

# Carry and Overflow

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- 1 "Self-service Linux: Mastering the Art of Problem Determination",

Mark Wilding

- 1 "Computer Architecture: A Programmer's Perspective", Bryant & O'Hallaron

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# Compiling 32-bit program on 64-bit gcc

- `gcc -v`
- `gcc -m32 t.c`
- `sudo apt-get install gcc-multilib`
- `sudo apt-get install g++-multilib`
- `gcc-multilib`
- `g++-multilib`
- `gcc -m32`
- `objdump -m i386`

- Carry flag and overflow flag
- Signed and unsigned computations
- Flags for an unsigned number
- Flags for a signed number
- Detecting errors in unsigned and signed arithmetic
- The verb to overflow v.s. the overflow flag

# Carry flag and overflow flag

- considering carry and overflow flags in **x86**
- do not confuse the **carry flag** with the **overflow flag** in integer arithmetic.
- the *ALU* always sets these flags appropriately when doing any integer math.
- these flags can occur on its *own*, or *both* together.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Signed and unsigned computations

- the CPU's ALU doesn't care or know whether **signed** or **unsigned** computations are performed;
- the ALU just performs integer arithmetic and sets the flags appropriately.
- It's up to the programmer to know which flag to check after the arithmetic is done.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Flags for an unsigned number

- if a word is treated as an **unsigned** number,
  - the **carry** flag must be used to check if the result is fit into  $n$ -bit or  $(n+1)$ -bit number
  - the **overflow** flag is *irrelevant* to an **unsigned** number arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Flags for a signed number

- if a word is treated as an **signed** number,
  - the **carry** flag is *irrelevant* to an **signed** number arithmetic
  - the **overflow** flag must be used to check if the result is wrong or not

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Detecting errors in unsigned and signed arithmetic (1)

---

**unsigned** integer  
arithmetic

---

**signed** integer  
arithmetic

---

**CF** Carry Flag

detects *overflows*  
extends an  $n$ -bit result  
into an  $(n+1)$ -bit result

---

**OF** Overflow Flag

detects *overflows*  
errors  
the result cannot be used

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Detecting errors in unsigned and signed arithmetic (2)

- **unsigned** integer arithmetic *overflow*  
is indicated by the **carry** flag
  - $P + P$  **CF=1** → carry out – the result is too large for an  $n$ -bit integer
  - $P - P$  **CF=1** → borrow in – the result is too small for an  $n$ -bit integer
- **signed** integer arithmetic *overflow*  
is indicated by the **overflow** flag
  - $P + P \rightarrow N$  **OF=1** → overflow – the result is not correct
  - $N + N \rightarrow P$  **OF=1** → overflow – the result is not correct
- $P$  (positive),  $N$  (negative)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Detecting errors in unsigned and signed arithmetic (3)

- **unsigned** integer arithmetic *overflow* is indicated by the **carry** flag
  - the *overflowed*  $n$ -bit result can be extended into  $(n+1)$ -bit result by using the carry flag
- **signed** integer arithmetic *overflow* is indicated by the **overflow** flag
  - the *overflowed*  $n$ -bit result cannot be used

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# The verb to overflow v.s. the overflow flag (1)

- Do not confuse the English verb *to overflow* with the **overflow flag** in the ALU.
- The verb *to overflow* is used casually to indicate that some math result doesn't fit in the number of bits available;
- it could be integer math, or floating-point math, or whatever.
- The **overflow flag** is set specifically by the ALU it isn't the same as the casual English verb "to overflow"

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## The verb to overflow v.s. the overflow flag (2)

- In English, we may say "the binary/integer math overflowed the number of bits available for the result, causing the carry flag to come on".
- Note how this English usage of the verb "to overflow" is **not** the same as saying the **overflow flag** is on".
- A math result can overflow (the verb) the number of bits available without turning on the ALU **overflow flag**

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing Carry and Overflow Flags

## CF (carry flag) and OF (overflow flag) computation

ADD (addition)	SUB (subtraction)
$CF = C_n$	$CF = \overline{C_n}$
$OF = C_n \oplus C_{n-1}$	$OF = C_n \oplus C_{n-1}$
a 2's complement addition $A + B = A + B + 0$	a transformed addition $A - B = A + \overline{B} + 1$
$\{C_n, S_{n-1}\} = a_{n-1} + b_{n-1} + c_{n-1}$	$\{C_n, S_{n-1}\} = a_{n-1} + \overline{b_{n-1}} + c_{n-1}$
$\{C_{n-1}, S_{n-2}\} = a_{n-2} + b_{n-2} + c_{n-2}$	$\{C_{n-1}, S_{n-2}\} = a_{n-2} + \overline{b_{n-2}} + c_{n-2}$

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

- Examples of signed and unsigned integer arithmetic
- Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag



# TOC: Examples of signed and unsigned integer arithmetic

- Examples of interpreting **signed** and **unsigned** numbers
- Examples of **signed** and **unsigned** integer arithmetic
- 2's complements
- **Unsigned** subtraction
- **Signed** subtraction
- Interpreting the result as a **signed** or an **unsigned** integer
- Summary of **signed** and **unsigned** subtractions
- Examples of **unsigned** integer overflows
- Examples of **signed** integer overflows

# Examples of interpreting **signed** and **unsigned** numbers (1)

- interpreting 0xFFFFBDC3

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as an **unsigned** (positive) number    +0xFFFFBDC3    +4294950339<sub>10</sub>

---

as a **signed** (negative) number    -0x0000423D    -16957<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting **signed** and **unsigned** numbers (2)

- interpreting 0xFFFFBDC3
  - as an **unsigned** (positive) number | +0xFFFFBDC3 | +4294950339<sub>10</sub> |

$$15 * 16^7 + 15 * 16^6 + 15 * 16^5 + 15 * 16^4 \\ + 11 * 16^3 + 13 * 16^2 + 12 * 16^1 + 3 * 16^0$$

- as a **signed** (negative) number | -0x0000423D | -16957<sub>10</sub> |

$$0 * 16^7 + 0 * 16^6 + 0 * 16^5 + 0 * 16^4 \\ + 4 * 16^3 + 2 * 16^2 + 3 * 16^1 + 13 * 16^0$$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting signed and unsigned numbers (3)

- the 2's complement of 0xFFFFBDC3 : 0x0000423D (= +16957<sub>10</sub>)

	F	F	F	F	B	D	C	3
0xFFFFBDC3	0x1111	1111	1111	1111	1011	1101	1100	0011
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1100
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1101
	0	0	0	0	4	2	3	D

- the 2's complement of 0x0000423D : 0xFFFFBDC3 (= -16957<sub>10</sub>)

	0	0	0	0	4	2	3	D
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1101
0x0000BDC2	0x1111	1111	1111	1111	1011	1101	1100	0010
0xFFFFBDC3	0x1111	1111	1111	1111	1011	1101	1100	0011
	F	F	F	F	B	D	C	3

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **signed** and **unsigned** integer arithmetic

- subtracting **0x0000618D** from **0x0000195D**

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**0x0000195D - 0x0000618D**      **unsigned** subtraction

subtraction by hand

---

**0x0000195D + (-0x0000618D)**      **signed** subtraction

the *transformed addition* using  
the 2's complement of subtrahend

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## 2's complements

- the 2's complement of **0x0000618D** : 0xFFFF8E73 (= -24973<sub>10</sub>)

		F	F	F	F	8	E	7	3	
0xFFFF9E73		0x1111_1111_1111_1111_1001_1110_0111_0011								
0x0000617C		0x0000_0000_0000_0000_0110_0001_1000_1100								(1's complement)
0x0000618D		0x0000_0000_0000_0000_0110_0001_1000_1101								(2's complement)
		0	0	0	0	6	1	8	D	

- the 2's complement of **0xFFFF8E73** : 0x0000618D (= +24973<sub>10</sub>)

		0	0	0	0	6	1	8	D	
0x0000618D		0x0000_0000_0000_0000_0110_0001_1000_1101								
0xFFFF9E72		0x1111_1111_1111_1111_1001_1110_0111_0010								(1's complement)
0xFFFF9E73		0x1111_1111_1111_1111_1001_1110_0111_0011								(2's complement)
		F	F	F	F	8	E	7	3	

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Unsigned subtraction

- **0x0000195D - 0x0000618D** : unsigned subtraction  
subtraction by hand

```

          0 0 0 0 1 9 5 D
0x0000195D  0x0000_0000_0000_0000_0001_1001_0101_1101
- 0x0000618D  0x0000_0000_0000_0000_0110_0001_1000_1101
          0 0 0 0 6 1 8 D
-----
0xFFFFB7D0  1 0x1111_1111_1111_1111_1011_0111_1101_0000 (hand subtraction)
          1   F   F   F   F   B   7   D   0
          .
          V borrow (CF=1) : unsigned integer overflow
```

- A **borrow** is indicated by the **carry** flag (CF=1)
  - whenever an **unsigned integer overflow** happened
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Signed subtraction

- $0x0000195D + (-0x0000618D)$  : signed subtraction  
the *transformed addition* using the 2's complement of subtrahend

```

                0  0  0  0  1  9  5  D
0x0000195D      0x0000_0000_0000_0001_1001_0101_1101 (+0x0000195D)
+ 0xFFFF9E73   0x1111_1111_1111_1111_1001_1110_0111_0011 (-0x0000618D)
                F  F  F  F  9  E  7  3
-----
0xFFFFB7D0    0  0x1111_1111_1111_1111_1011_0111_1101_0000 (hand addition)
0              F  F  F  F  B  7  D  0
-0x00004830    .  0x0000_0000_0000_0000_0100_1000_0011_0000 (2's complement)
.              0  0  0  0  4  8  3  0
V no carry in the transformed addition (Cn=0) --> (CF=1)
```

- signed integer overflow is indicated by the **overflow** flag (OF)
  - the **carry** flag is set by the **inverted** carry of a transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>



# Interpreting the result as a **signed** or an **unsigned** integer

- subtracting **0x0000618D** from **0x0000195D**  
the results of **unsigned** and **signed** subtractions have  
the same bit pattern **0xFFFFB7D0**

- the 2's complement of **0xFFFFB7D0** :  $0x00004830$  ( $= +18480_{10}$ )

	F	F	F	F	B	7	D	0
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							
0x0000482F	0x0000_0000_0000_0000_0100_1000_0010_1111							(1's complement)
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							(2's complement)
	0	0	0	0	4	8	3	0

- the 2's complement of **0x00004830** :  $0xFFFFB7D0$  ( $= -18480_{10}$ )

	0	0	0	0	4	8	3	0
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							
0xFFFFB7CF	0x1111_1111_1111_1111_1011_0111_1100_1111							(1's complement)
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							(2's complement)
	F	F	F	F	B	7	D	0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Summary of signed and unsigned subtractions (1)

- subtracting  $0x0000618D$  from  $0x0000195D$ 
  - $0x0000195D - 0x0000618D$  : unsigned integer subtraction  
hand subtraction
  - $0x0000195D + (-0x0000618D)$  : signed integer subtraction  
the *transformed addition* using the 2's complement of the subtrahend
  - the same result :  $0xFFFFB7D0$  (the same bit pattern)
    - interpreting as a unsigned integer  $4294948816_{10}$   
 $0xFFFFB7D0$  with a borrow (CF=1)
    - interpreting as a signed integer  $-18480_{10}$   
 $-0x00004830$  (meaningless CF=1)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Summary of **signed** and **unsigned** subtractions (2)

---

0xFFFFB7D0    the result of **unsigned** subtraction    4294948816<sub>10</sub>  
with CF=1    with **unsigned** integer overflow

---

-0x00004830    the result of **signed** subtraction    -18480<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **unsigned** integer overflows

- $0x0000195D - 0x0000618D$  : **unsigned** subtraction
  - there is an **unsigned** integer overflow  
so the **carry** flag will be set ( $CF=1$ ) to indicate a **borrow**
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$   
(unsigned integers can't be negative),

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of signed integer overflows

- $0x0000195D + (-0x0000618D)$  : signed subtraction
  - there is no signed integer overflow  
the overflow flag won't be set ( $OF=0$ )
  - signed overflow occurs , in the transformed addition,
    - two *positive* numbers are added and  
the result is a *negative*, ( $P + P \rightarrow N$ ), or
    - two *negative* numbers are added and  
the result is a *positive*, ( $N + N \rightarrow P$ )

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# TOC Carry flag in unsigned and signed computations

- 2's complement numbers : 4-bit
- Addend and augend in a  $n$ -bit addition
- Full adder operation in each bit position
- Internal and external carry bits
- Addition and Subtraction
- Using the Carry Flag as a borrow

## 2's complement numbers : 4-bit

---

0111	(+7)	1000	(-8)
0110	(+6)	1001	(-7)
0101	(+5)	1010	(-6)
0100	(+4)	1011	(-5)
0011	(+3)	1100	(-4)
0010	(+2)	1101	(-3)
0001	(+1)	1110	(-2)
0000	(0)	1111	(-1)

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addend and augend in a $n$ -bit addition

$n$	bits	addened	$A$	$\{a_{n-1}, a_{n-2}, \dots, a_1, a_0\}$
$n$	bits	augend	$B$	$\{b_{n-1}, b_{n-2}, \dots, b_1, b_0\}$
$(n+1)$	bits	carry bits	$C$	$\{C_n, C_{n-1}, C_{n-2}, \dots, C_1, C_0\}$
$n$	bits	sum bits	$S$	$\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$

external carry bits :  $C_n$  carry out,  $C_0$  carry in

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 & \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Full adder operation in each bit position

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full adder operation in the  $i^{\text{th}}$  bit position

$$\{C_{i+1}, S_i\} = a_i + b_i + C_i$$

---

$$\begin{array}{r} a_i \\ b_i \\ C_i \\ \hline C_{i+1} \quad S_i \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Internal and external carry bits

external carries  $C_n$  output,  $C_0$  input  
 internal carries  $\{C_{n-1}, C_{n-2}, \dots, C_2, C_1\}$  output / input  
 sum bits  $\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$  output

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \dots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \dots & b_1 & b_0 \\
 \hline
 C_n & C_{n-1} & C_{n-2} & \dots & C_1 & C_0 \\
 \hline
 & S_{n-1} & S_{n-2} & \dots & S_1 & S_0
 \end{array}$$

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \dots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \dots & b_1 & b_0 \\
 & & & & & C_0 \\
 \hline
 C_n & S_{n-1} & S_{n-2} & \dots & S_1 & S_0
 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addition and Subtraction

- addition

$$\{C_n, S\} = A + B = A + B + 0$$

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 & \\ \hline C_{n-1} & C_{n-2} & \cdots & C_1 & 0 & \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

- subtraction - transformed addition

$$\{C_n, S\} = A - B = A + \overline{B} + 1$$

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 & \\ \hline C_{n-1} & C_{n-2} & \cdots & C_1 & 1 & \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Using the Carry Flag as a borrow (1)

- a **borrow** (CF=1) occurs in the **subtraction**  $A - B$  when  $b$  is larger than  $a$  ( $A < B$ ) as unsigned numbers
- Computer hardware can detect a **borrow** (CF=1) in **subtraction** by looking at whether a carry out (Cn) occurred in the transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Using the Carry flag as a borrow (2)

- a **borrow** ( $CF=1$ ) occurs in the **subtraction**  $A - B$  ( $A < B$ ) as unsigned numbers
- a carry out ( $C_n$ ) in the transformed addition
  - If there is no **carry** ( $C_n=0$ ) then there is a **borrow** ( $CF=1$ )
  - If there is a **carry** ( $C_n=1$ ) then there is no **borrow** ( $CF=0$ )
  - **$CF = !C_n$**

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## Using the Carry Flag as a borrow (3)

- the same *addition* and *subtraction* instructions are used for both **unsigned** and **signed** integer arithmetic.
  - no special *addition* and *subtraction* instructions for **unsigned** and **signed** integer arithmetic
- the only difference is
  - which flags you *test* afterwards and
  - how you *interpret* the result

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# TOC Rules for the carry flag

- 2's complement numbers : 4-bit
- The 1st rule for setting the carry flag
- The 2nd rule for setting the carry flag
- Cases for clearing the carry flag
- Computing CF in unsigned additions and subtractions

## 2's complement numbers : 4-bit

---

0111	(+7)	1000	(-8)
0110	(+6)	1001	(-7)
0101	(+5)	1010	(-6)
0100	(+4)	1011	(-5)
0011	(+3)	1100	(-4)
0010	(+2)	1101	(-3)
0001	(+1)	1110	(-2)
0000	(0)	1111	(-1)

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# The 1st rule for setting the carry flag

- 1 **CF = 1** : **carry** in **unsigned addition**
  - the **carry flag** is set if the **addition** of two **unsigned** numbers causes a **carry** out of the most significant bits added.
  - unsigned integer overflow** in **unsigned addition**
  - hand addition rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# The 2nd rule for setting the carry flag

- ② **CF = 1 : borrow in unsigned subtraction**
  - the **carry flag** is also set if the **subtraction** of two **unsigned** numbers requires a **borrow** into the most significant bits subtracted.
  - **unsigned integer overflow in unsigned subtraction**
  - *hand subtraction rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (1)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=1, the same S=0000

unsigned addition		signed addition		signed subtraction
0111 ( 7)		0111 (+7)		0111 (+7)
+1001 +( 9)		+1001 +(-7)		-0111 -(+7)
-----		-----		-----
10000 (16)		10000 ( 0)		10000 ( 0)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn		

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the carry flag (2)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=0, the same S=1111

unsigned addition		signed addition		signed subtraction
0111 ( 7)		0111 (+7)		0111 (+7)
+1001 +( 9)		+1001 +(-7)		-0111 -(+7)
-----		-----		-----
10000 (16)		10000 ( 0)		10000 ( 0)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn		

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing CF in unsigned additions and subtractions

- Computing CF in an **unsigned** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow C_n$
- Computing CF in an **unsigned** subtraction
  - do the transformed **signed** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow !C_n$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: Method for computing the carry flag

- Carry flag computation

# Carry flag computation (1)

ADD (addition)	SUB (subtraction)
$CF = C_n$	$CF = \overline{C_n}$
normal carry of a 2's complement addition	<b>inverted</b> carry of a transformed addition
$A + B = A + B + 0$	$A - B = A + \overline{B} + 1$
$\{C_n, S_{n-1}\}$ $= a_{n-1} + b_{n-1} + c_{n-1}$	$\{C_n, S_{n-1}\}$ $= a_{n-1} + \overline{b_{n-1}} + c_{n-1}$

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

## Carry flag computation (2)

- In **unsigned** arithmetic,
  - the **carry flag** is used to detect *overflow*
  - the **carry flag** is used to extend *n-bit* result into *(n+1)-bit* result
  - for **addition**, the **carry flag** is a **carry out**
  - for **subtraction**, the **carry flag** is a **borrow in**
- In **signed** arithmetic,
  - the **carry flag** is useless
  - the **carry flag** neither detects overflow nor extends n-bit result

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Carry flag computation (3)

- In **unsigned** arithmetic,

---

Addition	<b>CF = 1</b> means <b>carry out</b>	when <b>C<sub>n</sub> = 1</b>
Subtraction	<b>CF = 1</b> means <b>borrow in</b>	when <b>C<sub>n</sub> = 0</b>

---

- **CF** - Carry Flag in x86
- **C<sub>n</sub>** - the normal carry out
  - the carry out of a 2's complement addition for **ADD**
  - the carry out of a *transformed* addition for **SUB**
- In **signed** arithmetic,
  - the **carry** flag is useless

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: More examples of the carry flag

- Summary I
- Summary II
- Cases for setting the carry flag
- Cases for clearing the carry flag

# Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
-1110	-(14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	( 5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
+0010	+( 2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	( 5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-( 2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
11011	(11)	(-16)	11011	(-5)		11011	(-5)		0	0

# Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	( 7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	( 9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
+0100	+( 4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	( 9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-( 4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	( 7) ( 0)		10111	(+7)		10111	(+7)		0	1

# Cases for setting the carry flag (1) CF=1, OF=0

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1101 (-3)		1101 (-3)
+1110 +(14) ADD		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (2) CF=1, OF=0

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 0101		S = 0101		S = 0101
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		
-----		-----		-----

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (3) CF=1, OF=1

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
+1100 +(12) ADD		+1100 +(-4) ADD		-0100 -(+4)
-----		-----		-----
10111 ( 7) (+16)		10111 (+7)		10111 (+7)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means +16		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=0111, OF=1 for all three interpretations

# Cases for setting the carry flag (4) CF=1, OF=1

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0101 ( 5)		0101 (+5)		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 ( 9) (-16)		01001 (-7)		01001 (-7)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1001		S = 1001		S = 1001
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)
-----		-----		-----

\* CF=1, S=1001, OF=1 for all three interpretations



# Cases for clearing the carry flag (1) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0011 ( 3)		0011 (+3)	0011 (+3)
+0010 +( 2) ADD		+0010 +(2) ADD	-1110 -(-2)
-----		-----	-----
00101 ( 5) (+ 0)		00101 (+5)	00101 (+5)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0		CF meaningless	CF meaningless
S = 0101		S = 0101	S = 0101
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (2) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1101 (-3)		1101 (-3)
-0010 -( 2) SUB		+1110 +(-2)		-0010 -(+2) SUB
-----		-----		-----
11011 (11) (-16)		11011 (-5)		11011 (-5)
CF=0		Cn=0 -> CF=0		Cn=0 -> CF=0
CF means 0		CF meaningless		CF meaningless
S = 1011		S = 1011		S = 1011
* think hand		* think Cn of the corresponding addition		
subtraction		CF <- Cn (for unsigned addition)		

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (3) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition		signed subtraction
0101 ( 5)		0101 (+5)		0101 (+5)
+0100 +( 4) ADD		+0100 +( +4) ADD		-1100 -(-4)
-----		-----		-----
01001 ( 9) (+ 0)		01001 (-7)		01001 (-7)
CF=0		Cn=0 -> CF=0		Cn=0 -> CF=0
CF means +0		CF meaningless		CF meaningless
S = 1001		S = 1001		S = 1001
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for clearing the carry flag (4) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
-0100 -( 4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 ( 7) ( 0)		10111 (+7)		10111 (+7)
CF=0		Cn=1 -> CF=0		Cn=1 -> CF=0
CF means 0		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: Overflow flag

- Overflow flag in unsigned and signed computations
- Rules for the overflow flag
- Method 1 for computing the overflow flag
- Method 2 for computing the overflow flag
- More examples of the overflow flag

- Overflow flag

# Overflow flag (1)

- only need to look at the **sign bits** (leftmost) of the three numbers

$$\begin{array}{rclcl} \text{augend} & + & \text{addend} & = & \text{sum} \\ \text{minuend} & - & \text{subrahend} & = & \text{difference} \end{array}$$

to decide if the **overflow** flag is turned on or off.

- overflow** flag is based on **signed** arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Overflow flag (2)

- in **signed** arithmetic,
  - watch the **overflow** flag to detect errors
  - **overflow** flag on means the result is wrong
  - errors can be detected by examining the sign of the result, in the 2's complement arithmetic
- in **unsigned** arithmetic,
  - the **overflow** flag tells you nothing interesting

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow flag (3)

- when two positive numbers are added
  - if the result is a negative, ( $P + P \rightarrow N$ ), then overflow
  - if the result is a positive, ( $P + P \rightarrow P$ ), then no overflow
- when two negative numbers are added
  - the result is a positive, ( $N + N \rightarrow P$ ), then overflow
  - the result is a negative, ( $N + N \rightarrow N$ ), then no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow flag (4)

- adding negative and positive numbers cannot be wrong, because the sum is between the addends.
  - mixed-sign addition never turns on the **overflow** flag.
  - opposite signed numbers are added, then no **overflow**
  - both of the addends lies in the allowable range of numbers, and their sum is between the addends, therefore the sum lies also in the allowable range
- $(P + N \rightarrow P)$  no overflow
- $(P + N \rightarrow N)$  no overflow
- $(N + P \rightarrow P)$  no overflow
- $(N + P \rightarrow N)$  no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Rules for the overflow flag

- the 1st rule for setting OF
- the 2nd rule for setting OF
- cases for clearing OF (1 ~ 6)

# Overflow flag setting and clearing conditions

ADD conditions      SUB conditions

OF=1	$P + P \rightarrow N$	$P - N \rightarrow N$	$C_n \oplus C_{n-1} = 1$
OF=1	$N + N \rightarrow P$	$N - P \rightarrow P$	$C_n \oplus C_{n-1} = 1$
OF=0	$P + P \rightarrow P$	$P - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + N \rightarrow N$	$N - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$
OF=0	$P + N \rightarrow P$	$P - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$P + N \rightarrow N$	$P - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + P \rightarrow P$	$N - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + P \rightarrow N$	$N - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$

$$+P = -(-P) = -N$$

$$+N = -(-N) = -P$$

# The 1st rule for setting the overflow flag

- 1 If the **sum** of two **signed** numbers with the sign bits off (0, 0) yields a result number with the sign bit on (1) the **overflow flag** is turned on ( $OF = 1 : P + P \rightarrow N$ )

signed addition

```
0100 carries
 0100 (+4)
+0100 +(4)
-----
01000 (-8)
```

signed subtraction

```
0100 (+4)
-1100 -(-4)
-----
01000 (-8)
```

unsigned addition

```
0100 ( 4)
+0100 +( 4)
-----
01000 ( 8)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 1 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## The 2nd rule for setting the overflow flag

- 2 If the **sum** of two numbers with the sign bits on (1, 1) yields a result number with the sign bit off (0) the **overflow flag** is turned on. ( $OF = 1 : N + N \rightarrow P$ )

signed addition

```
1001 carries
 1001 (-7)
+1001 +(-7)
-----
10010 ( 2)
```

signed subtraction

```
1001 (-7)
-0111 -(+7)
-----
10010 ( 2)
```

unsigned addition

```
1001 ( 9)
+1001 +( 9)
-----
10010 (18)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 0 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (1)

- **overflow flag** is turned off. ( $OF = 0 : P + P \rightarrow P$ )

signed addition

```
0011  carries
0011  (+3)
+0011  +(+3)
-----
00110  (+6)
```

signed subtraction

```
0011  (+3)
-1101  -(-3)
-----
00110  (+6)
```

unsigned addition

```
0011  ( 3)
+0011  +( 3)
-----
00110  ( 6)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the overflow flag (2)

- **overflow flag** is turned off. ( $OF = 0 : N + N \rightarrow N$ )

signed addition

```
1101 carries
 1101 (-3)
+1101 +(-3)
-----
11010 (-6)
```

signed subtraction

```
1101 (-3)
-0011 -(+3)
-----
11010 (-6)
```

unsigned addition

```
1101 (13)
+1101 +(13)
-----
11010 (26)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Cases for clearing the overflow flag (3)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow P$ )

signed addition

```
1100 carries
 0100 (+4)
+1101 +(-3)
-----
10001 (+1)
```

signed subtraction

```
0100 (+4)
-0011 -(+3)
-----
10001 (+1)
```

unsigned addition

```
0100 ( 4)
+1101 +(13)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (4)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow N$ )

signed addition

```
0000 carries
 0011 (+3)
+1100 +(-4)
-----
01111 (-1)
```

signed subtraction

```
0011 (+3)
-0100 -(+4)
-----
01111 (-1)
```

unsigned addition

```
0011 ( 3)
+1100 +(12)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (5)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow P$ )

signed addition

```
1100 carries
 1101 (-3)
+0100 (+4)
-----
10001 (+1)
```

signed subtraction

```
0011 (-3)
-1100 -(-4)
-----
10001 (+1)
```

unsigned addition

```
1101 (13)
+0100 +( 4)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (6)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow N$ )

signed addition

```
0000 carries
1100 (-4)
+0011 +(+3)
-----
01111 (-1)
```

signed subtraction

```
0100 (-4)
-1101 -(-3)
-----
01111 (-1)
```

unsigned addition

```
1100 (12)
+0011 +( 3)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 1 for computing the overflow flag

- Adding two numbers with the same sign
- Overflow conditions for additions and subtractions
- Overflow condition for an addition
- Overflow conditions for a subtraction
- Overflow in signed computations

# Adding two numbers with the same sign

- **overflow** can only happen when adding two numbers of the same sign results in a different sign ( $P + P \rightarrow N$ ,  $N + N \rightarrow P$ )

- $n$ -bit **signed** binary arithmetic  $A + B = C$

$$A = (a_{n-1}, \dots, a_1, a_0)$$

$$B = (b_{n-1}, \dots, b_1, b_0)$$

$$C = (c_{n-1}, \dots, c_1, c_0)$$

- to detect overflow
  - only the **sign** bits are considered
  - **msb** (most significant bit)  $a_{n-1}, b_{n-1}, c_{n-1}$
  - the other bits are ignored

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow conditions for additions and subtractions

- with two operands ( $A$  and  $B$ ) and one result ( $C$ ), three sign bits ( $a_{n-1}, b_{n-1}, c_{n-1}$ ) are considered  
→  $2^3 = 8$  possible combinations
- only two cases result in **overflow** for an addition
  - 0 0 1      ( $p + p \rightarrow n$ )
  - 1 1 0      ( $n + n \rightarrow p$ )
- only two cases are considered as **overflow** for an subtraction
  - 0 1 1      ( $p - n \rightarrow n$ )
  - 1 0 0      ( $n - p \rightarrow p$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow condition for an addition

- Overflow in an addition ( $A + B$ )

	$a_{n-1}$	$b_{n-1}$	$c_{n-1}$	
	0	0	0	$p + p \rightarrow p$
OVER	0	0	1	$p + p \rightarrow n$
	0	1	0	$p + n \rightarrow p$
	0	1	1	$p + n \rightarrow n$
	1	0	0	$n + p \rightarrow p$
	1	0	1	$n + p \rightarrow n$
OVER	1	1	0	$n + n \rightarrow p$
	1	1	1	$n + n \rightarrow n$

- adding two positives should be positive
- adding two negatives should be negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow conditions for a subtraction

- Overflow in a subtraction ( $A - B$ )

	$a_{n-1}$	$b_{n-1}$	$c_{n-1}$	
	0	0	0	$p - p \rightarrow p$
	0	0	1	$p - p \rightarrow n$
	0	1	0	$p - n \rightarrow p$
OVER	0	1	1	$p - n \rightarrow n$
OVER	1	0	0	$n - p \rightarrow p$
	1	0	1	$n - p \rightarrow n$
	1	1	0	$n - n \rightarrow p$
	1	1	1	$n - n \rightarrow n$

- subtracting a negative is the same as adding a positive
- subtracting a positive is the same as adding a negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in signed computations

- ALU might contain a small logic that sets the **overflow** flag to "1" if and only if any one of the above four **OV conditions** is met.
- in **signed** computations, adding two numbers of the same sign must produce a result of the same sign, otherwise overflow happened.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 2 for computing the overflow flag

- Carry into and carry out of the sign bit
- Overflow in 2's complement arithmetic
- Overflow flag =  $C_n \oplus C_{n-1}$
- Examples of 4-bit signed additions
- $C_n$  and  $C_{n-1}$  in a  $n$ -bit addition
- Overflow flag computation
- Examples of computing overflow flag
- Hexadecimal carry, octal carry, decimal carry
- No carry into the sign bit

# Carry into and carry out of the sign bit

- When adding two  $n$ -bit binary values, consider
  - the **carry coming into** the most significant bit (msb)  
 $C_{n-1}$  : **carry into** the **sign** bit
  - the **carry going out of** the most significant bit (msb)  
 $C_n$  : **carry out of** the **sign** bit  
this is the **carry flag (CF)** in the processor

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in 2's complement arithmetic

- **overflow** in 2's complement happens (**OF=1**) when
  - there is a **carry into** the **sign** bit ( $C_{n-1} = 1$ )  
but no carry out of the **sign** bit ( $C_n = 0$ )
  - there is no carry into the **sign** bit ( $C_{n-1} = 0$ )  
but a **carry out** of the **sign** bit ( $C_n = 1$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

$$\text{Overflow flag} = C_n \oplus C_{n-1}$$

- the **overflow** flag is the **XOR** ( $C_n \oplus C_{n-1}$ ) of
  - of the **carry coming into** the **sign** bit ( $C_{n-1}$ )
  - with the **carry going out of** the **sign** bit ( $C_n$ )
- **overflow** happens when the **carry in** ( $C_{n-1}$ ) does not equal to the **carry out** ( $C_n$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (1)

- 4-bit 2's complement addition examples

```
0000
 0100 (+4) (pos sign 0)
+1000 (-8) (neg sign 1)
=====
01100 (-4) (neg sign 1)
```

```
C4 carry out 0 (1+0+0)
C3 carry in 0 (0+1+0)
0 XOR 0 = NO OVERFLOW
```

```
1100
 1100 (-4) (neg sign 1)
+0100 (+4) (pos sign 0)
=====
10000 ( 0) (pos sign 0)
```

```
C4 carry out 1 (1+0+1)
C3 carry in 1 (1+1+0)
1 XOR 1 = NO OVERFLOW
```

```
0100
 0100 (+4) (pos sign 0)
+0100 (+4) (pos sign 0)
=====
01000 (-8) (neg sign 1)
```

```
C4 carry out 0 (0+0+1)
C3 carry in 1 (1+1+0)
0 XOR 1 = OVERFLOW!
```

```
1000
 1100 (-4) (neg sign 1)
+1000 (-8) (neg sign 1)
=====
10100 (+4) (pos sign 0)
```

```
C4 carry out 1 (1+1+0)
C3 carry in 0 (1+0+0)
1 XOR 0 = OVERFLOW!
```

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (2)

- same sign addition  $\rightarrow$  possible overflow

----- + +, - -----	----- - -, + -----	----- + +, + -----	----- - -, - -----
+5 +5	-5 -5	+5 +1	-5 -1
----- -6(OF)	----- +6(OF)	----- +6	----- -6

0101 0101 0101	1011 1011 1011	0001 0101 0001	1111 1011 1111
----- 01010	----- 10110	----- 00110	----- 11010
----- C4 = 0	----- C4 = 1	----- C4 = 0	----- C4 = 1
----- C3 = 1	----- C3 = 0	----- C3 = 0	----- C3 = 1
----- OF = 1	----- OF = 1	----- OF = 0	----- OF = 0

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Examples of 4-bit signed additions (3)

- mixed sign addition → no overflow

----- + -, + ----- +5 -1 ----- +4	----- + -, - ----- +5 -6 ----- -1	----- - +, + ----- -5 +6 ----- +1	----- - +, - ----- -5 +1 ----- -4
1111 0101 1111 ----- 10100	0000 0101 1010 ----- 01111	1110 1011 0110 ----- 10001	0011 1011 0001 ----- 01100
----- C4 = 1 C3 = 1 ----- OF = 0	----- C4 = 0 C3 = 0 ----- OF = 0	----- C4 = 1 C3 = 1 ----- OF = 0	----- C4 = 0 C3 = 0 ----- OF = 0

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# $C_n$ and $C_{n-1}$ in a $n$ -bit addition

## $(n-1)^{th}$ bit – MSB

- adding operations at the  $(n-1)$  bit position
- $\{C_n, S_{n-1}\} = a_{n-1} + b_{n-1} + c_{n-1}$

$$\begin{array}{r} \text{msb} \\ a_{n-1} \\ b_{n-1} \\ \hline C_{n-1} \\ \hline C_n \quad S_{n-1} \end{array}$$

- $C_n$  : carry coming out of the msb

## $(n-2)^{th}$ bit

- adding operations at the  $(n-2)$  bit position
- $\{C_{n-1}, S_{n-2}\} = a_{n-2} + b_{n-2} + c_{n-2}$

$$\begin{array}{r} \text{msb} \\ a_{n-2} \\ b_{n-2} \\ \hline C_{n-2} \\ \hline C_{n-1} \quad S_{n-2} \end{array}$$

- $C_{n-1}$  : carry coming into the msb

# Overflow flag computation

---

**ADD** (addition)

---

$$OF = C_n \oplus C_{n-1}$$

a 2's complement addition

$$A + B = A + B + \mathbf{0} \quad (C_0 = 0)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + b_{n-1} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + b_{n-2} + c_{n-2} \end{aligned}$$

---

**SUB** (subtraction)

---

$$OF = C_n \oplus C_{n-1}$$

the transformed addition

$$A - B = A + \overline{B} + \mathbf{1} \quad (C_0 = 1)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + \overline{b_{n-1}} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + \overline{b_{n-2}} + c_{n-2} \end{aligned}$$

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Hexadecimal carry, octal carry, decimal carry

- Note that this XOR method only works with the **binary** carry that goes into the sign **bit**.
- not works with **hexadecimal carry**  
**decimal carry**, **octal carry**
  - the carry doesn't go into the sign **bit**
  - can't XOR that non-binary carry with the outgoing carry.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# No carry into the sign bit

- Hexadecimal addition example  
(showing that XOR doesn't work for hex carry):

```
8Ah
+8Ah
====
114h
```

- The hexadecimal carry of 1 resulting from A+A does not affect the sign bit.
- If you do the math in binary, you'll see that there is **no** carry **into** the sign bit; but, there is carry out of the sign bit. Therefore, the above example sets OVERFLOW on. (The example adds two negative numbers and gets a positive number.)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
-1110	-(14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	( 5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
+0010	+( 2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	( 5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-( 2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
11011	(11)	(-16)	11011	(-5)		11011	(-5)		0	0

# Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	( 7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	( 9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
+0100	+( 4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	( 9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-( 4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	( 7) ( 0)		10111	(+7)		10111	(+7)		0	1

# Cases for setting the overflow flag (1) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned addition		* signed addition	signed subtraction
1011 (11)		1000	
+1100 +(12) ADD		1011 (-5)	1011 (-5)
-----		+1100 +(-4) ADD	-0100 -(+4)
10111 ( 7) (+16)		-----	-----
		10111 (+7)	10111 (+7)
OF=1		n + n -> p (OF=1)	n - p -> p (OF=1)
OF meaningless		-> incorrect S	-> incorrect S
S = 0111		S = 0111	S = 0111
* think hand		* OF <- C4 XOR C3 = 1 XOR 0 = 1	
addition		of signed addition	

\* CF=1, S=0111, OF=1 for all three interpretations



# Cases for setting the overflow flag (2) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned subtraction		signed addition		* signed subtraction
0101 (5)		0100		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 (9) (-16)		01001 (-7)		01001 (-7)
OF=1		p + p -> n (OF=1)		p - n -> n (OF=1)
OF meaningless		-> incorrect S		-> incorrect S
S = 1001		S = 1001		S = 1001
-----				
* think hand subtraction		* OF <- C4 XOR C3 = 0 XOR 1 = 1		of signed addition
-----				

\* CF=1, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (3) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned addition		* signed addition	signed subtraction
0101 ( 5)		0100	
+0100 +( 4) ADD		0101 (+5)	0101 (+5)
-----		+0100 +(4) ADD	-1100 -(-4)
01001 ( 9) (+ 0)		-----	-----
		01001 (-7)	01001 (-7)
OF=1		p + p -> n (OF=1)	p - n -> n (OF=1)
OF meaningless		-> incorrect S	-> incorrect S
S = 1001		S = 1001	S = 1001
* think hand		* OF <- C4 XOR C3 = 0 XOR 1 = 1	
addition		of signed addition	

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (4) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1000		1011 (-5)
-0100 -(4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 (7) (0)		10111 (+7)		10111 (+7)
OF=1		n + n -> p (OF=1)		n - p -> p (OF=1)
OF meaningless		-> incorrect S		-> incorrect S
S = 0111		S = 0111		S = 0111
* think hand subtraction		* OF <- C4 XOR C3 = 1 XOR 0 = 1		of signed addition

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (1) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1100		
+1110 +(14) ADD		1101 (-3)		1101 (-3)
-----		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
OF=0		n + n -> n (OF=0)		n - p -> n (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0000		S = 0000		S = 0000
* think hand		* OF <- C4 XOR C3 = 1 XOR 1 = 0		
addition		of signed addition		

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the overflow flag (2) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0010		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		p + p -> p (OF=0)		p - n -> p (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0101		S = 0101		S = 0101
* think hand subtraction		* OF <- C4 XOR C3 = 0 XOR 0 = 0		of signed addition

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (3) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
0011 (3)		0010		
+0010 +(2) ADD		0011 (+3)		0011 (+3)
-----		+0010 +(2) ADD		-1110 -(-2)
00101 (5) (+0)		-----		-----
		00101 (+5)		00101 (+5)
OF=0		p + p -> p (OF=0)		p - n -> p (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0101		S = 0101		S = 0101
* think hand		* OF <- C4 XOR C3 = 0 XOR 0 = 0		
addition		of signed addition		

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (4) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1100		
-0010 -(2) SUB		1101 (-3)		1101 (-3)
-----		+1110 +(-2)		-0010 -(+2) SUB
-----		-----		-----
11011 (11) (-16)		11011 (-5)		11011 (-5)
OF=0		n + n -> n (OF=0)		n - p -> n (OF=0)
OF meaningless		-> correct S		-> correct S
S = 1011		S = 1011		S = 1011
* think hand		* OF <- C4 XOR C3 = 1 XOR 1 = 0		
subtraction		of signed addition		

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# 4-bit addition table (1)

	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)
0000 (0)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)
0001 (+1)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8) OF
0010 (+2)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8) OF	1001 (-7) OF
0011 (+3)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8) OF	1001 (-7) OF	1010 (-6) OF
0100 (+4)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8) OF	1001 (-7) OF	1010 (-6) OF	1011 (-5) OF
0101 (+5)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8) OF	1001 (-7) OF	1010 (-6) OF	1011 (-5) OF	1100 (-4) OF
0110 (+6)	0110 (+6)	0111 (+7)	1000 (-8) OF	1001 (-7) OF	1010 (-6) OF	1011 (-5) OF	1100 (-4) OF	1101 (-3) OF
0111 (+7)	0111 (+7)	1000 (-8) OF	1001 (-7) OF	1010 (-6) OF	1011 (-5) OF	1100 (-4) OF	1101 (-3) OF	1110 (-2) OF

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# 4-bit addition table (2)

	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)
1000 <b>(-8)</b>	1000 <b>(-8)</b>	1001 <b>(-7)</b>	1010 <b>(-6)</b>	1011 <b>(-5)</b>	1100 <b>(-4)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>
1001 <b>(-7)</b>	1001 <b>(-7)</b>	1010 <b>(-6)</b>	1011 <b>(-5)</b>	1100 <b>(-4)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)
1010 <b>(-6)</b>	1010 <b>(-6)</b>	1011 <b>(-5)</b>	1100 <b>(-4)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)
1011 <b>(-5)</b>	1011 <b>(-5)</b>	1100 <b>(-4)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)	0010 (+2)
1100 <b>(-4)</b>	1100 <b>(-4)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)
1101 <b>(-3)</b>	1101 <b>(-3)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)
1110 <b>(-2)</b>	1110 <b>(-2)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)
1111 <b>(-1)</b>	1111 <b>(-1)</b>	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# 4-bit addition table (3)

	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)
0000 (0)	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)
0001 (+1)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)	0000 (0)
0010 (+2)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)	0000 (0)	0001 (+1)
0011 (+3)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)	0000 (0)	0001 (+1)	0010 (+2)
0100 (+4)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)
0101 (+5)	1101 (-3)	1110 (-2)	1111 (-1)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)
0110 (+6)	1110 (-2)	1111 (-1)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)
0111 (+7)	1111 (-1)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# 4-bit addition table (4)

	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)	1111 (-1)
1000 (-8)	0000 CF ( 0) OF	0001 CF (+1) OF	0010 CF (+2) OF	0011 CF (+3) OF	0100 CF (+4) OF	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF
1001 (-7)	0001 CF (+1) OF	0010 CF (+2) OF	0011 CF (+3) OF	0100 CF (+4) OF	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)
1010 (-6)	0010 CF (+2) OF	0011 CF (+3) OF	0100 CF (+4) OF	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)
1011 (-5)	0011 CF (+3) OF	0100 CF (+4) OF	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)	1010 CF (-6)
1100 (-4)	0100 CF (+4) OF	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)	1010 C (-6)	1011 CF (-5)
1101 (-3)	0101 CF (+5) OF	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)	1010 CF (-6)	1011 CF (-5)	1100 CF (-4)
1110 (-2)	0110 CF (+6) OF	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)	1010 CF (-6)	1011 CF (-5)	1100 CF (-4)	1101 CF (-3)
1111 (-1)	0111 CF (+7) OF	1000 CF (-8)	1001 CF (-7)	1010 CF (-6)	1011 CF (-5)	1100 CF (-4)	1101 CF (-3)	1110 CF (-2)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)