DM Week 01

20180310

AND (Conjunction) $P \wedge Q = f(P, Q)$

function

$P = T/F$ $Q = T/F$

$1/0$ $1/0$

P	Q	P ∧ Q
T	T	T
T	F	
F	T	
F	F	

all inputs are true

P	Q	P ∧ Q
T	T	
T	F	F
F	T	F
F	F	F

at least one input is false

	P	Q	P ∧ Q
Case 1	T	T	T
Case 2	T	F	F
Case 3	F	T	F
Case 4	F	F	F

all inputs are true

at least one input is false

All possible combination of inputs (P, Q, R)

T	T	T
T	T	F
T	F	T
T	F	T
F	T	T
F	T	F
F	F	T
F	F	T

T	T
T	F
F	T
F	F

T→1
F→0

	2^2	2^1	2^0			
7	1	1	1	T	T	T
6	1	1	0	T	T	F
5	1	0	1	T	F	T
4	1	0	0	T	F	F
3	0	1	1	F	T	T
2	0	1	0	F	T	F
1	0	0	1	F	F	T
0	0	0	0	F	F	F

Three input AND

2^2	2^1	2^0	
p	q	r	$p \wedge q \wedge r$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

$(p \wedge q) \wedge r$

$p \wedge (q \wedge r)$

$(p \wedge q) \wedge r$

$p \wedge (q \wedge r)$

associative law

	p	q	r	$p \wedge q \wedge r$
7	1	1	1	T
6	1	1	0	F
5	1	0	1	F
4	1	0	0	F
3	0	1	1	F
2	0	1	0	F
1	0	0	1	F
0	0	0	0	F

10^2	10^1	10^0
1	2	3

2^2	2^1	2^0
1	1	1

$$\boxed{\begin{array}{l} T \rightarrow 1 \\ F \rightarrow 0 \end{array}}$$

bit-wise AND/OR/NOT
logical AND/OR/NOT operators in C

$p \wedge q$ p && q = 0/1   p & q = any integer

$p \vee q$ p || q = 0/1   p | q = any integer

$\neg p$!p = 0/1   ~p = any integer

OR (Disjunction) $P \vee Q = f(P, Q)$

function

$P = T/F$ $Q = T/F$

$1/0$ $1/0$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

function

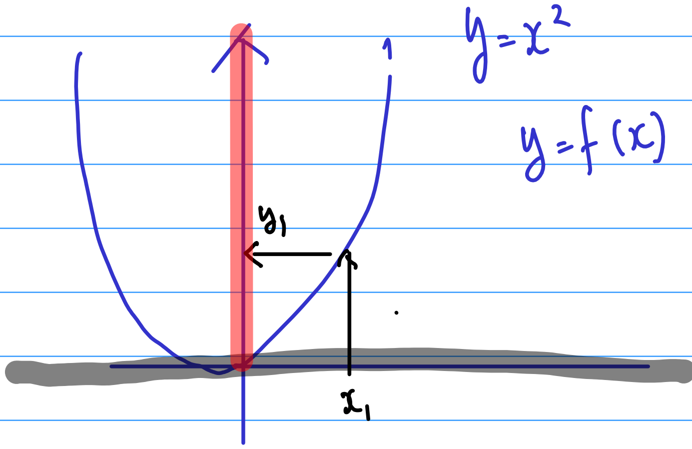
at least one input is true

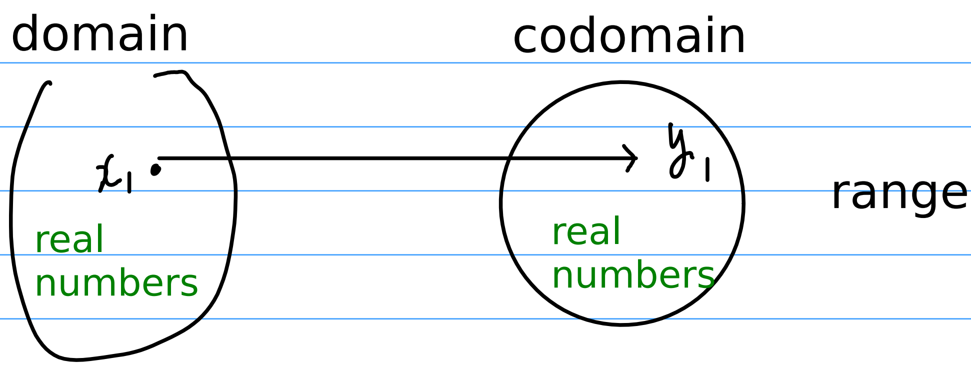
all inputs are false

all

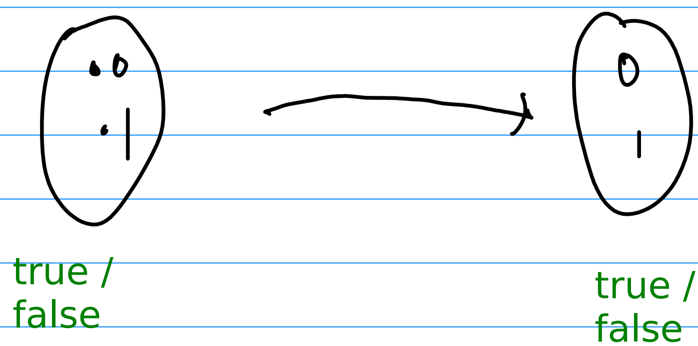
any

Function



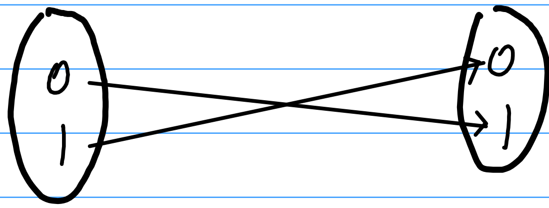


boolean function

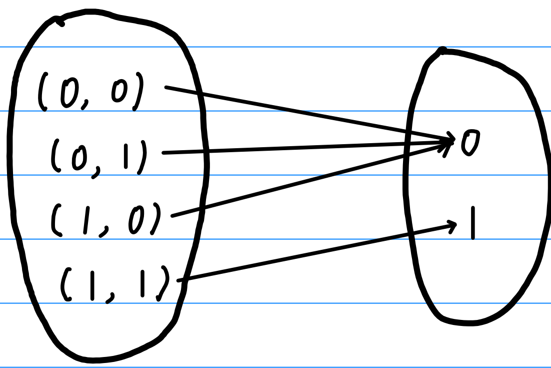


$$P \wedge Q = f(P, Q)$$

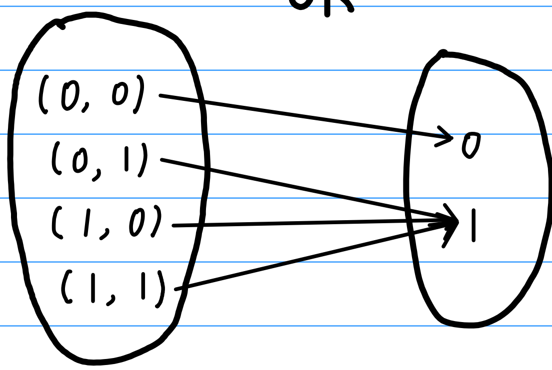
NOT



AND



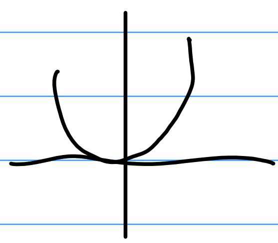
OR



1-variable function

$y = x^2$

$y = f(x)$

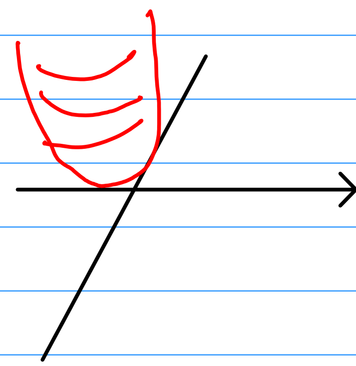


2-dimensional graph

2-variable function

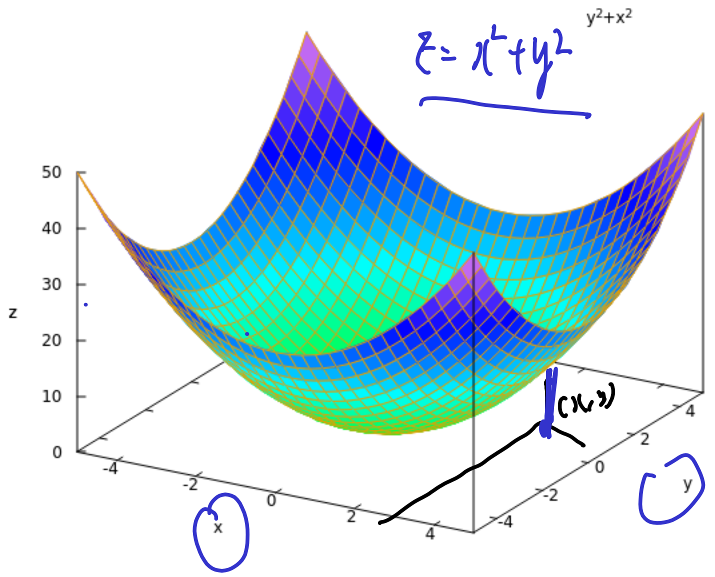
$z = x^2 + y^2$

$z = f(x, y)$



3-dimensional graph

2-variable real function



2-variable boolean function

$f(x, y)$

$f(P, Q) = P \wedge Q$

$P \vee Q$

0/1

0/1

0/1

	P	Q	
0	F	F	
1	F	T	
2	T	F	
3	T	T	

Material Implication

$\neg P \vee Q$

P Q $P \to Q$

P	$\to$	Q
hypothesis (가설)		conclusion (결론)
antecedent (전제)		consequence (결과)

수학과에 6만 달러를 지원한다 P

수학과에 신임교수를 한명 고용한다 Q

	P	Q	$\boxed{P \to Q}$	~P	Q	$\boxed{\sim P \vee Q}$
case 1	T	T	T	F	T	T
case 2	T	F	F	F	F	F
case 3	F	T	T	T	T	T
case 4	F	F	T	T	F	T

$p \to Q$ 동치 $\boxed{\text{logical equivalent}}$ $\equiv$

$\sim p \vee Q$

Material Implicaton Example

$$p \rightarrow q \equiv \neg p \lor q$$

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$(p \land q) \rightarrow r$$

$$\neg(p \land q) \lor r$$

$$(\neg p \lor \neg q) \lor r$$

$$\boxed{\neg p} \lor \boxed{\neg q} \lor \boxed{r} \Rightarrow F$$

$F \quad F \quad F$

$p = T \quad q = T \quad r = F$

false only when all inputs are false
true otherwise

De Morgan's Law

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\square \longrightarrow \bigcirc$$

$$\equiv \neg \square \lor \bigcirc$$

Verification by a truth table

$p \wedge q \to r$

$p = T \quad q = T \quad r = F$

	p	q	r	$p \wedge q$	r	$p \wedge q \to r$	$\neg p \vee \neg q \vee r$
case 1	0	0	0	0	0	1	1
case 2	0	0	1	0	1	1	1
...	0	1	0	0	0	1	1
	0	1	1	0	1	1	1
	1	0	0	0	0	1	1
	1	0	1	0	1	1	1
	1	1	0	1	0	0	0
	1	1	1	1	1	1	1

logical equivalence

Precedence

Order of precedence [edit]

As a way of reducing the number of necessary parentheses, one may introduce precedence rules: ¬ has higher precedence than ∧, ∧ higher than ∨, and ∨ higher than →. So for example, $P \vee Q \wedge \neg R \rightarrow S$ is short for $(P \vee (Q \wedge (\neg R))) \rightarrow S$.

Here is a table that shows a commonly used precedence of logical operators.[15]

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$p \wedge q \rightarrow r$

$(p \wedge q) \rightarrow r$

https://en.wikipedia.org/wiki/Logical_connective

Associativity

Truth functional connectives [edit]

Associativity is a property of some logical connectives of truth-functional propositional logic. The following logical equivalences demonstrate that associativity is a property of particular connectives. The following are truth-functional tautologies.

Associativity of disjunction:

$$((P \vee Q) \vee R) \leftrightarrow (P \vee (Q \vee R))$$
$$(P \vee (Q \vee R)) \leftrightarrow ((P \vee Q) \vee R)$$

Associativity of conjunction:

$$((P \wedge Q) \wedge R) \leftrightarrow (P \wedge (Q \wedge R))$$
$$(P \wedge (Q \wedge R)) \leftrightarrow ((P \wedge Q) \wedge R)$$

Associativity of equivalence:

$$((P \leftrightarrow Q) \leftrightarrow R) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow R))$$
$$(P \leftrightarrow (Q \leftrightarrow R)) \leftrightarrow ((P \leftrightarrow Q) \leftrightarrow R)$$

https://en.wikipedia.org/wiki/Associative_property

Commutativity

Truth functional connectives [edit]

Commutativity is a property of some logical connectives of truth functional propositional logic. The following logical equivalences demonstrate that commutativity is a property of particular connectives. The following are truth-functional tautologies.

Commutativity of conjunction

$$(P \land Q) \leftrightarrow (Q \land P)$$

Commutativity of disjunction

$$(P \lor Q) \leftrightarrow (Q \lor P)$$

Commutativity of implication (also called the law of permutation)

$$(P \to (Q \to R)) \leftrightarrow (Q \to (P \to R))$$

Commutativity of equivalence (also called the complete commutative law of equivalence)

$$(P \leftrightarrow Q) \leftrightarrow (Q \leftrightarrow P)$$

https://en.wikipedia.org/wiki/Commutative_property

Distributivity (1)

Truth functional connectives [edit]

Distributivity is a property of some logical connectives of truth-functional propositional logic. The following logical equivalences demonstrate that distributivity is a property of particular connectives. The following are truth-functional tautologies.

Distribution of conjunction over conjunction

$$(P \land (Q \land R)) \leftrightarrow ((P \land Q) \land (P \land R))$$

Distribution of conjunction over disjunction

$$(P \land (Q \lor R)) \leftrightarrow ((P \land Q) \lor (P \land R))$$

Distribution of disjunction over conjunction

$$(P \lor (Q \land R)) \leftrightarrow ((P \lor Q) \land (P \lor R))$$

Distribution of disjunction over disjunction

$$(P \lor (Q \lor R)) \leftrightarrow ((P \lor Q) \lor (P \lor R))$$

$$3 \cdot (4 + 7)$$

$$3 \cdot 4 + 3 \cdot 7$$

https://en.wikipedia.org/wiki/Distributive_property

Laws of logical equivalence (1)

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws

$(q \wedge r) \vee p \equiv (q \vee p) \wedge (r \vee p)$

$(q \vee r) \wedge p \equiv (q \wedge p) \vee (r \wedge p)$

https://en.wikipedia.org/wiki/Logical_equivalence

Laws of logical equivalence (2)

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

p	~p	p∨¬p
T	F	T
F	T	T

https://en.wikipedia.org/wiki/Logical_equivalence

Young Won Lim
3/1/18

argument
(논영)

bug는 module 17 또는 module 81에 있다 — $p \vee q$
bug는 numerical error이다 — r
module 81에는 numerical error가 없다 — $r \to \neg q$

p: bug는 module 17에 있다
q: bug는 module 81에 있다
r: bug는 numerical error이다

$$\frac{(p \vee q) \wedge (r) \wedge (r \to \neg q)}{p}$$

?

논법 argument

$\boxed{r} \to \neg q$

$\neg \boxed{r} \vee \neg q$

$(p \vee q) \wedge$

$(r \quad) \wedge$

$(r \to \neg q) \qquad \neg r \vee \neg q$

p	q	r	$p \vee q$	r	$\neg r \vee \neg q$
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	0	1
0	1	1	1	1	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	0

p: bug는 module 17에 있다
q: bug는 module 81에 있다
r: bug는 numerical error이다

$$\left(\begin{array}{ll} p \vee q. & \checkmark \\ r & \checkmark \\ r \to \neg q & \checkmark \end{array}\right)$$

p

논법
argument

H1 $p \vee q$
H2 r
H3 $r \to \neg q$

$$\frac{\begin{array}{c} p \vee q \\ r \\ r \to \neg q \end{array}}{p}$$

$\boxed{r} \to \neg q$

$\neg \boxed{r} \vee \neg q$

$$\frac{\begin{array}{c} p \vee q \\ r \\ \neg r \vee \neg q \end{array}}{\dfrac{\begin{array}{c} p \vee q \\ \neg q \end{array}}{p}}$$

$$\frac{\begin{array}{l}(A \vee B) \wedge \\ (\neg A)\end{array}}{?}$$

I if $(A \vee B) \wedge (\neg A)$ is true

$(A \vee B)$ must be True

A must be False

B must be True

II $(A \wedge (\neg A)) \vee (B \wedge (\neg A))$

$F \vee (B \wedge (\neg A))$

$(B \wedge (\neg A))$ must be true

A can be T or F

Case 1 $A = T$: $B \wedge F = F$ cannot be true

Case 2 $A = F$: $B \wedge T = B$ can be true

in order for $(B \wedge (\neg A))$ to be true

B must be true

III $A \lor B \equiv \neg A \rightarrow B$

$\neg A$

$$\frac{\begin{array}{c}\neg A \\ \neg A \rightarrow B\end{array}}{B}$$

IV

$$\frac{\begin{array}{l}A \lor B \\ \neg A\end{array}}{}$$

resolution

$$\frac{\begin{array}{l}\cancel{A} \lor B \\ \cancel{\neg A}\end{array}}{B}$$

Modus Ponens

Case 1: p is false.

Case 2: p is true.

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline q \vee r \end{array}$$

$$\begin{array}{c} F \vee q \equiv q \\ T \vee r \equiv T \\ \hline q \end{array}$$

$$\begin{array}{c} T \vee q \equiv T \\ F \vee r \equiv r \\ \hline r \end{array}$$

$(p \vee q) \wedge$
$(r) \wedge$
$(r \to \neg q)$ $\quad \neg r \vee \neg q$

$$\begin{array}{l} p \vee q \\ r \\ r \to \neg q \\ \hline ? \end{array} \qquad \begin{array}{l} p \vee q \\ \cancel{r} \\ \cancel{\neg r} \vee \neg q \\ \hline \end{array} \qquad \begin{array}{l} p \vee \cancel{q} \\ \cancel{\neg q} \\ \hline p \end{array}$$

resolution 방식