple Pole Forms | Geometric Series Forms

	Simple Pole Forms				Geometric Series Forms	
①	$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot\, a^{-1}$	id	$a_n = -a^{n}$	$(n \geq 0)$
	$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot\, a^{-1}$	id	$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
②	$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$	$\cdot\, a^{-1}$	id	$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n < 1)$
	$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot\, a^{-1}$	id	$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
③	$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot\, z$	id	$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
	$a_n = a^{n+1}$	$(n < 0)$	$\cdot\, a^{-1}$	S(c(n))	$a_n = a^{n}$	$(n < 1)$
④	$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot\, z^{-1}$	id	$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
	$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$	$\cdot\, a^{-1}$	S(c(n))	$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$
⑤	$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$	$\cdot\, a$	id	$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$
	$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot\, a$	id	$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
⑥	$a_n = -a^{n-1}$	$(n < 1)$	$\cdot\, a$	id	$a_n = -a^{n}$	$(n < 1)$
	$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot\, a$	id	$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
⑦	$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot\, z$	id	$z\,\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
	$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$	$\cdot\, a$	s(c(n))	$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 1)$
⑧	$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot\, z^{-1}$	id	$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
	$a_n = a^{n-1}$	$(n \geq 1)$	$\cdot\, a$	s(c(n))	$a_n = a^{n}$	$(n \geq 0)$

Simple Pole Forms				Geometric Series Forms	

1

$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot\, a$	id	$a_n = -a^{n}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot\, a$	id	$a^{-1} f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

$-a\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$-\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

2

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$	$\cdot\, a$	id	$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot\, a$	id	$a^{-1} f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

$-a\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-a\,\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

$-\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-\,\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

3

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot\, z^{-1}$	id	$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$	$\cdot\, a$	S(c(n))	$a_n = a^{n}$	$(n < 1)$

$-\,z^{-1}(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-\,(a^0 z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + \cdots)$

$-\,(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

4

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot\, z$	id	$z^{-1} g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$	$\cdot\, a$	S(c(n))	$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$-\,z\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,\left(\left(\tfrac{1}{a}\right)^0 z^1 + \left(\tfrac{1}{a}\right)^1 z^2 + \left(\tfrac{1}{a}\right)^2 z^3 + \cdots\right)$

$-\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$

Simple Pole Forms | Geometric Series Forms

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

$\cdot\, a^{-1}$ id

$\cdot\, a^{-1}$ id

$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$
$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

$-\left(\frac{1}{a}\right)\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^1 z^0 + \left(\frac{1}{a}\right)^2 z^1 + \left(\frac{1}{a}\right)^3 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

$\cdot\, a^{-1}$ id

$\cdot\, a^{-1}$ id

$a_n = -a^{n}$	$(n < 1)$
$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

$-a^{-1}\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

$-\left(a^{-1} z^0 + a^{-2} z^{-1} + a^{-3} z^{-2} + \cdots\right)$

$-\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\cdot\, z^{-1}$ id

$\cdot\, a^{-1}$ s(c(n))

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 1)$

$z^{-1}\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^{-1} + \left(\frac{1}{a}\right)^{-1} z^{-2} + \left(\frac{1}{a}\right)^{-2} z^{-3} + \cdots\right)$

$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

$\cdot\, z$ id

$\cdot\, a^{-1}$ s(c(n))

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n}$	$(n \geq 0)$

$z\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$\left(a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots\right)$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

neg(n) sym(n)

inv(z) inv(z)

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

neg(n) sym(n)

inv(z) inv(z)

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\cdot z$ id

$\cdot a$ s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\cdot z$ id

$\cdot a$ s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n<0)$

$\cdot\, z^{-1}$ id

$\cdot\, a$ s(c(n))

$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n}$	$(n<1)$

$$X(z) = \frac{1}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$a_n = a^n \qquad (n<1)$$

$$+\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right) \qquad n = 0, -1, -2, \cdots$$

$\bullet\, z^{-1}$ $\quad a^0 \quad a^{-1} \quad a^{-2} \quad \cdots$ $\quad \bullet\, a$

$$+\left(a^0 z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + \cdots\right) \qquad n = -1, -2, -3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$a_{n-1} = a^{n+1} \qquad (n<0)$$

$$\boxed{g(z) = \frac{z}{1-a^{-1}z}} \quad \boxed{|z|<a}$$
$$\boxed{a_n = \left(\frac{1}{a}\right)^{n-1}} \quad \boxed{(n \geq 1)}$$

$\cdot z$ id

$\cdot a^{-1}$ s(c(n))

$$\boxed{z^{-1}g(z) = \frac{1}{1-a^{-1}z}} \quad \boxed{|z|<a}$$
$$\boxed{a_n = \left(\frac{1}{a}\right)^{n}} \quad \boxed{(n \geq 0)}$$

$$X(z) = \frac{1}{1-a^{-1}z} \qquad |z|<a$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n \geq 0)$$

$$-\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right) \qquad n = 0, 1, 2, \cdots$$

$\bullet z$ $\qquad a^0 \quad a^{-1} \quad a^{-2} \quad \cdots \qquad \bullet a^{-1}$

$$-\left(a^0 z^1 + a^{-1} z^2 + a^{-2} z^3 + \cdots\right) \qquad n = 1, 2, 3, \cdots$$

$$zX(z) = \frac{z}{1-a^{-1}z} \qquad |z|<a$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n-1} \qquad (n \geq 1)$$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n<0)$

$\cdot z$ id

$\cdot a^{-1}$ s(c(n))

$z\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n<1)$

$$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > a$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n<1)$$

$\bullet z$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right) \qquad n = 0, -1, -2, \cdots$$

$$a^0 \quad a^1 \quad a^2 \quad \cdots$$

$\bullet a^{-1}$

$$-\left(a^0 z^{-1} + a^1 z^{-2} + a^2 z^{-3} + \cdots\right) \qquad n = -1, -2, -3, \cdots$$

$$z\,X(z) = \frac{z^{-1}}{1-az^{-1}} \qquad \lvert z\rvert > a$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n+1} \qquad (n<0)$$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

$\cdot\, z^{-1}$ id

$\cdot\, a^{-1}$ s(c(n))

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n}$	$(n \geq 0)$

$$X(z) = \frac{1}{1-az} \qquad \lvert z\rvert < a^{-1}$$

$$a_n = a^n \qquad (n \geq 0)$$

$\bullet\, z^{-1}$

$+\ (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} \quad)$ $\quad n = 0, 1, 2, \cdots$

$a^0 \quad a^1 \quad a^2 \quad \cdots$ $\quad \bullet\, a^{-1}$

$+\ (a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots)$ $\quad n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z}{1-az} \qquad \lvert z\rvert < a^{-1}$$

$$a_{n-1} = a^{n-1} \qquad (n \geq 1)$$

①

$a_n = -2^n$	$(n \geq 0)$
$0.5 f(z) = -\frac{1}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{2})^n$	$(n < 1)$
$0.5 f(z^{-1}) = -\frac{1}{1-2z^{-1}}$	$\lvert z\rvert > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

③

$z g(z^{-1}) = \frac{1}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

④

$z^{-1} g(z) = \frac{1}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = (\frac{1}{2})^n$	$(n \geq 0)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{2}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{2}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\frac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \frac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \frac{z}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{2}{1-2z}$	$\lvert z\rvert < 0.5$

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\frac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$

neg(n) sym(n)

inv(z) inv(z)

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{2}{1-2z}$	$\lvert z\rvert < 0.5$

③

$g(z^{-1}) = \frac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) sym(n)

inv(z) inv(z)

$g(z) = \frac{z}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

- z id
- a s(c(n))

$z^{-1}g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

④

$g(z) = \frac{z}{1-2z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

- z id
- a s(c(n))

$z^{-1}g(z) = \frac{1}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n}$	$(n \geq 0)$

$$\frac{3}{2}\ \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$|z| < 0.5 \qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$$|z| < 0.5 \qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

$$\text{ROC} \qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$$

$$\text{ROC} \qquad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\updownarrow z^{-1} \qquad \updownarrow z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad \downarrow -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$

① $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$ $|z|<0.5$

② $-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$ $|z|>2$

③ $+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ $|z|>2$

④ $+\frac{z}{1-0.5z}-\frac{z}{1-2z}$ $|z|<0.5$

$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$|z|<0.5$

$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n<1)$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$$

$|z|>2$

z^{-1} $-n$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$|z|<0.5$

$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$|z|>2$

z^{-1} $-n$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$$

$|z|<0.5$

$\cdot z$ $n-1$

$$+\frac{1}{1-0.5z}-\frac{1}{1-2z}$$

$|z|<0.5$

$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n\geq 1)$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$$

$|z|<0.5$

$\cdot z$ $n-1$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$|z|<0.5$

$z^{-1}X(z)$ Shifted Sequence

$$|z|>1 \qquad\qquad |z|>2$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \qquad n \qquad n = 0, 1, 2, \cdots$$

• z^{-1}

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \qquad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

$$\cdot z \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$\bullet z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \qquad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \qquad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $X(z)$ $a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n <$

$|z| > 2$ $X(z)$ $b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a$ $X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ a^{n+1} $n \geq 0$ $n \geq 1$ $n < 0$ $n <$

$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $-n$

$|z| > a$ $X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ a^{-n+1} $n < 0$ $n < 1$ $n \geq 0$ n

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az}$ $\sum_{n=0}^{\infty} a^{n+1} z^n$ $\frac{z}{1-az}$ $\sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $-\sum_{n=0}^{-\infty} a^{n-1} z^n$

$z\,X(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \quad (|z|>2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$\cdot z^{-1}$

$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \quad)$ n $n = 0, 1, 2, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ $n-1$ $n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

z

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ n $n = 1, 2, 3, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1}

$(1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots)$ $-n$ $n = -1, -2, -3, \cdots$

$$z\,X(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n<0)$$

$a_{-(n+1)}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\| \quad \| \quad = \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\qquad \downarrow -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \quad z^{-1} \quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \quad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$

$\boxed{2z}$ $\boxed{2z^{-1}}$ $\boxed{2^{-1}z^{-1}}$ $\boxed{2^{-1}z}$

$\boxed{|z|<0.5}$ $\boxed{|z|>2}$ $\boxed{|z|>0.5}$ $\boxed{|z|<2}$

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
-\dfrac{2}{1-2z} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, shift right, "\cdot\frac{(2z)^{-1}}{(2z)^{-1}}"'] & -\dfrac{2}{1-2z^{-1}} \arrow[d, shift right, "\cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}}"'] \\
+\dfrac{z^{-1}}{1-0.5z^{-1}} \arrow[u, shift right, "\cdot\frac{(2z)}{(2z)}"'] \arrow[r, leftrightarrow, "z^{-1}"] & +\dfrac{z}{1-0.5z} \arrow[u, shift right, "\cdot\frac{(2z^{-1})}{(2z^{-1})}"']
\end{tikzcd}
$$

z-Transform $(n \rightarrow -n)$

(5) $-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z| > a^{-1}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{1}z^{-1} + (\frac{1}{a})^{2}z^{-2} + (\frac{1}{a})^{3}z^{-3} + \cdots)$

$-a^{-n}\ u(-(-n)-1)$	$(-n < 0)$
$-(\frac{1}{a})^{n}\ u(n-1)$	$(n \geq 1)$

(6) $-\dfrac{az^{-1}}{1-az^{-1}}$ $\quad |z| > a$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

$-(\frac{1}{a})^{-n}\ u(-(-n)-1)$	$(-n < 0)$
$-a^{n}\ u(n-1)$	$(n \geq 1)$

(7) $+\dfrac{az}{1-az}$ $\quad |z| < a^{-1}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$((\frac{1}{a})^{-1}z^{1} + (\frac{1}{a})^{-2}z^{2} + (\frac{1}{a})^{-3}z^{3} + \cdots)$

$a^{-n}\ u((-n)-1)$	$(-n \geq 1)$
$(\frac{1}{a})^{n}\ u(-n-1)$	$(n < 0)$

(8) $+\dfrac{a^{-1}z}{1-a^{-1}z}$ $\quad |z| < a$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

$(\frac{1}{a})^{-n}\ u((-n)-1)$	$(-n \geq 1)$
$a^{n}\ u(-n-1)$	$(n < 0)$

Laurent Series vs. z-Transform (n → -n)

(5)

$$\boxed{-\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}} \qquad \boxed{|z|>a^{-1}}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$$

Laurent	$-a^{n}$	$u(-n-1)$	$(n<0)$
z-Trans	$-\left(\frac{1}{a}\right)^{n}$	$u(n-1)$	$(n\geq 1)$

(6)

$$\boxed{-\frac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}} \qquad \boxed{|z|>a}$$

$$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$$

Laurent	$-\left(\frac{1}{a}\right)^{n}$	$u(-n-1)$	$(n<0)$
z-Trans	$-a^{n}$	$u(n-1)$	$(n\geq 1)$

(7)

$$\boxed{+\frac{\boxed{az}}{1-\boxed{az}}} \qquad \boxed{|z|<a^{-1}}$$

$$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$$

Laurent	a^{n}	$u(n-1)$	$(n\geq 1)$
z-Trans	$\left(\frac{1}{a}\right)^{n}$	$u(-n-1)$	$(n<0)$

(8)

$$\boxed{+\frac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}} \qquad \boxed{|z|<a}$$

$$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$$

Laurent	$\left(\frac{1}{a}\right)^{n}$	$u(n-1)$	$(n\geq 1)$
z-Trans	a^{n}	$u(-n-1)$	$(n<0)$